SENIOR 2011 — G1 (Basic)

Titolo nota 05/09/2011

① GEOMETRIA SINTETICA G3

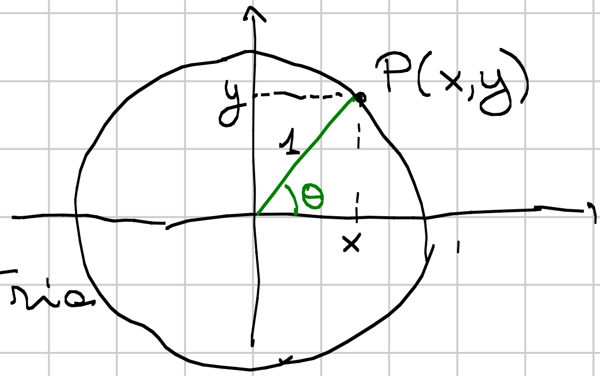
② ALGEBRIZZAZIONI (vettori, numeri complessi, geometria analitica) G2

③ CALCOLO TUTTO (trigonometria).

GONIOMETRIA

$$\begin{cases} x = \cos\theta \\ y = \sin\theta \end{cases}$$

$$x^2+y^2=1$$



⇒ Formula fond. della goniometria

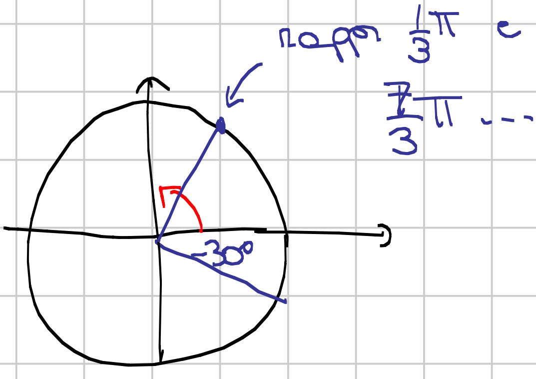
$$\cos^2\theta + \sin^2\theta = 1.$$

Gradi vs radianti $\theta :=$ lunghezza dell'arco.

$$\frac{\theta^\circ}{360^\circ} = \frac{\theta^{rad}}{2\pi}$$

Angoli con segno

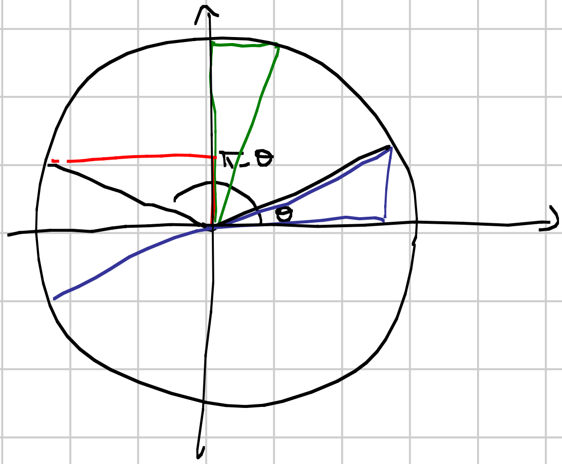
$$\frac{1}{3}\pi + 2\pi = \frac{1}{3}\pi$$



Periodicità e simmetrie.

$$\sin\theta + 2\pi = \sin\theta$$

$$\theta\ ,\ \pi-\theta\ ,\ \pi+\theta\ ,\ \frac{\pi}{2}\pm\theta$$



$$\cos\pi-\theta = -\cos\theta$$

$$\sin\pi-\theta = \sin\theta$$

$$\cos\pi+\theta = -\cos\theta$$

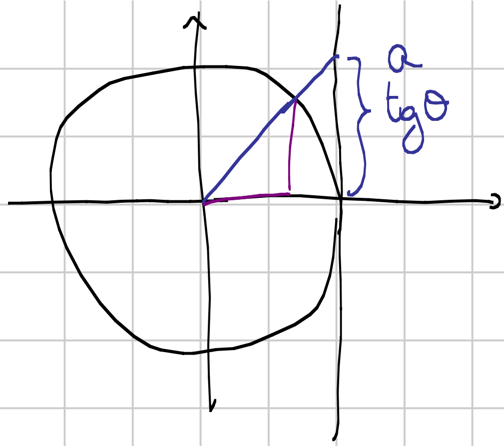
$$\sin\pi+\theta = -\sin\theta$$

$$\begin{cases} \cos\frac{\pi}{2}-\theta = \sin\theta \\ \sin\frac{\pi}{2}-\theta = \cos\theta \end{cases}$$

Altre funzioni trigonometriche

$\tan\theta = tg\theta := \dfrac{\sin\theta}{\cos\theta}$

$cotg\,\theta := \dfrac{\cos\theta}{\sin\theta}$



$\dfrac{a}{1} = \dfrac{\sin\theta}{\cos\theta}$

Periodo di $tg\theta$?

sin e cos hanno periodo 2π.

$\Rightarrow$ tg ha periodo $\mid 2\pi$

$tg\,\theta+\pi = \dfrac{\sin\theta+\pi}{\cos\theta+\pi} = \dfrac{\cancel{-}\sin\theta}{\cancel{-}\cos\theta} = tg\theta$

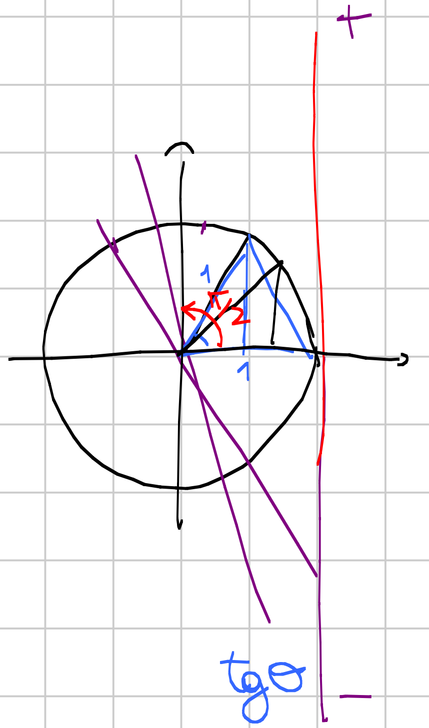
Ex: tg ha periodo ESATTAM. π. (hint: segni).

