G4 - 2 ADVANCED

Titolo nota 05/09/2012

AA_4, BB_4, CC_4 NON concorrono su Γ

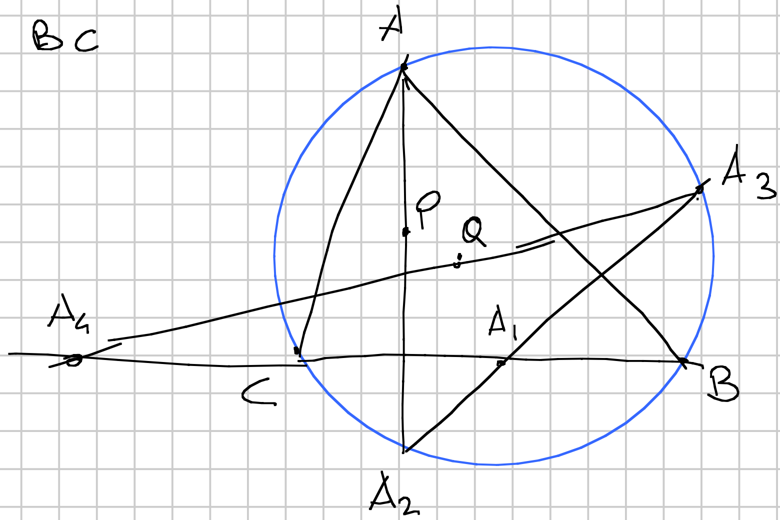
$\overset{\triangle}{ABC}$, Γ circoscritta, P e Q isog. coniug.

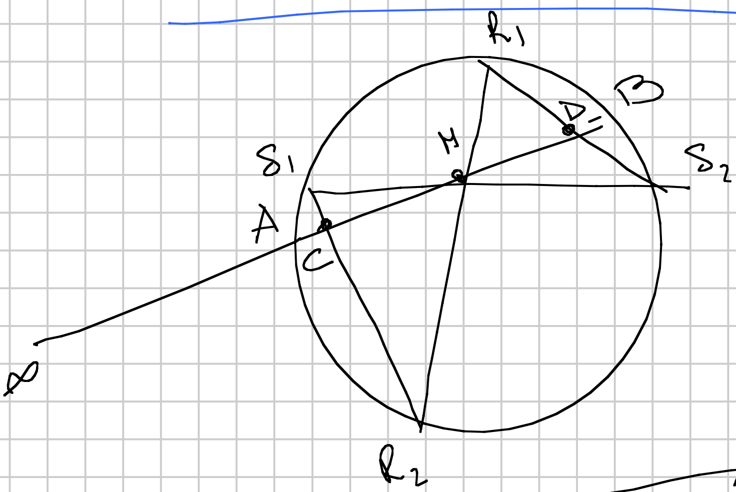
A_1 pto medio di BC

$A_2 = AP \cap \Gamma$

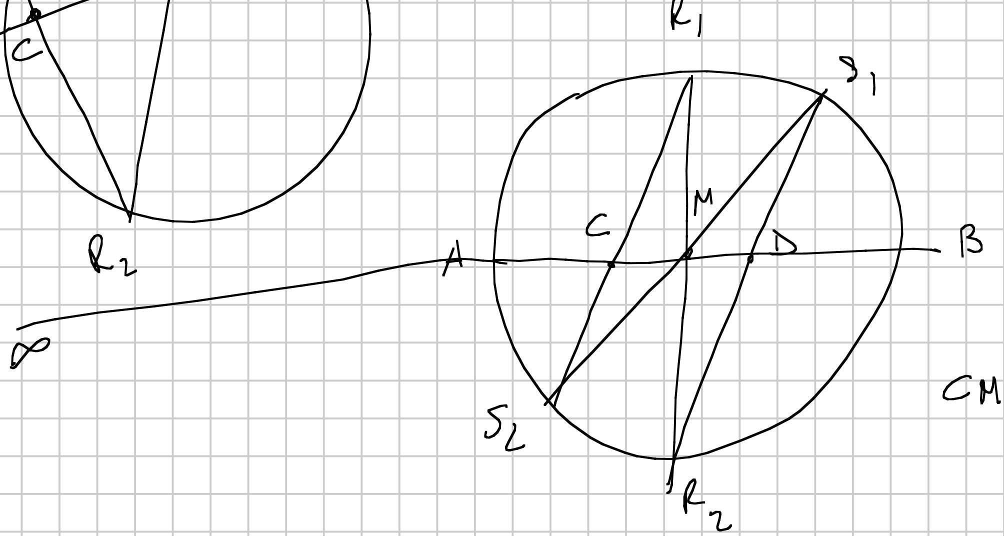
$A_3 = A_2A_1 \cap \Gamma$

$A_4 = A_3Q \cap BC$



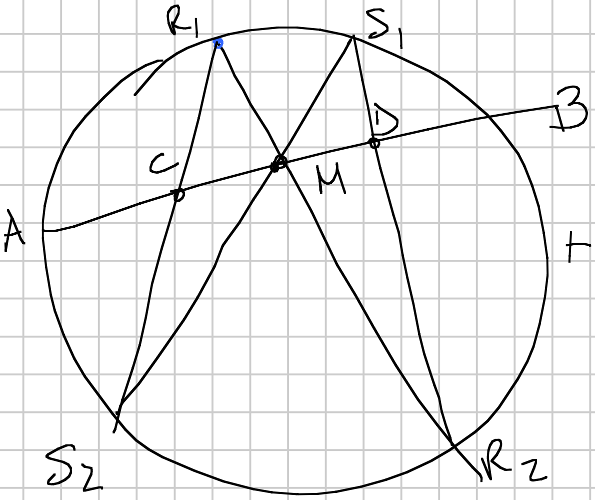


M pto medio di CD



$CM = MD$

$-1 = (ABM\infty)$ $\qquad CDM\infty = -1$



$$A \quad C \quad M \quad B$$

$$\|$$

$$(R_1A,\ R_1S_2,\ R_1R_2,\ R_1B)$$

$$\| \ R_1$$

$$(A,\ S_2,\ R_2,\ B)$$

$$\| \ S_1$$

$$(S_1A,\ S_1S_2,\ S_1R_2,\ S_1B)$$

$$\|$$

$$(A \quad M \quad D \quad B)$$

$$(ACMB) = (AMDB)$$

$$\frac{\cancel{AM}\cdot CB}{\cancel{AB}\cdot CM} = \frac{AD\cdot \cancel{MB}}{\cancel{AB}\cdot MD}$$

$$\frac{CB}{CM} = \frac{AD}{MD} \qquad CB = CM + MB \qquad AD = AM + MD$$

$$\cancel{1+}\ \frac{MB}{CM} = \cancel{1+}\ \frac{AM}{MD} \qquad \boxed{MD = CM}$$

