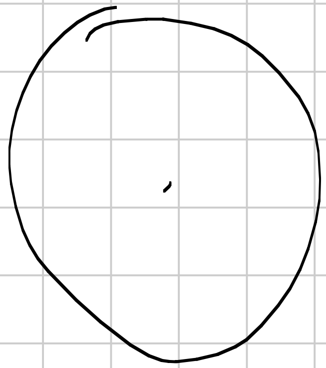
G3 - BASIC

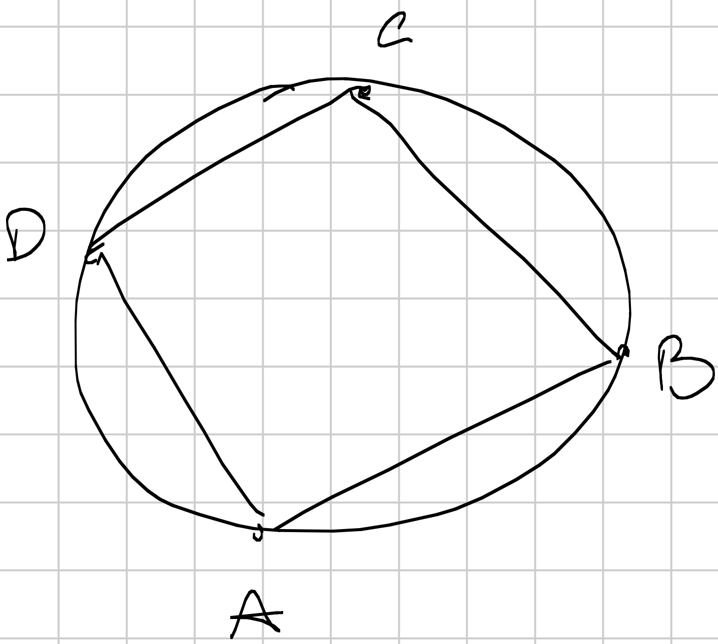
53 the best (Julian)

Titolo nota

08/09/2011

Circonferenza = Luogo dei punti P equidistanti da un punto dato





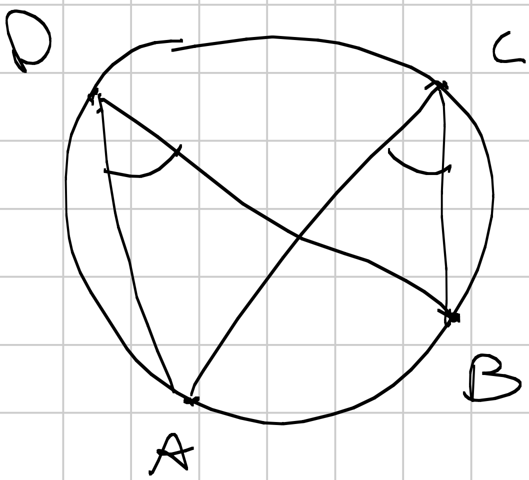
ABCD ciclico

$\Updownarrow$

i 4 punti stanno su una circ.

$\Updownarrow$

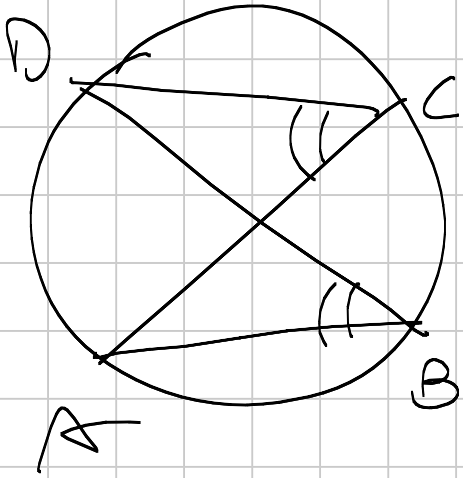
$D\hat{A}B = \pi - D\hat{C}B$



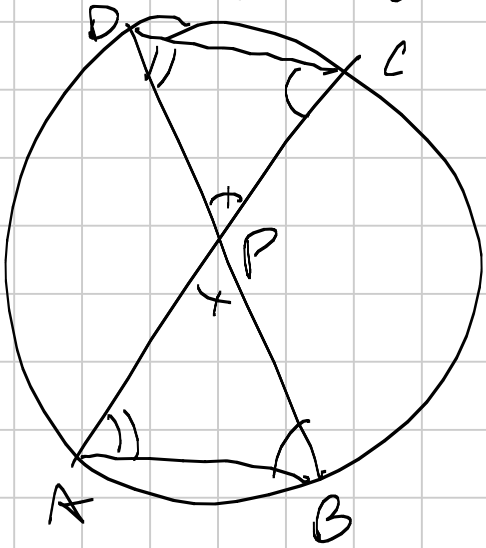
$A\hat{D}B = A\hat{C}B$

$\Updownarrow$

ABCD ciclico



Teorema delle corde



ABCD ciclico

$PA \cdot PC \qquad P = AC \cap DB$

$=$

$PB \cdot PD$

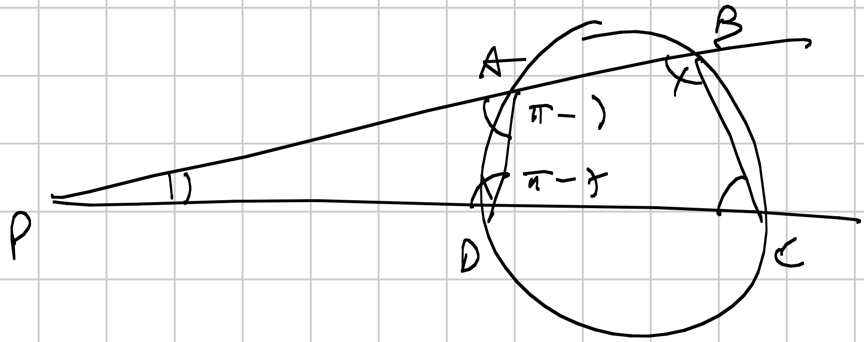
$P\hat{A}B \sim P\hat{D}C$

per il secondo criterio

$\Downarrow$

$$\frac{PA}{PB} = \frac{PD}{PC} \Rightarrow PA \cdot PC = PB \cdot PD$$

Teorema delle secanti



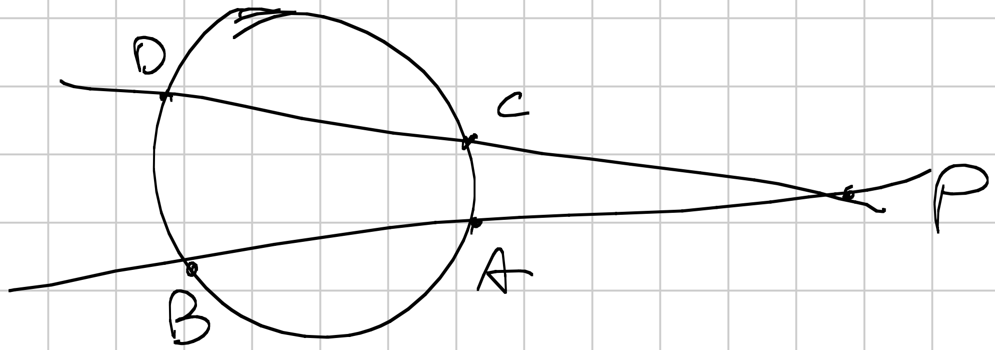
$PA \cdot PB$

$=$

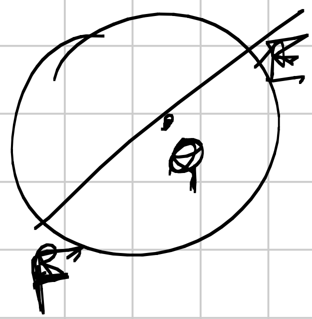
$PD \cdot PC$

$\hat{P A D} \sim \hat{P C B} \Rightarrow \frac{PD}{PA} = \frac{PB}{PC} \Rightarrow \underline{Th}$

Potenza

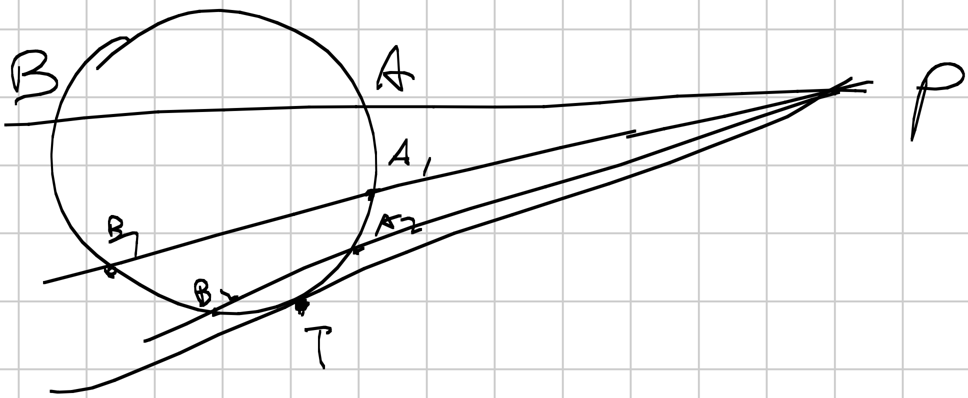


$Pow_{\Gamma}(P) = PA \cdot PB$



$PC \cdot PD = PA \cdot PB$

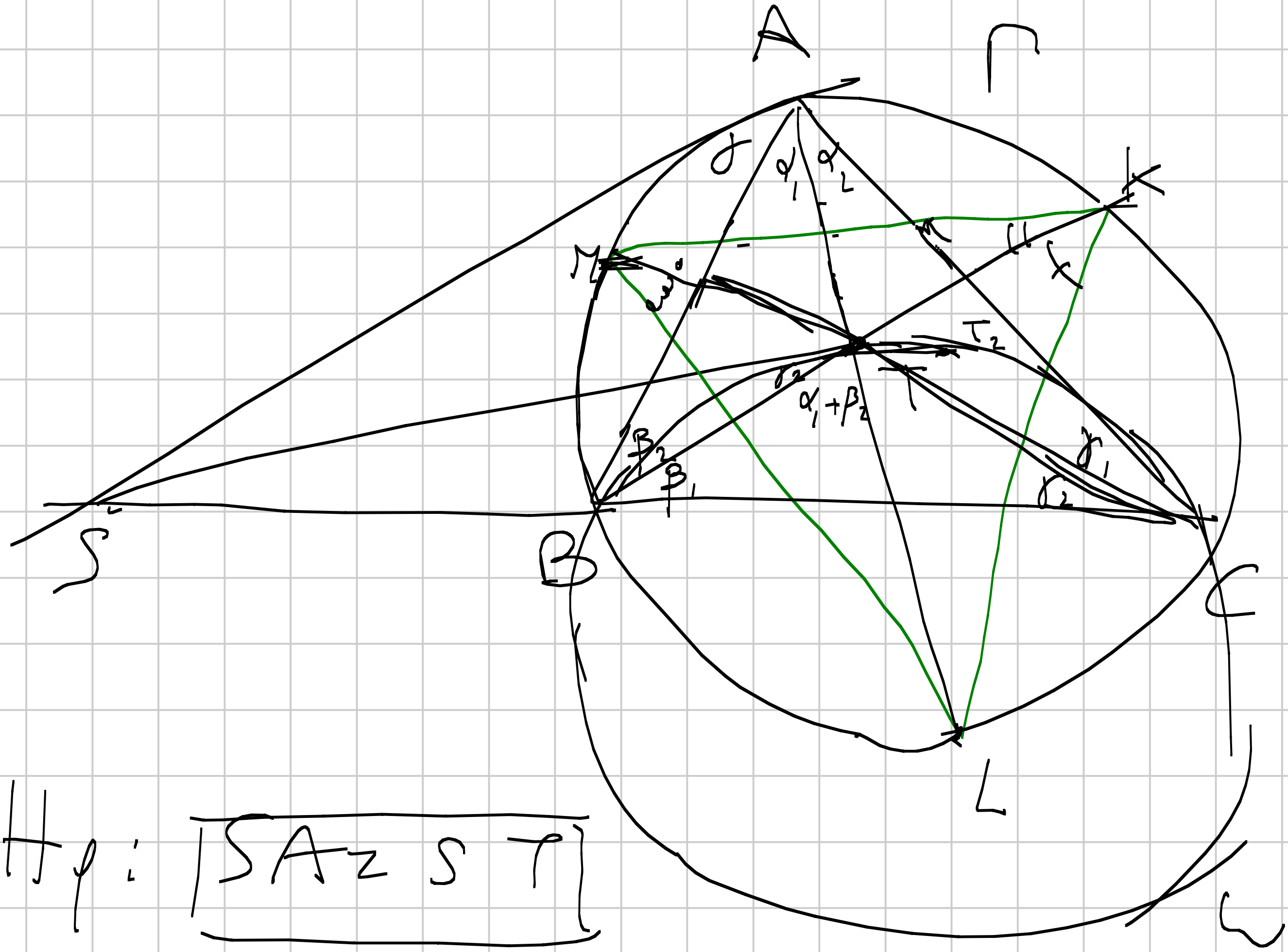
$Pow_{\Gamma}(Q) = QE \cdot QF$



$PA \cdot PB = PA_1 \cdot PB_1 = PA_2 \cdot PB_2 \longrightarrow PT^2$

$\mathrm{Pow}_\Gamma(P) = PT^2$

IMO 2010 - 4



Hp: $\boxed{SA = ST}$

Th: $ML = KL$

$$SA^2 = SB \cdot SC = \mathrm{Pow}_\Gamma(S)$$

$\parallel$

ST^2

$$ST^2 = SB \cdot SC$$

Prendiamo la circ. ω che passa per B, C, T. Vogliamo dimostrare che ω tange ST. Perché se ci fosse $T \neq T_2 = \omega \cap ST$

$$\Rightarrow \cancel{ST} \cdot ST_2 = SB \cdot SC = Pow_{\omega}(S)$$
$$\shortparallel$$
$$ST^2$$

$$\Downarrow$$

$ST = ST_2 \Rightarrow$ assurdo

$S\hat{T}B = T\hat{C}B$ (insistono sullo stesso arco di ω)

$T\hat{C}B = \gamma_2 \Rightarrow S\hat{T}B = \gamma_2$

$B\hat{T}L = \alpha_1 + \beta_2$ (teo. dell'ang esterno)

$$S\hat{A}T = S\hat{A}L = \alpha_1 + \underbrace{S\hat{A}B}_{\gamma_1+\gamma_2} = \alpha_1 + \gamma_1 + \gamma_2$$

(insistono sullo stesso arco)

$$S\hat{T}A \overset{Hp}{=} S\hat{A}T = \alpha_1 + \gamma_1 + \gamma_2$$

$$S\hat{T}A + S\hat{T}B + B\hat{T}L = \pi$$

$$(\alpha_1+\gamma_1+\gamma_2)+(\gamma_2)+(\alpha_1+\beta_2) = \pi$$

$$\boxed{2\alpha_1+\beta_2+\gamma_1+2\gamma_2=\pi}$$

Lavoro sulla tesi

$$\text{Th} \iff \underbrace{M\hat{K}L}_{\gamma_2+\alpha_1} = \underbrace{K\hat{M}L}_{\beta_1+\alpha_2} \iff \gamma_2+\alpha_1=\beta_1+\alpha_2$$

Hp: $2\alpha_1+\beta_2+\gamma_1+2\gamma_2=\pi \Rightarrow \gamma_2+\alpha_1=\beta_1+\alpha_2$