<u>Valori notevoli</u>

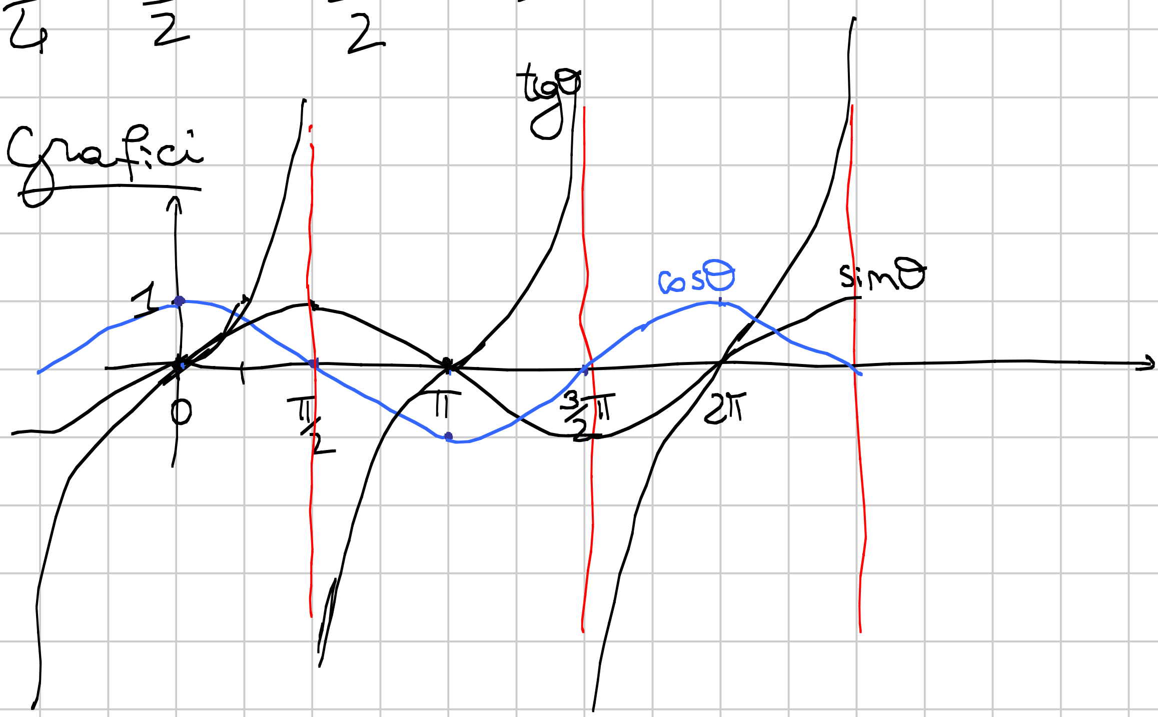
0, π, $\pi/2$, $\pi/3$ e $\pi/6$, $\frac{\pi}{4}$

	cos	sin	tg
$\frac{\pi}{2}$	0	1	----
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1

$\left(\frac{\pi}{6}\right)$



<u>Grafici</u>



$tg\theta$

$0 \to 0$

$\frac{\pi}{2}^- \to$ diverge $+\infty$

$\frac{\pi}{2}^+ \to$ diverge $-\infty$

Funzioni inverse:

$\text{tg}\,\theta : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow (-\infty, +\infty) = \mathbb{R}$

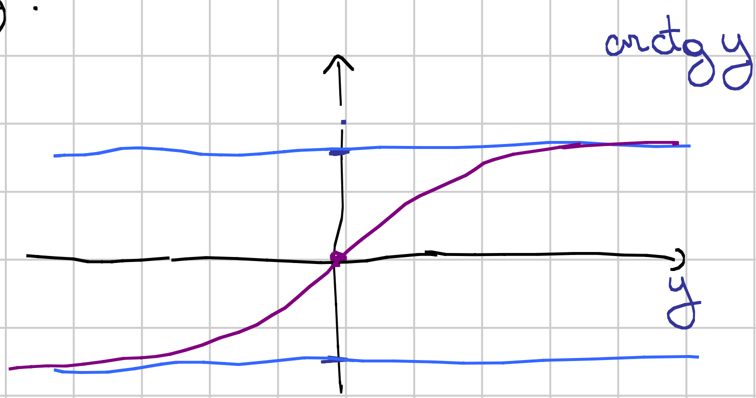
è biunivoca.

$\text{arctg}\, y :=$ l'angolo θ tra $-\frac{\pi}{2}$ e $\frac{\pi}{2}$ t.c.

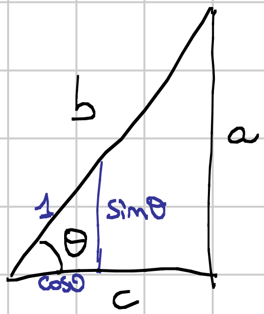
$$\text{tg}\,\theta = y.$$

Grafico arctg y?

$y \longrightarrow +\infty$



Trigonom del triangolo rettangolo



a in funzione di b e θ?