$\overset{\triangle}{ABC}$, Γ circoscritta, P e Q isog. coniug.

A_1 pt medio di BC

$A_2 = AP \cap \Gamma$

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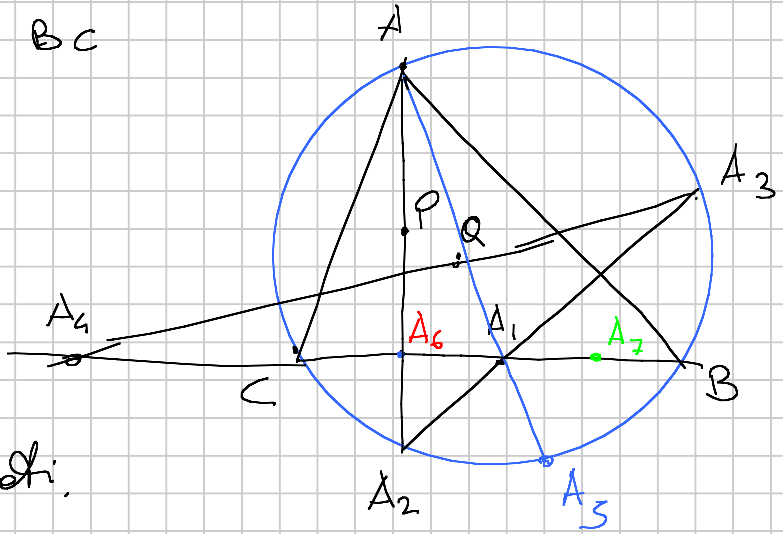


A_5, A_7, A_3 allineati.

$P = [u:v:w]$.

$A_6 = [0:v:w] \qquad A_7 = [0:w:v]$

$A_5 = [-a^2 : b^2+c^2 : b^2+c^2]$

$A_5 A_7 = [(b^2+c^2)(w-v) : -a^2 v : a^2 w]$

$A_3 = \left[\frac{a^2}{w-v} : \frac{b^2}{v} : -\frac{c^2}{w}\right]$

$Q = \left[\frac{a^2}{u} : \frac{b^2}{v} : \frac{c^2}{w}\right]$ $\qquad QA_3 = [m : n : \sigma]$

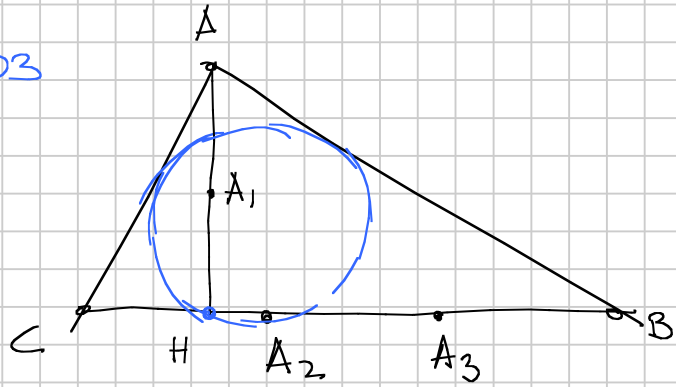
$[0 : \sigma : -n]$

$\frac{\sigma}{-n} \cdot \text{arc} = 1$

$A_3Q = \left[\cdots : \frac{\widetilde{w-v+u}}{\underset{\sim}{b^2 w}} : \frac{\widetilde{u-w+v}}{\underset{\sim}{c^2 v}}\right]$

Vietnam TST 2003

Dimostrare A_1A_2 e ciclide concorrono su



A_1, A_2, Ex_A allineati

Omotetia di centro A che manda inscritta in exinscritta.

$A_2 \to$ va nel diametralmente opposto e $A_3 = A_4$

$A A_2 A_4$ allineati

$A\hat{H}A_2$ omotetico $A_2A_3A_4$

$A_1 \to$ pt medio di $A_3A_4 = Ex_A$

$\to A_1, A_2, Ex_A$ allineati.

Il triangolo degli excentri è omotetico al triangolo di contatto. (lati paralleli)

Quindi A, Ex_A e cicliche concorrono. (1° parte)

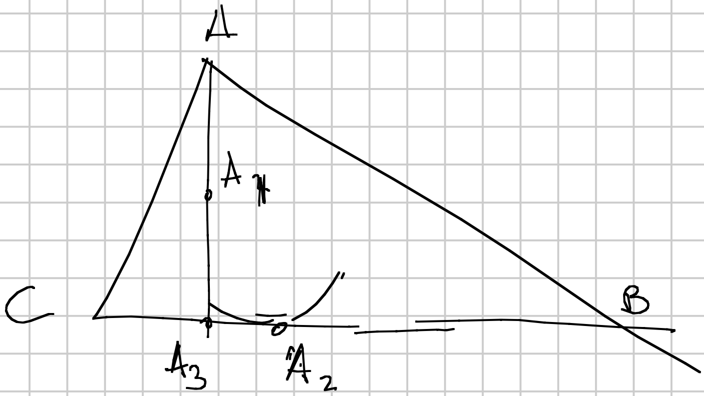
$S =$ L'ortocentro del triangolo excentrico è l'incentro

Il circocentro " " di contatto è l'incentro

$S \to$ ortocentro del triangolo di contatto.