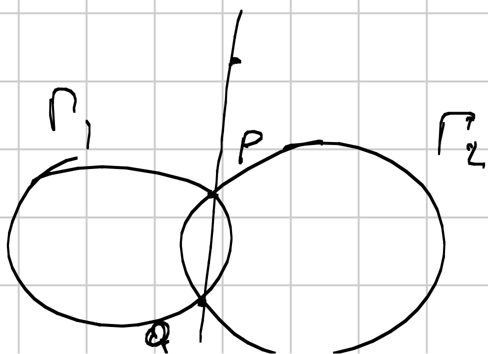
$$2\alpha_1+\beta_2+\gamma_1+2\gamma_2=\pi$$

$$\cancel{2}\alpha_1-\cancel{\alpha_1}+\cancel{\beta_2}+\cancel{\gamma_1}+\cancel{2}\gamma_2=\pi-\alpha_1=\cancel{\alpha_1}+\alpha_2+\beta_1+\cancel{\beta_2}+\cancel{\gamma_1}+\cancel{\gamma_2}$$

$$\Downarrow$$

$$\alpha_1+\gamma_2=\beta_1+\alpha_2 \Rightarrow \text{Win}$$

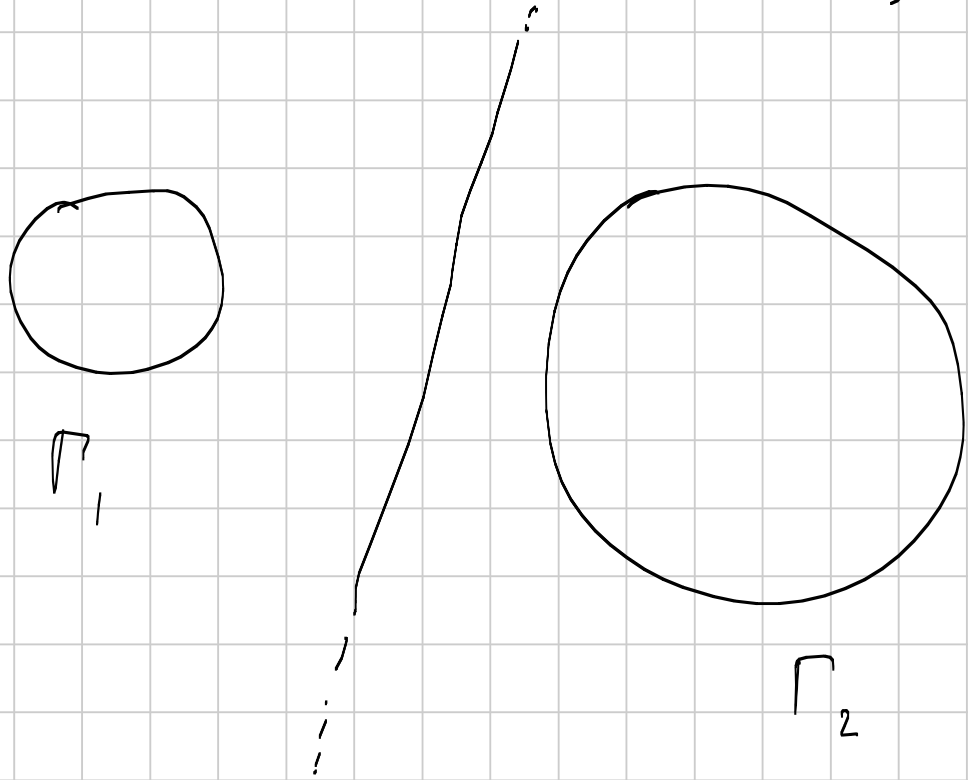
Asse radicale



L'asse radicale r di Γ_1 e Γ_2 := la retta PQ

$$A \in r \iff \underset{\substack{\parallel \\ PA \cdot QA}}{Pow_{\Gamma_1}(A)} = \underset{\substack{\parallel \\ PA \cdot QA}}{Pow_{\Gamma_2}(A)}$$

(scelgo PQ come secante) (scelgo PQ come secante)



<u>Def.</u> generale = Luogo dei punti P t.c

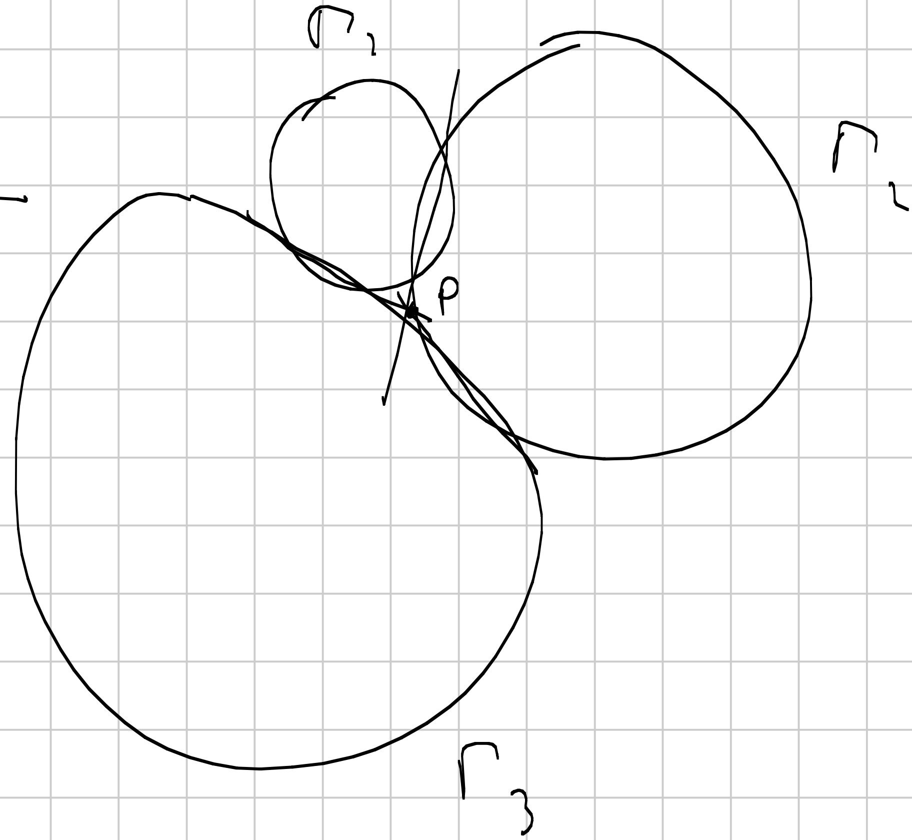
$$Pow_{\Gamma_1}(P) = Pow_{\Gamma_2}(P)$$

<u>Fatto 1</u> Asse $\perp O_1 O_2$

dove O_1 = centro di Γ_1

e O_2 = centro di Γ_2

Fatto 2



$Asse(\Gamma_1, \Gamma_2)$, $Asse(\Gamma_2, \Gamma_3)$, $Asse(\Gamma_3, \Gamma_1)$

concorrono.

$P' = Asse(\Gamma_1, \Gamma_2) \cap Asse(\Gamma_2, \Gamma_3)$

$$Pow_{\Gamma_1}(P') = Pow_{\Gamma_2}(P')$$

ma P' sta anche sul secondo asse

$$\Rightarrow Pow_{\Gamma_2}(P') = Pow_{\Gamma_3}(P')$$

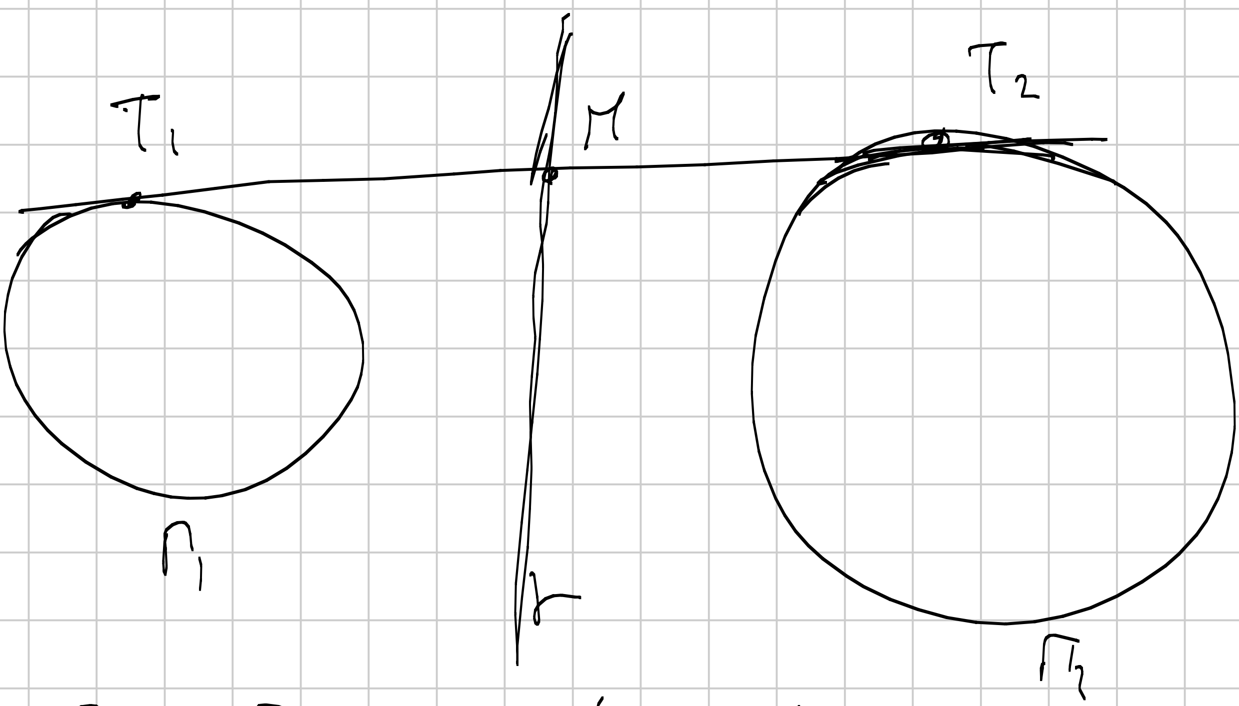
Rasse $\Rightarrow$ $Pow_{\Gamma_1}(P') = Pow_{\Gamma_2}(P') = Pow_{\Gamma_3}(P')$

$$\Rightarrow Pow_{\Gamma_1}(P') = Pow_{\Gamma_3}(P')$$

⇓ def. di asse radicale

P' sta sull'asse radicale di Γ_1 e Γ_3.

⇒ Win



r: asse di Γ_1 e Γ_2 $\quad s =$ tangente in comune a Γ_1 e Γ_2

$T_1 = s \cap \Gamma_1 \qquad T_2 = s \cap \Gamma_2 \qquad M = r \cap T_1T_2$

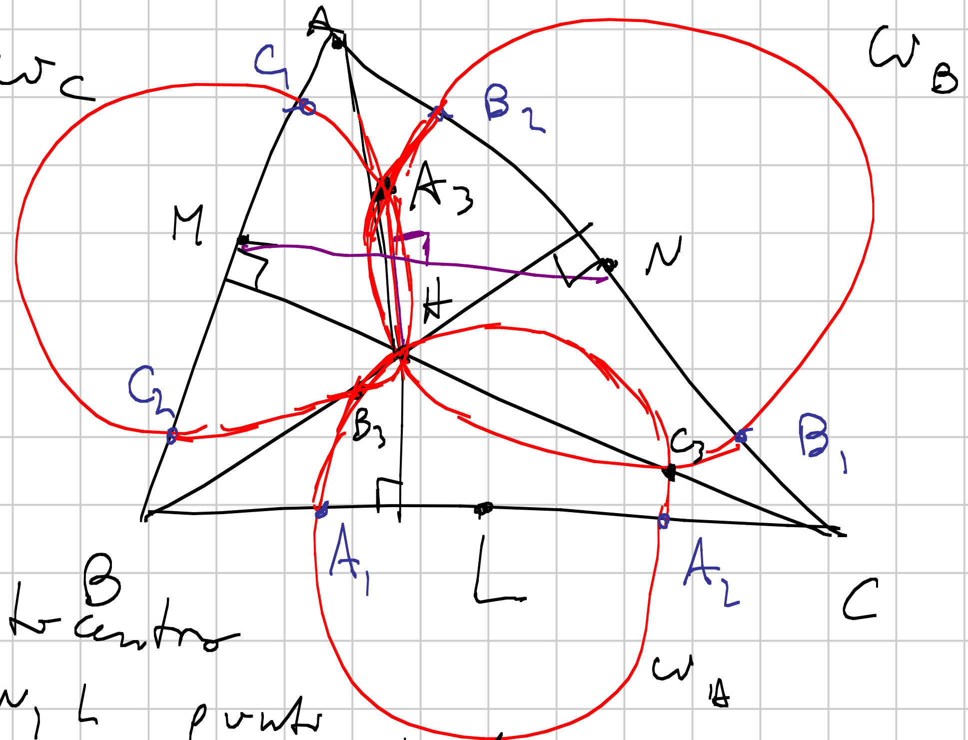
$\Rightarrow M =$ p.to medio di T_1T_2

$$\underset{\displaystyle MT_1^2}{\overset{\shortparallel}{Pow_{\Gamma_1}(M)}} \overset{def}{=} \underset{\displaystyle MT_2^2}{\overset{\shortparallel}{Pow_{\Gamma_2}(M)}}$$

⇓

$MT_1^2 = MT_2^2 \Rightarrow MT_1 = MT_2 \Rightarrow M$ p.to medio

IMO 1 - 2008



H ortocentro

M, N, L punti medi

ω_A centro in L passante per H

ω_B " " N " " "

ω_C " " M " " "

Th: $A_1, A_2, B_1, B_2, C_1, C_2$ stanno su una circ.

—

$\omega_B \cap \omega_C = A_3$ e cyc

Vogliamo dimostrare che $A_3 \in AH$

Fatto 1: A_3H = asse radicale di ω_B e ω_C

$\Rightarrow$

Fatto 2 $\quad MN \parallel BC$

Fatto 1 + Fatto 2 = Fatto 3

$A_3H \perp MN \parallel BC \Rightarrow A_3H \perp BC$

$\Rightarrow A_3H \equiv AH$ (retta perp. per A é unica)

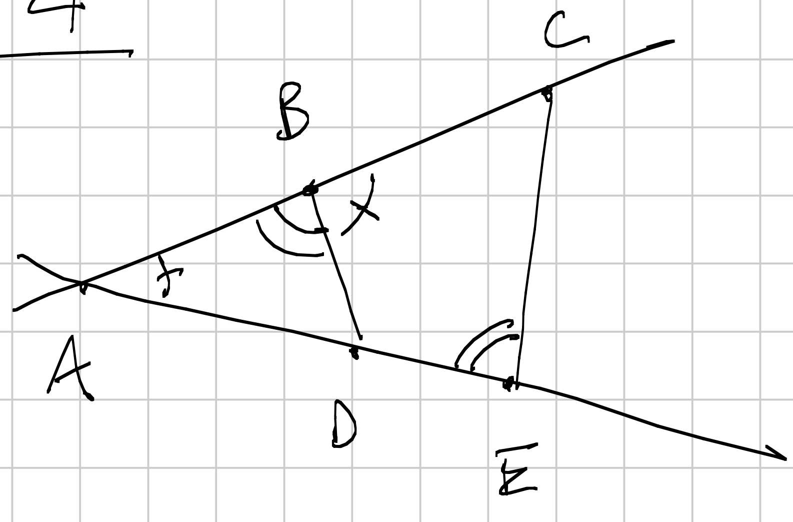
$\Rightarrow A_3 \in AH \Rightarrow AH$ é asse radicale

di ω_B e $\omega_C \Rightarrow Pow_{\omega_B}(A) = Pow_{\omega_C}(A)$

def.