$a = b \cdot \sin\theta$

$c = b \cos\theta$

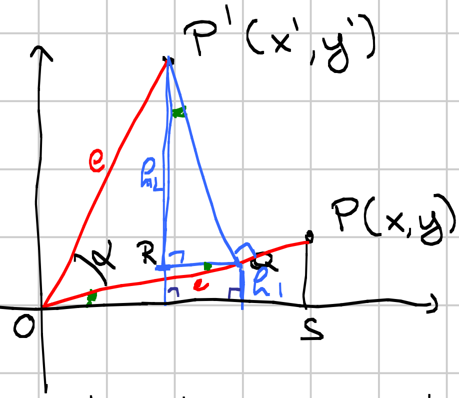
$b^2 = a^2 + c^2 \iff 1 = \cos^2\theta + \sin^2\theta$

FORMULE

Rotazione nel piano

$(x',y') = (x,y)$ ruotato di α.

Scriviamo in funzione di x, y, α.



Oss 1: $x'^2 + y'^2 = x^2 + y^2 = e^2$ È sufficiente trovare y'.

$\overline{OQ} = e\cos\alpha$ $\qquad \dfrac{h_1}{\cancel{\overline{OQ}}} = \dfrac{y}{e}\,\overline{OQ} =$

$h_1 = y\cos\alpha$

$\overset{\triangle}{P'QR}$ e $\overset{\triangle}{OPS}$ sono simili.

$$\frac{h_2}{P'Q} = \frac{OS}{OP} = \frac{x}{e}$$

$P'Q = e\sin\alpha$

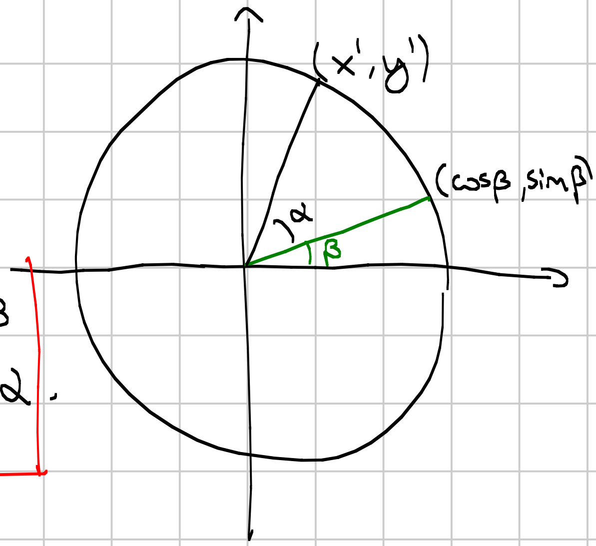
$h_2 = x\sin\alpha$.

$$\begin{cases} x' = x\cos\alpha - y\sin\alpha \\ y' = x\sin\alpha + y\cos\alpha \end{cases}$$ (ex: ricavare con oss 1).

Formula di addizione

$\cos\alpha+\beta = ?$

$\sin\alpha+\beta = ?$



rotazione

$$\boxed{\begin{aligned} x' &= \cos\alpha+\beta = \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ y' &= \sin\alpha+\beta = \sin\alpha\cos\beta + \sin\beta\cos\alpha \end{aligned}}$$.

$$\begin{aligned} \cos(\alpha-\beta) &= \cos(\alpha + (-\beta)) \\ &= \cos\alpha\cos(-\beta) - \sin\alpha\sin(-\beta) \\ &= \cos\alpha\cos\beta + \sin\alpha\sin\beta \end{aligned}$$

$$\sin \alpha-\beta = \sin\alpha \cos\beta - \sin\beta \cos\alpha .$$

Formule di duplicazione

formule fond.

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1-2\sin^2\alpha = 2\cos^2\alpha - 1 \quad (*)$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha .$$

Formule di bisezione

$$\cos\frac{\beta}{2} = ? \qquad\qquad \frac{\beta}{2}=\alpha$$

Sostituisco $\alpha = \beta/2$ in $(*)$

$$\cos\beta = 2\cos^2 \beta/2 - 1$$

$$\cos \beta/2 = \pm\sqrt{\frac{\cos\beta+1}{2}} \qquad\qquad \cos\beta \geq -1 \Rightarrow \text{ ok radice.}$$

$$\sin \beta/2 = \pm\sqrt{\frac{1-\cos\beta}{2}}$$

$\sin^2 \beta/2 + \cos^2 \beta/2 = 1 \Rightarrow$ ricavo $\sin \beta/2$

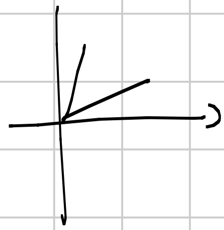
$$\operatorname{tg} \alpha+\beta = \frac{\sin\alpha+\beta}{\cos\alpha+\beta} = \frac{\sin\alpha\cos\beta+\sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{1-\frac{\sin\alpha}{\cos\alpha}\frac{\sin\beta}{\cos\beta}} = \frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}$$

Fatto: $\alpha,\beta,\gamma \in (0,\pi)$. Allora

$\alpha+\beta+\gamma = \pi$ se e solo se

$$\operatorname{tg}\frac{\alpha}{2}\operatorname{tg}\frac{\beta}{2} + \operatorname{tg}\frac{\beta}{2}\operatorname{tg}\frac{\gamma}{2} + \operatorname{tg}\frac{\gamma}{2}\operatorname{tg}\frac{\alpha}{2} = 1$$



Dim:

$$\Rightarrow)\ \operatorname{tg}\gamma/2 = \operatorname{tg}\frac{\pi-\alpha-\beta}{2} =$$

$$= \operatorname{tg}\frac{\pi}{2}-\frac{\alpha+\beta}{2}$$

$$\operatorname{tg}\frac{\pi}{2}-\theta = \frac{1}{\operatorname{tg}\theta}$$

$$\frac{\sin\frac{\pi}{2}-\theta}{\cos\frac{\pi}{2}-\theta} = \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\operatorname{tg} \frac{\alpha+\beta}{2}}$$

$$= \frac{1 - \operatorname{tg}\frac{\alpha}{2} \operatorname{tg}\beta/2}{\operatorname{tg}\frac{\alpha}{2} + \operatorname{tg}\beta/2}. \qquad \text{ok}.$$

