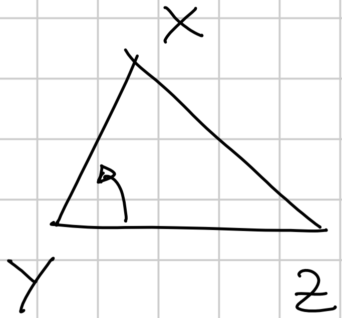
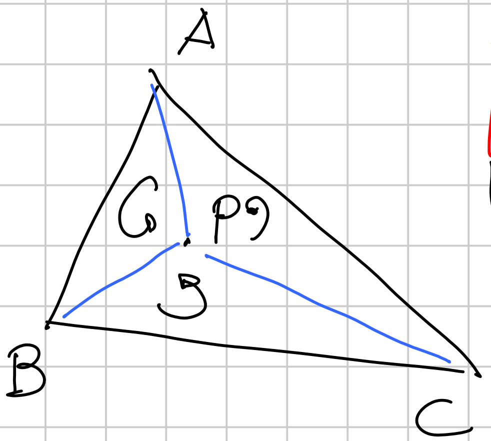
Coordinate Baricentriche e Trilineari

$$[X\ Y\ Z] = (\text{area di } X\hat{Y}Z)\cdot \begin{cases} 1 & \text{antiorario} \\ -1 & \text{orario} \end{cases}$$

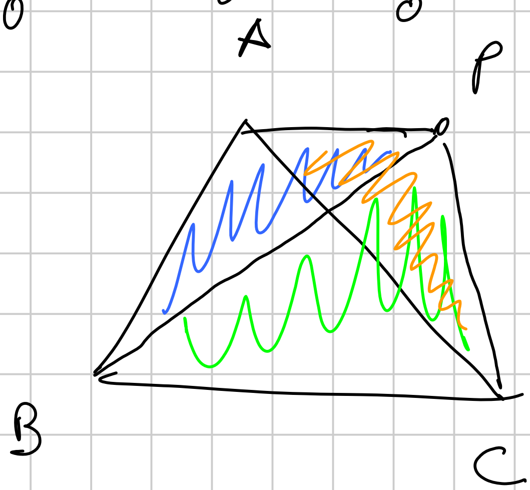
$$= \frac{1}{2}(Z-Y)\times(X-Y) = \frac{1}{2}\, ZY \cdot XY \cdot \text{sen}\, Z\hat{Y}X$$





$$[ABP] + [BCP] + [CAP] = [ABC]$$

$[ABP] > 0$, $[BCP] > 0$, $[CAP] < 0$



Oss: $[ABC] = [BCA] = [CAB] = -[ACB] = -[CBA] = -[BAC]$

$$[BPA] + [APC] + [CPB] = \frac{1}{2}\left\{(A-P)\times(B-P) + (C-P)\times(A-P) + (B-P)\times(C-P)\right\} =$$

$$= \frac{1}{2}\left\{A\times B - P\times B - A\times P + C\times A - P\times A - C\times P + B\times C - P\times C - B\times P\right\} = \frac{1}{2}\left\{A\times B + B\times C + C\times A\right\} =$$

$$= \frac{1}{2}\left\{(C-B)\times(A-B)\right\} = [ABC]$$

$$P \to \left[\frac{[CPB]}{[ABC]}, \frac{[APC]}{[ABC]}, \frac{[BPA]}{[ABC]} \right]$$ Coord. baric. esatte di P

$$[\lambda, \mu, \nu] \qquad \lambda + \mu + \nu = 1$$

$$\vec{P'} = \lambda \vec{A} + \mu \vec{B} + \nu \vec{C}$$

$$2[CP'B] = (B-P')\times(C-P') = [-\lambda A + (1-\mu)B - \nu C] \times [-\lambda A - \mu B + (1-\nu)C]$$