$\downarrow$

La retta di Eulero del triangolo di contatto passa per il centro omotetico (che per quanto dimostrato prima è il punto di concorrenza) ma la retta di Eulero del triangolo di contatto è OI.



$$A_2 = [0 : a+b-c : a-b+c]$$

$$A_3 = [0 : b\cos\gamma : c\cos\beta] \doteq \left[0 : \cancel{b}\frac{a^2+b^2-c^2}{2a\cancel{b}} : \cancel{c}\frac{c^2+a^2-b^2}{2a\cancel{c}}\right]$$

$$[0 : a^2+b^2-c^2 : c^2+a^2-b^2].$$

$$\vec{A_3} = \vec{B}\cdot\left(\frac{a^2+b^2-c^2}{2a^2}\right) + \vec{C}\cdot\left(\frac{c^2+a^2-b^2}{2a^2}\right)$$

$$\vec{A} = 1\cdot\vec{A}$$

$$\vec{A}\,\frac{1}{2} + \vec{B}\,\frac{a^2+b^2-c^2}{4a^2} + \vec{C}\,\frac{c^2+a^2-b^2}{4a^2}$$

$$A_2 = [2a^2 : a^2+b^2-c^2 : \underbrace{c^2+a^2-b^2}]$$

Chiamo $\alpha = b+c-a$

$A_1A_2 = [\underline{\alpha(c-b)} : \beta a : -\gamma a]$

$\| u : v : w \|$ $\qquad$ $\| a : c : b \| = \begin{Vmatrix} a & c & b \\ c & b & a \\ b & a & c \end{Vmatrix}$

$\| \alpha(c-b) \quad \beta a \quad -\gamma a \| = 0$

$\|$

$\underbrace{\| \alpha c \quad \beta e \quad -\gamma a \|}_{\|} - \underbrace{\| \alpha b \quad \beta a \quad -\gamma a \|}\ \Delta$

Sia p il semiperimetro

$\frac{\alpha}{2} = p - a$

$\| \frac{\alpha}{2} c \quad \frac{\beta}{2} a \quad -\frac{\gamma}{2} a \|$

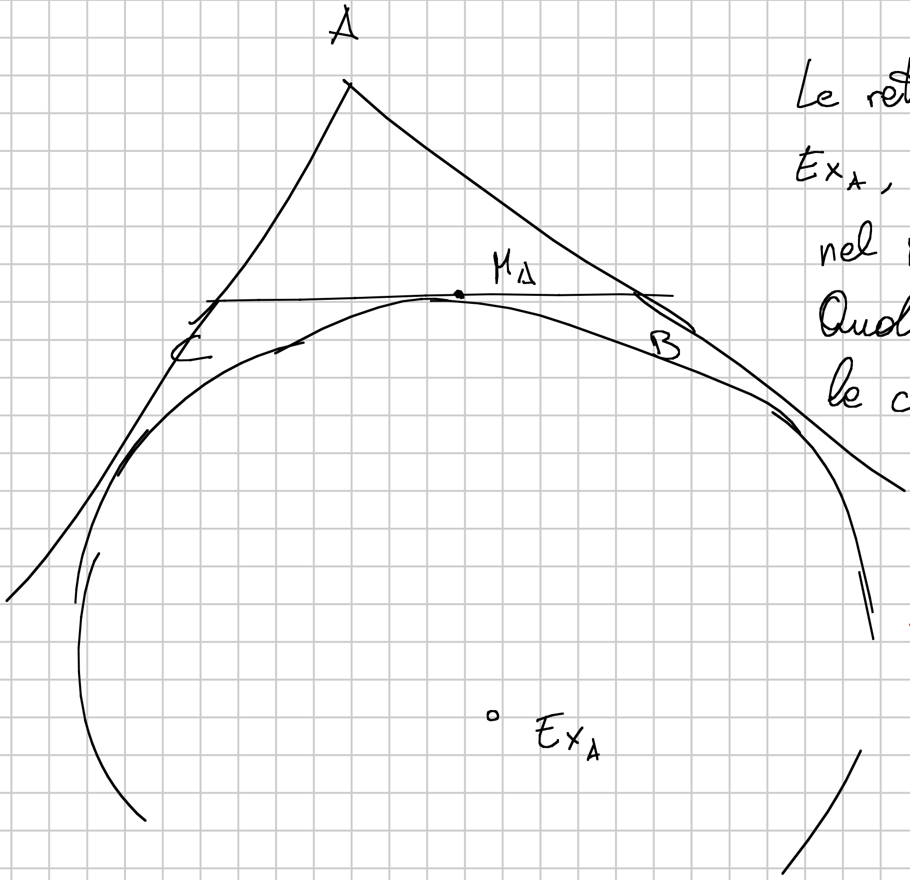
$\|$

$\| (p-a)c \quad (p-b)a \quad -(p-c)a \|$

$\|$

$\cancel{\| pc : pe \quad -pe \| - \| ac \quad ab \quad -ac \|}$

0



Le rette

Ex_A, M_A concorrono

nel MITTEN PUNKT

Quali sono

le coordinate del

MITTENPUNKT?