$\Leftarrow)$ $\alpha, \beta, \pi-\alpha-\beta = \theta$ verificano la relazione
Per hp α, β, γ verificano.

$$\operatorname{tg}\frac{\gamma}{2} = \operatorname{tg}\frac{\pi-\alpha-\beta}{2}$$

$$\gamma = \pi - \alpha - \beta$$

Ex: concludere questa uguaglianza

Ex (difficile): $a, b, c \in (0,1)$ t.c $ab + bc + ca = 1$
Allora

$$\sum_{a,b,c} \frac{a}{1-a^2} \geq \frac{3}{4} \sum_{a,b,c} \frac{1-a^2}{a}.$$

Idea:

1. $a = \operatorname{tg}\frac{\alpha}{2}$ con $\alpha \in [0, \frac{\pi}{2})$
   b, c
2. $ab + bc + ca = 1 \iff \alpha + \beta + \gamma = \pi$
3. Riscriviamo la disug
   $$\frac{a}{1-a^2} = \frac{\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}^2\frac{\alpha}{2}} = \dots = \frac{\operatorname{tg}\alpha}{2}$$
4. In generale, se $\alpha + \beta + \gamma = \pi$ allora
   $$\operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma = \operatorname{tg}\alpha \operatorname{tg}\beta \operatorname{tg}\gamma$$

Formula di bisezione per tg θ

$$\operatorname{tg}\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\cos^2\theta}{(1+\cos\theta)^2}}$$

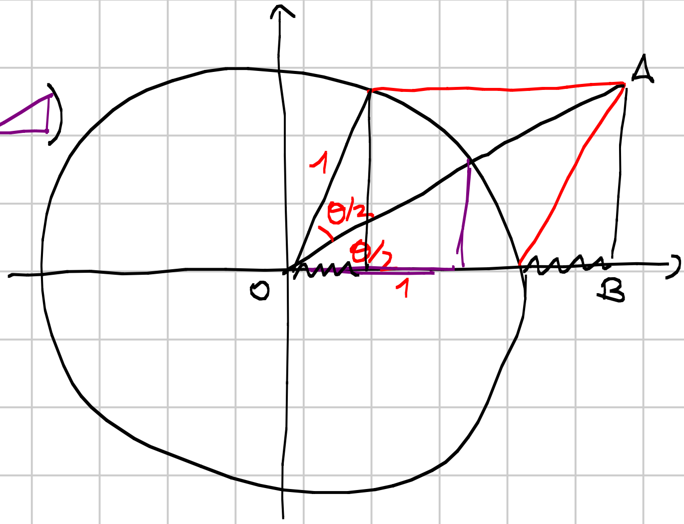
$$= \sqrt{\frac{\sin^2\theta}{(1+\cos\theta)^2}} = \frac{\sin\theta}{1+\cos\theta}.$$

Altro modo.

$\operatorname{tg}\frac{\theta}{2} = \frac{AB}{OB}$ (similitudine con)

$AB = \sin\theta$

$OB = 1 + \cos\theta$



Formule parametriche

$t = \operatorname{tg}\frac{\theta}{2}$. Allora vale

$$\sin\theta = \frac{2t}{1+t^2} \quad (1) \qquad\qquad \cos\theta = \frac{1-t^2}{1+t^2} \quad (2)$$

Esempio: $5\cos\theta + 2\sin\theta = 1$

Sostituendo, diventa $5 \cdot \frac{2t}{1+t^2} + 2\,\frac{1-t^2}{1+t^2} = 1$, eq di 2° grado

...

Verifichiamo (1).

$$\frac{2t}{1+t^2} = \frac{2\operatorname{tg}\theta/2}{1+\operatorname{tg}^2\frac{\theta}{2}} = \frac{2\,\frac{\sin\theta}{1+\cos\theta}}{1+\frac{\sin^2\theta}{(1+\cos\theta)^2}} = \frac{2\sin\theta\,(1+\cos\theta)}{(1+\cos\theta)^2+\sin^2\theta}$$

$$= \cancel{\frac{\cancel{2}\sin\theta\,\cancel{(1+\cos\theta)}}{2\cancel{+2\cos\theta}}}$$

Verifichiamo (2).

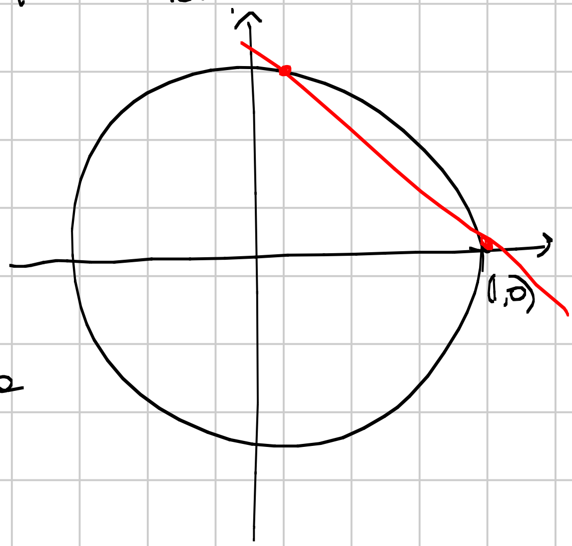
$$\cos\theta = \sqrt{1-\sin\theta} = \sqrt{1-\left(\frac{2t}{1+t^2}\right)^2} = \sqrt{\frac{1+t^4+2t^2-4t^2}{(1+t^2)^2}} = \cancel{\sqrt{\frac{(1-t^2)^{\cancel{2}}}{(1+t^2)^{\cancel{2}}}}}.$$

↑ occhio ai segni... ora assumo $\cos\theta > 0$

Oss: esistono ∞ pti a coordinate razionali sulla circonferenza unitaria.