$\Rightarrow AB_1 \cdot AB_2 = AC_1 \cdot AC_2$

Fatto 4



Hp: $AB \cdot AC = AD \cdot AE$

Th: DECB ciclico

Dim: $AB \cdot AC = AD \cdot AE \Rightarrow \dfrac{AB}{AD} = \dfrac{AE}{AC}$

$\Rightarrow$ $A\hat{B}D$ e $A\hat{E}C$ sono simili per il 1° criterio ($\hat{A}$ in comune) e $\frac{AB}{AD} = \frac{AE}{AC}$

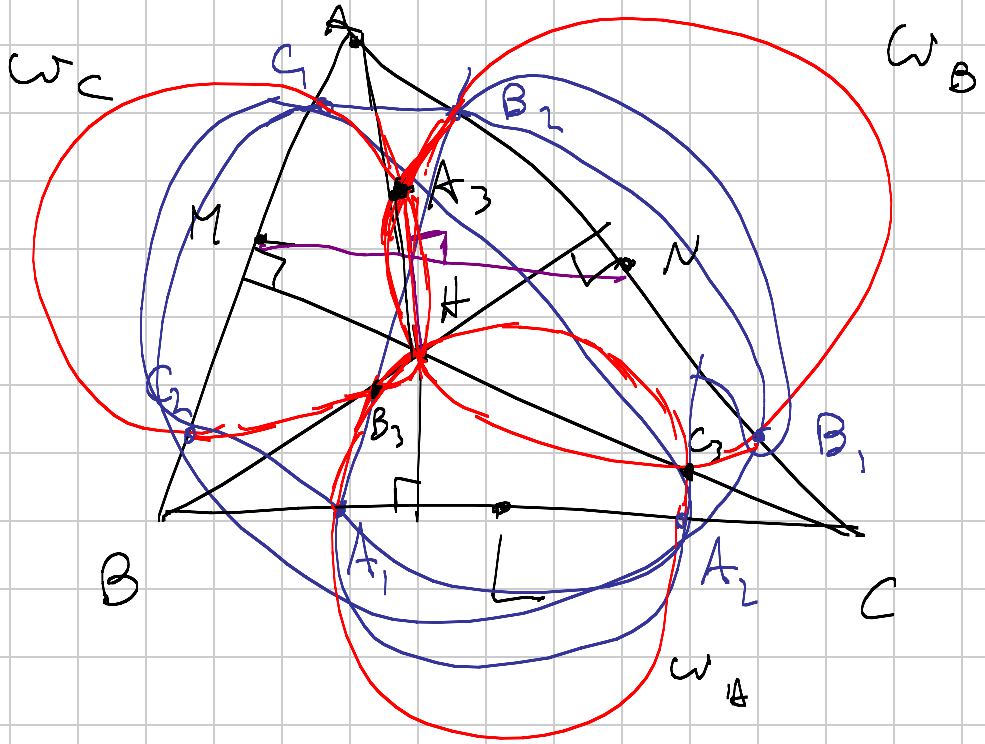
$\Rightarrow A\hat{B}D = D\hat{E}C \Rightarrow \underbrace{\pi - A\hat{B}D}_{D\hat{B}C} = \pi - D\hat{E}C$

$\Rightarrow D\hat{B}C + D\hat{E}C = \pi \Rightarrow$ Th

Torno al problema

$AB_1 \cdot AB_2 = AC_1 \cdot AC_2 \Rightarrow$ $B_1 B_2 \; C_1 C_2$ ciclico

Ragionando in modo uguale su B e su C dimostro che $A_1 A_2 \; B_1 B_2$ e $A_1 A_2 \; C_1 C_2$ sono ciclici ma queste 3 potrebbero essere distinte



Asse $(A_1A_2, B_1B_2, A_1A_2, C_1C_2) = A_1A_2$

Asse $(B_1B_2\ C_1C_2,\ A_1A_2\ C_1C_2) = C_1C_1$

Asse $(A_1A_2\ B_1B_2,\ B_1B_2\ C_1C_2) = B_1B_2$

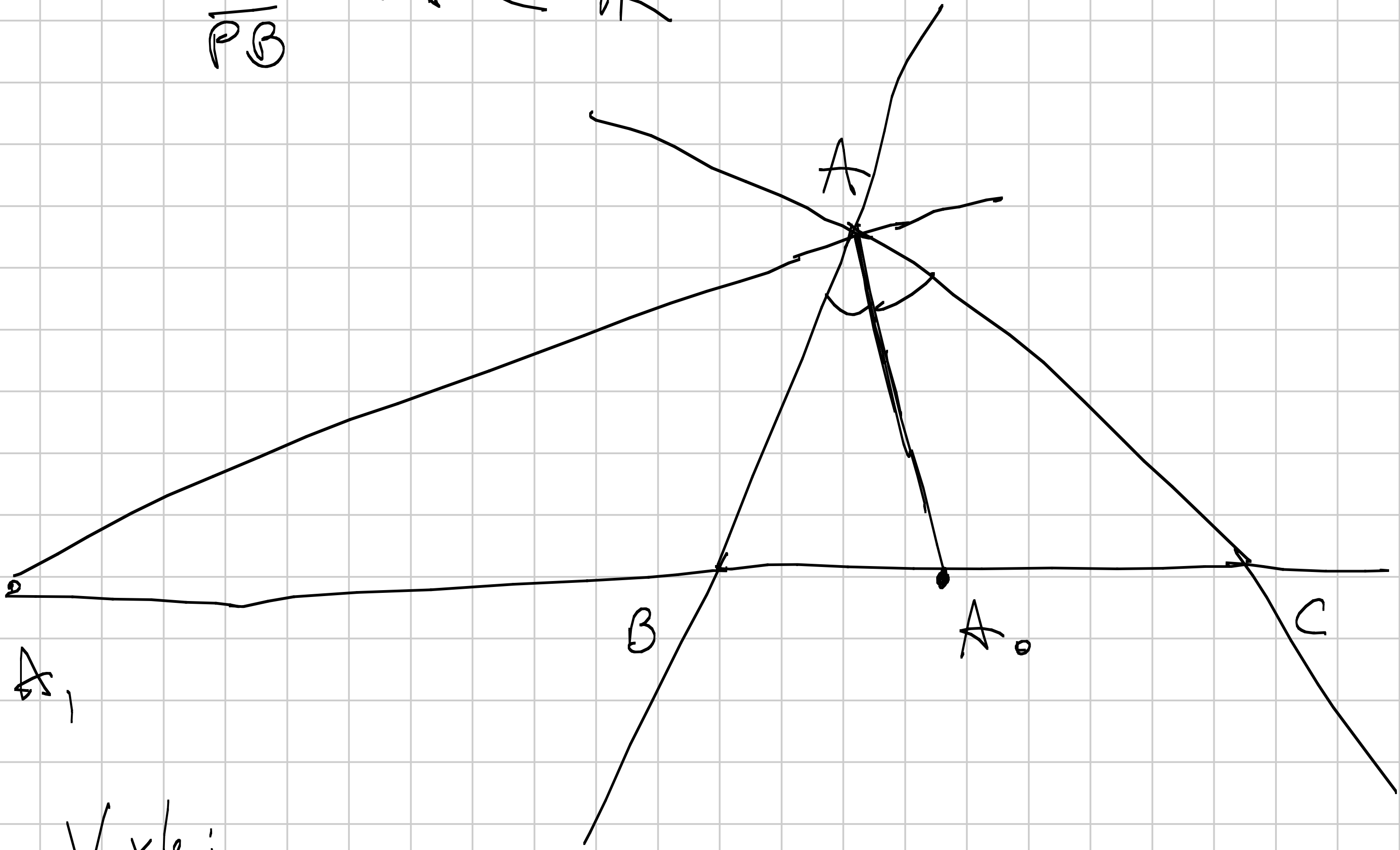
Se le 3 circ. fossero distinte A_1A_2, B_1B_2 e C_1C_2 concorrerebbero ma sono i lati del triangolo $\Rightarrow$ assurdo.

$\Rightarrow A_1A_2B_1B_2C_1C_2$ ciclico

Circonferenze di Apollonio

Def Luogo dei punti P t.c.

$$\frac{\overline{PA}}{\overline{PB}} = k \in \mathbb{R}^+$$



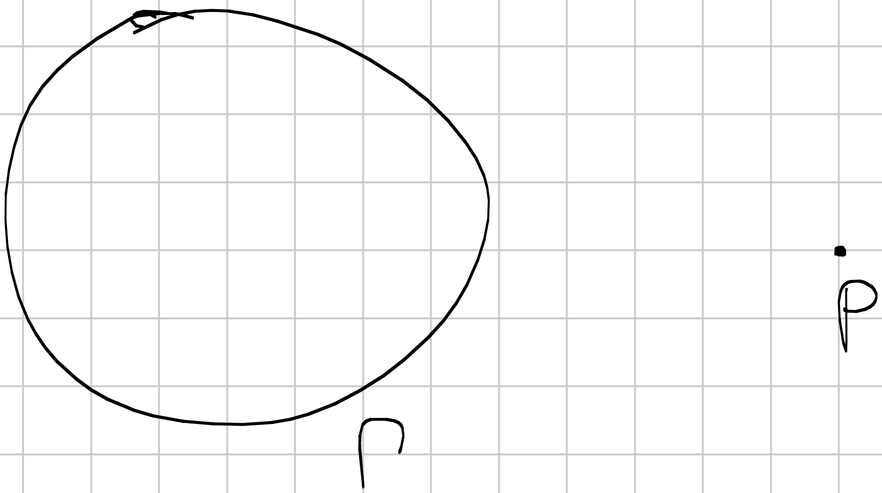
Vale:

$$\frac{A_0 B}{A_0 C} = \frac{AB}{AC} \qquad \frac{A_1 B}{A_1 C} = \frac{AB}{AC}$$

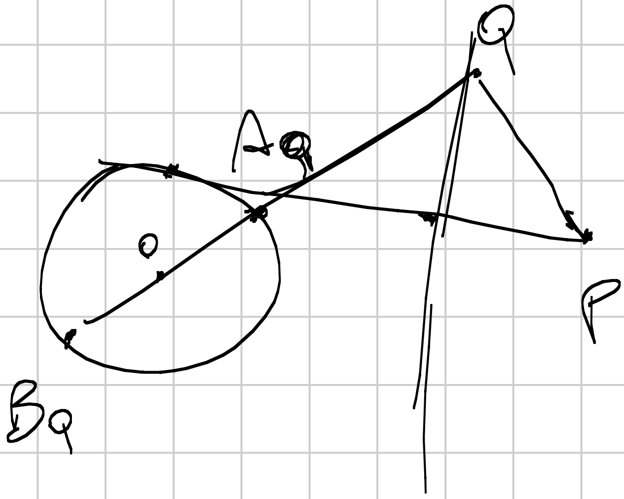
(esercizio per casa, dimostratelo)

$\Rightarrow$ A_0, A, A_1 stanno su una circonferenza di Apollonio. Circonferenza di Apollonio del triangolo (ce ne sono 3, una per vertice).

Torniamo agli assi radicali

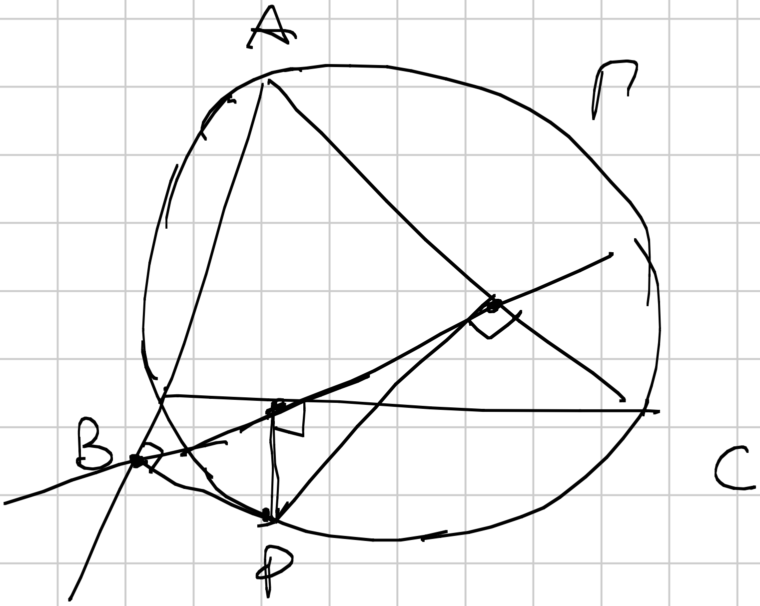


∃ l'asse anche di Γ e P (perché P la potete vedere come una circ. di r=0)



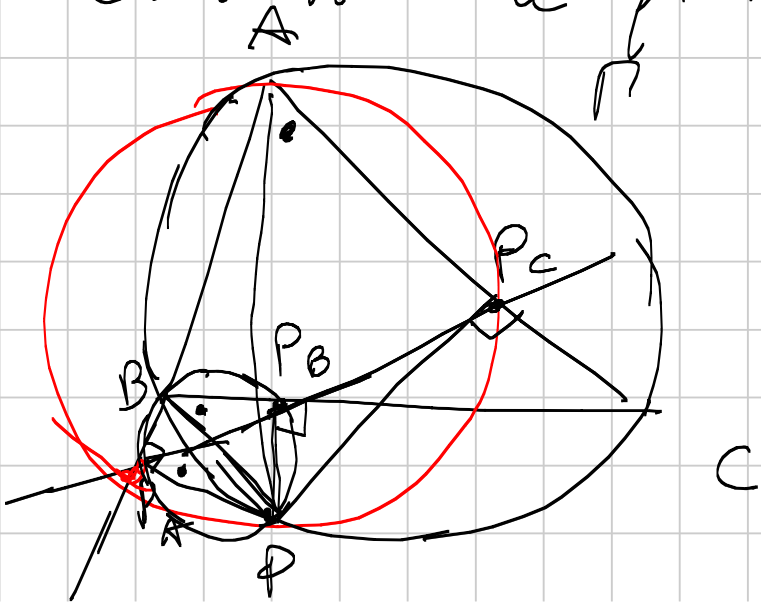
$\mathrm{Pow}_P(Q) = QP^2$

Retta di Simson



P sta su $\Gamma \iff$ le proiezioni di P sui lati sono allineate

Dim Chiamiamo le proiezioni P_A, P_B, P_C



PP_AP_BB è ciclico

$(\underset{\frac{\pi}{2}}{PP_AB} = \pi - \underset{\pi - \frac{\pi}{2}}{PP_B B})$

PP_BP_CC è ciclico anche lui

e pure PP_AAP_C

([redacted]

Quando ho $ABCD$ con 2 angoli retti è sempre ciclico (perché $\frac{\pi}{2} = \frac{\pi}{2}$ e $\frac{\pi}{2} + \frac{\pi}{2} = \pi$))

