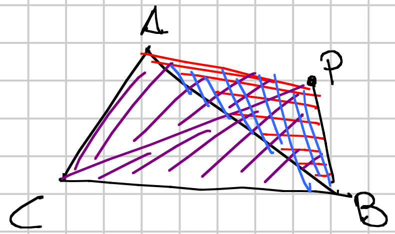
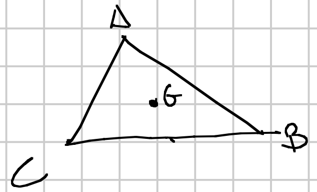
SENIOR 2012 — Advanced — G

Titolo nota 03/09/2012

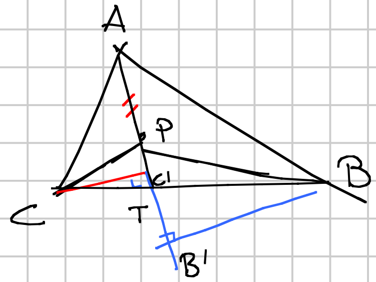


$$\left[\frac{[PBC]}{[ABC]} : \frac{[APC]}{[ABC]} : \frac{[ABP]}{[ABC]}\right]$$



$$G = \left[\frac{1}{3} : \frac{1}{3} : \frac{1}{3}\right]$$

$CTC' \sim BTB'$



$P = [x : y : z]$

$T = AP \cap BC$

$$\frac{y}{z} = \left|\frac{CC'}{BB'}\right| = \left|\frac{CT}{TB}\right|$$

$\boxed{[x : y : z]}$

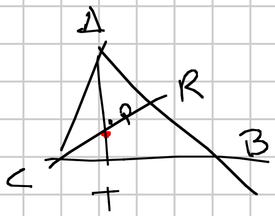
Sia $\vec{P} = x\vec{A} + y\vec{B} + z\vec{C}$

$$\vec{P} = x\vec{A} + \underset{(1-x)}{(y+z)}\underbrace{\left(\frac{y}{y+z}\vec{B} + \frac{z}{y+z}\vec{C}\right)}_{\vec{T}}$$

$\vec{B}, \vec{C}, \vec{T}$ allineati

$$\frac{CT}{BT} = \frac{y}{z}$$

P, A, T allineati



$P = [x : y : z]$ coincide con $x\vec{A} + y\vec{B} + z\vec{C}$

$\boxed{[kx : ky : kz]} \equiv [x : y : z]$

NORMALIZZATA

$[1 : 1 : 1] \to \left[\frac{1}{3} : \frac{1}{3} : \frac{1}{3}\right]$

Una retta in baricentriche è della forma $ux + vy + wz = 0$

3 punti M, N, O allineati $\iff$

$$\begin{vmatrix} x_M & y_M & z_M \\ x_N & y_N & z_N \\ x_O & y_O & z_O \end{vmatrix} = 0$$

$u x_M + v y_M + w z_M = 0$ $\quad$ $M, N, O \in \; ux + vy + wz = 0$

$\vdots$ e cyc

Date 3 rette m, n, o concorrono sse $\begin{vmatrix} x_m & y_m & z_m \\ & \vdots & \end{vmatrix} = 0$

$\boxed{\underbrace{ux + vy + wz = 0}} \iff x + y + z = 1$

$\underbrace{\qquad\qquad\qquad\qquad}$

$(u, v, w) \sim (1, 1, 1)$

$[x : y : z]$ con somme 1

$\exists P$ con quelle coordi note.

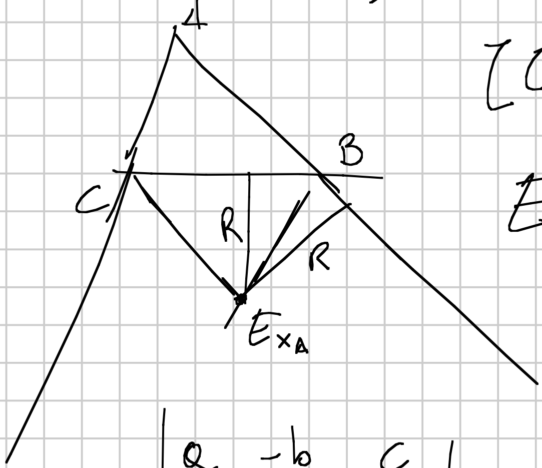
$u \quad v \quad w$

$\boxed{x + y + z = 0}$ retta all'infinito

Se 2 rette si intersecano in x_0, y_0, z_0 con $\underbrace{x_0 + y_0 + z_0 = 0}$

Vale $m // n \iff m \cap n \in$ retta all'infinito.

Ex_B, Ex_C, Δ sono allineati.



$[[ABC] : 0 : 0]$ $\quad$ $[1 : 0 : 0]$

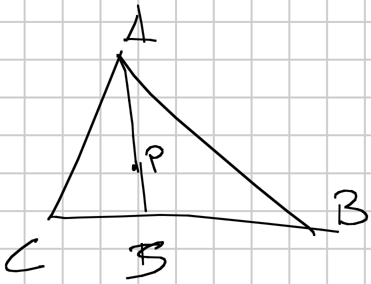
$AC\,Ex_A$

$Ex_A = [-BC \cdot R : AC \cdot R : AB \cdot R]$

$[-a : b : c]$

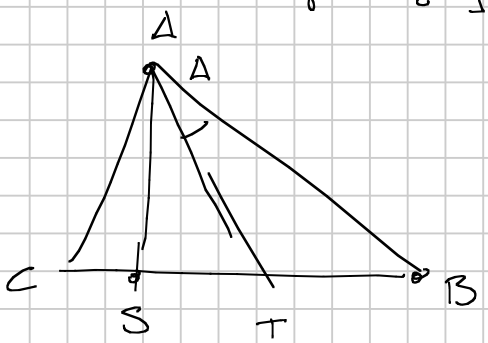
$$\begin{vmatrix} a & -b & c \\ a & b & -c \\ 1 & 0 & 0 \end{vmatrix} = 0 \rightsquigarrow bc - bc = 0$$

