GEOMETRIA 2 - Medium

Titolo nota 07/09/2011

(Metodi Algebrici)

1) Coordinate cartesiane

- $$\begin{cases} x^2+y^2+\alpha x+\beta y+\gamma=0 \\ x^2+y^2+\alpha' x+\beta' y+\gamma'=0 \end{cases}$$

$$\begin{cases} x^2+y^2+\alpha x+\beta y+\gamma=0 \\ \underbrace{(\alpha'-\alpha)x+(\beta'-\beta)y+\gamma'-\gamma=0}_{\text{asse radicale}} \end{cases}$$

— . —

- Se $\mathcal{C}$ e $\mathcal{C}'$ sono due parabole congruenti ottenute l'una dall'altra per una rotazione di $\frac{\pi}{2}$, allora $\mathcal{C}\cap\mathcal{C}'$ è fatto di punti conciclici.

$$\begin{cases} y=x^2+ax+b \\ x=y^2+cy+d \end{cases}$$

$y=p(x)$ $\deg p(x)=2$

$$\begin{cases} y=kx^2+ax+b & (\circ) \\ x=ky^2+cy+d & (\circ\circ) \end{cases}$$

$$\begin{cases} y=kx^2+ax+b \\ kx^2+ky^2+\underbrace{(a-1)x+(c-1)y+b+d}=0 \end{cases}$$

- Classificazione delle coniche

$$3x^2+4y^2-28xy+2x-3y+1=0$$

$$\cancel{3\left(x-\frac{14}{3}y\right)^2-\frac{184}{3}y^2}+2x-3y+1=0$$

$$\frac{196}{9}y^2$$

$2x^2+y^2=1$

$x^2-y^2=1$

$x+y^2=1$

$$3\left(x-\frac{14}{3}y+\frac{1}{3}\right)^2-\frac{184}{3}y^2+\frac{19}{3}y+\frac{2}{3}$$

$$-\frac{28}{3}y$$

$$\begin{cases} x'=ax+by+c \\ y'=dx+ey+f \end{cases}$$

$$3\square-\frac{184}{3}\square=k$$

$$\alpha x^2+2\beta xy+\gamma y^2+\dots$$

$$\left(\sqrt{\alpha}x+\frac{\beta}{\sqrt{\alpha}}y+k\right)^2+y^2\left(\gamma-\frac{\beta^2}{\alpha}\right)+\dots$$

$$\gamma-\frac{\beta^2}{\alpha}=\frac{\alpha\gamma-\beta^2}{\alpha}=\frac{4\alpha\gamma-(2\beta)^2}{4\alpha}$$

>0 ellisse

$=0$ parabola

<0 iperbole

2) Vettori

- Baricentro di ABC $=\dfrac{\vec{A}+\vec{B}+\vec{C}}{3}$ vale per ogni origine

- Ortocentro di ABC $=\vec{OA}+\vec{OB}+\vec{OC}=\vec{A}+\vec{B}+\vec{C}$ con origine in $O=$ circocentro

• Centro della sf. di Feuerbach $= \dfrac{\vec{O}+\vec{H}}{2} = \dfrac{\vec{A}+\vec{B}+\vec{C}}{2}$

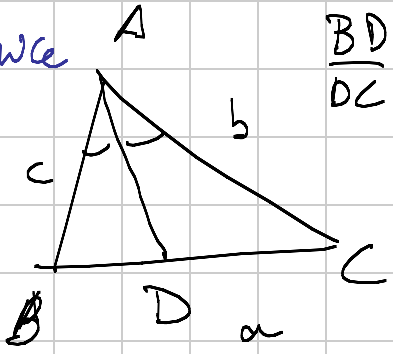
vero sempre

con origine in O.

• Incentro di ABC $= \dfrac{a\vec{A}+b\vec{B}+c\vec{C}}{a+b+c}$ vale per ogni scelta dell'origine

Perché?