$$= +\lambda\mu\, A\times B - \lambda(1-\nu)\, A\times C - \lambda(1-\mu)\, B\times A + (1-\mu)(1-\nu)\, B\times C$$

$$+ \lambda\nu\, C\times A + \mu\nu\, C\times B = \lambda A\times B(\mu + 1 - \mu) + \lambda C\times A(\nu + 1 - \nu) +$$

$$+ B\times C(\underbrace{1-\mu-\nu}_{\lambda} + \mu\nu - \mu\nu) = \lambda \cdot 2[ABC]$$

$$\Rightarrow P' = P.$$

Coord. baricentriche di $P = (l : m : n)$ t.c. $l:m:n = [CPB]:[APC]:[BPA]$

$$P = [p_1, p_2, p_3] \qquad Q = [q_1, q_2, q_3]$$

pt. medio di $PQ = \dfrac{p_1 A + p_2 B + p_3 C + q_1 A + q_2 B + q_3 C}{2} =$

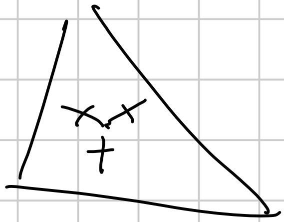
$$= \frac{p_1 + q_1}{2} A + \ldots \Rightarrow \left[\frac{p_1+q_1}{2}, \frac{p_2+q_2}{2}, \frac{p_3+q_3}{2} \right]$$

K t.c. $\dfrac{PK}{KQ} = \dfrac{\lambda_1}{\lambda_2}$ $\qquad \dfrac{\lambda_2 P + \lambda_1 Q}{\lambda_2 + \lambda_2} = K$

$$\left[\frac{\lambda_2 p_1 + \lambda_1 q_1}{2}, \ldots \right]$$

Es: $G = (1:1:1)$

$I = (a:b:c)$



$A = (1:0:0) \quad B = (0:1:0) \quad C = (0:0:1)$

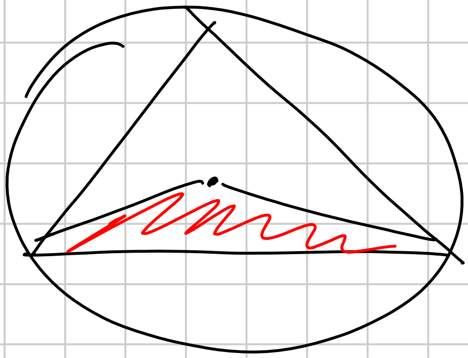
Excentri $= (-a:b:c), (a:-b:c), (a:b:-c)$

$O = (\sin 2\alpha : \sin 2\beta : \sin 2\gamma) =$

$= (\sin\alpha\cos\alpha : \_\_\_) =$

$= \left(a\dfrac{b^2+c^2-a^2}{2bc} : \_\_\_\right) =$

$= (a^2(b^2+c^2-a^2) : \_\_\_)$



$\frac{1}{2}R^2 \sin 2\alpha$

$s = \dfrac{a+b+c}{2}$

Ex: centro di simil. interno tra (O), (I)

S_i

$$\frac{R}{r} = \frac{abc}{4S}\cdot\left(\frac{2S}{a+b+c}\right)^{-1} = \frac{(a+b+c)\,abc}{8S^2} = \frac{s\,abc}{4S^2}$$

$$\frac{OS_i}{S_iI} = \frac{R}{r} = \frac{s\,abc}{4S^2}$$

$I = (a:b:c)\frac{1}{2s}$

$O = (a^2(b^2+c^2-a^2) : \_\_\_)\frac{1}{16S^2}$

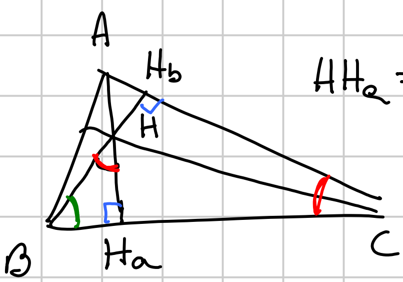
$2(a^2b^2+b^2c^2+c^2a^2) - a^4 - b^4 - c^4$

