Geometria 3 - Adv

Titolo nota — Sem — 06/09/2012

Es 1: ABCD quad. ciclico di circocentro O

$\Gamma_{ABO} \cap \Gamma_{CDO} = \{O, P\}$ P interno a $D\hat{A}O$.

Sia Q sulla semiretta OP oltre P,

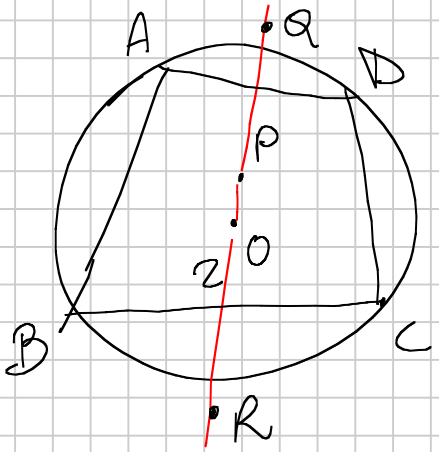
sia R sulla semiretta PO oltre O (R, O, P, Q)

$\Rightarrow Q\hat{A}P = O\hat{B}R \iff P\hat{D}Q = R\hat{C}O$.

Idee: 1) • Inversione in Γ_{ABCD}

2) • Angoli uguali $\iff$ stesse ciclicità

1) Invertiamo in Γ_{ABCD}



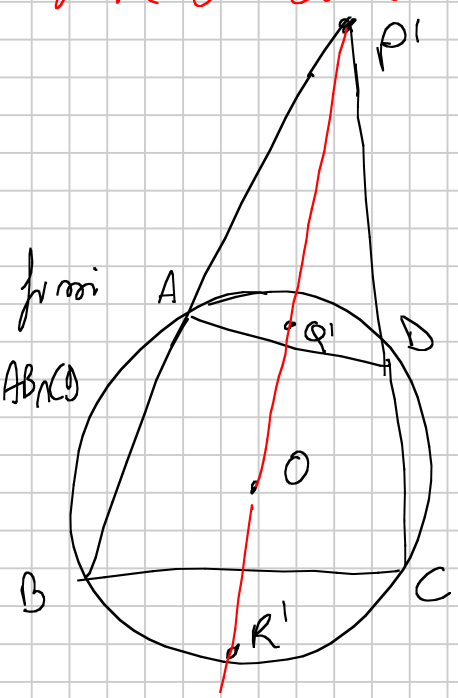
A, B, C, D fissi

$P \to P' = AB \cap CD$

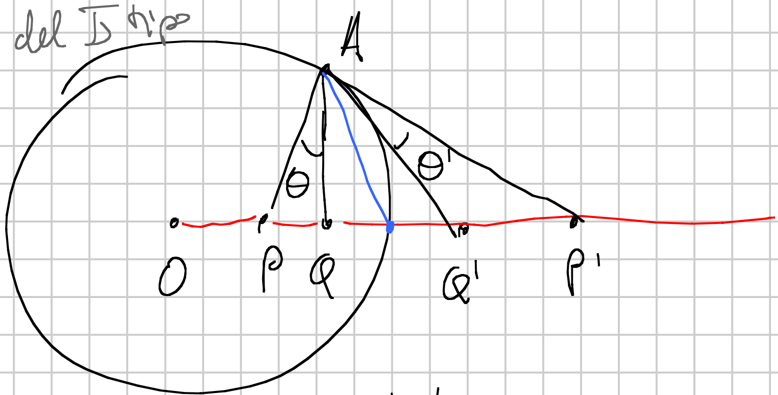
$R \to R$

$Q \to Q'$

$R \to R'$



Angoli del I tipo



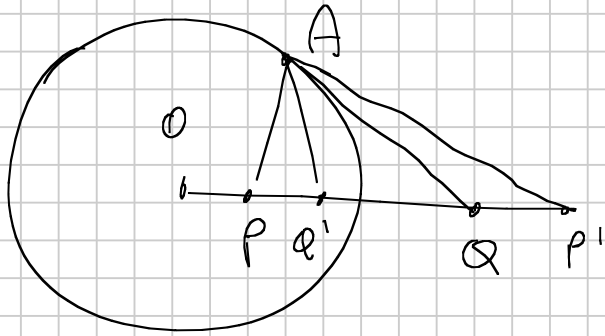
$P'Q' = \dfrac{R^2}{OP \cdot OQ} \cdot PQ$

$AP' = \dfrac{R}{OP} \cdot AP$

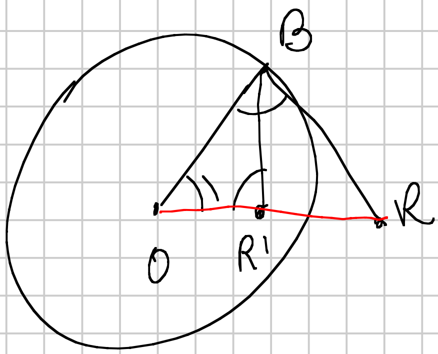
$AQ' = \dfrac{R}{OQ} \cdot AP$

$\sin \theta' = \dfrac{h \cdot Q'P'}{AP' \cdot AQ'} =$

$$= h \cdot \frac{QP}{AP \cdot AQ} \cdot \frac{\frac{R^2}{OP \cdot OQ}}{\frac{R}{OP} \cdot \frac{R}{OQ}} = \frac{h \, QP}{AP \cdot AQ} = \sin\theta$$