1) Bisettrice



$$\vec{D} = \frac{b\vec{B}+c\vec{C}}{b+c}$$

$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$$

$$\Downarrow$$

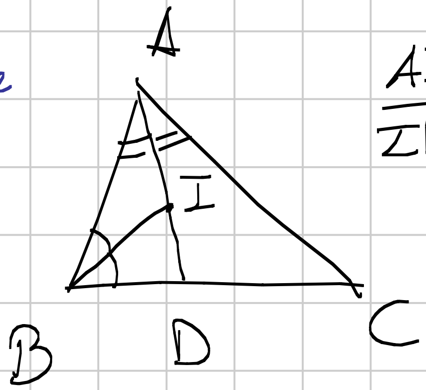
$$\frac{DC}{BD}+1 = \frac{b}{c}+1$$

$$\frac{BC}{BD} = \frac{b+c}{c}$$

$$\Downarrow$$

$$BD = \frac{ca}{b+c}$$

2) Un'altra bisettrice



$$\frac{AI}{ID} = \frac{AB}{BD} = \frac{\cancel{c}}{\cancel{c}a}(b+c)$$

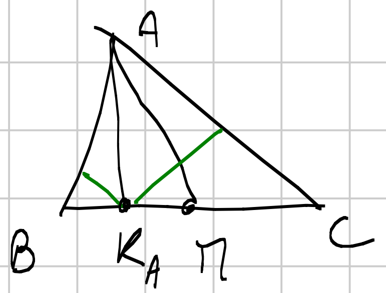
$$\frac{AI}{ID} = \frac{b+c}{a}$$

$$\vec{I} = \frac{a\vec{A}+(b+c)\vec{D}}{a+b+c} = \frac{a\vec{A}+b\vec{B}+c\vec{C}}{a+b+c}$$

SCARPA GIOACCHINO

• Punto di Lemoine di ABC = ?

1) Simmediana



AK_A simm. $\dfrac{d(K_A, AB)}{d(K_A, AC)} = \dfrac{d(M, AC)}{d(M, AB)} = \dfrac{2\cdot\cancel{[AMC]}/AC}{2\cancel{[AMB]}/AB} = \dfrac{AB}{AC} = \dfrac{c}{b}$

$$\frac{BK_A}{K_AC}$$

$$\parallel$$

$$\parallel$$

$$\frac{[ABK_A]}{[ACK_A]} = \frac{AB \cdot d(K_A, AB)}{AC \cdot d(K_A, AC)} = \frac{c}{b} \cdot \frac{c}{b} = \frac{c^2}{b^2}$$

2) Come per l'incentro $\Rightarrow \vec{K} = \dfrac{a^2\vec{A} + b^2\vec{B} + c^2\vec{C}}{a^2+b^2+c^2}$

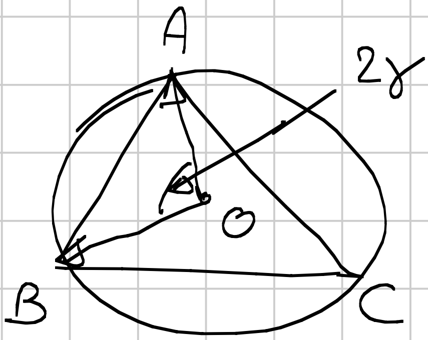
Prodotto scalare: $\langle \vec{A}, \vec{B} \rangle \quad (\vec{A}, \vec{B}) \quad \vec{A} \cdot \vec{B}$

$$OH^2 = \vec{OH} \cdot \vec{OH} \underset{\substack{\uparrow \\ \text{con origine} \\ \text{in } O}}{=} (\vec{A}+\vec{B}+\vec{C}) \cdot (\vec{A}+\vec{B}+\vec{C}) =$$

$$= \underbrace{\vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + \vec{C}\cdot\vec{C}}_{3R^2} + 2(\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{C} + \vec{C}\cdot\vec{A})$$

1. def. di prod. scalare

$$X \cdot Y = \|X\| \cdot \|Y\| \cdot \cos X\hat{O}Y$$



$$\cos A\hat{O}B = \cos 2\gamma = \frac{2R^2 - c^2}{2R^2}$$

$$A \cdot B = R^2 \cdot \frac{2R^2 - c^2}{2R^2} = R^2 - \frac{c^2}{2}$$

2. proprietà del prodotto scalare

$$c^2 = (A-B)\cdot(A-B) = A\cdot A + B \cdot B - 2A\cdot B = 2(R^2 - A\cdot B)$$

$$A \cdot B = R^2 - \frac{c^2}{2}$$

$$\Rightarrow OH^2 = 3R^2 + 2(A\cdot B + B\cdot C + C \cdot A) = 3R^2 + 2\left(3R^2 - \frac{a^2+b^2+c^2}{2}\right) =$$

$$= 9R^2 - a^2 - b^2 - c^2.$$

• $OI^2 = (O-I)\cdot(O-I) = \underset{\substack{\uparrow \\ \text{origine in } O}}{I\cdot I} = \frac{1}{p^2}(aA+bB+cC)\cdot(aA+bB+cC) =$

$$= \frac{1}{p^2}\Big(\underbrace{(a^2+b^2+c^2)R^2 + (2ab+2bc+2ca)R^2}_{R^2(a+b+c)^2} - abc(a+b+c)\Big) =$$