$X + y + z = 0$ non esiste (ci si intersecano)



$P[x : y : z]$

$Q\left[\frac{a^2}{x} : \frac{b^2}{y} : \frac{c^2}{z}\right]$



$$\frac{CS}{BS} = \frac{y}{z}$$

$$\frac{BS}{\sin BAS} = \frac{AS}{\sin \beta}$$

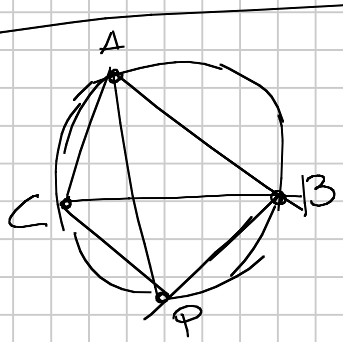
$$\frac{BT}{\sin BAT} = \frac{AT}{\sin \beta} \qquad \frac{CT}{\sin CAT} = \frac{AT}{\sin \gamma}$$

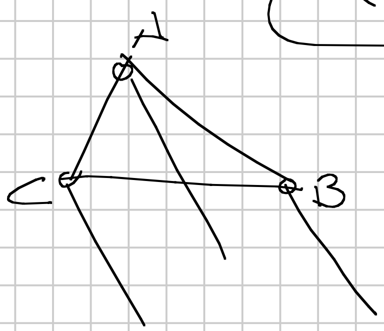
$$\frac{BT}{CT} = \frac{\sin B\hat{A}T}{\cancel{\sin C\hat{A}T}} \cdot \frac{\sin \gamma}{\sin \beta} = \frac{\sin B\hat{A}T}{\sin C\hat{A}T} \cdot \frac{c}{b}$$

$$\frac{BS}{CS} = \frac{\sin BAS}{\sin CAS} \cdot \frac{c}{b}$$

$$\left\{ \frac{BS}{CS} = \frac{1}{\frac{BT}{CT}} \qquad \frac{c^2}{b^2} \right.$$

$$\frac{z}{y} = \frac{CT}{BT} \cdot \frac{c^2}{b^2} \iff \frac{CT}{BT} = \frac{\left(\frac{b^2}{y}\right)}{\left(\frac{c^2}{z}\right)}$$





• $X+Y+Z=0$

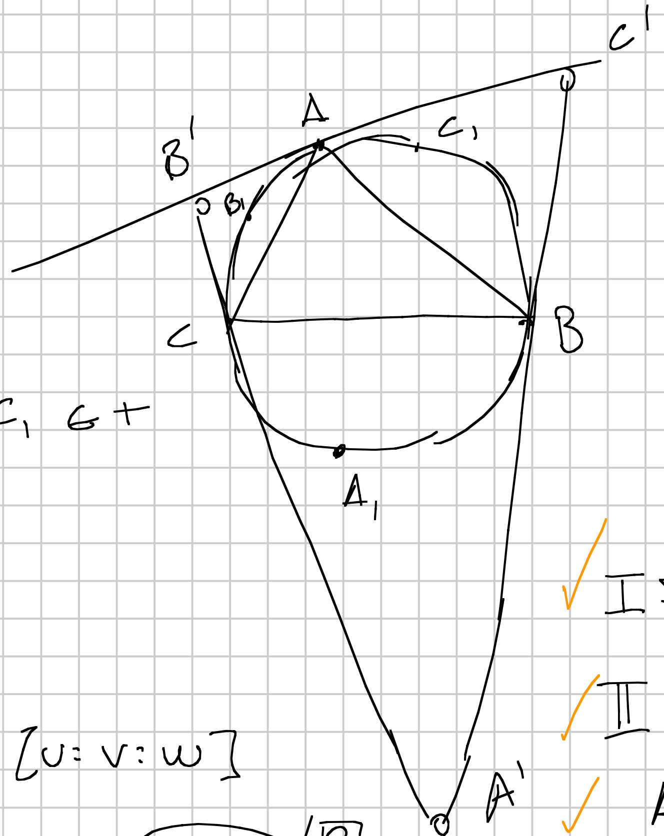
→ Coniugato isogonale della circoscritta è la retta all'∞

Coniug. isog. della ∞ = circoscritta!

→ $[x:y:z] \in t \iff \left[\frac{a^2}{x}:\frac{b^2}{y}:\frac{c^2}{z}\right] \in \infty \iff$

$$\boxed{\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z}=0} \qquad \left(a^2yz+b^2xz+c^2xy=0\right)$$

Teorema di Steinbert



$\triangle ABC$ persp $\triangle A_1B_1C_1$

$\Downarrow$

$\triangle A'B'C'$ persp $\triangle A_1B_1C_1$

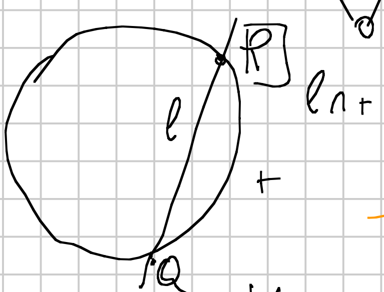
$A_1, B_1, C_1 \in t$

I Fatto: $A' = [-a^2 : b^2 : c^2]$

II Idea: Scelgo P e def.

$A_1 = AP \cap t$ e circ.