<u>Dim 1</u>
Prendo t numero razionale. $\sin\vartheta$ e $\cos\vartheta$ corrisp sono numeri razionali.
Ex: a t diversi corrispondono pti diversi?

<u>Dim 2</u>
$y = k(x-1)$ $\qquad k \in \mathbb{Q}$

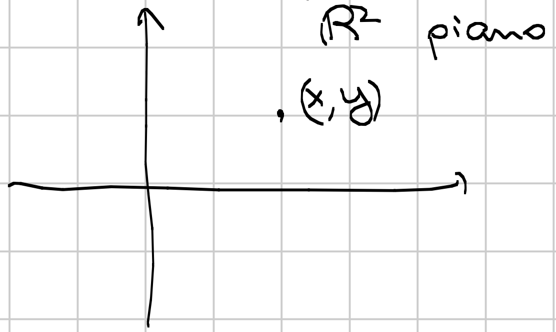


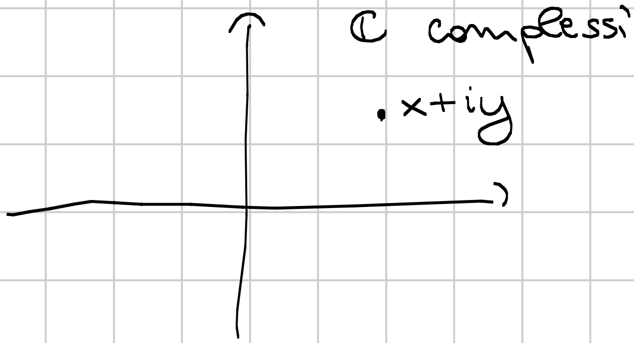
$$\begin{cases} y = k(x-1) & ① \\ x^2+y^2=1 & ② \end{cases}$$ altra intersezione retta circonferenza

x e y sono razionali?
Da ①, basta verif che x sia razionale.

② diventa
$$x^2 + k^2(x-1)^2 = 1$$
$x=1$ è soluzione, è razionale $\Rightarrow$
anche l'altra sol dell'eq di 2° grado è razionale.

Numeri complessi.





Moltiplicazione?

In $\mathbb{R}$, prodotto scalare

$(x_1, y_1) \cdot (x_2, y_2) := x_1 x_2 + y_1 y_2$

vett $\cdot$ vett $=$ numero reale

Ok per perpendicolarità!

Ex: vettori ortogonali $\Leftrightarrow$ prod scal $= 0$

In $\mathbb{C}$, moltiplicazione

$(x+iy)(u+iv) := xu - yv + i(xv + yu)$

compl $\cdot$ compl $=$ complesso.

$i^2 = -1$

$$(\cos\alpha + i\sin\alpha)(x+iy) = x\cos\alpha - y\sin\alpha + i(x\sin\alpha + y\cos\alpha)$$

Moltiplicare per $\cos\alpha + i\sin\alpha$ equivale a ruotare di α!

$x+iy = \cos\beta + i\sin\beta$

$$(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = \cos\alpha+\beta + i\sin\alpha+\beta. \quad (*)$$

Definizione: $e^{i\alpha} := \cos\alpha + i\sin\alpha$.

Abbiamo mostrato $(*)$

$$e^{i(\alpha+\beta)} = e^{i\alpha} \cdot e^{i\beta}$$

Ex: $\cos 3\alpha = \cos(2\alpha + \alpha) = \ldots$

$\cos 6\alpha = \ldots$

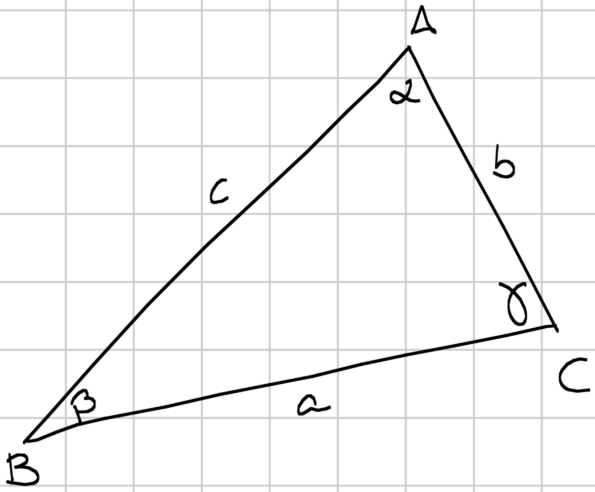
$$\begin{aligned}\cos 6\alpha &= Re\left(e^{i6\alpha}\right)\\ &= Re\big(\underbrace{e^{i\alpha} \cdot e^{i\alpha} \cdot \ldots \cdot e^{i\alpha}}_{6 \text{ volte}}\big)\\ &= Re\left((e^{i\alpha})^6\right)\\ &= Re\left((\cos\alpha + i\sin\alpha)^6\right)\end{aligned}$$

$$= (\cos\alpha)^6 + \binom{6}{2} \cos^2\alpha \sin^4\alpha - \binom{6}{4} \cos^4\alpha \sin^2\alpha$$
$$- \sin^6\alpha$$

TRIGONOMETRIA

- 3 lati
- 2 lati e un angolo
- 1 lato e 2 angoli

Un triangolo ha 3 gradi di libertà



Teorema dei seni

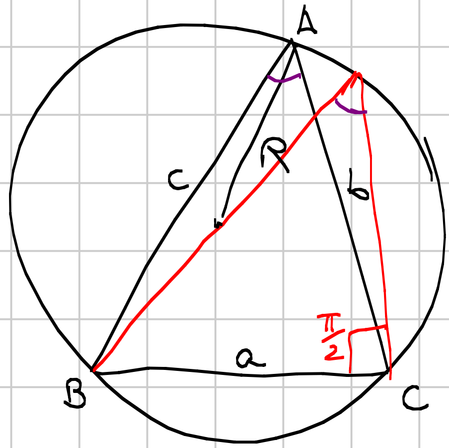
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$

Dim:

$a = 2R\sin\alpha$ (guardando $\triangle$)

$$2R = \frac{a}{\sin\alpha}$$

Stessa cosa sugli altri lati...