$$= \frac{1}{p^2}\left(R^2p^2 - abc\cdot p\right) = R^2 - \frac{abc}{p} = R^2 - \underbrace{\frac{abc}{4S}}_{R}\cdot 2\cdot \underbrace{\frac{2S}{p}}_{r} =$$

$$= R^2 - 2Rr \qquad OI = \sqrt{R(R-2r)}$$

Formula di Eulero

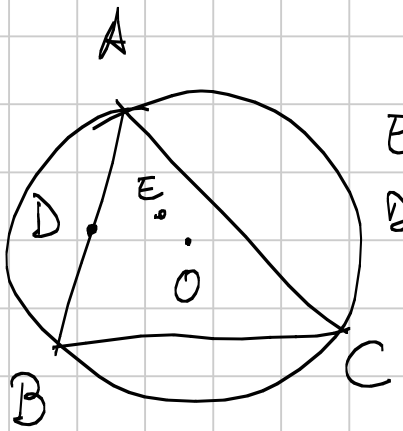
$r < \frac{R}{2}$

$OH = \sqrt{9R^2 - a^2 - b^2 - c^2}$

$4R^2\left(\frac{9}{4} - \sum \sin^2\alpha\right)$

$\frac{9}{4} \geq \sum \sin^2\alpha$

Es:



E = baricentro di $\overset{\triangle}{ACD}$

D = pt medio di AB

$CD \perp OE \iff AB = AC$

Sol (forse): Origine in O. La Tesi diventa

$$(C-D)\cdot E = 0 \iff \|A-B\| = \|A-C\|$$

(i) $D = \frac{A+B}{2}$

(ii) $E = \frac{A+C+D}{3} = \frac{A+C+\frac{A+B}{2}}{3} =$

$= \frac{3A+2C+B}{6}$

$(A-B)\cdot(A-B) = (A-C)\cdot(A-C)$

$\cancel{A\cdot A} + \cancel{B\cdot B} - 2A\cdot B = \cancel{A\cdot A} + \cancel{C\cdot C} - 2A\cdot C$

origine in O

$A\cdot(B-C) = 0.$

$$(C-D)\cdot E = \underbrace{\left(C-\frac{A+B}{2}\right)}_{\frac{2C-A-B}{2}}\cdot\left(\frac{3A+2C+B}{6}\right) =$$

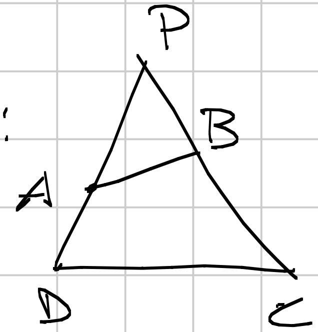
$$= \frac{1}{12}\Big(6C\cdot A + \cancel{4C\cdot C} + \cancel{2C\cdot B} - \cancel{3A\cdot A} - 2A\cdot C - A\cdot B - 3A\cdot B - \cancel{2B\cdot C} - \cancel{B\cdot B}\Big) =$$

$$= \frac{1}{12}\big(A\cdot(6C-2C-B-3B)\big) = \frac{1}{12}A\cdot(4C-4B) =$$

$$= \frac{1}{3}A\cdot(C-B)$$

$$\frac{1}{3}A\cdot(C-B)=0 \iff A\cdot(B-C)=0 \qquad \square$$

ES (perfidamente) per casa:



O_1, H_1 circo-orto-centri di PAB

O_2, H_2 circo/orto-centri di PDC

E_i pt. med. di O_iH_i $\quad i=1,2$

la perp. da E_1 a CD
la perp. da E_2 a AB $\Longleftarrow$
H_1H_2

concorrono.

Inizio: $P =$ origine

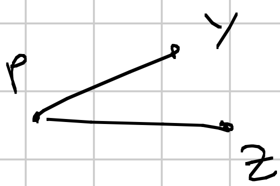
$X_1 = (\text{perp. da } E_1 \text{ a } CD)\cap H_1H_2$

$X_2 = (\text{perp. da } E_2 \text{ a } AB)\cap H_1H_2$

Se trovo che $\left.\begin{array}{l} X_1\cdot Y = X_2\cdot Y \\ X_1\cdot Z = X_2\cdot Z\end{array}\right\}$ $\quad Y, Z$ non allineati