$\sqrt{s(s-a)(s-b)(s-c)}$

$S_i = [a^2(b+c-a), \_\_\_]$

$H \to$ asse

$H \to \dfrac{HG}{GO} = 2$



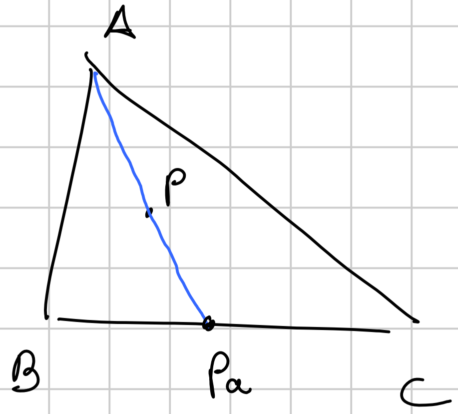
$$HH_a = CH_b \cdot \frac{BH_a}{H_bB} =$$

$$= \frac{a\cos\gamma \cdot c\cos\beta}{a\cdot\sin\gamma}$$

$(a\cos\gamma\cos\beta : \_\_\_) =$

$= \left(\frac{a}{\cos\alpha} : \_\_\_\right) = \left(\tan\alpha : \_\_\_\right) = \left(\frac{1}{b^2+c^2-a^2} , \_\_\_\right)$

H = centro delle cf. di F. $= [a\cos(\beta-\gamma) : \_\_\_]$



$P = (p_1 : p_2 : p_3)$

$P_a = (0 : p_2 : p_3)$

P_b ___

P_c ___

$(x, y, z) \in \mathbb{R}^3 \qquad x+y+z \neq 0.$

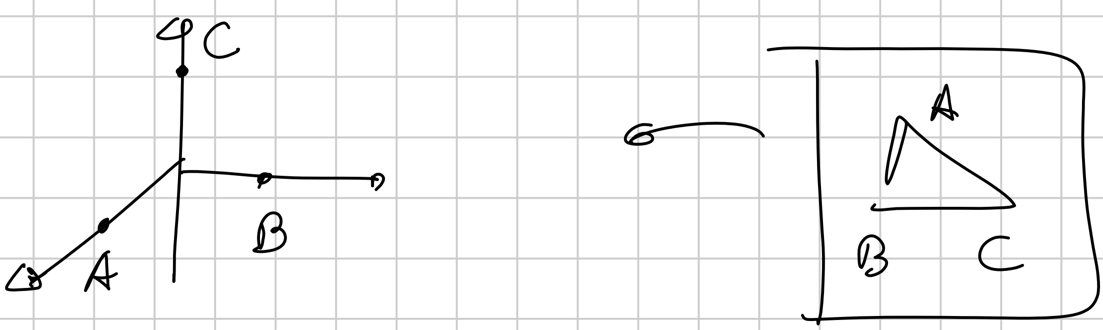
$\downarrow$

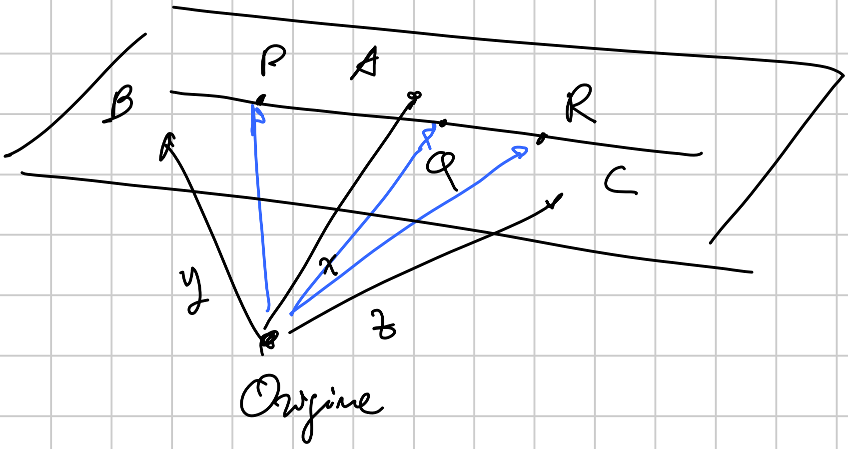
$\left(\frac{x}{x+y+z}, \frac{y}{x+y+z}, \frac{z}{x+y+z}\right) \in \{x+y+z=1\}$