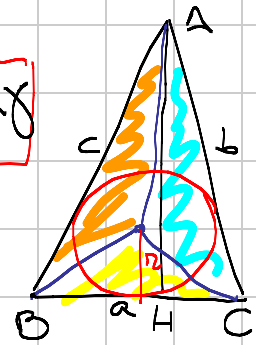


Formule per l'area

$$[ABC] = \frac{\overline{BC}\cdot\overline{AH}}{2} = \frac{a\cdot b\sin\gamma}{2} = \boxed{\frac{1}{2}ab\sin\gamma}$$

$$= \boxed{\frac{abc}{4R}}$$

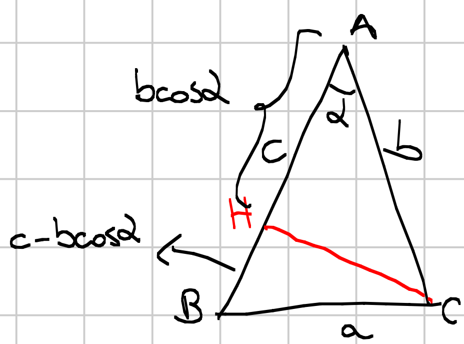
$$[ABC] = \frac{a+b+c}{2}\,r$$



Teorema di Carnot

Noti: b, c, α, quanto vale a?

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

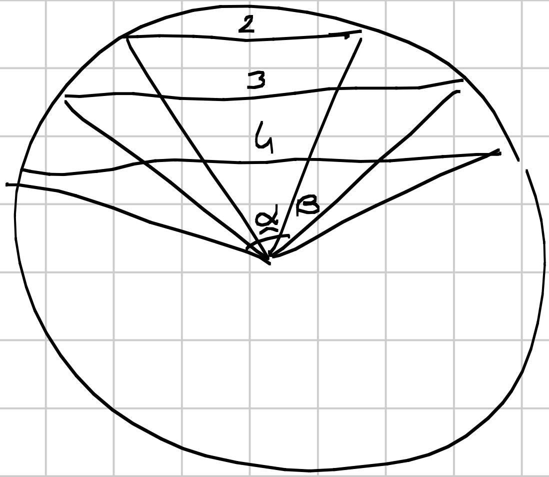


$$a^2 = \overline{BH}^2 + \overline{HC}^2 = (c - b\cos\alpha)^2 + b^2\sin^2\alpha$$
$$= c^2 - 2bc\cos\alpha + \underbrace{b^2\cos^2\alpha + b^2\sin^2\alpha}_{= b^2}$$
$$= b^2 + c^2 - 2bc\cos\alpha .$$

Ex libretto

Corde lunghe 2, 3, 4 insistono su $\alpha, \beta, \alpha+\beta$.

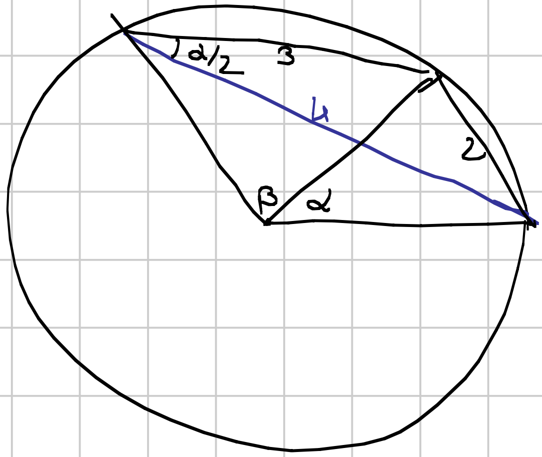
$\cos\alpha = ?$



Per il teo di Carnot

$$2^2 = 3^2 + 4^2 - 2\cdot 3\cdot 4\cdot \cos\frac{\alpha}{2}$$
$$4 = 25 - 24\cdot\cos\frac{\alpha}{2}$$
$$24\cos\frac{\alpha}{2} = 21$$
$$\cos\alpha/2 = \frac{7}{8}$$
$$\cos^2\alpha/2 = \frac{1+\cos\alpha}{2}$$
$$\left(\frac{7}{8}\right)^2 = \frac{1+\cos\alpha}{2}$$

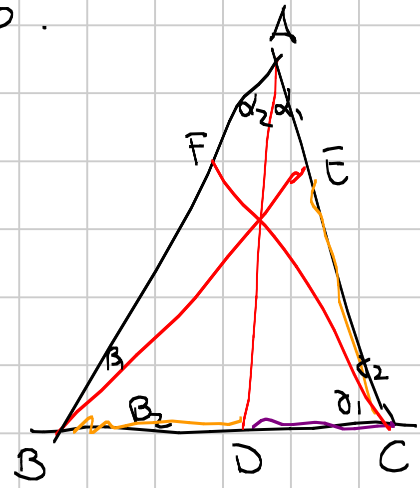
Risolvendo $\cos\alpha = \dfrac{17}{32}$ (forse).



Ex: teorema di Ceva trigonometrico.

Si ha che

$$\frac{\sin\alpha_1}{\sin\alpha_2}\cdot\frac{\sin\beta_1}{\sin\beta_2}\cdot\frac{\sin\gamma_1}{\sin\gamma_2}=1.$$



Teorema di Ceva:

$$\frac{BD}{DC}\cdot\frac{CE}{EA}\cdot\frac{AF}{FB}=1 \qquad (*)$$

Dim: vedi G3.