$\Downarrow$

$X_1 = X_2$



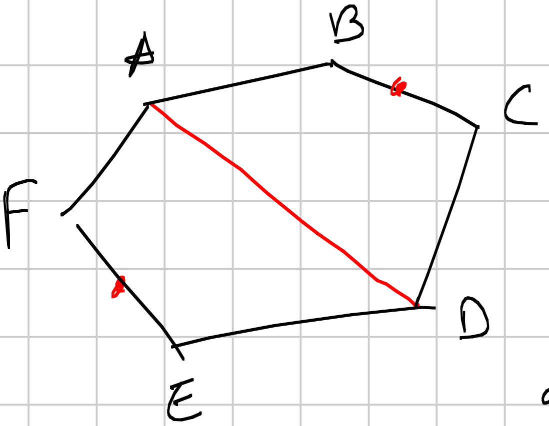
$X_2\cdot Y = \alpha$

$X_2\cdot Z = \beta$

$X_2 = \lambda Y + \mu Z$

λ, μ unici!

Es:



$$AD = BC + EF$$
$$BE = CD + FA \implies \frac{AB}{DE} = \frac{EF}{BC} = \frac{CD}{FA}$$
$$CF = AB + DE$$

disug. triangolare

$$\|A-D\| = \|B-C\| + \|F-E\| \geq \|B-C+F-E\|$$
$$\|B-E\| = \|C-D\| + \|F-A\| \geq \|C-D+A-F\|$$
$$\|C-F\| = \|A-B\| + \|D-E\| \geq \|A-B-D+E\|$$

$X = A - D \qquad \|X\| \geq \|Y - Z\|$

$Y = B - E \qquad \|Y\| \geq \|Z - X\|$

$Z = C - F$

$\|Z\| \geq \|X - Y\|$

$$L = \frac{Y+Z-X}{2} \qquad Y - Z = L + N - (L+M) = N - M$$

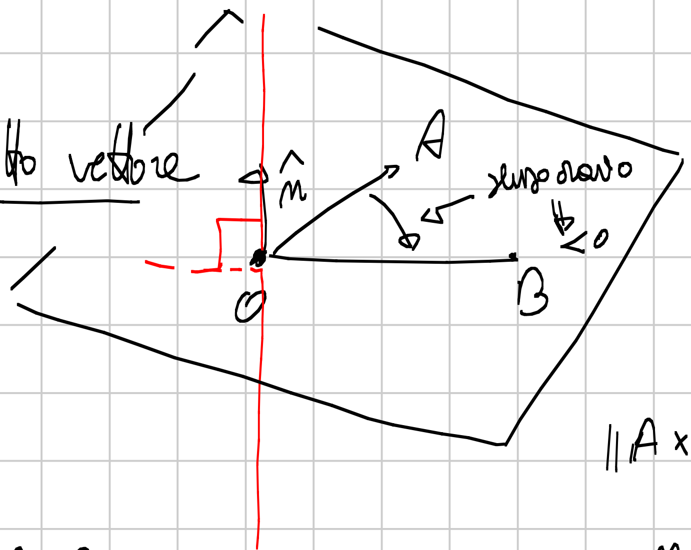
$$M = \frac{X+Z-Y}{2} \qquad \|M+N\| \geq \|M-N\|$$

$$\|L+N\| \geq \|L-N\|$$

$$N = \frac{X+Y-Z}{2} \qquad \|L+M\| \geq \|L-M\|$$

To be continued

Prodotto vettore



$$A \times B = \|OA\| \cdot \|OB\| \cdot \sin \widehat{AOB} \cdot \hat{n}$$
$$\| \quad\quad \uparrow$$
$$OA \times OB \quad\quad \text{angolo orientato}$$

$$\|A \times B\| = 2 \cdot [OAB]$$

$A \times B = 0 \iff A, O, B$ sono allineati.