$$P\hat{P_A}P_B = P\hat{B}P_B = P\hat{A}C = P_C\hat{P_A}P$$

$$\| $$

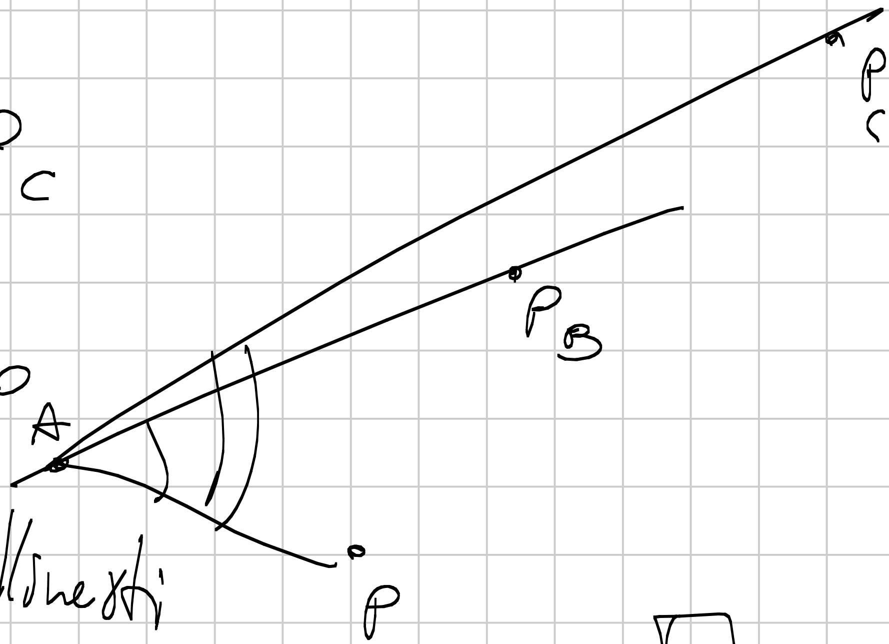
$$P\hat{P_A}P_C$$

$$\Downarrow$$

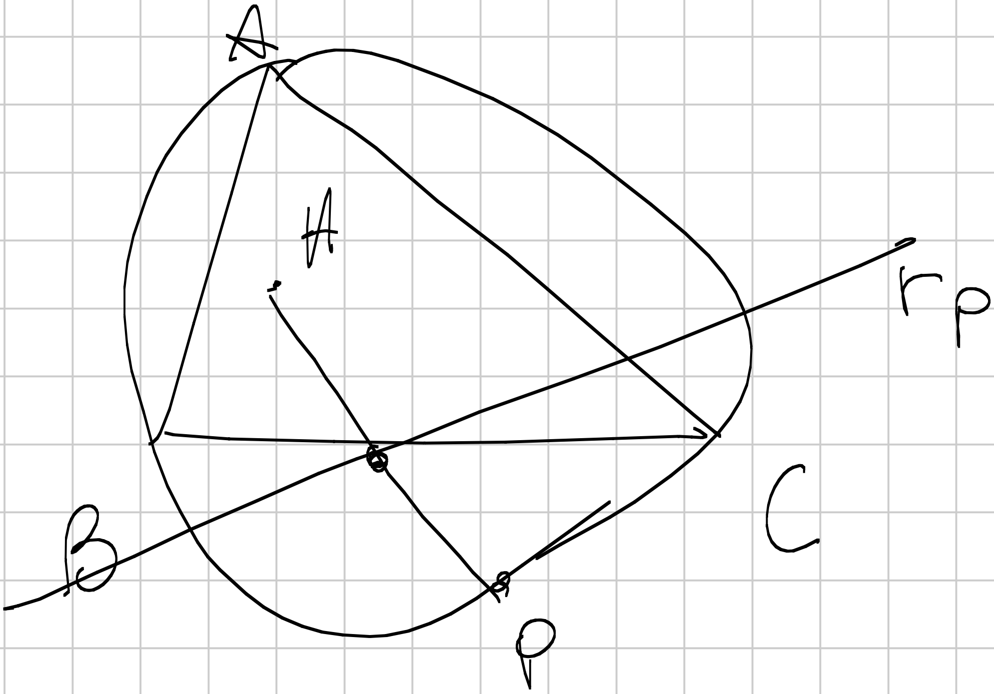
$$P\hat{P_A}P_B = P\hat{P_A}P_C$$

$$\Downarrow$$

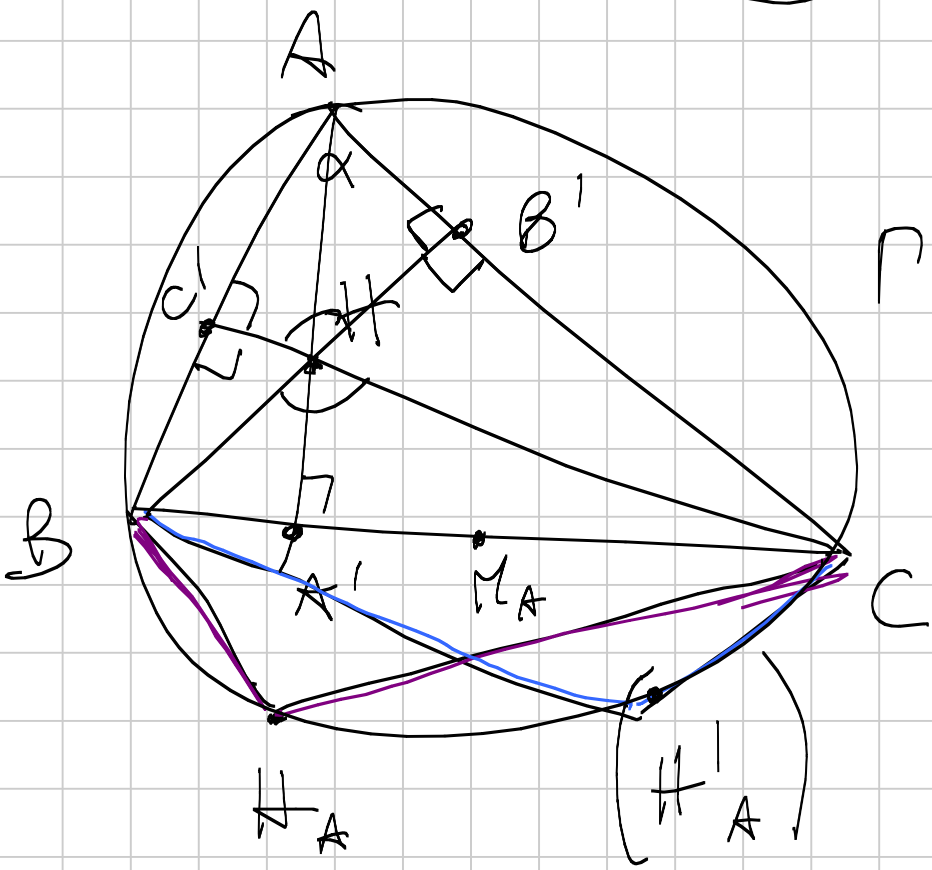
P_A, P_B, P_C allineati



$\blacksquare$



r_P biseca PH



H ortocentro

Th: simmetrico di H rispetto a BC ∈ Γ

<u>Dim</u>

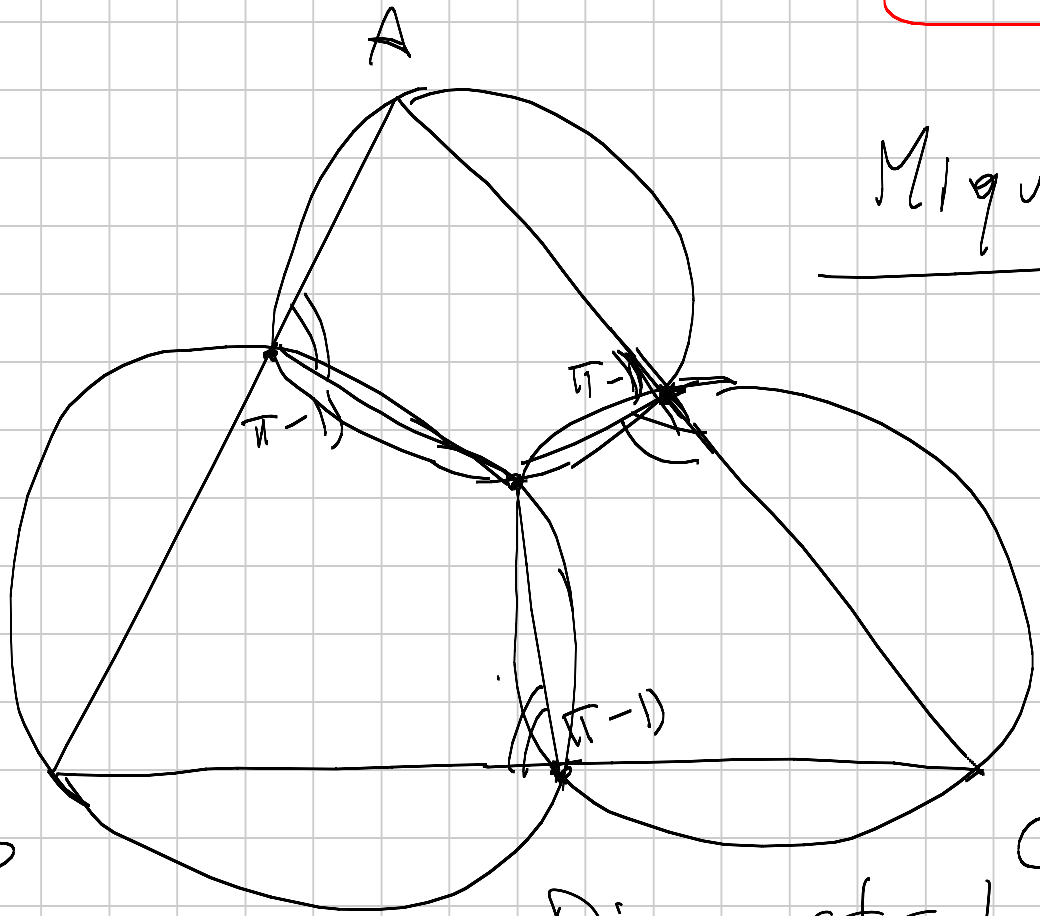
$AC'B'H$ è ciclico (ha 2 angoli retti)

$$B\hat{H}C = B'\hat{H}C' = \pi - \alpha$$

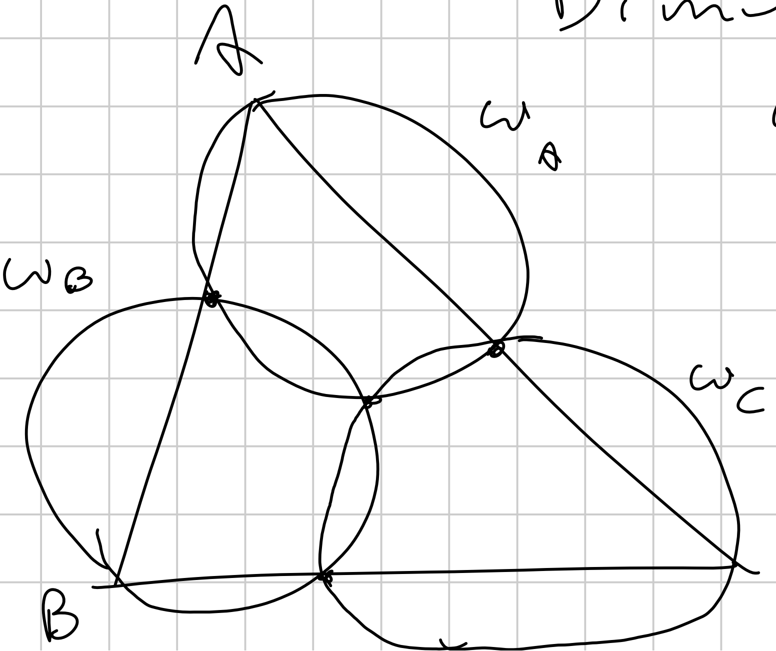
$$B\hat{H_A}C = B\hat{H}C = \pi - \alpha = \pi - B\hat{A}C$$

$\Rightarrow BACH_A$ ciclico $\Rightarrow$ $\boxed{H_A \text{ sta su } \Gamma}$

Miquel



Dimostrate a casa



$\omega_A \cap \omega_B \in AB$

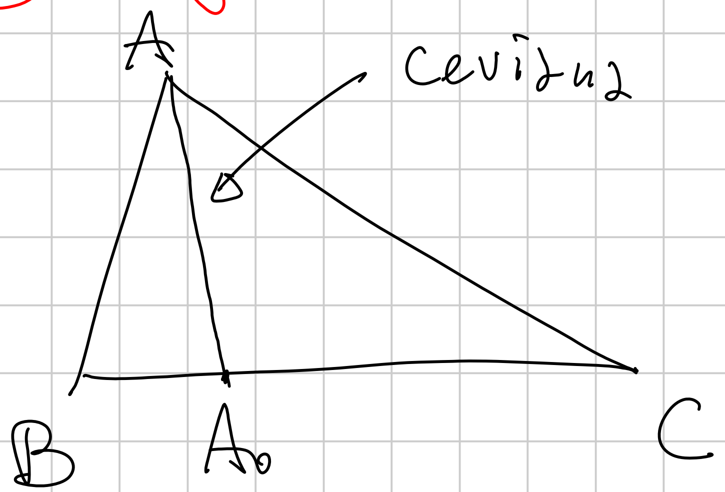
e cyc

$\Downarrow$

$\omega_A, \omega_B, \omega_C$

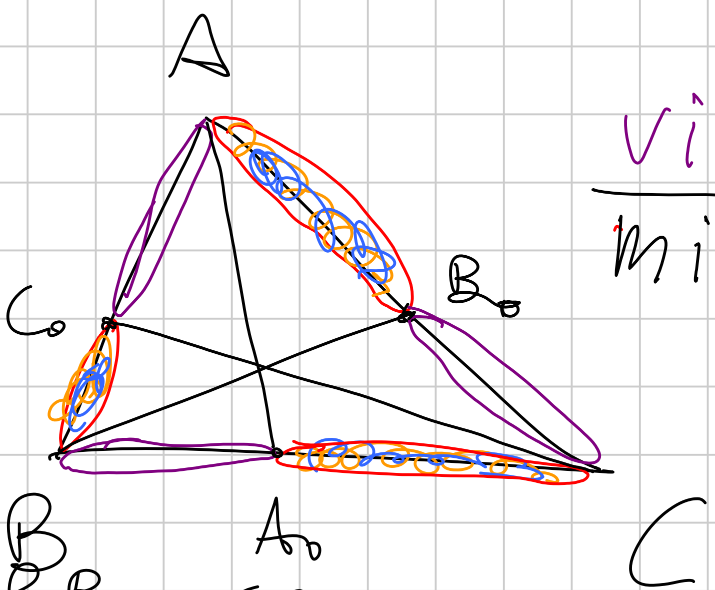
concorrono

Ceva



AA_0 ceviana (qualsiasi retta passante x un vertice)

Teo



$$\frac{\text{viola}}{\text{mix}} = 1$$

Ae: AA_0, BB_0, CC_0 ceviane

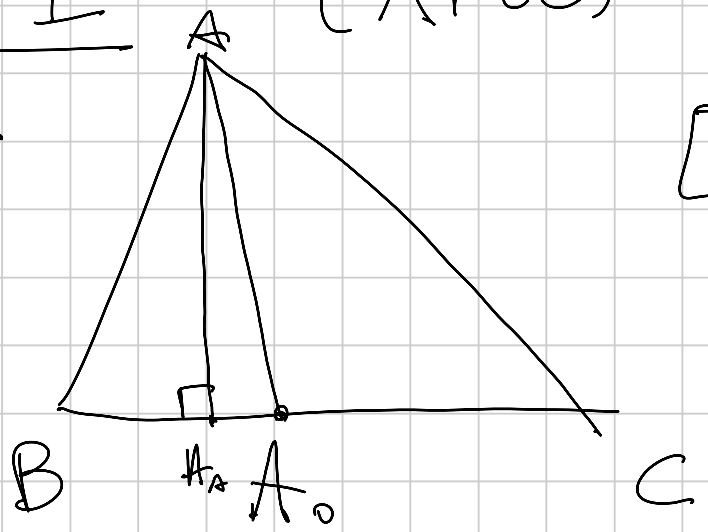
Th: AA_0, BB_0, CC_0 concorrono

$\Updownarrow$

$$\frac{BA_0}{A_0C} \cdot \frac{CB_0}{B_0A} \cdot \frac{AC_0}{C_0B} = 1$$

Dim 1 (Aree)

Lemma



$[ABC]$

$$\frac{[ABA_0]}{[AA_0C]} = \frac{BA_0}{A_0C}$$

Dim

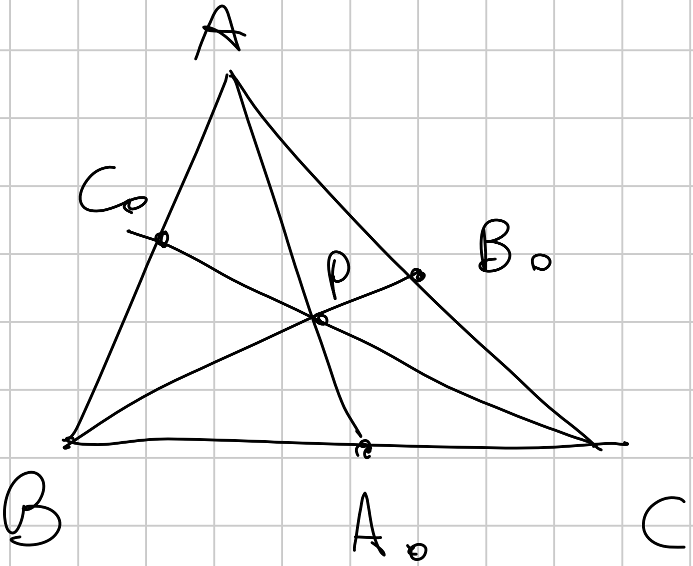
Sia AH_A l'altezza da A

$\Rightarrow [ABA_0] = \frac{1}{2} A_0B \cdot AH_A$

$[AA_0C] = \frac{1}{2} A_0C \cdot AH_A$

$$\Rightarrow \frac{[ABA_0]}{[AA_0C]} = \frac{\cancel{\frac{1}{2}} A_0B \cdot \cancel{AH_A}}{\cancel{\frac{1}{2}} A_0C \cdot \cancel{AH_A}} = \frac{A_0B}{A_0C}$$

$\square$



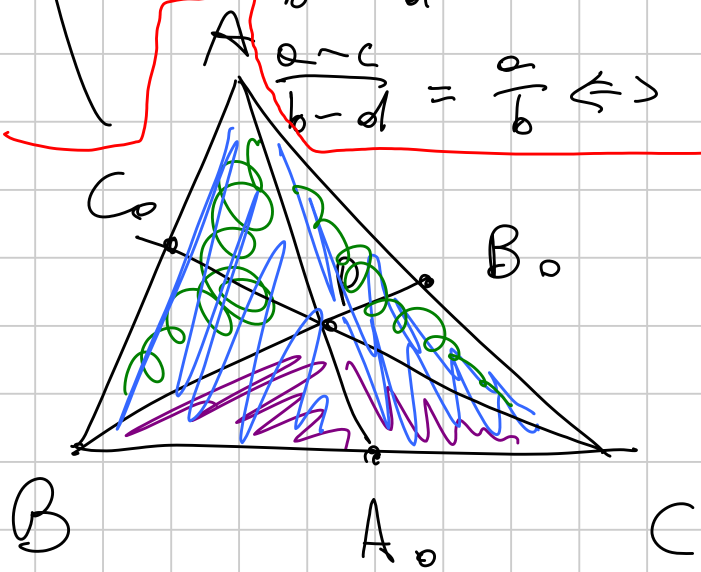
$$\frac{BA_0}{A_0C} \overset{\text{lemma}}{=} \frac{[ABA_0]}{[AA_0C]} \qquad \frac{BA_0}{A_0C} = \frac{[PBA_0]}{[PA_0C]}$$

Cosa sulle proporzioni

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-c}{b-d} = \frac{a}{b} = \frac{c}{d}$$