$\|$

GERGONNE

DEL TRIANGOLO MEDIALE

$Ex_A = [-a : b : c]$

$M_A = [0 : 1 : 1]$

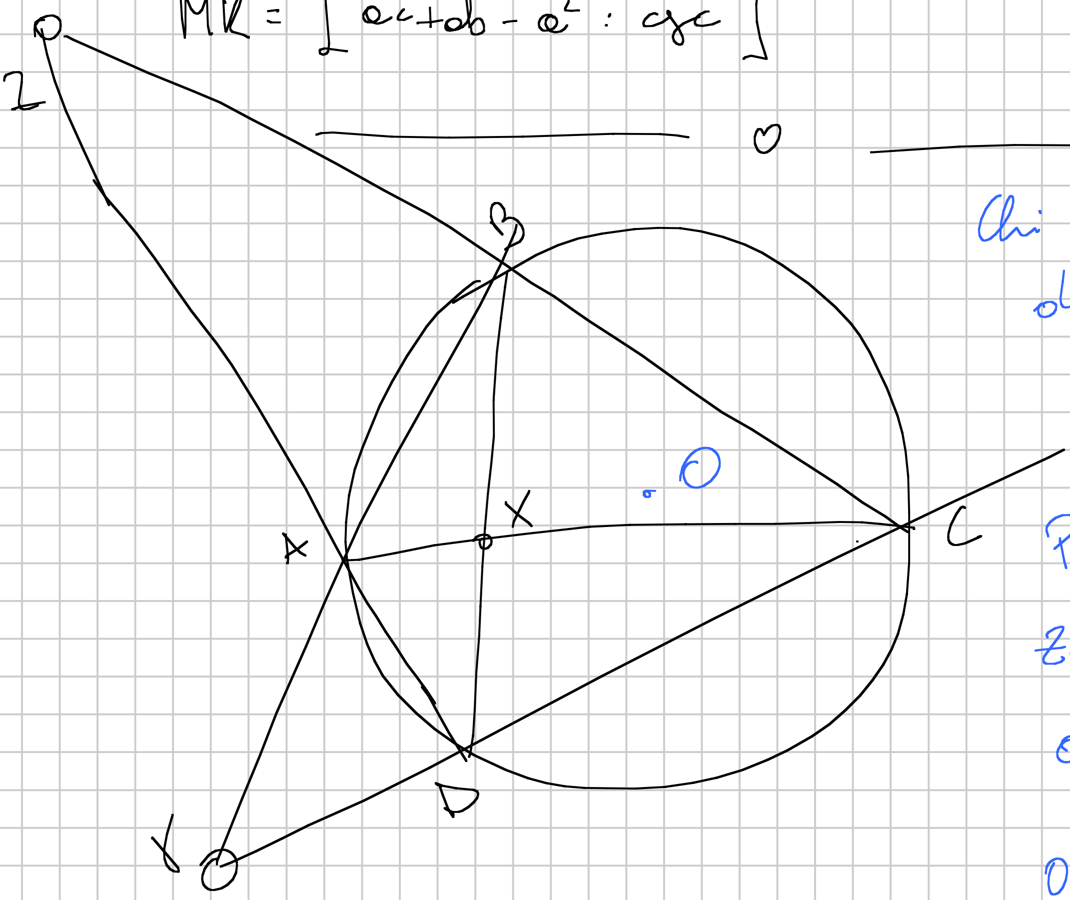
$$\begin{vmatrix} -a & b & c \\ 0 & 1 & 1 \\ x & y & z \end{vmatrix} =$$

$$x(+b-c) + y(+a) + z(-a) = 0$$

$Ex_A M_A = [b-c : a : -a]$

$Ex_B M_B = [-b : c-a : +b]$

$MK = [ac+ab-a^2 : \ldots]$



Chi è l'ortocentro di XYZ?

$Pol(Z) = XY$

$ZO \perp XY$

$OY \perp XZ$

$\Downarrow$

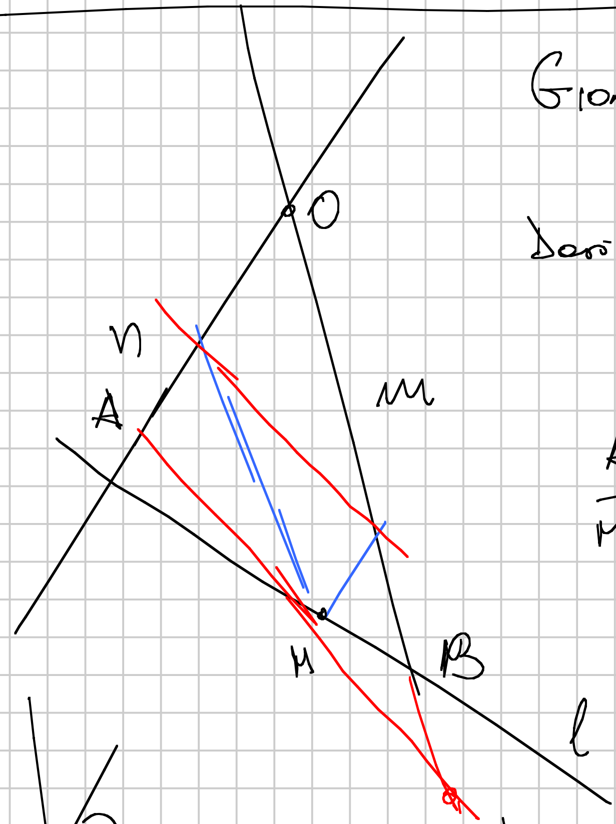
O ortocentro di XYZ

Coniug. del $MK = \left[\frac{a}{-a+b+c} : \cdots \right]$

$I = [a : b : c]$

$O = [a^2(-a^2+b^2+c^2) : - - -]$

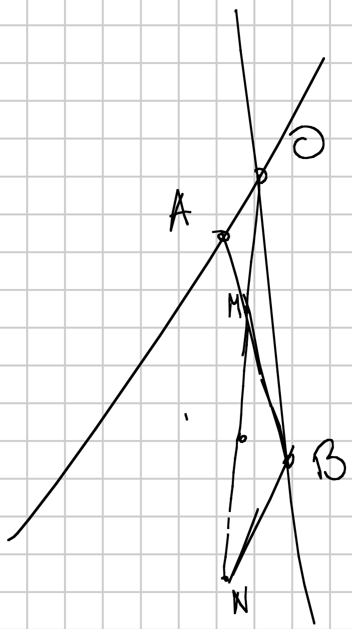
$$\left[\begin{matrix} \frac{1}{-a+b+c} & \\ & \text{cyc} \\ \frac{1}{a(-a^2+b^2+c^2)} & \end{matrix} \right] = 0$$