Dim di Ceva trigo dato Ceva:

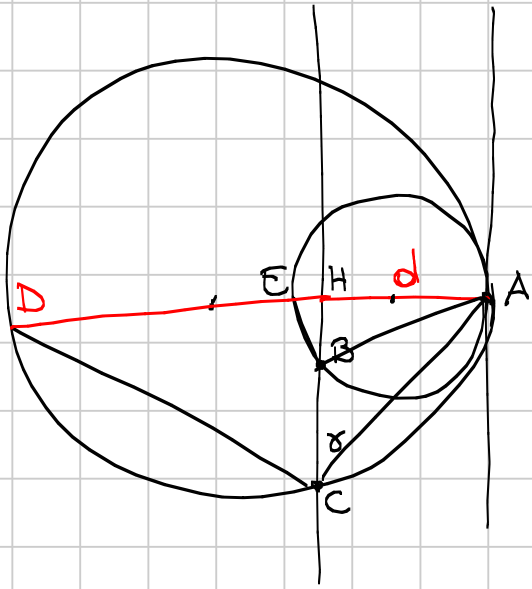
Dal teorema dei seni su ABD

$$\frac{BD}{\sin\alpha_2}=\frac{AD}{\sin\beta} \Rightarrow BD=\frac{\sin\alpha_2}{\sin\beta}\cdot AD$$

Ripetiamo sui 6 triangolini e sostituiamo in $(*)$

Ex: vederlo.

Ex 10 test iniziale.
Calcolare il raggio della circo circoscritta ad ABC (viene indipendente da d).



$\overset{\triangle}{ABE}$, teorema di Euclide

$AB^2 = AH \cdot AE$

$AB = \sqrt{2d\,r}$

$AC^2 = AH \cdot AD$

$AC = \sqrt{2dR}$

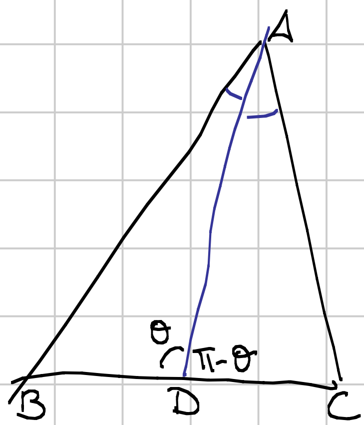
raggio cercato $= \dfrac{AB}{2\sin\gamma}$ (per il teo dei seni)

$\sin\gamma = \dfrac{d}{AC}$ (vedi triangolo $\overset{\triangle}{ACH}$)

raggio $= \dfrac{\sqrt{\cancel{2}\cancel{d}r}\,\sqrt{\cancel{2}\cancel{d}R}}{\cancel{2}\,\cancel{d}} = \sqrt{rR}$.

Ex: teorema della bisettrice

$\dfrac{BD}{DC} = \dfrac{AB}{AC}$



Dim:

Teorema dei seni su $\overset{\triangle}{ABD}$ dice

$\dfrac{AB}{\sin\theta} = \dfrac{BD}{\sin\frac{\alpha}{2}}$

$\Rightarrow \boxed{\dfrac{BD}{AB}} = \dfrac{\sin\alpha/2}{\sin\theta} = (*)$

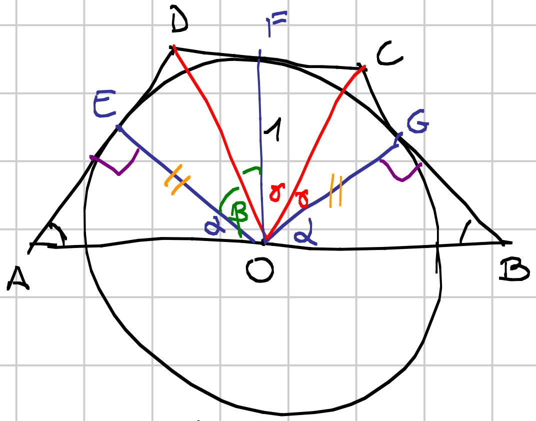
Teorema dei seni su $\overset{\triangle}{ADC}$

$$\frac{AC}{\sin\theta} = \frac{DC}{\sin\frac{\alpha}{2}}$$

$$\boxed{\frac{DC}{AC}} = \frac{\sin\frac{\alpha}{2}}{\sin\theta} = (*)$$

Es 9 libretto.

AD, DC, BC tangenti al cerchio centrato nel pto medio di AB.



Tesi: $AB^2 = 4\, BC \cdot AD$

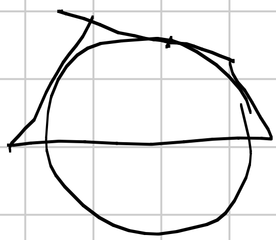
Oss: $\hat{A} = \hat{B}$.

I triangoli $A\hat{E}O$ e $G\hat{O}B$ sono congruenti.