Dove $P=[u:v:w]$



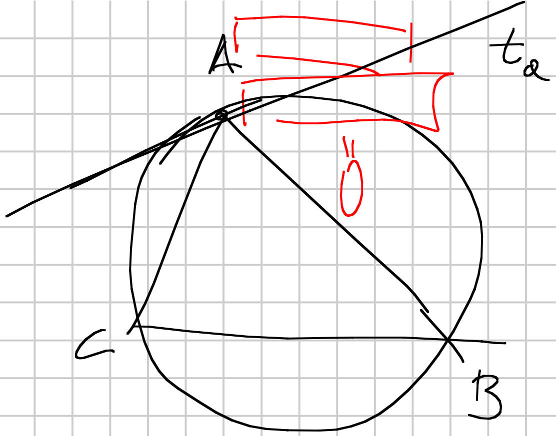
III

$$A_1 = \left[-a^2 : \frac{wb^2+vc^2}{w} : \frac{wb^2+vc^2}{v}\right]$$

$$\left\{A'A_1 = \underbrace{\left[v^2c^4 - w^2b^4 : -w^2a^2b^2 : v^2a^2c^2\right]}_{\text{circ}}\right\}$$

$$A'A_1 = \left[a^2v^2c^4 - a^2b^4w^2 : -a^4b^2w^2 : a^4v^2c^2\right]$$

$$A' = [-a^2 : b^2 : c^2]$$



t_a: $u y + v z = 0$

$$\boxed{c^2 y + b^2 z = 0}$$

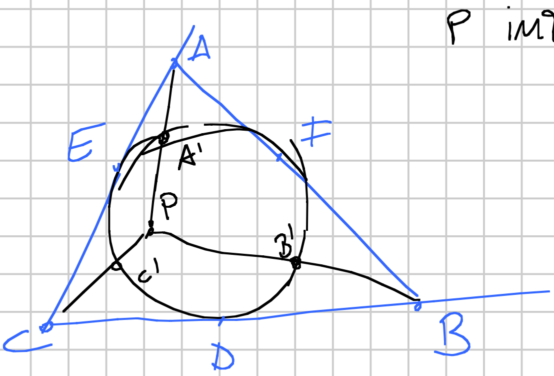
$$\boxed{\frac{c^2}{z} + \frac{b^2}{y} = 0}$$

$$\boxed{\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} = 0}$$

$$\frac{a^2}{x} = 0 \quad \text{NO}$$

Almeno 1 tra y e $z = 0$ $\quad$ $y = z = 0$
$[1 : 0 : 0]$

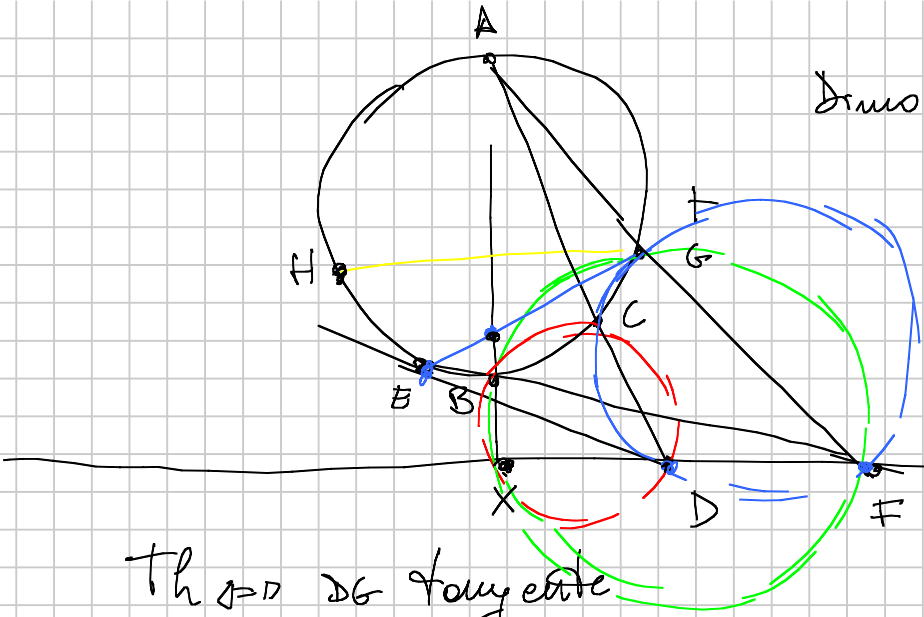
$$\begin{cases} t_B = c^2 x + a^2 z = 0 \\ t_C = b^2 x + a^2 y = 0 \end{cases} \qquad A' = [-a^2 : b^2 : c^2]$$



P interno all'inscritta.
DA', EB', FC' concorrono

Sia α il piano ref. Sia γ il piano ortogonale ad α per ℓ
e ω " " " " " per AB

Sia $P \in \omega \cap \gamma \setminus X$. Sia ε un piano $\perp AB$, $\alpha \perp_P \varepsilon$



Dimostrare F, C, H allineati

$\boxed{\mathrm{Pol}\,F \ni AB \cap EG}$

$\gamma = \mathrm{Pol}\ \ell$

$\gamma \in AB \quad \gamma \in \mathrm{Pol}(F)$

$\gamma = AB \cap \mathrm{Pol}(F)$

ℓ

Th $\iff$ DG tangente

1) BXCD ciclico

2) BXGF ciclico $\longrightarrow$ CGFD ciclico

$D\hat{C}F = D\hat{G}F$ Th $\iff$ $D\hat{C}F = A\hat{C}H = \boxed{A\hat{G}H}$

$\parallel$

$\boxed{D\hat{G}F}$

Th $\iff$ DG Tange

DE Tange Th $\iff$ $\mathrm{Pol}(D) = EG$

$\mathrm{Pol}(\ell) \in EG \implies \mathrm{Pol}(EG) \in \ell$

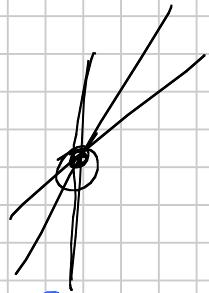
$\mathrm{Pol}(EG) \in EE$ $\mathrm{Pol}\ EG = \ell \cap EE \implies$