• proprietà: $(A+B)\times C = A\times C + B\times C$

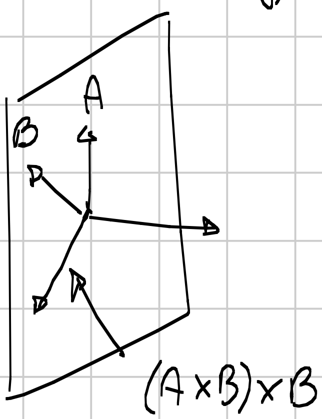
$$A\times B = -B\times A$$

$$(\lambda A)\times B = \lambda(A\times B)$$

$$A\times A = 0$$

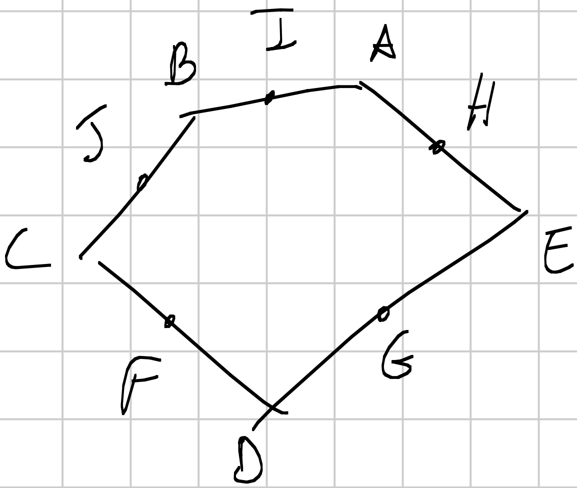
$A\times B\times C \qquad (A\times B)\times C \qquad A\times(B\times C)$

$B = C$ $\qquad 0 \qquad 0$



$A\times(B\times B) = 0.$

Es:



F, G, H, I, J p.ti medi dei lati

AF, BG, CH, DI concorrono in P

$\Downarrow$

anche EJ passa per P

$$(A-P)\times(F-P) = 0$$
$$(B-P)\times(G-P) = 0$$
$$(C-P)\times(H-P) = 0$$
$$(D-P)\times(I-P) = 0$$

$\Longrightarrow (E-P)\times(J-P) = 0$

$$0 = (A-P)\times\left(\frac{C+D}{2}-P\right) = \boxed{\frac{A\times C}{2}} + \frac{A\times D}{2} - A\times P - \boxed{\frac{P\times C}{2}} - \frac{P\times D}{2} +$$

$$0 = \frac{B\times D}{2} + \boxed{\frac{B\times E}{2}} - \boxed{B\times P} - \frac{P\times D}{2} - \frac{P\times E}{2} +$$

$$0 = \boxed{\frac{C\times E}{2}} + \boxed{\frac{C\times A}{2}} - \boxed{C\times P} - \frac{P\times E}{2} - \frac{P\times A}{2} +$$

$$0 = \frac{D\times A}{2} + \frac{D\times B}{2} - D\times P - \frac{P\times A}{2} - \frac{P\times B}{2} +$$

$$0 = 0+0+0+\frac{C\times E}{2}+\frac{B\times E}{2}+0+0-\frac{C\times P}{2}-\frac{B\times P}{2}-P\times E =$$

$$\Downarrow$$

$$0 = \frac{E\times B}{2}+\frac{E\times C}{2}-E\times P-\frac{P\times B}{2}-\frac{P\times C}{2} \qquad \boxed{\text{ok}}$$

— ∘ —

Fissato un triangolo ABC, per ogni P del piano esiste un'unica terna (α,β,γ) di numeri reali t.c. $\alpha+\beta+\gamma=1$ e $P=\alpha A+\beta B+\gamma C$

Dim: $P=\alpha A+\beta B+\gamma C \qquad \alpha+\beta+\gamma=1$

$$2\cdot[PAB] = \|(B-P)\times(A-P)\| = \|(-\alpha A+(1-\beta)B-\gamma C)\times((1-\alpha)A-\beta B-\gamma C)\| =$$

$$= \|\alpha\beta\, A\times B + \alpha\gamma\, A\times C + \underbrace{(1-\alpha)(1-\beta)}^{1-\alpha-\beta+\alpha\beta} B\times A - \cancel{\gamma}(1-\beta)\, B\times C - \gamma(1-\alpha)\, C\times A + \gamma\beta\, C\times B\| =$$

$$= \|A\times B(\cancel{\alpha\beta}-\cancel{\alpha\beta}-\overbrace{(1-\alpha-\beta)}^{-\gamma}) + B\times C(\overbrace{-\gamma}^{-\gamma}+\cancel{\gamma\beta}-\cancel{\gamma\beta}) +$$

$$+C\times A\left(-\cancel{\alpha\gamma}\overset{-\gamma}{-\gamma}+\cancel{\alpha\gamma}\right)\| = |\gamma|\,\|A\times B+B\times C+C\times A\| =$$

$$= |\gamma|\,\|(C-B)\times(A-B)\| = |\gamma|\cdot 2\,[ABC]$$

$$\frac{[PAB]}{[ABC]} = |\gamma|$$

se usiamo le aree orientate $[ABC]>0$ se A,B,C sono in senso antiorario $\Rightarrow \frac{[PAB]}{[ABC]} = \gamma$