Angoli del [illegible] tipo:



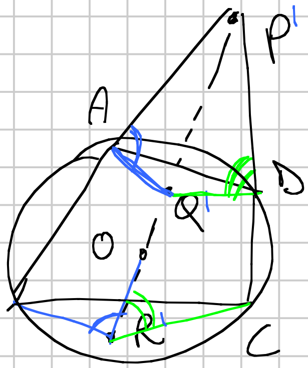
$$O\hat{B}R = O\hat{R'}B$$

$$OR \cdot OR' = OB^2$$

$$\frac{OR}{OB} = \frac{OB}{OR'}$$

$$Q\hat{A}P = P'\hat{A}Q'$$
$$P\hat{D}Q = Q'\hat{D}P'$$
$$O\hat{B}R = O\hat{R'}B$$
$$R\hat{C}O = C\hat{R'}O$$

$$P'\hat{A}Q' = O\hat{R'}B \iff Q'\hat{D}P' = C\hat{R'}O.$$



$$AQ'R'B \text{ ciclico} \overset{?}{\iff} Q'DCR' \text{ ciclico.}$$

$$PQ' \cdot PR' = PA \cdot PB \overset{!}{\iff} PQ' \cdot PR' = PC \cdot PD$$

$$PA \cdot PB = PC \cdot PD$$

2) Stesse cose senza inversione, passando per il centro radicale per spostare le potenze

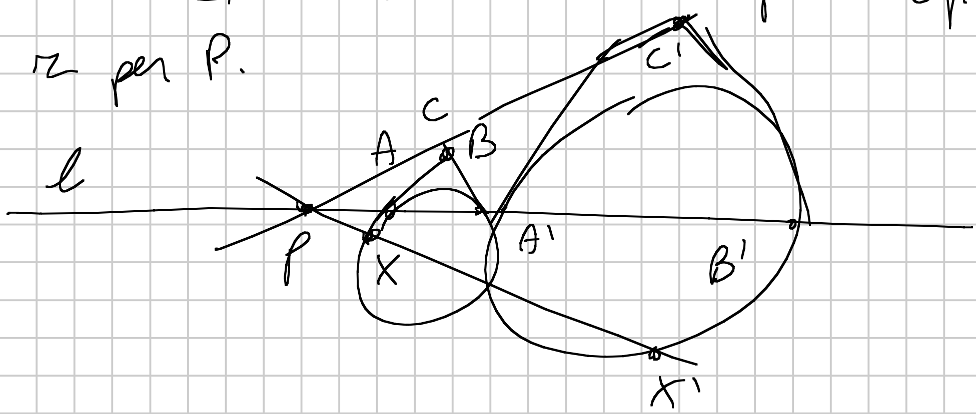
Es 2: ℓ retta, γ, γ' circonferenze

ℓ incontra γ in A, B, ℓ incontra γ' in A', B'
le tg a γ in A, B si incontrano in C, le tg a γ' in A', B' si incontrano in C'.

$\ell \cap CC' = P$. r retta variabile per P.

X un pt di intersez. tra r e γ, X' tra r e γ'.

Dim che $CX \cap C'X'$ sta su una fissata cf al variare di r per P.



1) Definire la cf. con $\ell = r$ ($AC \cap A'C'$ etc...)
cercare di lavorare con le potenze (passando per C', C)
per dim che anche $CX \cap C'X'$ sta sulla cf.

2) Determinare le pot di $CX \cap C'X'$ risp. alle due cf.
e sperare che tali potenze siano proporzionali

Oss: Γ_1, Γ_2 $\{ pow_{\Gamma_1}(P) = pow_{\Gamma_2}(P) \}$ retta

$\{ pow_{\Gamma_1}(P) = k \cdot pow_{\Gamma_2}(P) \}$ circonferenza

Parentesi algebrica: Γ_1 Γ_2 intersecano

$\Gamma_1 = \{ (x-x_0)^2 + (y-y_0)^2 = r^2 \}$ $\{ \lambda [(x-x_0)^2 + (y-y_0)^2 - r^2] + \mu [(x-a)^2 + (y-b)^2 -$
$\Gamma_2 = \{ (x-a)^2 + (y-b)^2 = R^2 \}$ $- R^2] = 0 \}$

2*) Osservo che se $\gamma \cap \gamma' \neq \{0\} \Rightarrow$ la ch che cerco è sicuramente coassiale a γ, γ'.