Th $\iff$ $\mathrm{Pol}(\ell) \in EG$ $\mathrm{Pol}(\ell) \in AB$

Pol(F) passa per $AB \cap EG$

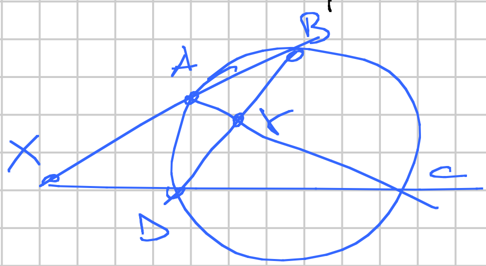
$\mathrm{Pol}(\ell) \in \mathrm{Pol}(F)$ $\mathrm{Pol}(\ell) \in AB$

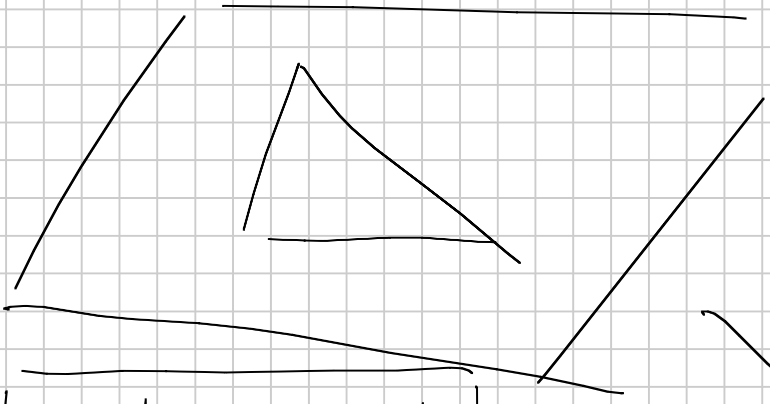
$\mathrm{Pol}(\ell) \in EG.$ $\mathrm{Pol}(\ell)$