$\Rightarrow$ dati α,β,γ P è sempre univocamente determinato

dato P, ho $\alpha = \frac{[PBC]}{[ABC]}, \dots$

<u>Menelao</u>: A,B,C D,E,F su lati

$$\frac{BD}{DC} = \frac{\lambda_2}{\lambda_1} \qquad \frac{CE}{EA} = \frac{\mu_2}{\mu_1} \qquad \frac{AF}{FB} = \frac{\nu_2}{\nu_1}$$

$$\lambda_1+\lambda_2 = 1 \qquad \mu_1+\mu_2 = 1 \qquad \nu_1+\nu_2 = 1$$

$$\lambda_2\mu_2\nu_2 = -\lambda_1\mu_1\nu_1$$

$\Updownarrow$

D,E,F allineati.

$$D = \lambda_1 B + \lambda_2 C$$

$$E = \mu_1 C + \mu_2 A$$

$$F = \nu_1 A + \nu_2 B$$

Vorremmo $F = \alpha D + \beta E$ $\alpha+\beta = 1$

$\Updownarrow$

$$\alpha\lambda_1 B + \alpha\lambda_2 C + \beta\mu_1 C + \beta\mu_2 A = \nu_1 A + \nu_2 B$$

$$(\alpha\lambda_1 - \nu_2)B + (\alpha\lambda_2+\beta\mu_1)C + (\beta\mu_2-\nu_1)A = 0$$

Oss: $(\alpha\lambda_1 - \nu_2) + (\alpha\lambda_2 + \beta\mu_1) + (\beta\mu_2 - \nu_1) =$

$$= \alpha(\lambda_1+\lambda_2) + \beta(\mu_1+\mu_2) - (\nu_2+\gamma_2) =$$

$$= \alpha + \beta - 1 = 0.$$

Oss 2: Se ogni coeff. è 0

$$\begin{cases} \alpha\lambda_1 = \nu_2 \\ \beta\mu_2 = \nu_1 \\ \alpha\lambda_2 + \beta\mu_1 = 0 \end{cases} \qquad \beta = \frac{\nu_1}{\mu_2} \qquad \longrightarrow \alpha = -\beta\frac{\mu_1}{\lambda_2}$$

$$-\beta = \frac{\nu_2\lambda_2}{\mu_1\lambda_1} \qquad \frac{\mu_2\nu_2\lambda_2}{\mu_1\nu_2\lambda_1} = -1 \iff \exists\, \alpha, \beta$$

Oss 3: So che $\exists\, x, y, z$ t.c. $x+y+z=1$

$$O = xA + yB + zC.$$

Se $\exists\, u, v, w$ t.c. $u+v+w=0$

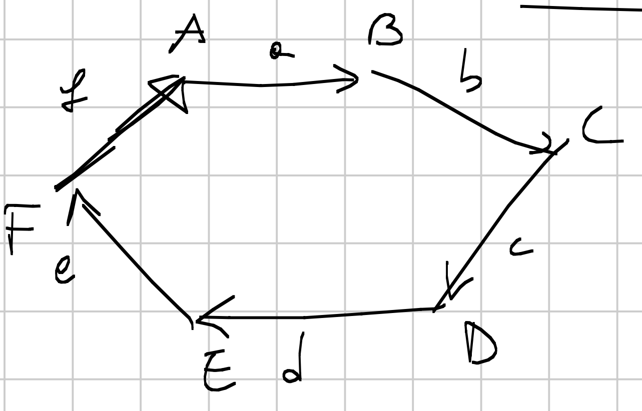
$$O = uA + vB + wC$$

$$\Downarrow$$

$$r+s+t = \begin{matrix}(x+y+z)\\+\\(u+v+w)\end{matrix} = 0+1 = 1 \qquad O = \underbrace{(x+u)}_{r}A + \underbrace{(y+v)}_{s}B + \underbrace{(z+w)}_{t}C$$

$$\Rightarrow \begin{matrix} r = x \\ s = y \\ t = z \end{matrix} \Rightarrow u = v = w = 0.$$