(volendo $AE = GB$ perchè A e B hanno la stessa potenza)

$\alpha + 2\beta + 2\gamma + \alpha = \pi$ ovvero

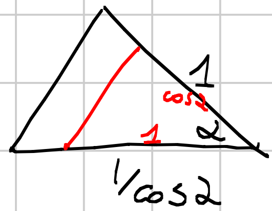
$\alpha + \beta + \gamma = \pi/2$.



$$AB = 2\, AO = \frac{2}{\cos\alpha}$$

$$BC = BG + GC = \operatorname{tg}\alpha + \operatorname{tg}\gamma$$

$$AD = AE + ED = \operatorname{tg}\alpha + \operatorname{tg}\beta$$



Dobbiamo dimostrare

$$\frac{1}{\cos^2\alpha} = \underbrace{(\operatorname{tg}\alpha + \operatorname{tg}\gamma)(\operatorname{tg}\alpha + \operatorname{tg}\beta)}_{RHS}$$

1° modo $\gamma = \frac{\pi}{2} - \alpha - \beta$, sviluppare $\operatorname{tg}\gamma = \dots$

2° modo

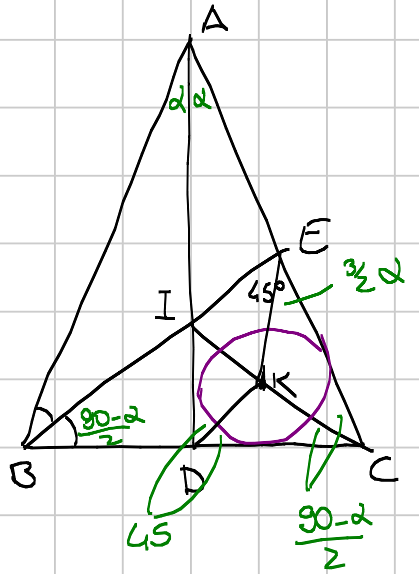
$$RHS = \operatorname{tg}^2\alpha + \operatorname{tg}\alpha\operatorname{tg}\gamma + \operatorname{tg}\beta\operatorname{tg}\alpha + \operatorname{tg}\beta\operatorname{tg}\gamma$$
$$= \operatorname{tg}^2\alpha + 1$$

↑ fatto visto prima con α al posto di $\frac{\alpha}{2}$
(il vincolo diventa $\alpha+\beta+\gamma=\frac{\pi}{2}$)

Resta da dim

$$\cancel{\cos^2\alpha}\,\frac{1}{\cancel{\cos^2\alpha}} = (\operatorname{tg}^2\alpha+1)\cos^2\alpha = \sin^2\alpha+\cos^2\alpha$$

$$\boxed{\frac{1}{\cos^2\alpha} = \operatorname{tg}^2\alpha + 1}$$



Ex IMO 09 - 4

$\hat{B}=\hat{C}$

BE bisettrice di $\hat{B}$

K centro della "incirconf" di ADC

Trovare $B\hat{A}C$.

Oss: IKC allineati (sulla bisettr di $\hat{C}$)

$$K\hat{E}C = 180 - \frac{90-\alpha}{2} - (90-\alpha) - 45 = \frac{3}{2}\alpha$$

$$E\hat{I}K = 180 - \frac{3}{2}\alpha - 45 - \frac{90-\alpha}{2} = 90-\alpha$$

Idea: calcolo $\frac{IK}{KC}$ in 2 modi diversi.

Dal teo dei seni su $\overset{\triangle}{EIK}$ $\quad \frac{IK}{\sin 45} = \frac{EK}{\cos\alpha}$

" " " " " $\overset{\triangle}{EKC}$ $\quad \frac{KC}{\sin\frac{3}{2}\alpha} = \frac{EK}{\sin\frac{90-\alpha}{2}}$

Quindi

$$\frac{IK}{KC} = \frac{\sin 45 \ \sin\frac{90-\alpha}{2}}{\cos\alpha \ \sin\frac{3}{2}\alpha}$$

D'altra parte

$$\frac{IK}{KC} = \frac{ID}{DC} = \operatorname{tg}\frac{90-\alpha}{2}$$

α deve soddisfare

$$\operatorname{tg}\frac{90-\alpha}{2} = \frac{\sin 45 \ \sin\frac{90-\alpha}{2}}{\cos\alpha \ \sin\frac{3}{2}\alpha}$$

$$\frac{\sin \frac{90-\alpha}{2}}{\cos \frac{90-\alpha}{2}}$$

ovvero

$$\cos\alpha \sin \tfrac{3}{2}\alpha = \sin 45 \cos \frac{90-\alpha}{2}.$$

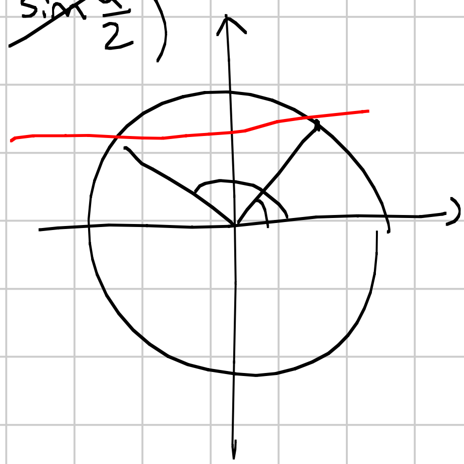
Fatto generale:

$$\sin x+y = \sin x \cos y + \sin y \cos x$$

$$\sin x-y = \sin x \cos y - \sin y \cos x$$

$$\Rightarrow \sin x \cos y = \frac{\sin x+y + \sin x-y}{2}$$

$$\cancel{\tfrac{1}{2}}\left(\sin \tfrac{5}{2}\alpha + \cancel{\sin \tfrac{\alpha}{2}}\right) = \cancel{\tfrac{1}{2}}\left(\sin 90-\tfrac{\alpha}{2} + \cancel{\sin \tfrac{\alpha}{2}}\right)$$



$$\sin \tfrac{5}{2}\alpha = \sin 90-\tfrac{\alpha}{2}$$

Ci sono 2 casi

$$\begin{cases} \frac{5}{2}\alpha = 90-\frac{\alpha}{2} + 2\cdot k\cdot 180 \\ \text{oppure} \\ \frac{5}{2}\alpha = 180-(90-\frac{\alpha}{2}) + 2k\,180 \end{cases}$$

$\alpha \in (0, 90)$

$\alpha = 30$ (dalla 1ª con $k=0$)

$\alpha = 45$.