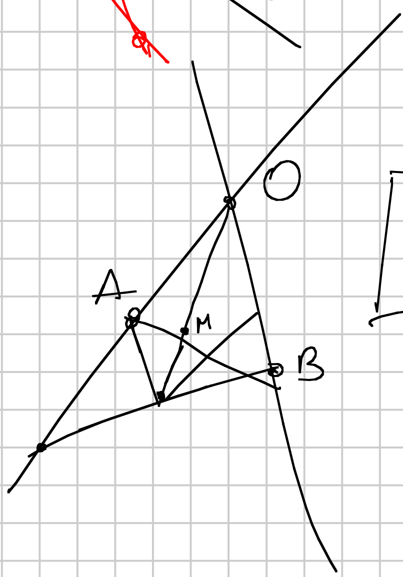


GIOACCHINO: So trovare ℓ t.c. $AM = MB$

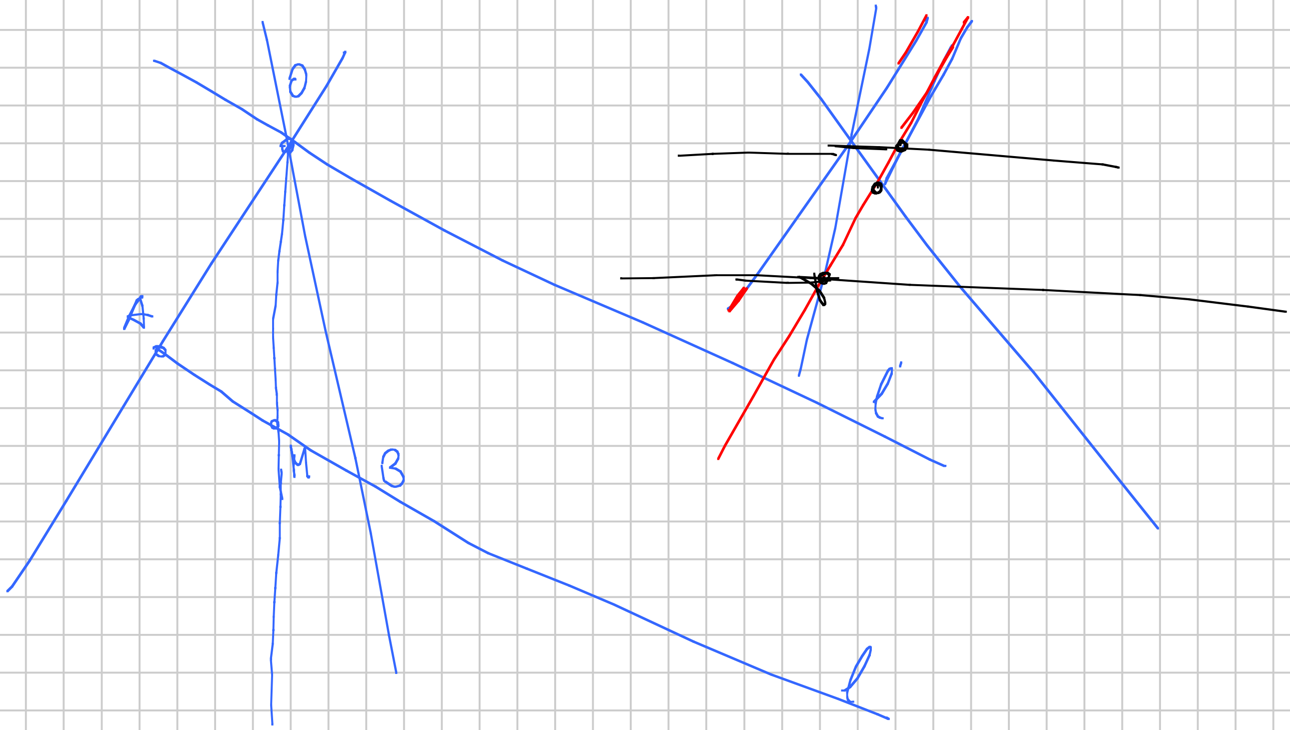
Dario: Vettori

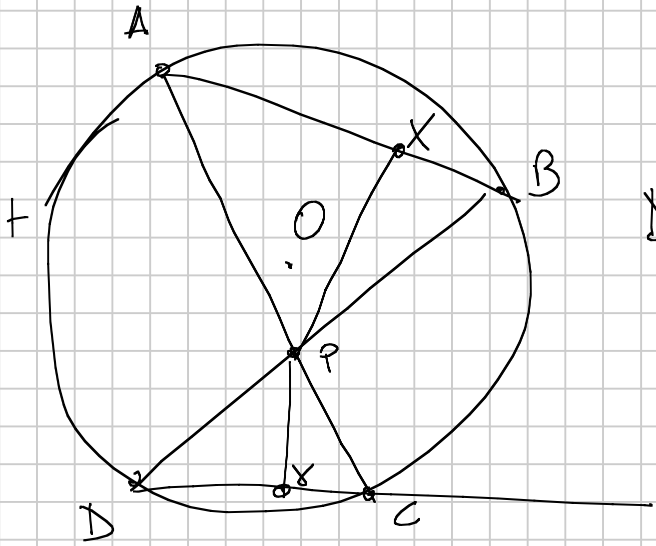
$$\frac{AM}{MB} = 2$$





$\frac{2011}{2012}$





$Z = \mathrm{Inv}_{\Gamma}(P)$

Dimostrare

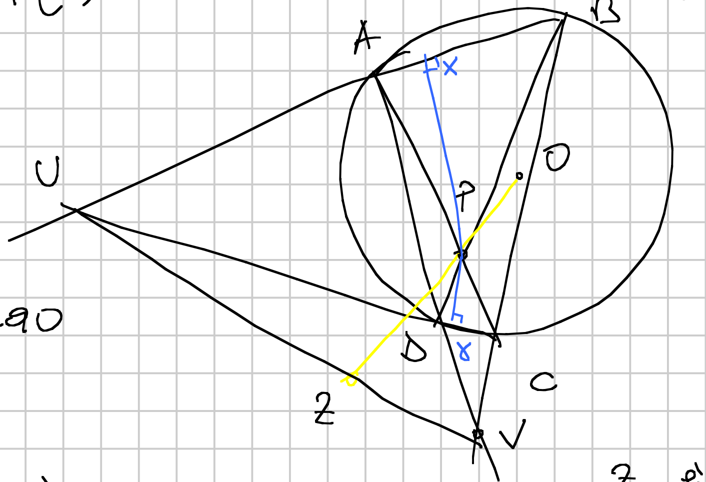
$XPYZ$ quadrilatero armonico

$AB \cap CD \in \mathcal{C}(XPY)$

$\mathrm{Inv}(P) = \mathrm{Pol}_{\Gamma}(P) \cap OP$

$\mathrm{Pol}_{\Gamma}(P) = (AB \cap CD)(AD \cap BC)$

$XPYZ$



Diametro UP

$PZ \perp UV \to U\hat{Z}P = 90$

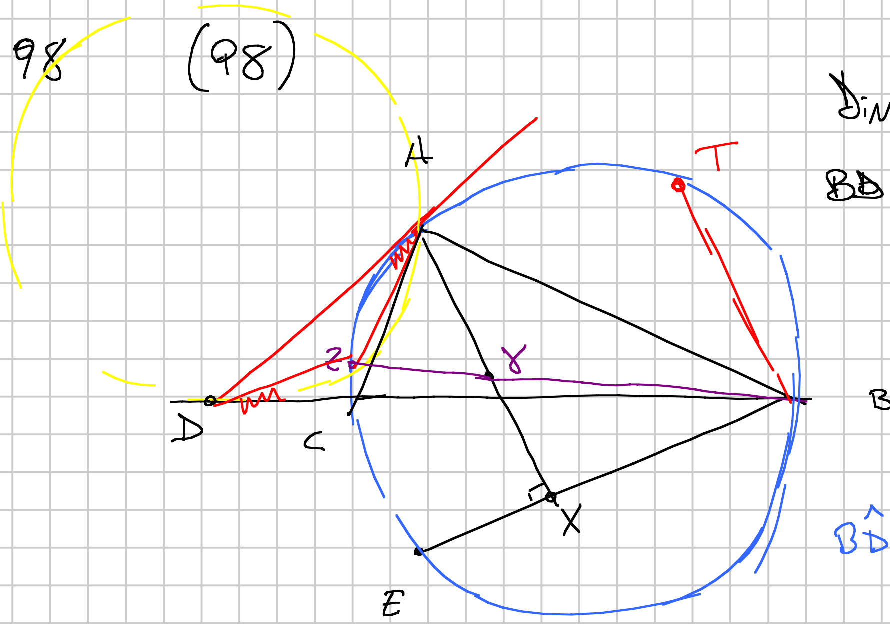
$XPYZO$ ciclico.

$(UX, UP, UY, UZ) = -1$

Z è il pt di Miquel di $ABCD$

GIDA SZEMW

SL 98 (98)



Dimostrare che BD tange $C(AZD)$

D, Z, T allineati

$AT // BC$

$B\hat{D}Z = B\hat{D}T = A\hat{T}D = D\hat{A}Z$

Idea I: Invertire da qualche parte

Idea II: Pt medio $\overset{?}{\to}$ Quaterne armoniche

Fatto 3: Traccio la parallela ad AX per B, che interseca Γ in T. Allora AZET armonico, ma per simmetria $Pol(AE) = D \to$ D, Z, T allineati

Fine per angle-chasing

Idea I: M pt medio = birapporti

Idea II:

birapporto non cambia tracciando le 4 perpendicolari

Idea III: $OR // AD$

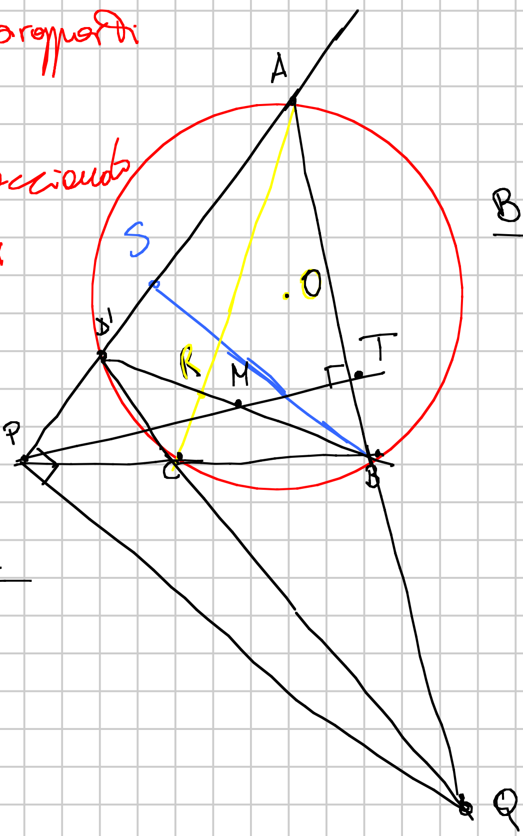
e $OM \perp BD$

Idea IV: Trigonometria

$PB \cdot PC = AQ \cdot BT$

$Pow(P) = AQ \cdot BT$

$PA \cdot PD$



$A\hat{P}Q = 90$

$BM = MD$

$R = AC \cap BD$

$T = PM \cap AB$

$A\overset{\triangle}{T}P \sim A\hat{P}Q$

$$\boxed{\frac{AP}{AQ} = \frac{BT}{PD}}$$

$$\frac{AP}{AQ} = \frac{PB}{PD} \cdot \cos(\beta)$$

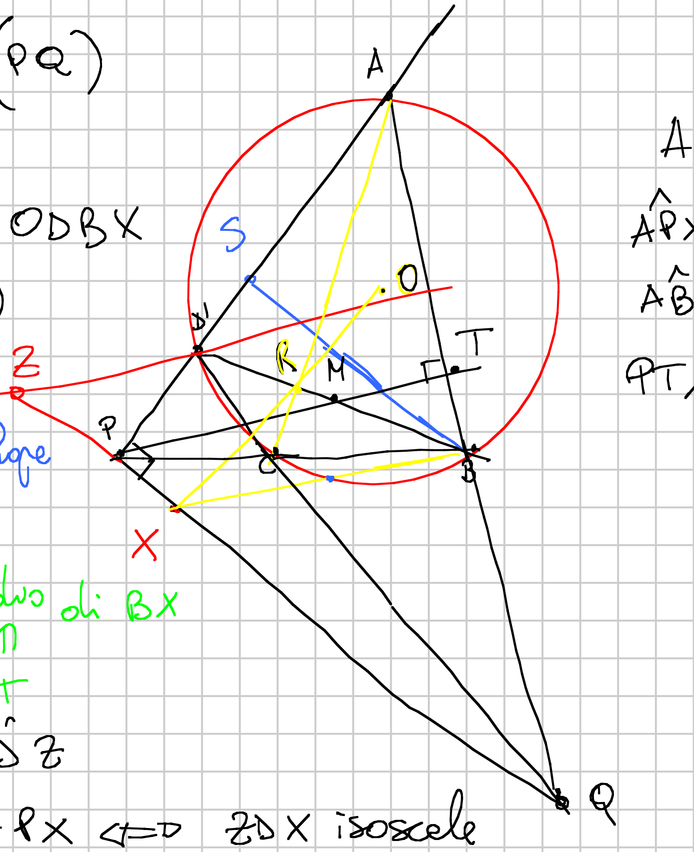
$BT = PB \cdot \cos(\beta)$ $\qquad$ $PC = AQ \cos\beta$

$C(ORM) = \mathrm{Inv}(PQ)$

$X = OR \cap PQ$

PDCX, ADQX, ODBX

(MIQUEL)