— o —



$B - A = a$ $\qquad A - F = f$

$C - B = b$

$D - C = c$ $\qquad a+b+c+d+e+f = 0$

$E - D = d$

$F - E = e$

$$AD = a+b+c = -d-e-f = \frac{1}{2}(a+b+c-d-e-f)$$

$$AD = D-A$$

$$x = a-d \qquad y = e-b \qquad z = c-f$$

$$\frac{1}{2}(a+b+c-d-e-f) = \frac{x-y+z}{2}$$

$$AD = BC+EF$$

$$\left|\frac{x-y+z}{2}\right| = |b|+|e| \geq |b-e| = |y|$$

$$\left|\frac{y+z-x}{2}\right| \geq |x|$$

$$\left|\frac{x+y-z}{2}\right| \geq |z|$$

$$\frac{x-y+z}{2} = l$$

$$\frac{y+z-x}{2} = m$$

$$\frac{x+y-z}{2} = n$$

$$|l| \geq |m+n|$$

$$|m| \geq |n+l|$$

$$|n| \geq |l+m|$$

$$l+m+n = \frac{1}{2}(x+y+z)$$

$$|l+m+n|^2 = |l|^2+|m|^2+|n|^2+2(lm+nl+mn)$$

$$\Rightarrow |l|^2 \geq |m+n|^2 = |m|^2+|n|^2+2m\cdot n$$

$$|m|^2 \geq |n|^2+|l|^2+2ln$$

$$|n|^2 \geq |m|^2+|l|^2+2lm$$

$$\cancel{|l|^2+|m|^2+|n|^2} \geq 2(|l|^2+|m|^2+|n|^2)+2(lm+ln+nm)$$

$$0 \geq \|l+m+n\|^2 \Longrightarrow l+m+n=0$$

$$\Rightarrow x+y+z=0$$

$\Rightarrow$ parallelismo $\Rightarrow \ldots \square$

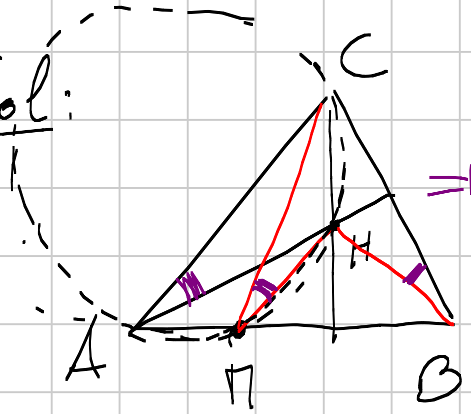
APMO – 2010

ABC Triangolo acutangolo, $AB > BC$, $AC > BC$.

O, H. Circonf. circo a AHC interseca AB in $M \neq A$.
Circonf. circo a AHB interseca AC in $N \neq A$.

$\Rightarrow$ Circocentro di MNH sta su OH

Sol:



$\Rightarrow M$ simm. di B risp. ad HC.
N simm. di C risp. ad HB

O = origine, cfr. circo ad AB è unitaria
$h = a + b + c$

$$x \to x - c \to (x-c)\frac{\overline{h}-\overline{c}}{|\overline{h}-\overline{c}|} \to (\overline{x}-\overline{c})\frac{h-c}{|h-c|} \to (\overline{x}-\overline{c})[\;] \to +c$$

$$m = a + c - \frac{ab}{c}$$

$$n = a + b - \frac{ac}{b}$$

Oss: Circocentro di O, x, y
$\overset{\shortparallel}{z}$

$$\begin{cases} \dfrac{z - \frac{x}{2}}{x/2} = -\dfrac{\overline{z} - \frac{\overline{x}}{2}}{\overline{x}/2} \\[2ex] \dfrac{z - y/2}{y/2} = -\dfrac{\overline{z} - \overline{y}/2}{\overline{y}/2} \end{cases} \qquad \begin{cases} \dfrac{2z}{x} - 1 = 1 - \dfrac{2\overline{z}}{\overline{x}} \\[2ex] \dfrac{2z}{y} - 1 = 1 - \dfrac{2\overline{z}}{\overline{y}} \end{cases}$$

$$z\left(\frac{\bar{x}}{x}-\frac{\bar{y}}{y}\right)=\bar{x}-\bar{y}$$

$$z=\frac{xy(\bar{x}-\bar{y})}{\bar{x}y-x\bar{y}}$$

<u>Oss</u>: $h \to 0$ $\quad m \to \underset{\underset{x}{\shortparallel}}{m-h}$ $\quad n \to \underset{\underset{y}{\shortparallel}}{n-h}$

$$m-h=-\frac{ab}{c}-b = -\frac{b}{c}(a+c) \qquad n-h=-\frac{ac}{b}-c = -\frac{c}{b}(a+b)$$

$$w=\frac{-(a+c)(a+b)\left(\frac{c}{b}\left(\frac{1}{a}+\frac{1}{c}\right)-\frac{b}{c}\left(\frac{1}{a}+\frac{1}{b}\right)\right)}{\frac{c}{b}\frac{c}{b}\left(\frac{1}{a}+\frac{1}{c}\right)(a+b)-\frac{b}{c}\frac{b}{c}(a+c)\left(\frac{1}{a}+\frac{1}{b}\right)}=$$