$f(P) = (\lambda, \mu, \nu)$ coord. bar. esatte

$f(\lambda P + (1-\lambda) Q) = \lambda f(P) + (1-\lambda) f(Q)$

$f(P), f(Q), f(R)$ allineati $\Longleftrightarrow$ P, Q, R allineati





$x+y+z=1$

$F(e_1)=OP$

$F(e_2)=OQ$

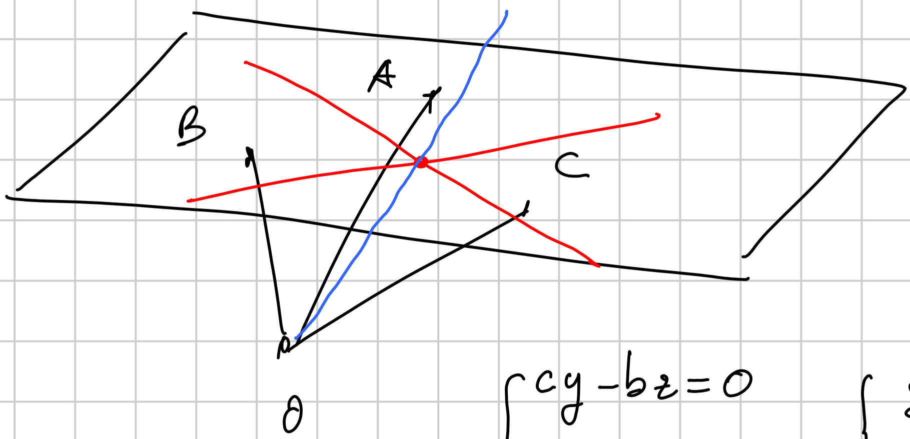
$F(e_3)=OR$

$P, Q\ R$ all $\Longleftrightarrow$ F non sing $\Longleftrightarrow$

$\det M = 0 \qquad M = \left(OP \,|\, OQ \,|\, OR \right)$

3 punti P, Q, R di coord bar. (p_i), (q_i), (r_i)

sono all $\Longleftrightarrow$ $\det \begin{pmatrix} p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \\ p_3 & q_3 & r_3 \end{pmatrix} = 0$



$\begin{cases} cy - bz = 0 \\ x - y = 0 \end{cases} \qquad \begin{cases} y = \frac{b}{c} z \\ x = y \end{cases}$