ABPX ciclico

$\hat{APX} = 90 \Rightarrow$

$\hat{ABX} = 90$

$PT \parallel BX$

$PC = AQ \cdot \cos\beta$ }
$\mathrm{Pow}(P) = AQ \cdot BT$ } hope

Claim: Punto medio di BX

$\Updownarrow$

T

$XB \parallel PT \parallel \hat{D}Z$

Th $\Leftrightarrow$ $ZP = PX$ $\Leftrightarrow$ ZDX isoscele

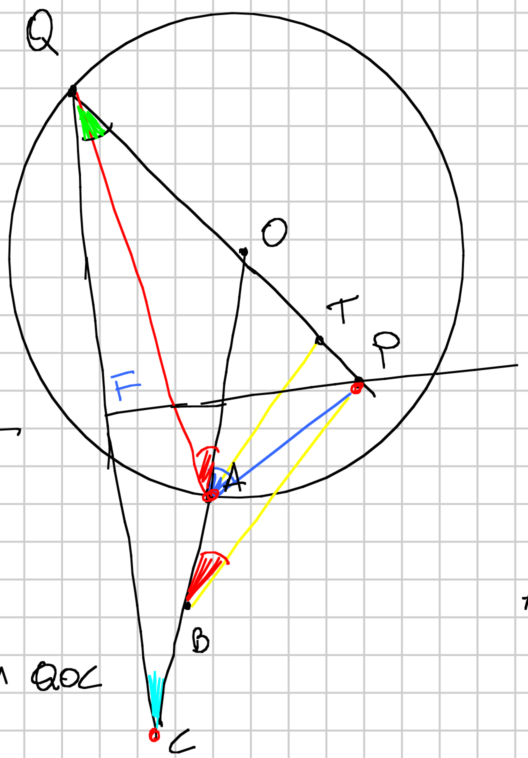
$ZD = DX$

$Z\hat{D}P = 90 - D\hat{A}B$

PDCX cic $\Rightarrow$ $P\hat{D}X = P\hat{C}X$

$P\hat{C}X = 90 - P\hat{C}D$

$P\hat{C}D = D\hat{A}B$



$OA = AC$

$AB = BC$

$P\hat{O}B = 2\,P\hat{B}O$

Th $\Leftrightarrow$ $P\hat{B}O = A\hat{Q}P$

$\Updownarrow$

AO biseca QA,

$Q\hat{O}A = \alpha$

$QC = \sqrt{5 - 4\cos\alpha}$

Trovo $QF = \frac{QC}{2}$

So tutti i seni degli angoli in QOC

Passo a $Q\hat{F}P$ QF ok   So dimostrare

Trovo QP

/ $OP = QP - QO$

$OB = \frac{3}{2}$ $P\hat{O}B = 180 - \alpha$

Arrivo a ragionare tutto in $\underline{OPB}$

$O = (0,0)$ $Q = (a,b)$ $a^2 + b^2 = 1$