$$=\frac{-(a+c)(a+b)\left(\frac{c}{ba}+\frac{1}{b}-\frac{b}{ca}-\frac{1}{c}\right)}{\frac{c^2}{b^2a}(a+b)+\frac{c}{b^2}(a+b)-\frac{b^2}{c^2a}(a+c)-\frac{b}{c^2}(a+c)}=$$

$$\frac{c^2+ac-b^2-ab}{abc} = (c+b)(c-b)+a(c-b) = (c-b)(a+b+c)$$

$$\frac{c^2}{b^2}+\frac{c^2}{ab}+\frac{ac}{b^2}+\frac{c}{b}-\frac{b^2}{c^2}-\frac{b^2}{ac}-\frac{ab}{c^2}-\frac{b}{c}=$$

$$=\frac{ac^4+c^4b+a^2c^3+abc^3-ab^4-b^4c-a^2b^3-ab^3c}{ab^2c^2}=$$

$$=\frac{c^4(a+b)+a^2(c^3-b^3)-b^4(a+c)+abc(c^2-b^2)}{ab^2c^2}=$$

$$=\frac{a(c^4-b^4)+cb(c^3-b^3)+a^2(c^3-b^3)+abc(c^2-b^2)}{ab^2c^2}$$

$$-(a+c)(a+b)\cancel{(c-b)}\,(a+b+c)\frac{1}{\cancel{abc}}$$

$$\frac{\cancel{(c-b)}\left[(c^2+b^2)(c+b)a+(cb+a^2)(a^2+bc+b^2)+abc(c+b)\right]}{\cancel{ab^2c^2}}$$

$$c^3a+c^2ba+b^2ca+b^3a+abc^2+ab^2c$$

$$ca(c^2+cb+b^2)+ab(b^2+c^2+bc)$$

$$=\frac{-\cancel{(a+c)}\cancel{(a+b)}(a+b+c)\,bc}{\cancel{(a^2+ab+ac+cb)}(c^2+bc+b^2)}=-\frac{bc}{c^2+bc+b^2}\underset{h}{(a+b+c)}=\mu$$

$$\frac{h}{\mu}=\frac{\bar h}{\bar\mu}\qquad \Rightarrow h,\mu,0 \text{ allineati}$$

$$\frac{\mu}{h}=\frac{\bar\mu}{\bar h}\qquad\qquad \frac{\mu}{h}=\frac{-bc}{c^2+bc+b^2}$$

$$\frac{\bar\mu}{\bar h}=\frac{-\frac{1}{bc}}{\frac{1}{c^2}+\frac{1}{bc}+\frac{1}{b^2}}=\frac{-\frac{1}{\cancel{bc}}\,bc}{\frac{b^2+bc+c^2}{\cancel{b^2c^2}}}=-\frac{bc}{c^2+bc+b^2}$$

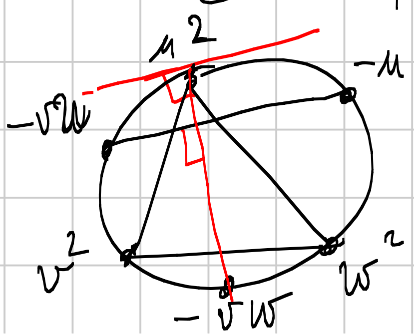
•) distanza tra I e il centro della cf. dei 9 punti

$O=$ origine

<u>Claim</u>: $\exists\ u,v,w$ numeri complessi sulla cf. unitaria

t.c. $A=u^2\quad B=v^2\quad C=w^2$

e $-uv,\ -vw,\ -uw$ sono le ulteriori intersez. di AI, BI, CI con la cf. unitaria



$$A'=-vw\quad B'=-uw\quad C'=-uv$$

$$\Rightarrow I=\text{ortocentro di } A'B'C'$$

$$I = -uv - vw - uw \qquad F = \frac{u^2+v^2+w^2}{2}$$

$$|IF| = |F - I| = \left| \frac{u^2+v^2+w^2+2uv+2vw+2uw}{2} \right| =$$

$$= \left| \frac{(u+v+w)^2}{2} \right| \qquad |OI| = |I| = |-uv-vw-uw| =$$

$$= |uvw| \cdot \left| \frac{1}{w} + \frac{1}{v} + \frac{1}{u} \right| = |uvw| \, |\bar{w} + \bar{v} + \bar{u}| =$$

$$= |uvw| \, |\overline{u+v+w}| = \underbrace{|uvw|}_{1} \, |u+v+w|$$

$$R = 1$$

$$IF = \frac{IO^2}{2} = \frac{R^2 - 2Rr}{2} = \frac{1-2r}{2} = \frac{1}{2} - r$$

raggio della cfr di Feuerbach