Dim $\frac{a}{b} = \frac{c}{d} \iff ad = bc$

$$\frac{a-c}{b-d} = \frac{a}{b} \iff \cancel{ab} - cb = \cancel{ab} - da \iff ad = bc \quad \square$$



$$\frac{BA_0}{A_0C} \overset{\text{lemma}}{=} \frac{[ABA_0]}{[AA_0C]} \qquad \frac{BA_0}{A_0C} \overset{\text{lemma}}{=} \frac{[PBA_0]}{[PA_0C]}$$

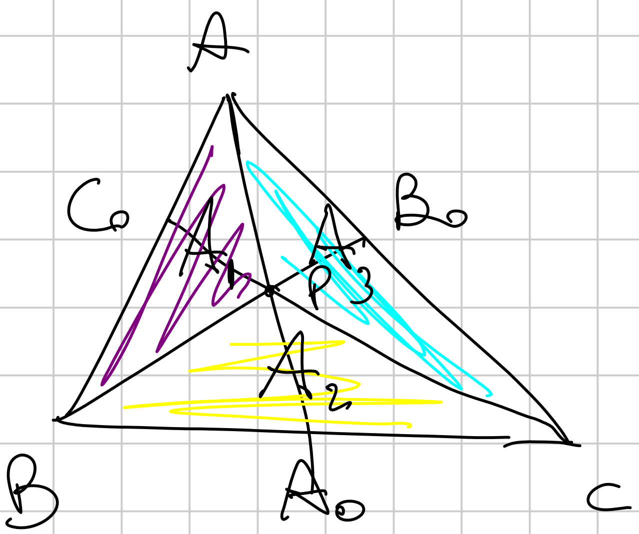
$$\frac{BA_0}{A_0C} = \frac{[ABA_0]}{[AA_0C]} = \frac{[PBA_0]}{[PA_0C]}$$

$\Downarrow$

$$\frac{BA_0}{A_0C} = \frac{[ABA_0]-[PBA_0]}{[AA_0C]-[PA_0C]} = \frac{[PAB]}{[PAC]}$$

Analogamente:

$$\frac{CB_0}{B_0A} = \frac{[PBC]}{[PBA]} \quad e \quad \frac{AC_0}{C_0B} = \frac{[PCA]}{[PCB]}$$



$[PAB] = A_1$

$[PAC] = A_3$

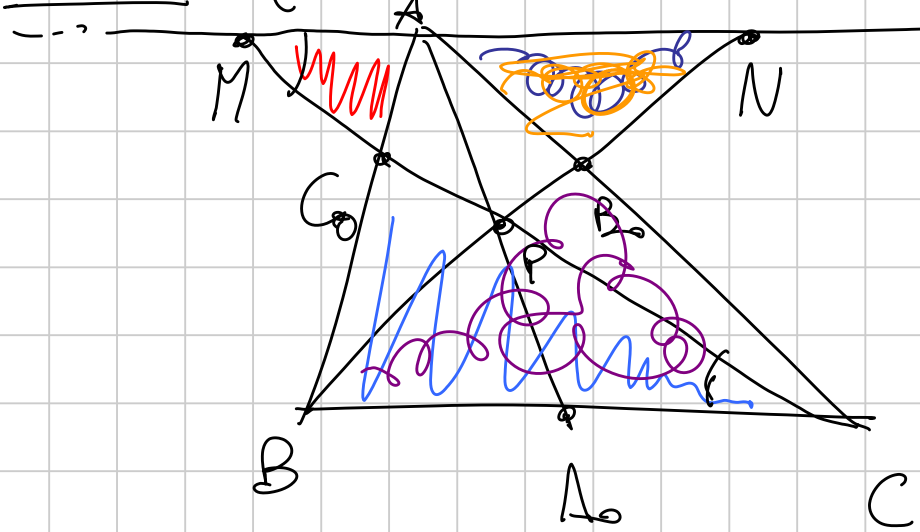
$[PBC] = A_2$

$$\frac{BA_0}{A_0C} \cdot \frac{CB_0}{B_0A} \cdot \frac{AC_0}{C_0B} = \frac{\cancel{A_1}}{\cancel{A_3}} \cdot \frac{\cancel{A_2}}{\cancel{A_1}} \cdot \frac{\cancel{A_3}}{\cancel{A_2}}$$

$$= 1$$

$\square$

Dim. 2 $(r \parallel BC)$



$\overset{\triangle}{CC_0B} \sim \overset{\triangle}{MC_0A}$

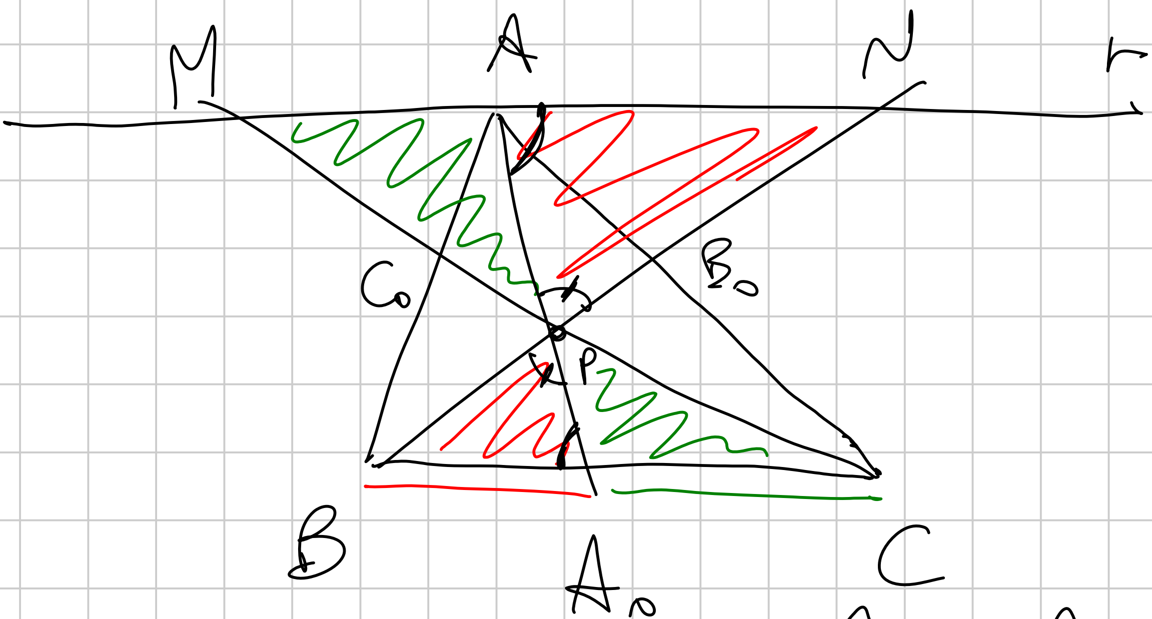
- $A\hat{M}C_0 = C_0\hat{C}B$
- $C_0\hat{B}C = M\hat{A}C_0$

(II criterio)

$$\Rightarrow \frac{AC_0}{C_0B} = \frac{AM}{BC}$$

Analogamente: $P\overset{\triangle}{B_0}C \sim N\overset{\triangle}{B_0}A$

$$\Rightarrow \frac{CB_0}{B_0A} = \frac{BC}{AN}$$



$P\overset{\triangle}{B}A_0 \sim P\overset{\triangle}{N}A$ $\left(\begin{array}{l} P\hat{A_0}B = P\hat{A}N \\ B\hat{P}A_0 = N\hat{P}A \\ \text{II criterio} \end{array} \right)$

$$\Rightarrow \frac{BA_0}{AN} = \frac{PA_0}{PA} \quad (\times \text{ similitudine})$$

Analogamente: $\dfrac{A_0C}{AM} = \dfrac{PA_0}{PA}$

$$\Rightarrow \boxed{\frac{BA_0}{AN} = \frac{A_0C}{AM}} \Rightarrow \frac{BA_0}{A_0C} = \frac{AN}{AM}$$

$$\frac{BA_0}{A_0C} \cdot \frac{CB_0}{B_0A} \cdot \frac{AC_0}{C_0B} = \frac{\cancel{AN}}{\cancel{AM}} \cdot \frac{\cancel{BC}}{\cancel{AN}} \cdot \frac{\cancel{AM}}{BC}$$

$$= 1$$

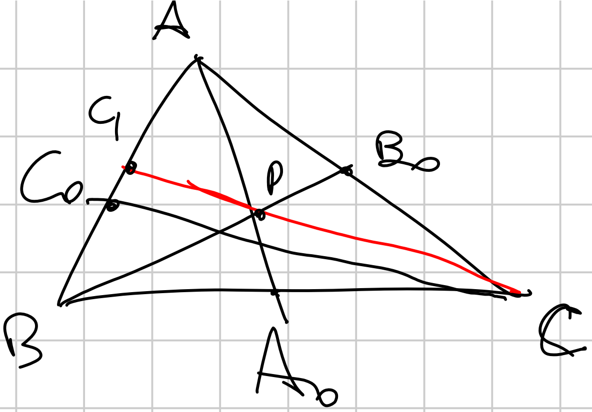
$\square$

AA_0, BB_0, CC_0 concorrono

$\Updownarrow$

$$\frac{BA_0}{A_0C} \cdot \frac{CB_0}{B_0A} \cdot \frac{AC_0}{C_0B} = 1$$

Freccia blu:



Supponiamo x assurdo $\boxed{\frac{AB_0}{B_0C}\cdot\frac{CA_0}{A_0B}}\cdot\frac{BC_0}{C_0A}=1$

ma $\cancel{AA_0}$, BB_0, CC_0

non concorrono

Sia $P = \cancel{AA_0} \cap BB_0$

e $C_1 = CP \cap AB$

$$\Rightarrow \boxed{\frac{AB_0}{B_0C}\cdot\frac{CA_0}{A_0B}}\cdot\frac{BC_1}{C_1A}=1$$

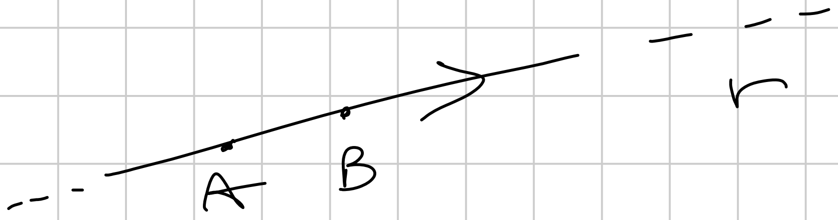
$$\Rightarrow \frac{BC_1}{C_1A}=\frac{BC_0}{C_0A} \Rightarrow \frac{BC_1}{C_1A}+1=\frac{BC_0}{C_0A}+1$$

$$\Rightarrow \frac{BC_1+C_1A}{C_1A}=\frac{BC_0+C_0A}{C_0A}\Rightarrow \frac{\cancel{BA}}{C_1A}=\frac{\cancel{BA}}{C_0A}$$

$\Rightarrow C_1A = C_0A \Rightarrow$ assurdo.

— o — o — o —

Cos'è un segmento orientato?

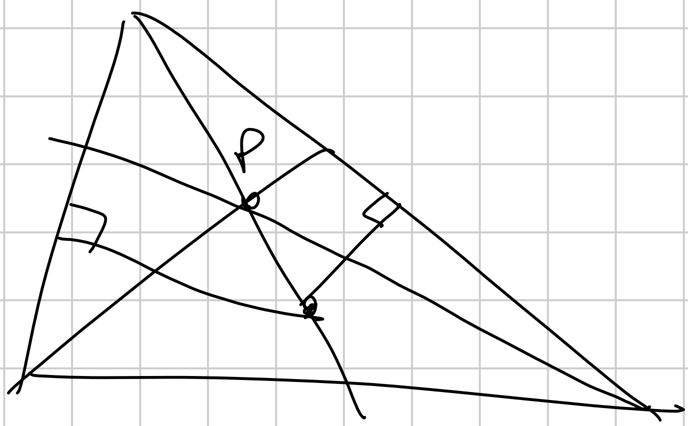


$\overline{AB}>0$ se B viene dopo A

e $\overline{AB} < 0$ se A viene dopo B

$$\overline{AB} = -\overline{BA}$$

$\_\_\_\_ \circ \_\_\_\_ \circ \_\_\_\_$

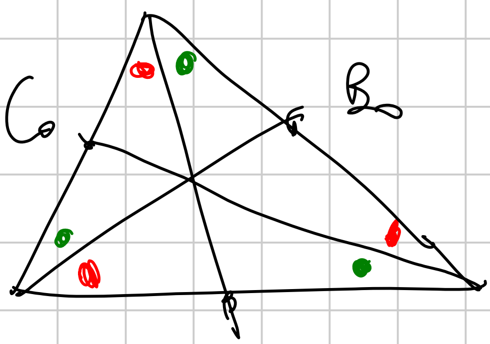


Esercizio per casa: ridimostrare

Ceva

$\_\_\_\_ \circ \_\_\_\_ \circ \_\_\_\_$

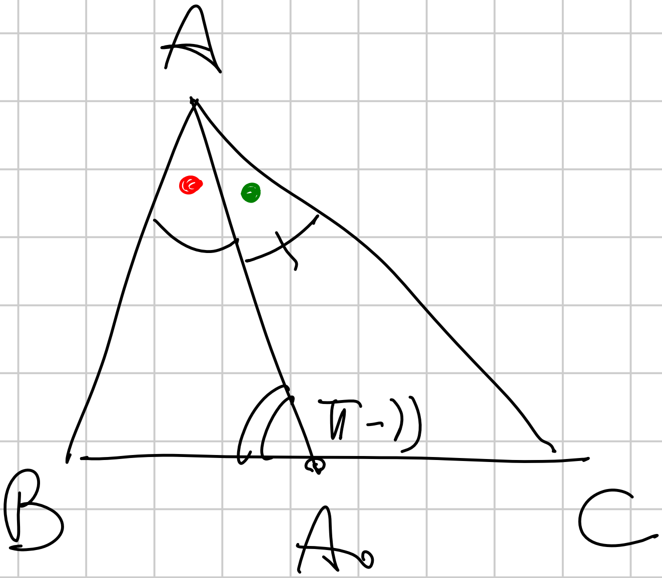
Ceva trigonometrico



$$\frac{\prod \sin \text{rossi}}{\prod \sin \text{verdi}} = 1$$

$$\frac{\sin B\hat{A}A_0}{\sin A_0\hat{A}C} \cdot \frac{\sin A\hat{C}C_0}{\sin C_0\hat{C}B} \cdot \frac{\sin C\hat{B}B_0}{\sin B_0\hat{B}A} = 1$$

Lemma



$$\frac{BA_0}{A_0C} = \frac{AB}{AC}\cdot\frac{\sin)}{\sin +}$$

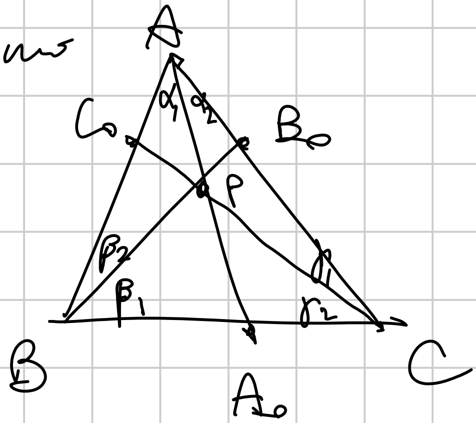
Dim.

$$\frac{BA_0}{BA} = \frac{\sin)}{\sin))}$$

$$\frac{A_0C}{CA} = \frac{\sin +}{\sin(\pi -))} = \frac{\sin +}{\sin))}$$

$$\Rightarrow \frac{\dfrac{BA_0}{BA}}{\dfrac{A_0C}{CA}} = \frac{\dfrac{\sin)}{\cancel{\sin))}}}{\dfrac{\sin +}{\cancel{\sin))}}} \Rightarrow \boxed{\frac{BA_0}{A_0C} = \frac{BA}{CA}\cdot\frac{\sin)}{\sin +}}$$

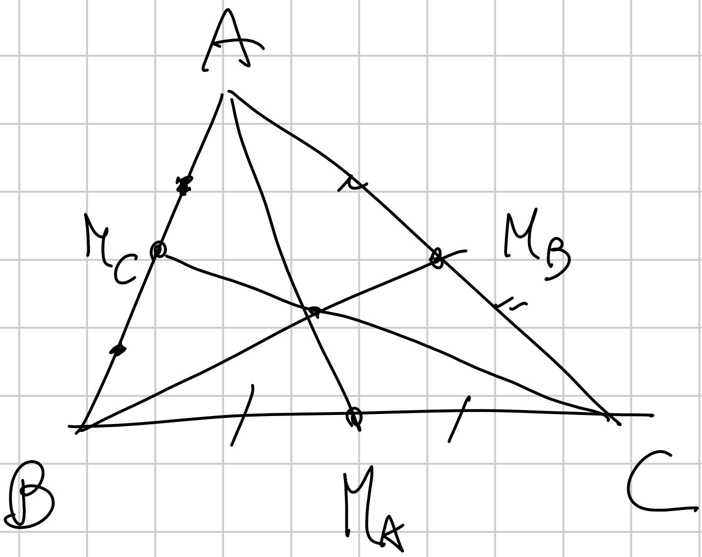
Teorema Ceva trig.