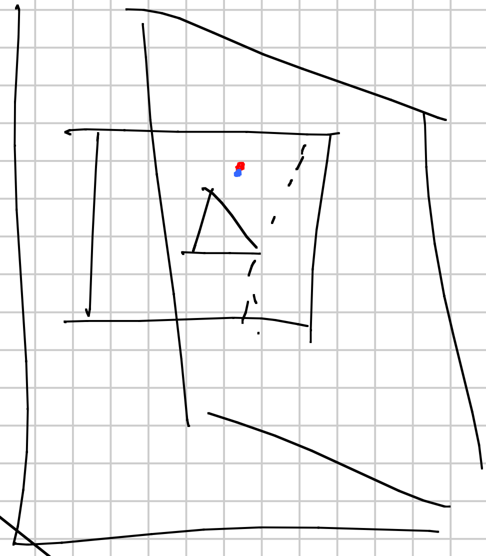


$X \in \mathrm{Pol}(Y) \iff Y \in \mathrm{Pol}(X)$

$Y \in \mathrm{Pol}(X)$





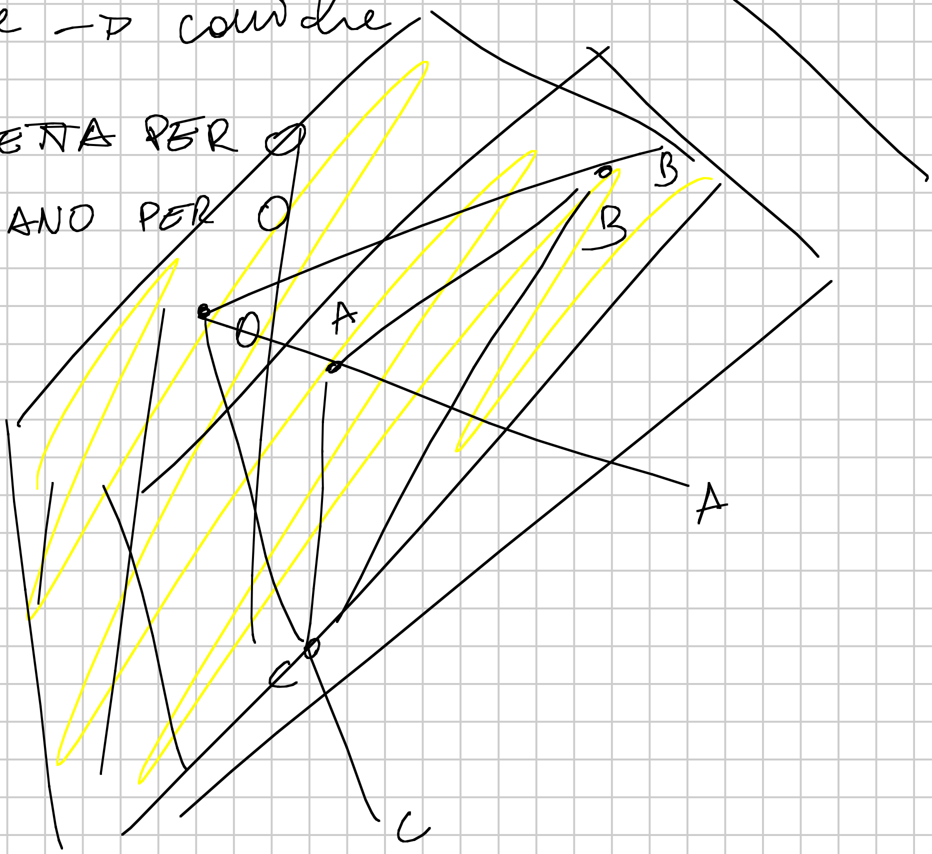


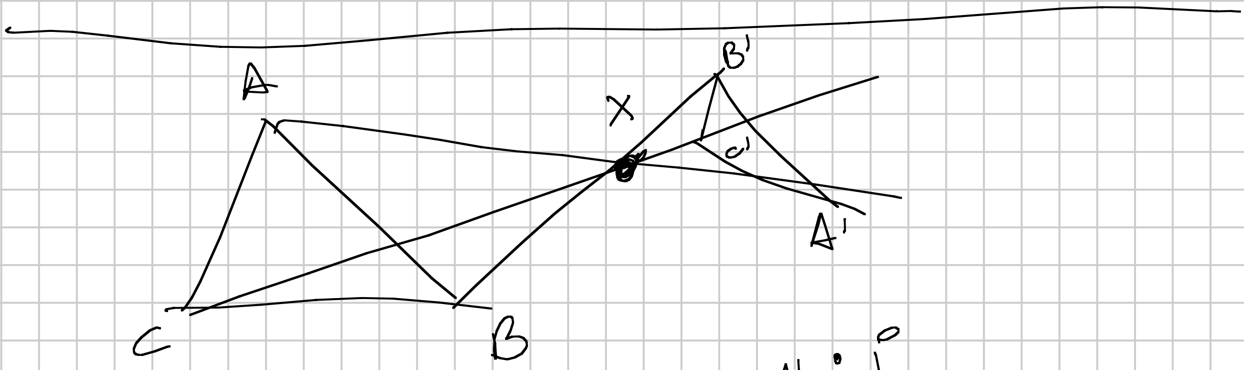
punti → punti

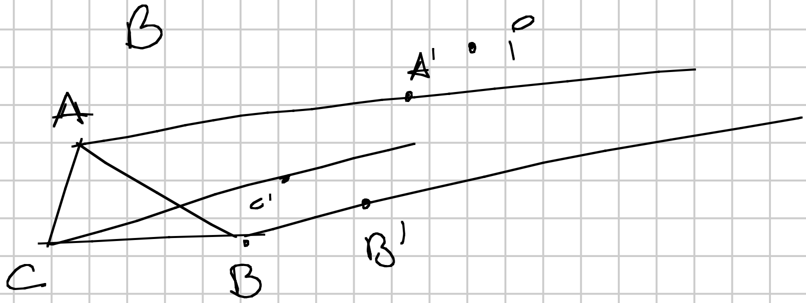
rette → rette

circonferenze → coniche

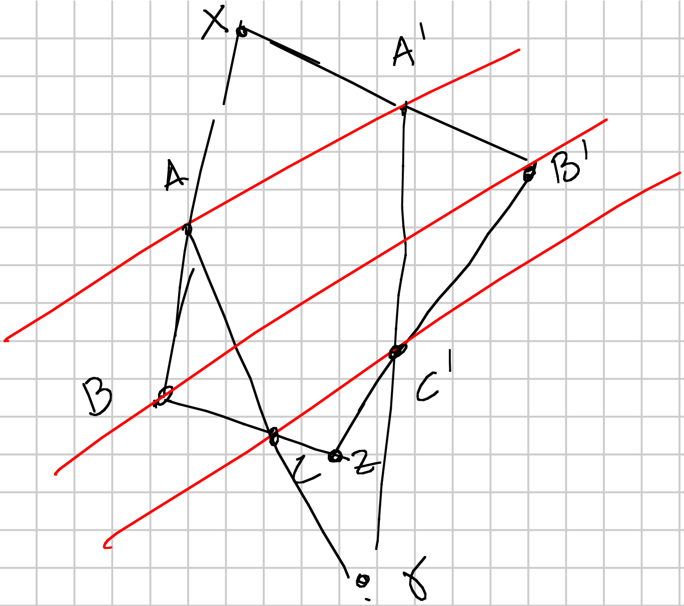
$$\begin{cases} \text{PUNTO} = \text{RETTA PER } O \\ \text{RETTA} = \text{PIANO PER } O \end{cases}$$