$\left(\frac{b}{c} : \frac{b}{c} : 1 \right)$

Piano: $l_1 x + m_1 y + n_1 z = 0$

$(l_1, m_1, n_1) \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$

l_2 - -

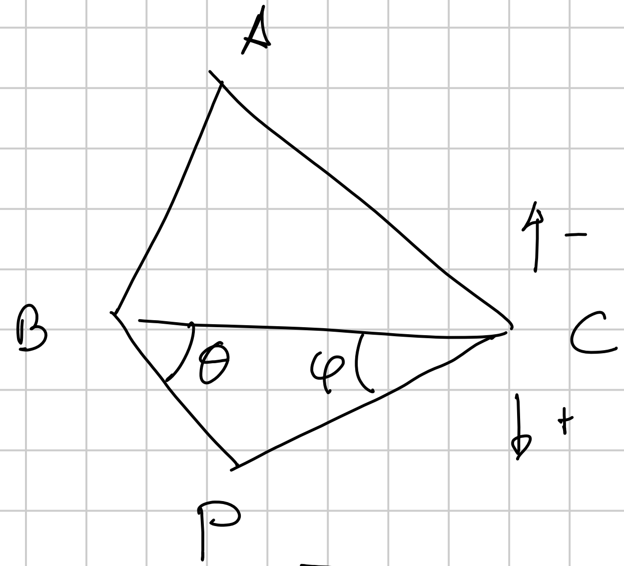
l_3 - - -

$\exists \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ t.c. $\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Conclusione: le rette $l_1 x + m_1 y + n_1 z = 0$
$l_2 x + m_2 y + n_2 z = 0$ concorrono
$l_3 x + m_3 y + n_3 z = 0$

sse $\det\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix} = 0.$

———— ∘ ————



$2[PCB] = -T$

$\frac{T}{\sin\theta} = BP \cdot BC \qquad BP = \frac{T}{a \sin\theta}$

$\frac{T}{\sin\varphi} = CP \cdot BC \qquad CP = \frac{T}{a \sin\varphi}$

$$2[PAB] = c \cdot \frac{T}{a\sin\theta} \cdot \sin(\beta+\theta) = \frac{cT}{a}\left(\frac{\sin\beta\cos\theta}{\sin\theta} + \frac{\cos\beta\sin\theta}{\sin\theta}\right) =$$

$$= \frac{cT}{a}\sin\beta\,(\cot\theta + \cot\beta)$$

$$2[PCA] = \frac{bT}{a}\sin\gamma(\cot\varphi + \cot\gamma)$$

Formule di CONWAY

$$P = \left(-a^2 : 2[ABC](\cot\theta + \cot\beta) : 2[ABC](\cot\varphi + \cot\gamma)\right)$$

$S_\theta = 2[ABC] \cdot \cot\theta$ Notazione di CONWAY

$S_{\theta\varphi} = S_\theta \cdot S_\varphi$

$$P = (-a^2 : S_\theta + S_B : S_\varphi + S_C)$$

i) $S_B + S_C = a^2$

$S_A = bc \cdot \sin\alpha \cdot \frac{\cos\alpha}{\sin\alpha} = \frac{b^2+c^2-a^2}{2bc} bc =$

ii) $S_{AB} + S_{BC} + S_{CA} = S^2 = 4[ABC]^2$

$= \frac{b^2+c^2-a^2}{2}$

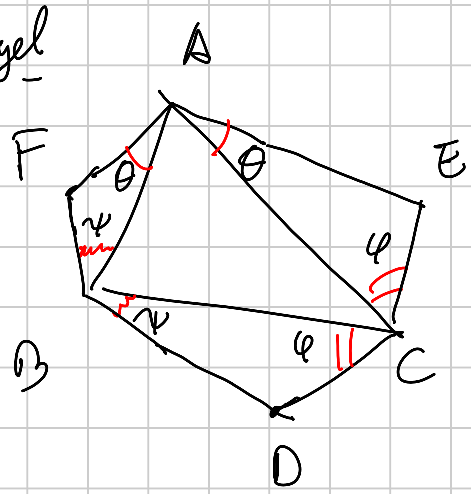
$\cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha = 1$