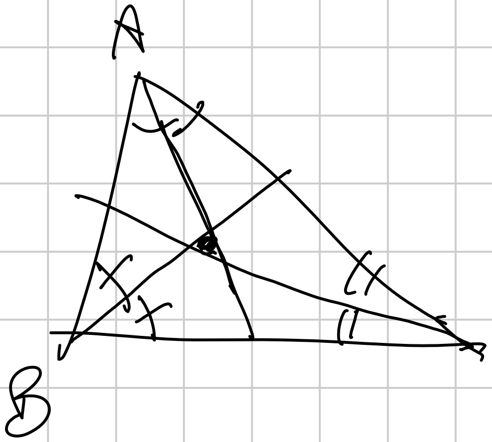
$AA_0,\ BB_0,\ CC_0$ concurrent $\iff \frac{BA_0}{A_0C}\cdot\frac{CB_0}{B_0A}\cdot\frac{AC_0}{C_0B}=1$

$$\iff \left(\frac{\cancel{AB}}{\cancel{AC}}\cdot\frac{\sin\alpha_1}{\sin\alpha_2}\right)\left(\frac{\cancel{AC}}{\cancel{BC}}\cdot\frac{\sin\beta_1}{\sin\beta_2}\right)\left(\frac{\cancel{BC}}{\cancel{AB}}\cdot\frac{\sin\gamma_1}{\sin\gamma_2}\right)=1$$

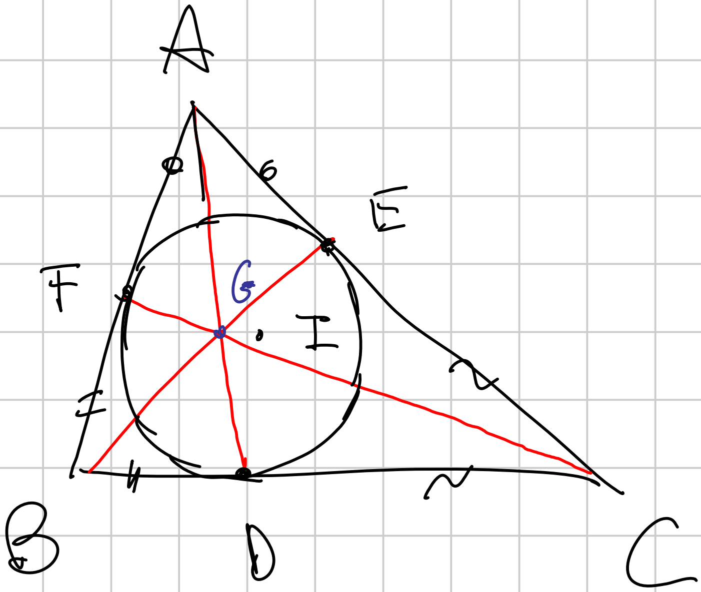
$\iff$ Win $\square$



$$\frac{BM_A}{\cancel{M_AC}}\cdot\frac{\cancel{CM_B}}{\cancel{M_BA}}\cdot\frac{\cancel{AM_C}}{\cancel{M_CB}}=1$$



$$\frac{\cancel{\sin\alpha_1}}{\cancel{\sin\alpha_2}}\cdot\frac{\cancel{\sin\beta_1}}{\cancel{\sin\beta_2}}\cdot\frac{\cancel{\sin\gamma_1}}{\cancel{\sin\gamma_2}}=1$$

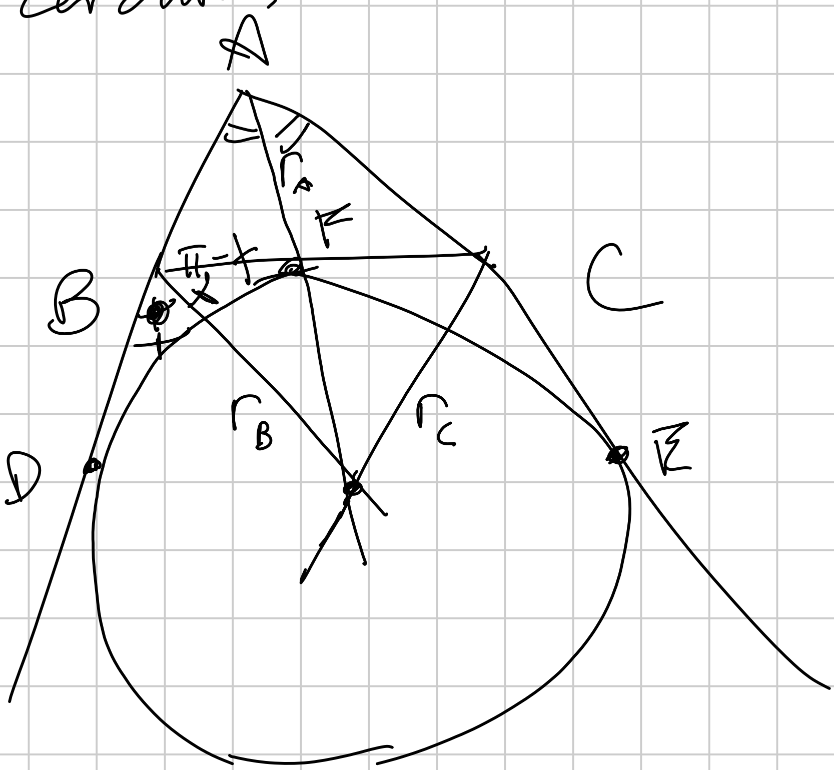


Th: AD, BE, CF } concorrono

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

G = Gergonne (dove concorrono)

Ex-cerchio:

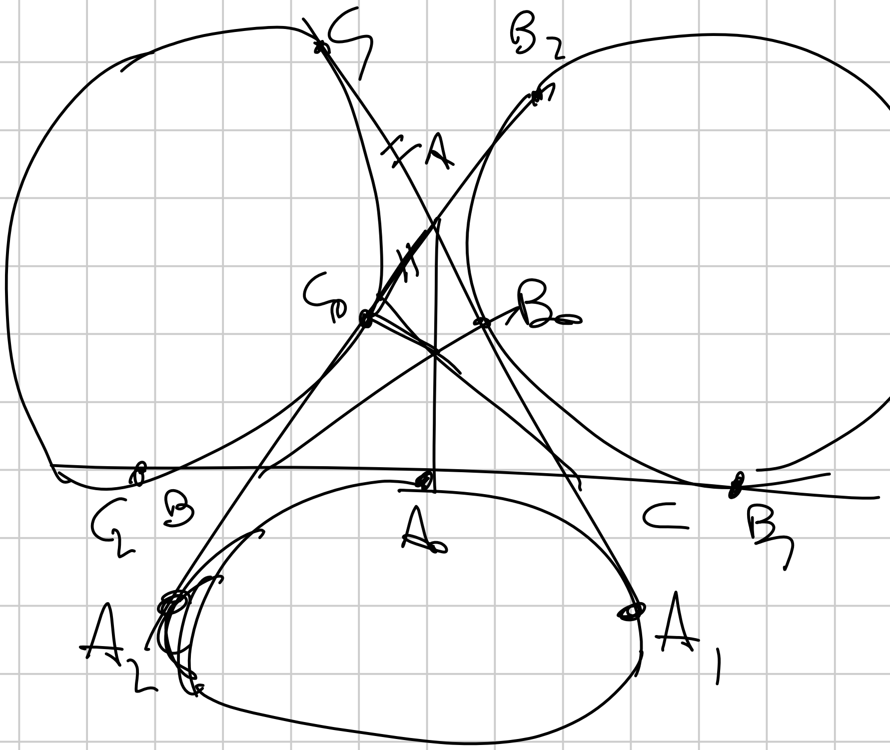


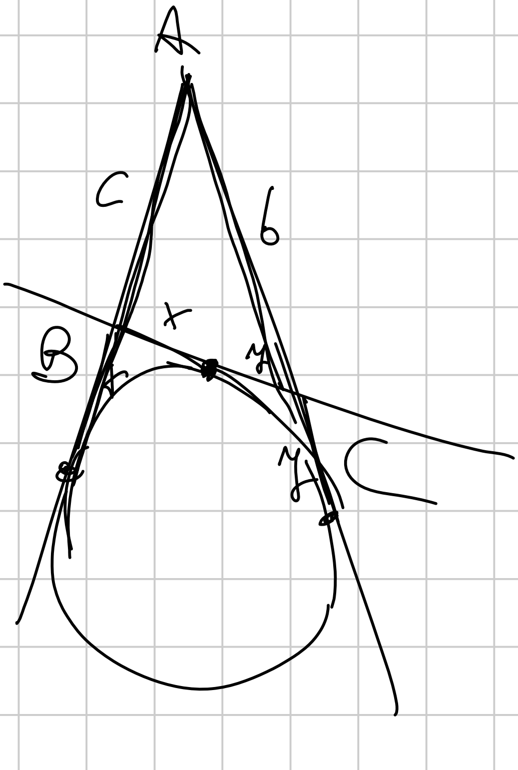
$r_A \cap r_B = I_A$ $\qquad d(I_A, AB) \overset{r_B}{=} d(I_A, BC)$

$$d(I_A, AB) \overset{r_A}{=} d(I_A, AC)$$

$$\Downarrow$$

$$d(I_A, AC) = d(I_A, BC) \Rightarrow$$ sta su r_C □





$c + x = b + y$ (tangenti da A)

$(c+x)+(b+y) = 2p$

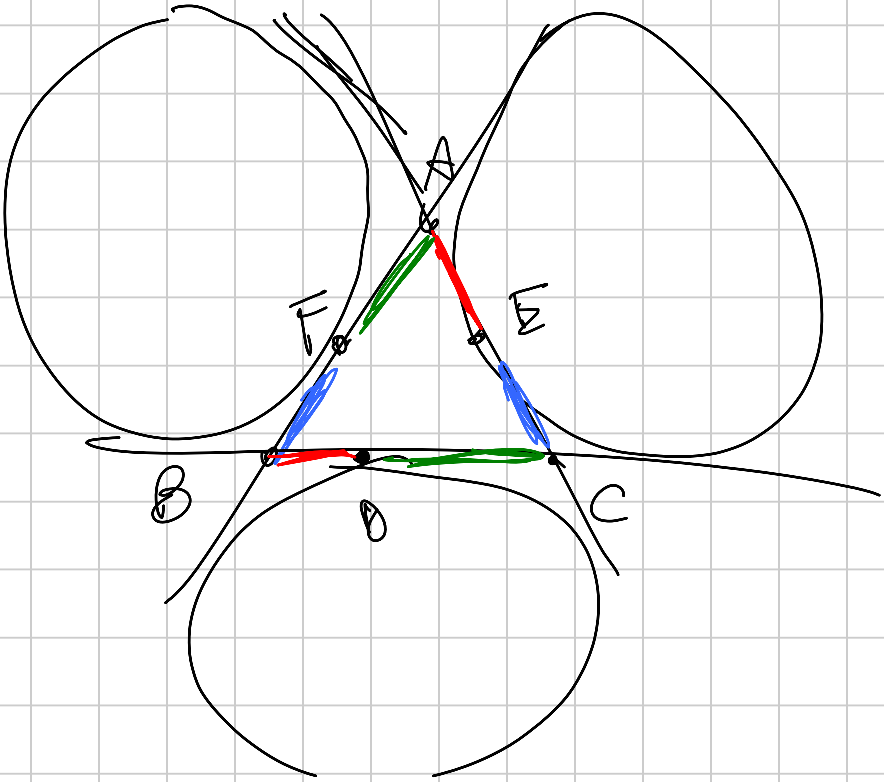
$$\Downarrow$$

$\cancel{2}(c+x) = \cancel{2}p$

$x = p - c$

$x = p - c \qquad y = p - b$

Torniamo a prima



$AE = p - c$

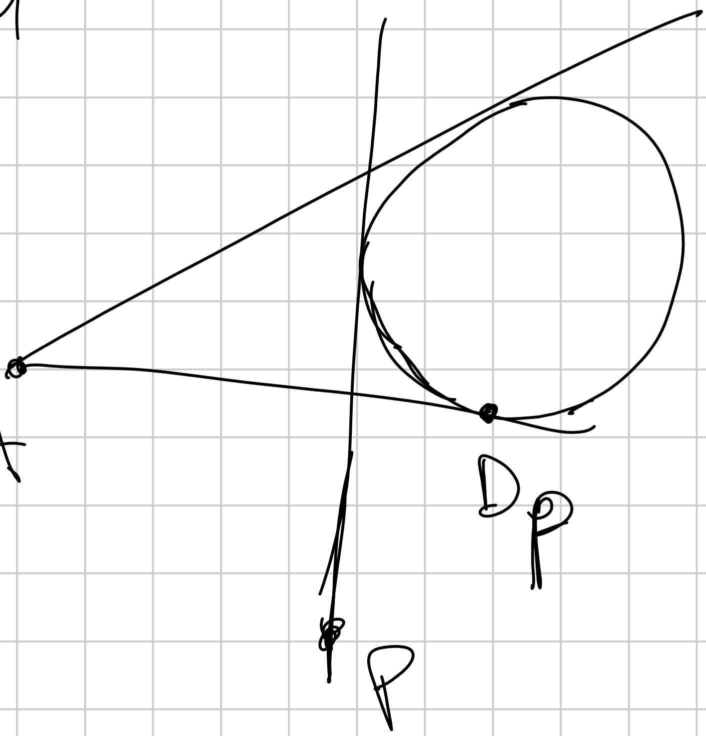
$BD = p - c$

$AF = p - b = DC$

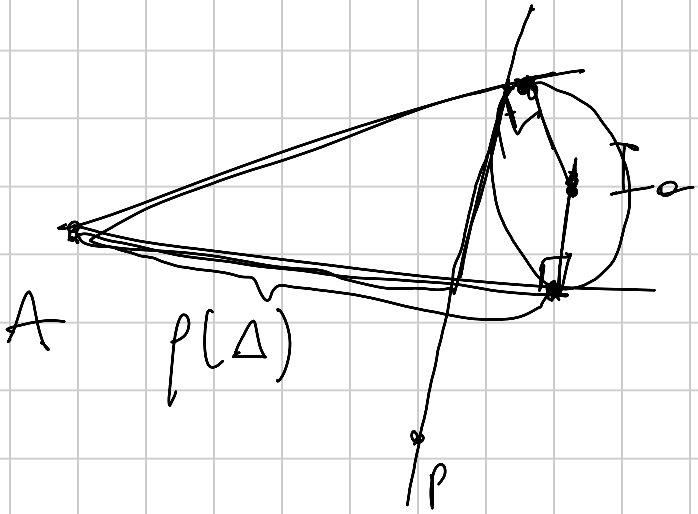
$EC = p - a = BF$

$$\Rightarrow \frac{\cancel{BD}}{\cancel{DC}} \cdot \frac{\cancel{EC}}{\cancel{AE}} \cdot \frac{\cancel{AF}}{\cancel{FB}} = 1$$