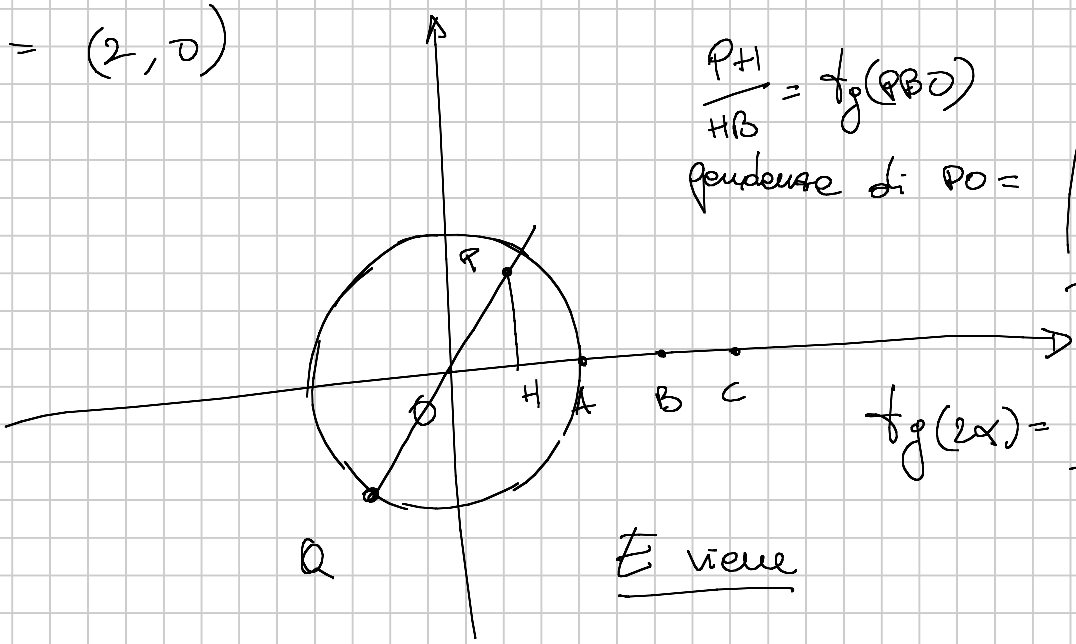
$A = (1,0)$ $P = \left(\frac{3a}{4a-2}, \frac{3b}{4a-2}\right)$

$B = \left(\frac{3}{2}, 0\right)$

$C = (2,0)$

$\frac{PH}{HB} = tg(PBO)$

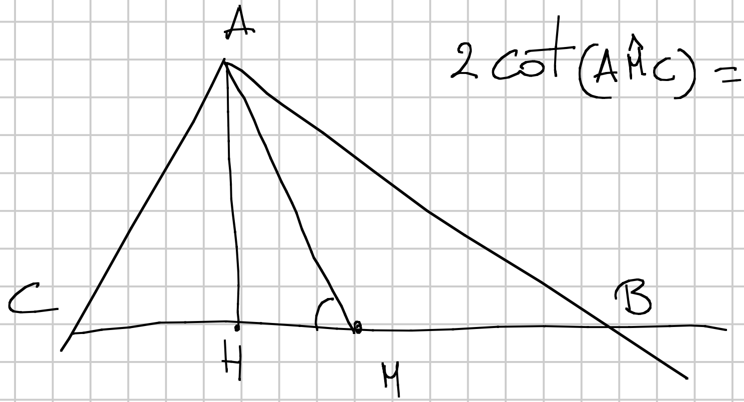
pendenza di $PO = \boxed{\frac{b}{a}}$



$tg(2\alpha) = \frac{2 tg\alpha}{1 - tg^2(\alpha)}$

$\underline{\text{E viene}}$

$2\cot(A\hat{M}C) = \cot(B) - \cot(C)$



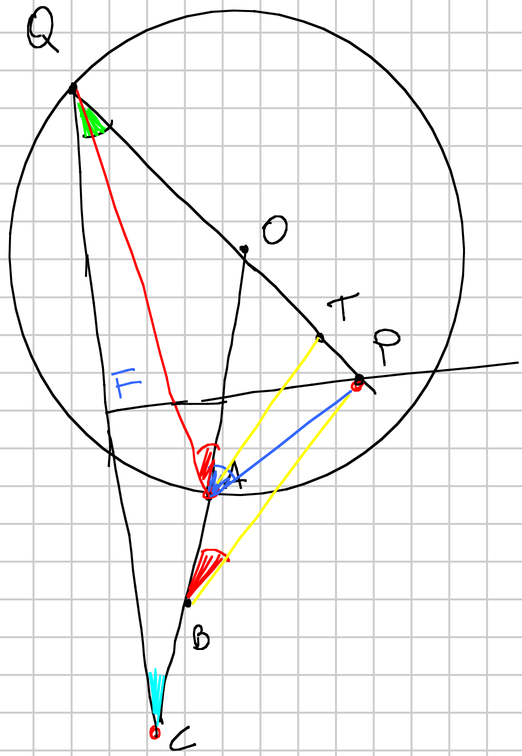
OA biseca

$$\frac{AQ}{QO} = \frac{AT}{TO}$$

$$\frac{AT}{TO} = \frac{BP}{PO}$$

$$A\hat{O}Q = \alpha$$

$$AQ^2 = 2(1-\cos\alpha)$$



$$\frac{BP^2}{OP^2}$$

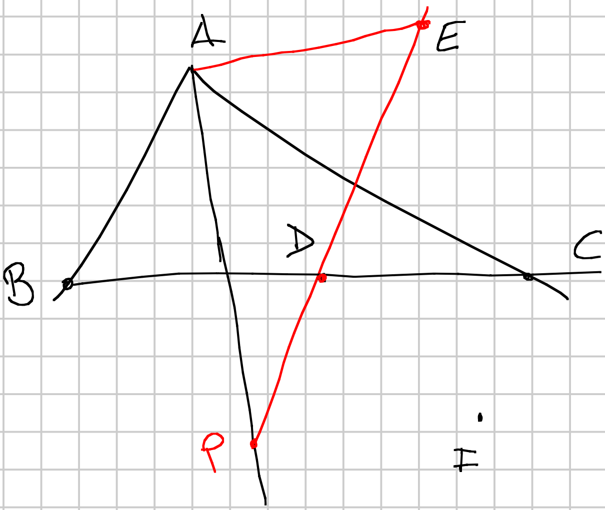
$$QC = \sqrt{5-4\cos\alpha}$$

$$\cos(FQP) = \frac{1-2\cos\alpha}{\sqrt{5-4\cos\alpha}}$$

$$QP = \frac{QF}{\cos FQP} = 1 + \frac{3}{2-4\cos\alpha}$$

$$OP = \frac{3}{2-4\cos\alpha}$$

$$PB^2 = \frac{9}{2}\left(\frac{1-\cos\alpha}{(1-2\cos\alpha)^2}\right)$$



$BD = DC$

AEFP rettangolo

FP biseca $B\hat{F}C$