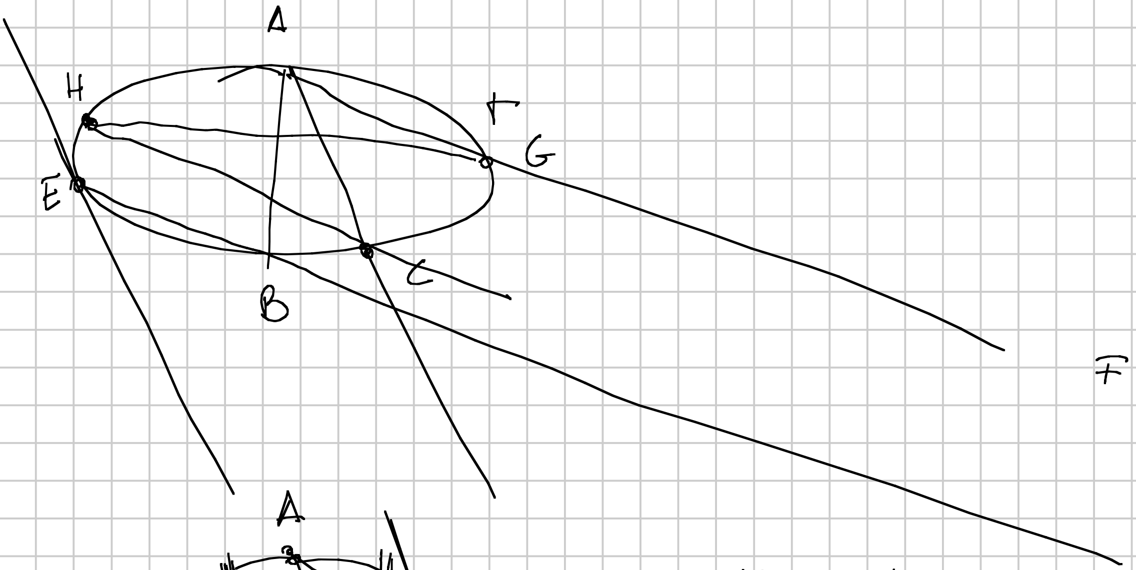




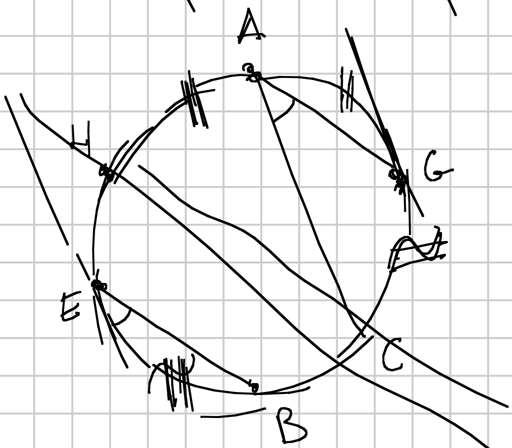


$AB \cap A'B'$ e qe sono



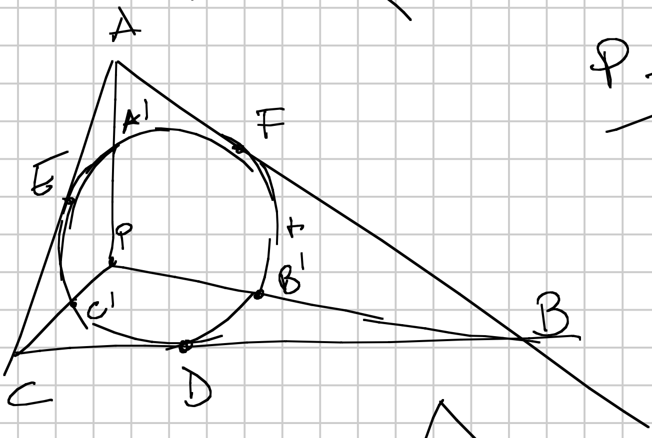


I



$CH \parallel AG \parallel EB$

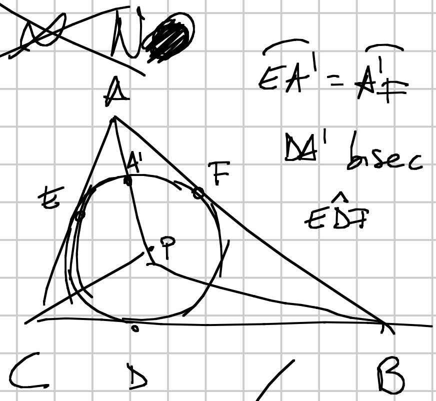
!! D



P – D ~~Ø~~ N

$\widehat{EA'} = \widehat{A'F}$

AA' bisec $\widehat{EDF}$



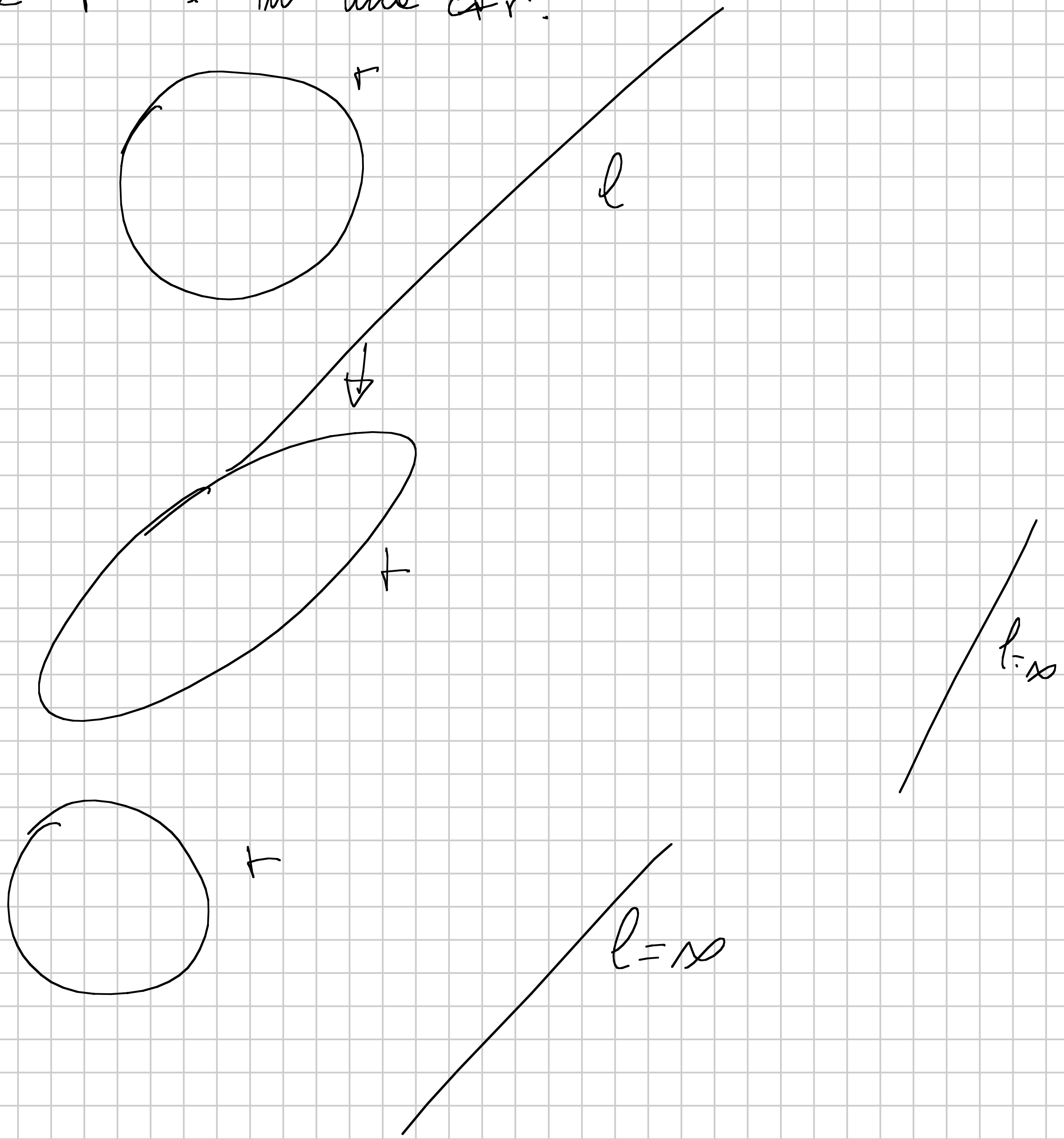
In realtà

bisogna controllare
le varie config

Data una retta ℓ e una circonferenza γ che non si intersecano $\exists$ una proiettività che:

manda $\ell \to \infty$

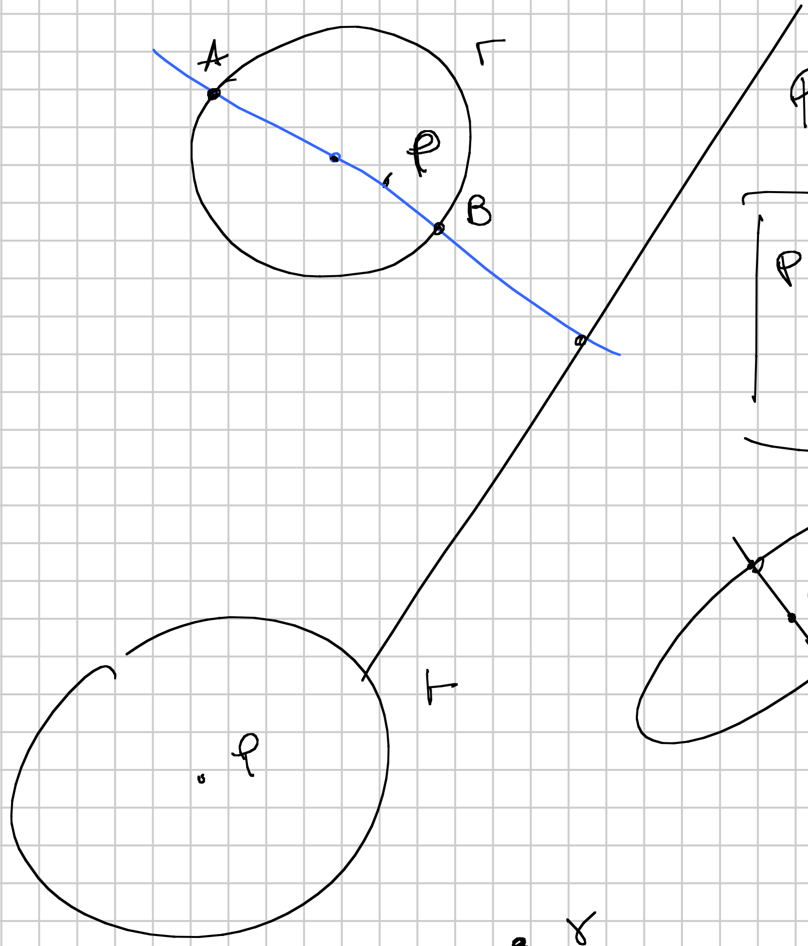
manda $\gamma \to$ in una cfr.



Sia P interno a γ $\exists$ ----.

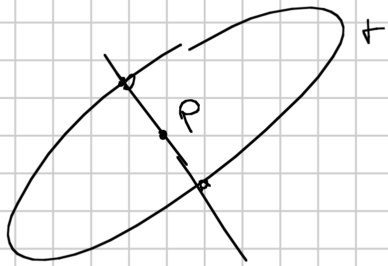
$\gamma \to$ in una cfr

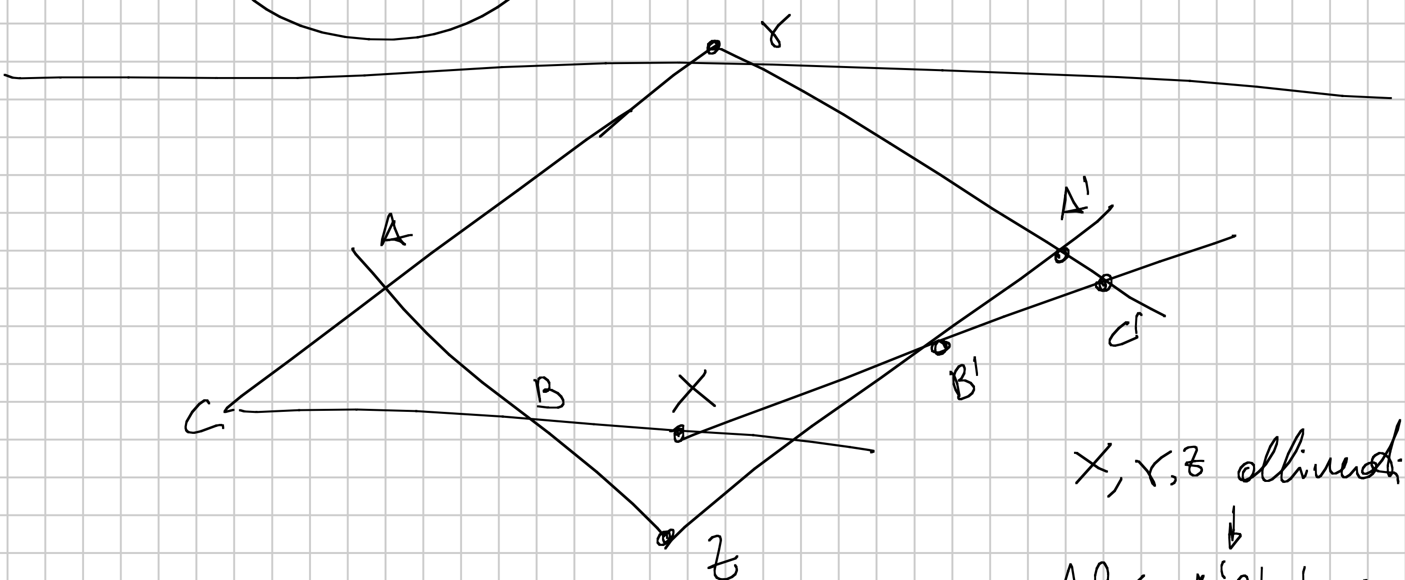
$P \to$ nel suo centro



Pol(P)