Il punto in cui concorrono si chiama Nagel



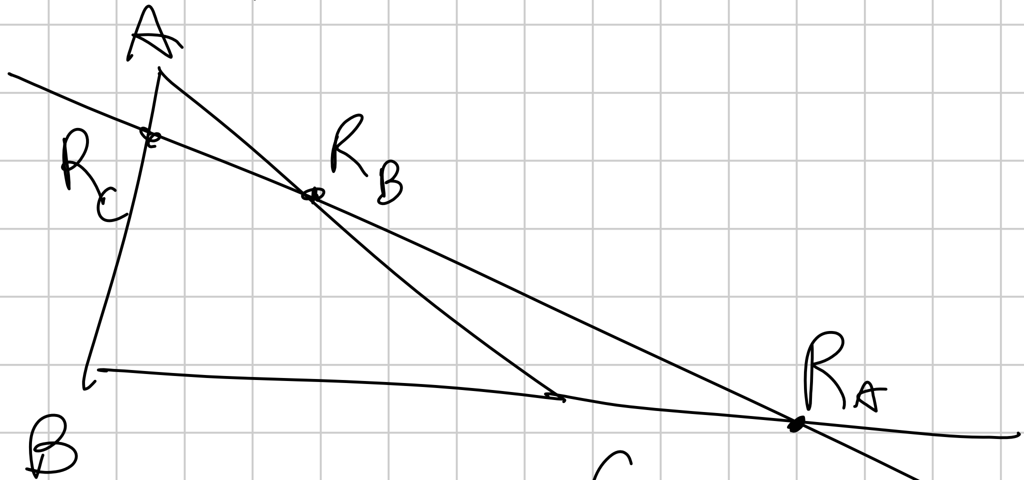
$AD_p = p(\Delta)$



$\rho(\Delta)$ ce l'ho già

$\square$

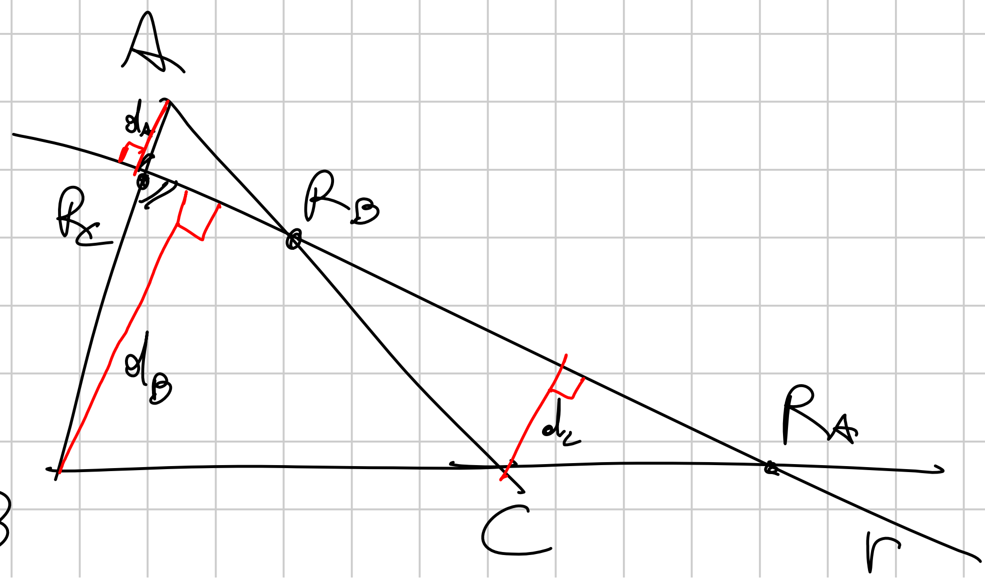
Menelao



R_A, R_B, R_C allineati $\Longleftrightarrow$

$$\frac{AR_B}{R_BC}\cdot\frac{BR_C}{R_CA}\cdot\frac{CR_A}{R_AB}=-1$$

Dim

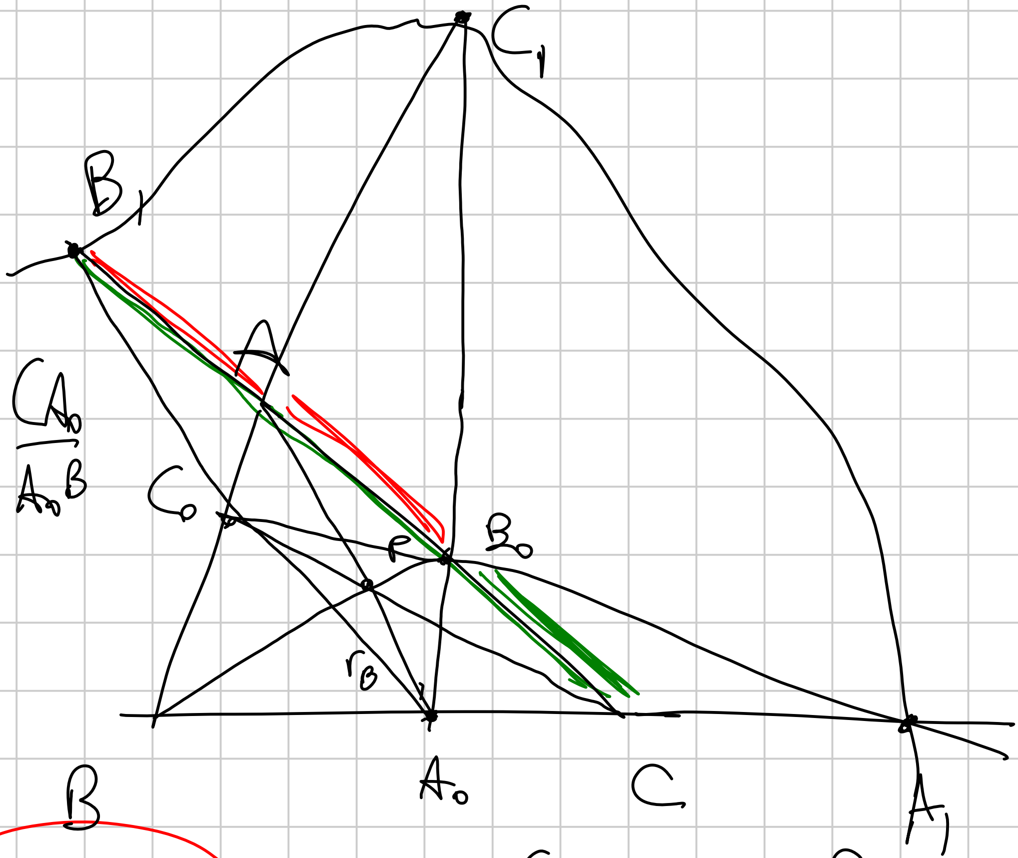


$$\frac{AR_C}{R_CB} = \frac{d_A}{d_B} \qquad \frac{CR_B}{R_BA} = \frac{d_C}{d_A}$$

$$\frac{BR_A}{R_AC} = \frac{d_B}{d_C}$$

$$\frac{AR_C}{R_CB} \cdot \frac{CR_B}{R_BA} \cdot \frac{BR_A}{R_AC} = \frac{\cancel{d_A}}{\cancel{d_B}} \cdot \frac{\cancel{d_C}}{\cancel{d_A}} \cdot \frac{\cancel{d_B}}{\cancel{d_C}} = (1)$$

L'altra freccia la dimostro come ho fatto con Ceva



Menelao su r_B

$$\left(\frac{AB_1}{B_1C}\right)\cdot\frac{BC_0}{C_0A}\cdot\frac{CA_0}{A_0B} = -1$$

$$\Downarrow$$

$$\frac{AB_1}{B_1C} = -\frac{C_0A}{BC_0}\cdot\frac{A_0B}{CA_0}$$

Ceva su P:

$$\frac{BA_0}{A_0C}\cdot\left(\frac{CB_0}{B_0A}\right)\cdot\frac{AC_0}{C_0B} = 1$$

$$\frac{B_0A}{CB_0} = \frac{AC_0}{C_0B}\cdot\frac{BA_0}{A_0C} = -\frac{AB_1}{B_1C}$$

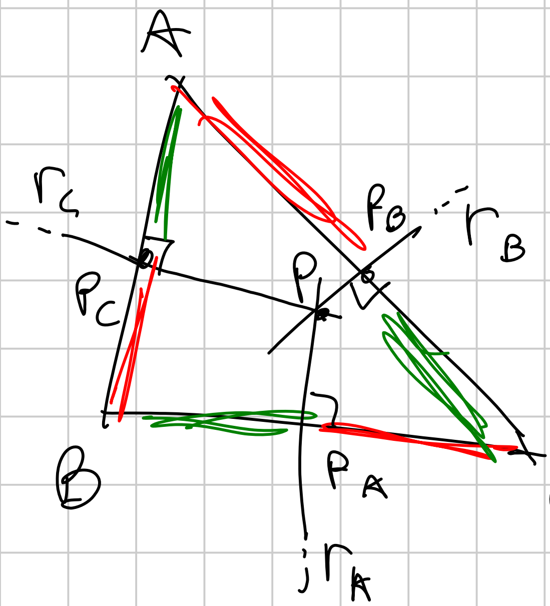
$$q_B = \frac{AB_1}{B_1C} = -\left(\frac{B_0A}{CB_0}\right) = -\rho_B$$

$$q_B = -\rho_B \qquad q_A = -\rho_A \qquad q_C = -\rho_C$$

$$\Rightarrow \quad q_A q_B q_C \overset{?}{=} (-1)^3 \underbrace{\rho_A \rho_B \rho_C}_{\substack{\shortparallel \\ 1}} = -1$$

$\Rightarrow$ <u>Win</u> $\square$

Carnot



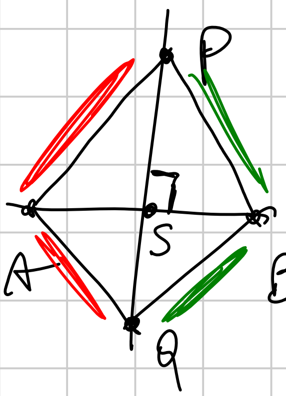
r_A, r_B, r_C Concorrono

$$\Updownarrow$$

$$P_A B^2 - P_A C^2 + CP_B^2 - P_B A^2 + AP_C^2 - P_C B^2 = 0$$

$\sum \text{rossi}^2$ $=$ $\sum \text{verdi}^2$

Lemma



$$AP^2 - PB^2 = AQ^2 - QB^2 \qquad AP^2 + QB^2 = PB^2 + QA^2$$

$$\Updownarrow$$

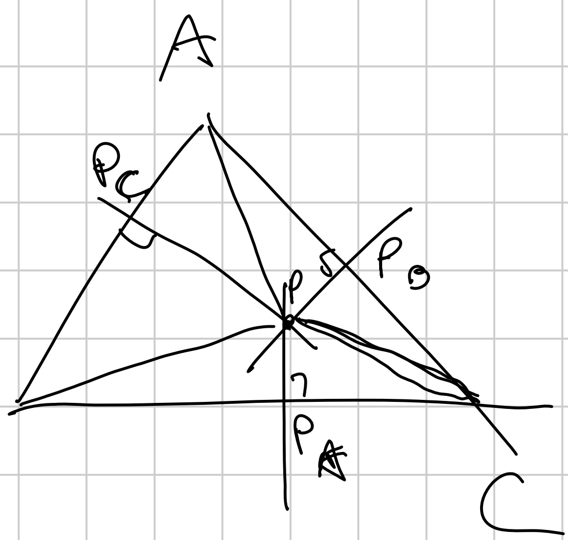
$$PQ \perp AB$$

$$AP^2 = AS^2 + PS^2 \qquad PB^2 = BS^2 + PS^2$$

$$AQ^2 = AS^2 + QS^2 \qquad QB^2 = BS^2 + QS^2$$

$$AP^2 - PB^2 = AS^2 + \cancel{PS^2} - BS^2 \cancel{- PS^2}$$

$$= AS^2 + QS^2 - BS^2 - QS^2 = AQ^2 - QB^2$$



$$+\begin{cases} BP_A^2 - P_AC^2 = \cancel{BP}^2 - \cancel{PC}^2 \\ CP_B^2 - P_BA^2 = \cancel{PC}^2 - \cancel{PA}^2 \\ AP_C^2 - P_CB^2 = \cancel{AP}^2 - \cancel{PB}^2 \end{cases}$$

$$\underset{\text{LHS}}{\parallel} \qquad\qquad \underset{0 = \text{RHS}}{\parallel}$$

<u>Win</u>

Trasformazioni geometriche (del piano)

ISOMETRIE (conservano tutto)

↳ SIMILITUDINI (OMOTETIE) (conservano tutto - le lunghezze)

↳ AFFINITÀ

(PROIETTIVITÀ che conservano allineamenti, concorrenze e birapporto $(ABCD) = \dfrac{\frac{CA}{CB}}{\frac{DA}{DB}}$)

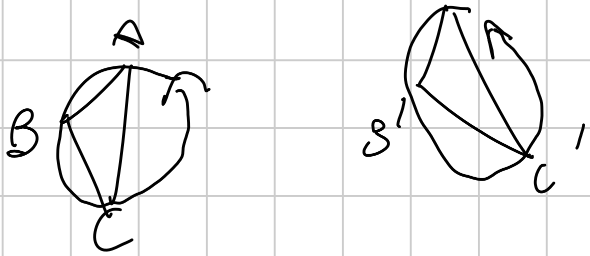
ISOMETRIE

Def Un'isometria è una trasformazione che conserva le distanze

Isometria = Rotazione + simm. assiale / trasl.

= 3 simm. ass.

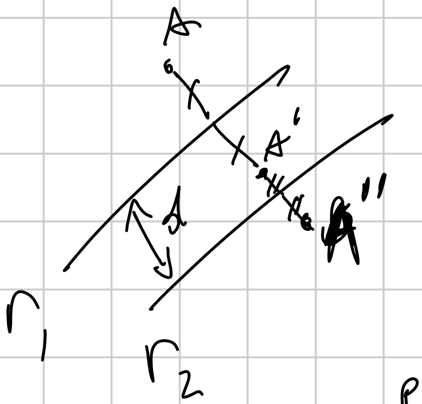
Segno dell'isometria:



Se ABC e A'B'C' girano nello stesso verso il segno è positivo, altrimenti è negativo.

Fatti interessanti:

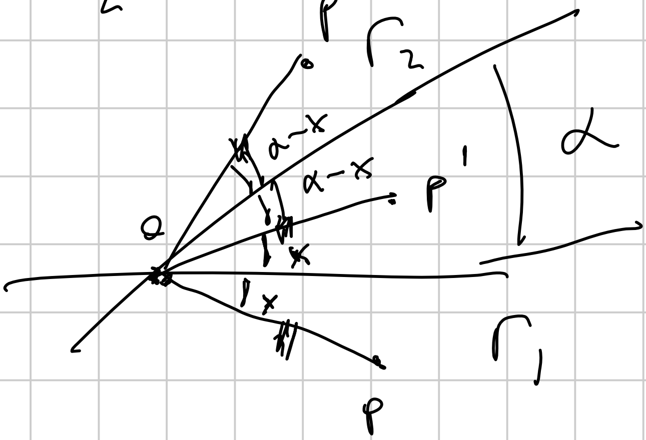
Simmetria assiale + simm. assiale = rotaz/trasl



$|AA''| = 2d$

$\Downarrow$

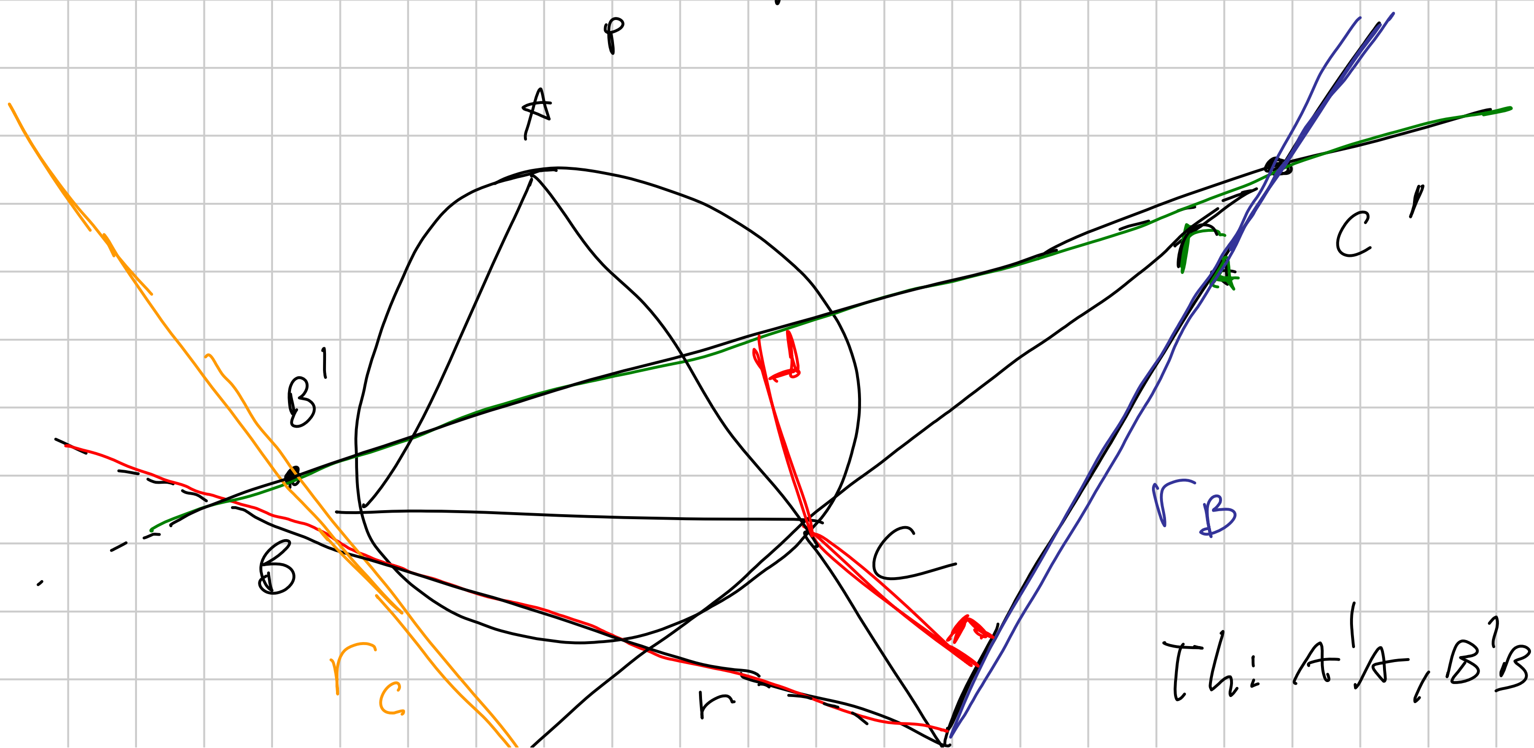
se $r_1 \parallel r_2$ simm + simm. = trasl