$$H = \left(\frac{1}{S_A} : \frac{1}{S_B} : \frac{1}{S_C}\right)$$

$$O = (a^2 S_A : b^2 S_B : c^2 S_C) = (S_A(S_B+S_C) : \text{———})$$

$$N = (S^2 + S_{BC} : \text{———})$$

Teo di Nagel



$\Rightarrow AD, BE, CF$ concorrono

$$D = (-a^2 : S_B + S_\psi : S_C + S_\varphi)$$

$$AD \cap BC = (0 : S_B + S_\psi : S_C + S_\varphi)$$

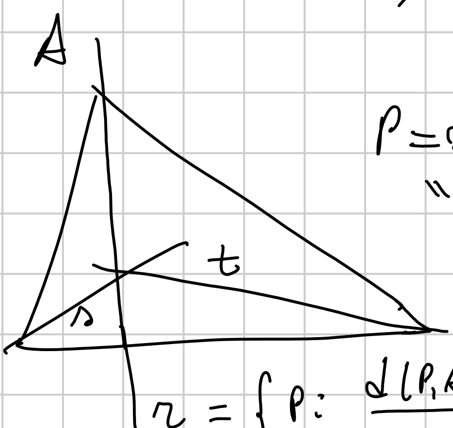
$$BE \cap AC = (S_A + S_\theta : 0 : S_C + S_\varphi)$$

$$CF \cap AB = (S_A + S_\theta : S_B + S_\psi : 0)$$

———∘———

Passaggio alle Trilineari: $\mathrm{Tri}(P) = (d(P;BC) : d(P;AC) : d(P,AB))$

$$\mathrm{bari}(P) = (\lambda, \mu, \nu) \longrightarrow \left(\frac{\lambda}{a}, \frac{\mu}{b}, \frac{\nu}{c}\right) = \mathrm{Tri}(P)$$



$P = r \cap s \cap t$ " $[p : q : r]$

$P' = r' \cap s' \cap t'$ " $\left(\frac{1}{p} : \frac{1}{q} : \frac{1}{r}\right)$

$$r = \left\{ P : \frac{d(P,AB)}{d(P,AC)} = k \right\} \qquad r' = \left\{ \text{---} = \frac{1}{k} \right\}$$

in Baricentriche: $(\lambda : \mu : \nu) \to \left(\frac{\lambda}{a} : \frac{\mu}{b} : \frac{\nu}{c}\right) \to \left(\frac{a}{\lambda} : \frac{b}{\mu} : \frac{c}{\nu}\right)$

$$\left(\frac{a^2}{\lambda} : \frac{b^2}{\mu} : \frac{c^2}{\nu}\right) \longleftarrow$$

Es: $H = \left(\frac{1}{S_A}, \dots\right)$ $O = (a^2 S_A, \dots)$

$K = (a^2 : b^2 : c^2)$ punto di Lemoine

Fatto: Coniug. isog. della cf. circoscritta non esiste

$P = (\lambda : \mu : \nu)$ t.c. $\frac{a^2}{\lambda} + \frac{b^2}{\mu} + \frac{c^2}{\nu} = 0$

$a^2 \mu\nu + b^2 \lambda\nu + c^2 \lambda\mu = 0$

$\{ a^2 yz + b^2 xz + c^2 xy = 0 \} = \Gamma$

cf. di F = immagine di Γ tramite l'omot. di centro G e fattore $-\frac{1}{2}$.

$P \in$ cf di F $P = [\lambda; \mu; \nu]$ $\lambda + \mu + \nu = 1$

Omot.

$-2(P-G) + G = -2\left[\left(\lambda - \frac{1}{3}\right), \mu - \frac{1}{3}, \nu - \frac{1}{3}\right] + \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right] =$

$= [-2\lambda + 1, -2\mu + 1, -2\nu + 1]$

$a^2(-2\mu + 1)(-2\nu + 1) + b^2(-2\lambda + 1)(-2\nu + 1) + c^2(-2\lambda + 1)(-2\mu + 1) = 0$

$4\mu\nu a^2 + 4\lambda\nu b^2 + 4\lambda\mu c^2 - 2a^2\mu - 2a^2\nu - 2b^2\lambda - 2b^2\nu - 2c^2(\lambda + \mu)$
$+ a^2 + c^2 + b^2 = 0$

$4(a^2 yz + b^2 xz + c^2 xy) + (x+y+z)\left[(a^2+b^2+c^2)(x+y+z) - 2a^2(y+z) - 2b^2(x+z) - 2c^2(x+y) \right]$

$K(a^2 yz + \dots) + (x+y+z)[\quad]$