P nel punto medio di AB

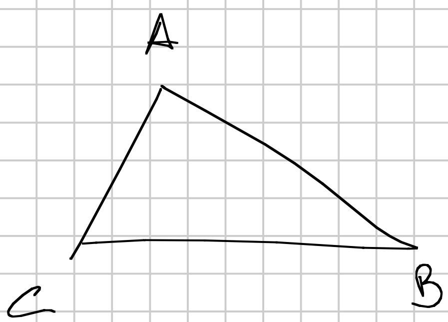




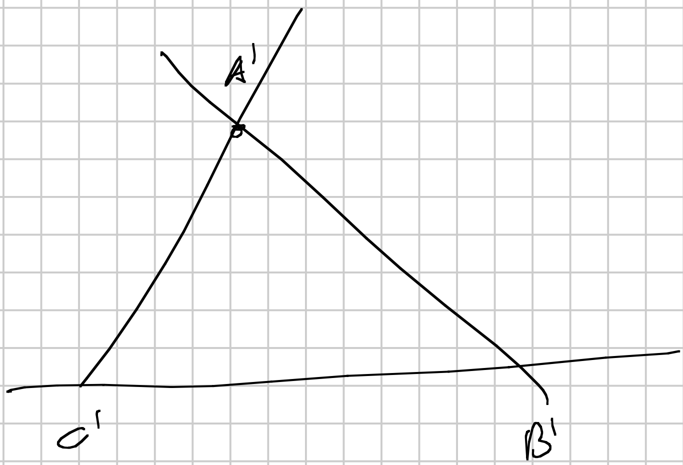
X, Y, Z allineati

↓

ABC, A'B'C' persp.



Desargues ✓

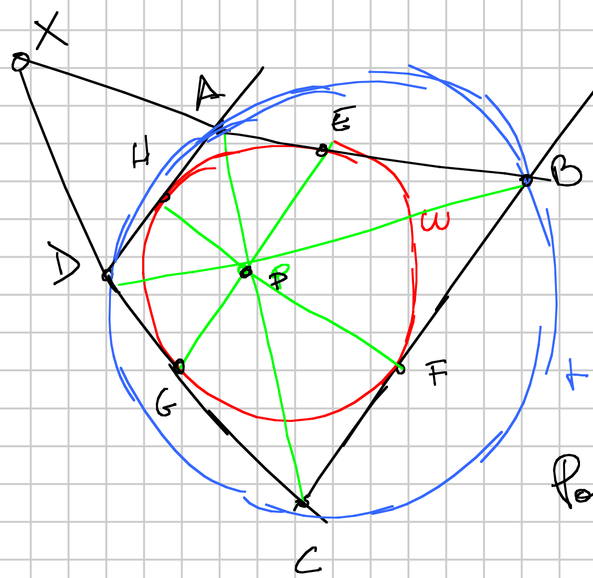


Th Newton (sl 199x)

Un quad bicentrico e' inscrittibile e circoscrittibile.

Dim. Centri delle 2 circonferenze e intersezione delle diagonali allineati.

Th Sole: Stessa cosa vale per un esagono bicentrico.



Brianchon degenere su A E B C G D

AEBCGD

$\boxed{\text{POLARI}}$

$Pol_\omega(P) = (HG \cap EF)(HE \cap GF)$

$DC \cap AB = X$

$AD \cap BC = Y$

$XY = Pol_\Gamma(P)$

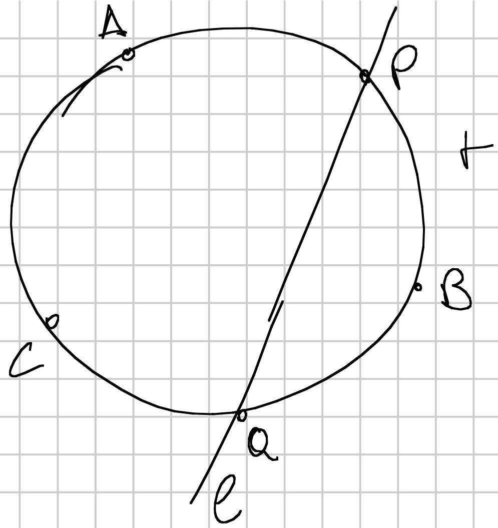
$Pol_\omega(X) = EG$

$Pol_\omega(Y) = HF$ $\to$ $Pol_\omega(XY) = P$

$\boxed{Pol_\omega(P) = XY} = Pol_\Gamma(P)$

$OP \perp \ell$

$O'P \perp \ell$ $\to$ O, O', P allineati $\boxed{\ell}$



$P, Q = l \cap t$

l NON comune

$P = [u : v : w]$

$l = [h : g : k]$

Quali sono le coordinate di Q?

$$\boxed{Q = \left[\frac{a^2}{uh} : \frac{b^2}{vg} : \frac{c^2}{wk}\right]}$$

$$uh + vg + wk = 0$$

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} = 0$$

$$\frac{a^2}{\left(\frac{a^2}{uh}\right)} + \frac{b^2}{\left(\frac{b^2}{vg}\right)} + \frac{c^2}{\left(\frac{c^2}{wk}\right)} = 0 \qquad Q \in t$$

$$\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$$

$$h\left(\frac{a^2}{uh}\right) + g\left(\frac{b^2}{vg}\right) + k\left(\frac{c^2}{wk}\right) = 0$$

$Q \in l$.

ABC triangolo. t circoscritta. P, Q coniugati isogonali

Sia A_1 pt medio di BC.

$A_2 = AP \cap t$

$A_3 = A_2A_1 \cap t$

$A_4 = A_3Q \cap BC$.

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Dimostrare AA_4, BB_4, CC_4 concorrono (su OI)