$P\hat{O}P''$

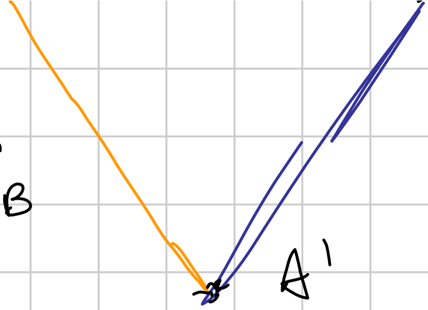
$\parallel$

$\cancel{2x} + 2\alpha - \cancel{2x} = 2\alpha$



Th: A'A, B'B

$$r_A \xrightarrow{BC} r \xrightarrow{CA} r_B$$



$C'C$ concorrono

$$r_A \xrightarrow{BC\cap CA = C} r_B$$

$$\Downarrow$$

$d(C, r_A) = d(C, r_B) \Rightarrow C'C$ è bisettrice

$\Rightarrow A'A,\ B'B,\ C'C$ concorrono nell'incentro di $A'B'C'$.

$\square$

Similitudini

Def Una sim. mantiene i rapporti tra lunghezze. $\Rightarrow$ Mantiene tutto tranne le lunghezze

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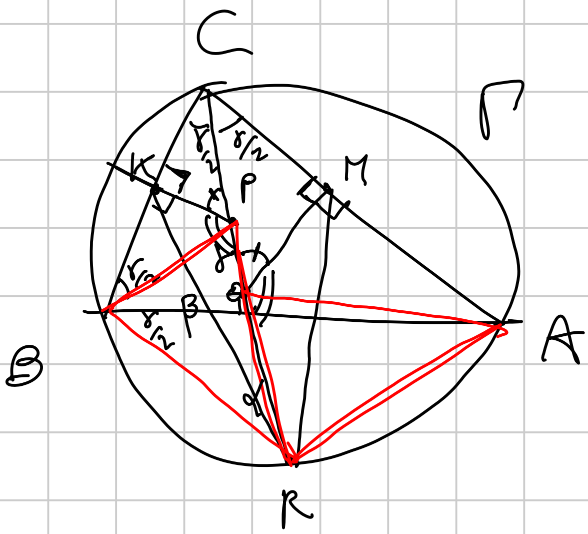
IMO 4 - 2007



$R\hat{B}A = \frac{\gamma}{2}$

$P\hat{B}R = \frac{\beta}{2}$

$$[RQM] = \frac{1}{2} QR \cdot QM \cdot \sin \hat{Q}$$

$$\parallel$$

$$[RPK] = \frac{1}{2} RP \cdot PK \cdot \sin \hat{P}$$

$$\text{Th} \iff QR \cdot QM = RP \cdot PK$$

$$PK = PC \sin \frac{\gamma}{2}$$

$$QM = QC \sin \frac{\gamma}{2}$$

$$QR \cdot QC \cdot \cancel{\sin \frac{\gamma}{2}} = RP \cdot PC \cdot \cancel{\sin \frac{\gamma}{2}}$$

$QR = x$

$CP = y$

$$\underbrace{QR \cdot QC}_{x(CR-x)} = \underbrace{RP \cdot PC}_{y(CR-y)}$$

$$\Updownarrow$$

$$x = y$$

$P\hat{B}R \sim C\hat{B}A$ analogamente

$Q\hat{R}A \sim C\hat{B}A$

$$\Downarrow$$

$$P\hat{B}R \sim Q\hat{R}A \Rightarrow \frac{PR}{PB} = \frac{AQ}{QR} \quad AQ = QC, \quad PB = PC$$

$$\Rightarrow \frac{PR}{PC} = \frac{QC}{QR} \qquad PC = y \qquad QR = x$$

$$\frac{CR - y}{y} = \frac{CR - x}{x}$$

$$\Downarrow$$

$$\cancel{\frac{CR}{y} - 1} = \cancel{\frac{CR}{x} - 1} \Rightarrow y = x$$

Omotetia

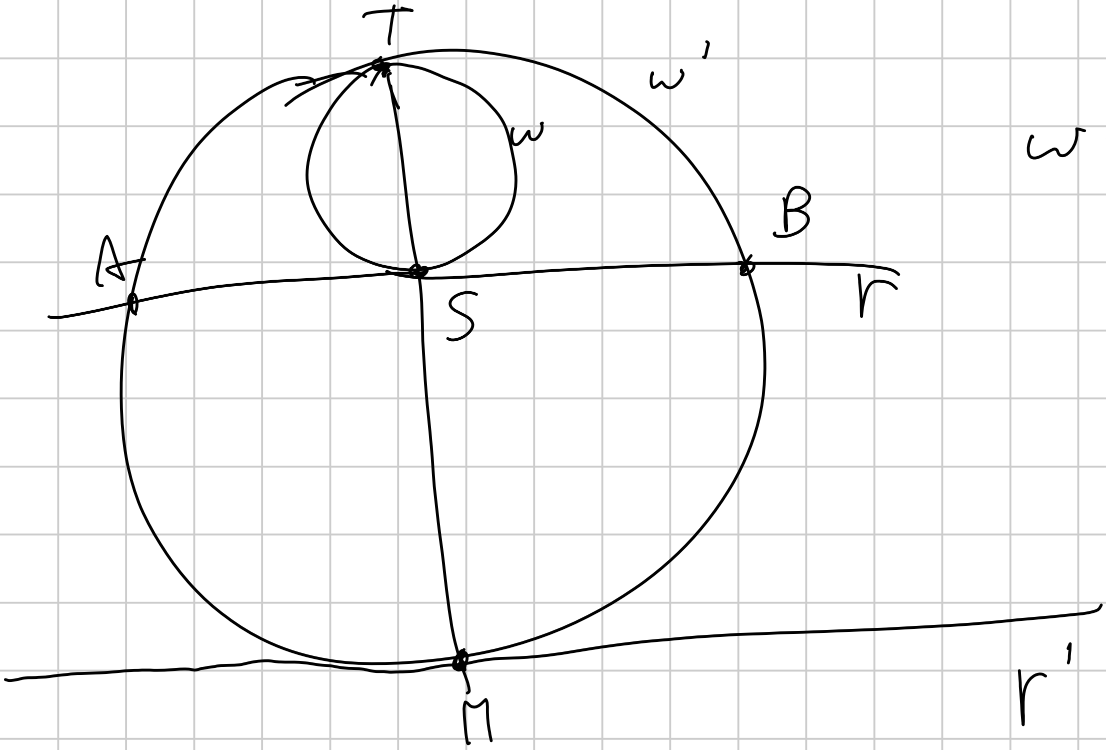
Dato un punto O si chiama omotetia di fattore k e centro O quella trasformazione che manda P in P' t.c.

$$\vec{OP'} = k\,\vec{OP}$$

2 fatti utili:

1. omotetia $\circ$ omotetia = omotetia di fattore = prodotto dei fattori e centro allineato con gli altri 2 centri di omotetia

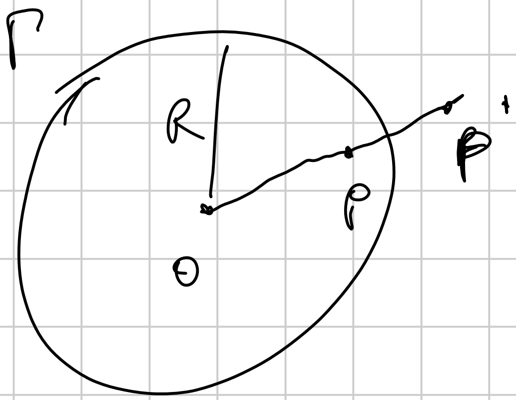
(2)



$w \to w'$

Inversione

È una trasformazione che è definita così



$P \to P'$

t.c.

$\to OP \cdot OP' = R^2$

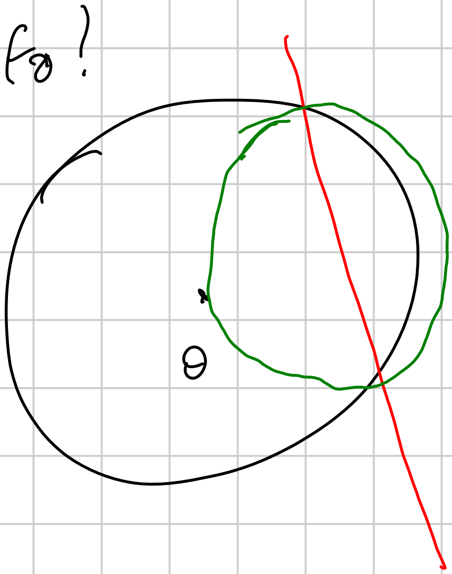
e O, P, P' al.

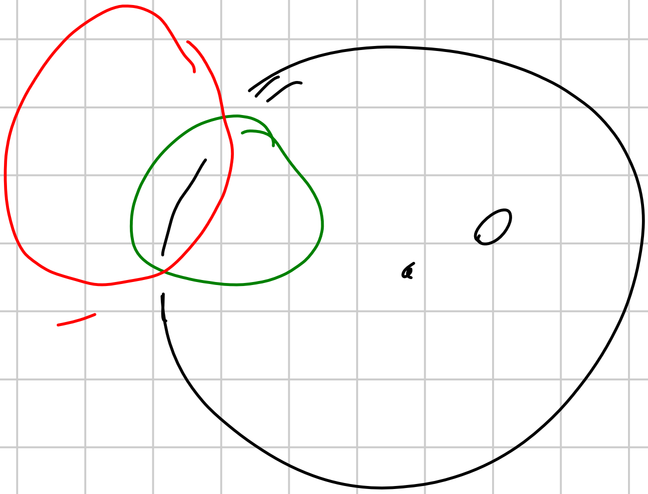
Cosa fa?

rossa

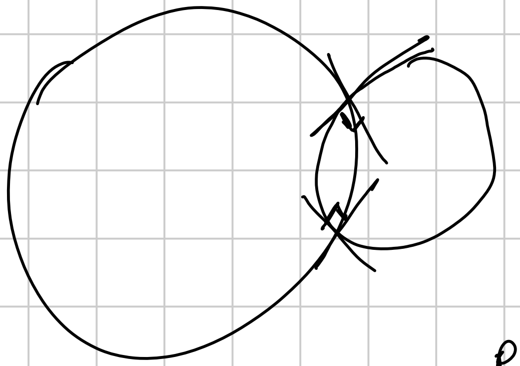
↓

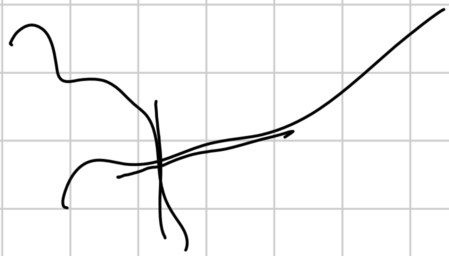
verde



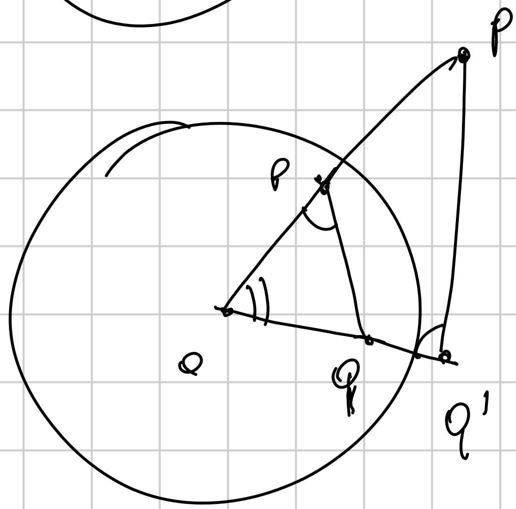


Mantiene fisse le circ. ortogonali a Γ





Tolomeo



Fatto 1

$PQP'Q'$ ciclico

$OP \cdot OP' = OQ \cdot OQ' = R^2$

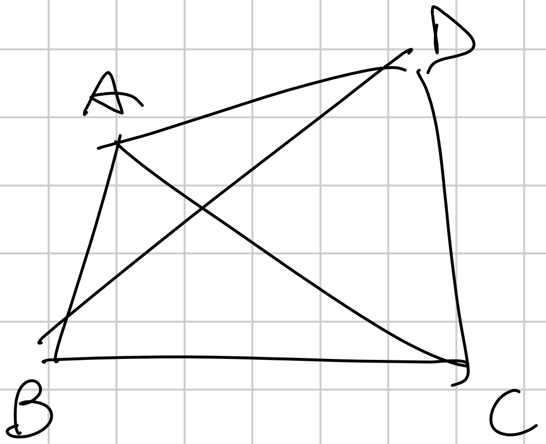
$$\Rightarrow \widehat{OPQ} = \pi - \widehat{QPP'} = \widehat{QQ'P'}$$

$$\Rightarrow \overset{\triangle}{OPQ} \sim \overset{\triangle}{OQ'P'}$$

$$\Rightarrow \frac{P'Q'}{PQ} = \frac{OQ'}{OP} = \frac{\frac{R^2}{OQ}}{OP} = \frac{R^2}{OP \cdot OQ}$$

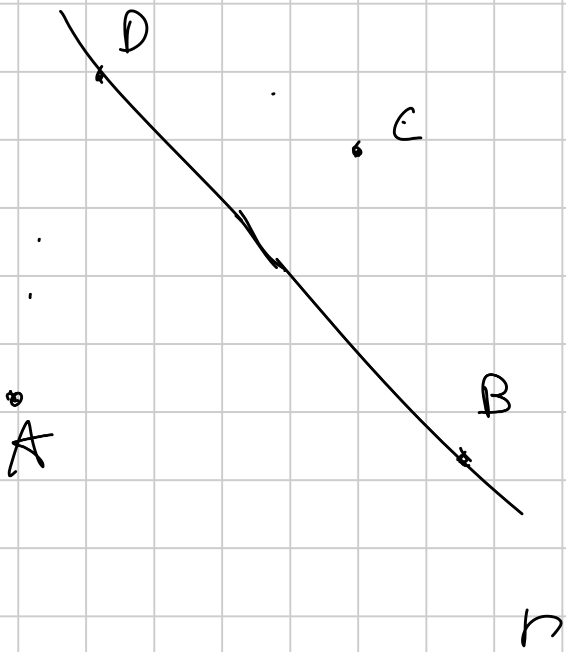
$$\Rightarrow P'Q' = \frac{PQ \cdot R^2}{OP \cdot QO}$$

$\boxed{\text{Tolomeo}}$



$$\boxed{BD \cdot AC \leq AB \cdot DC + AD \cdot BC}$$

Dim



$DC \longrightarrow D'C'$

$CB \longrightarrow C'B'$

$DB \longrightarrow D'B'$

$$D'C' + C'B' \geq D'B'$$

$$\frac{DC \cdot \cancel{R^2}}{AD \cdot AC} + \frac{CB \cdot \cancel{R^2}}{AC \cdot AB} \geq \frac{DB \cdot \cancel{R^2}}{AD \cdot AB}$$

$$DC \cdot AB + CB \cdot AD \geq DB \cdot AC \Longrightarrow \text{cvd}$$