3) Geometria analitica

$\gamma \quad x^2+y^2-1=0 \qquad\qquad y=k$

$\gamma' \quad (x-x_0)^2+(y-y_0)^2-r^2=0$

$$C=\left(0,\frac{1}{k}\right), \quad C'=\left(x_0,\ y_0+\frac{r^2}{k-y_0}\right)$$

$$CC': \frac{x}{y-\frac{1}{k}} = \frac{x_0}{y_0+\frac{r^2}{k-y_0}-\frac{1}{k}} \qquad P=\left(\frac{x_0\left(k-\frac{1}{k}\right)}{y_0+\frac{r^2}{k-y_0}-\frac{1}{k}},\ k\right)$$

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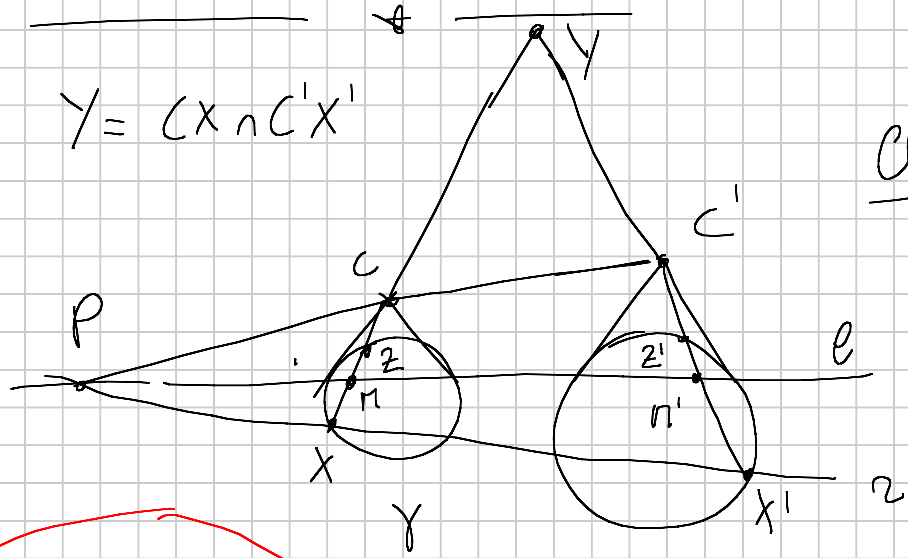
2) $Y = CX \cap C'X'$



Claim: P, Z, Z' allineati

$(C, M; Z, X) = -1$

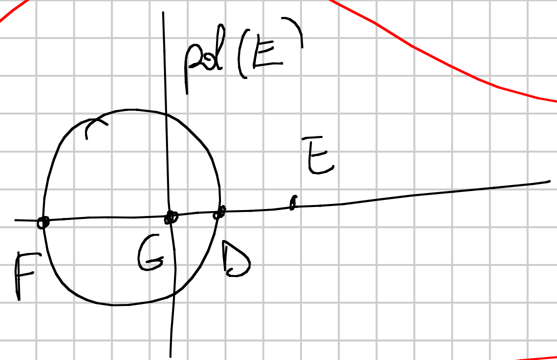
$(C', M'; Z', X') = -1$

$CC' \cap MM' \cap XX' = P$

$\Downarrow$

P, Z, Z' allineati

OK



$(F, D; G, E) = -1$

Vorrei calcolare $\dfrac{YZ \cdot YX}{YZ' \cdot YX'} = \dfrac{YZ}{YZ'} \cdot \dfrac{YX}{YX'}$

Nel tri $\overset{\Delta}{YCC'}$ considero le trasversali XX' e ZZ':

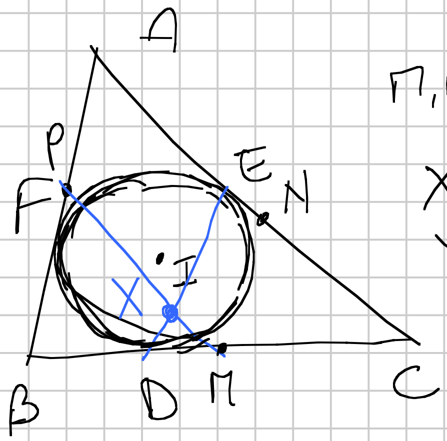
$$\frac{YX}{XC} \cdot \frac{CP}{PC'} \cdot \frac{C'X'}{X'Y} = -1$$

$$\frac{YZ}{ZC} \cdot \frac{ZP}{PC'} \cdot \frac{C'Z'}{Z'Y} = -1$$

$$\frac{YX}{X'Y'} \cdot \frac{YZ}{Z'Y'} = \frac{C'P}{PC} \cdot \frac{XC}{X'C'} \cdot \frac{C'P}{PC} \cdot \frac{CZ}{C'Z'} =$$

$$= \left(\frac{C'P}{PC}\right)^2 \cdot \frac{CA^2}{C'A'^2} = \text{costante.} \quad \square$$

Es 3:



M, N, P pt medi.

$X = MP \cap DE$
$Y = MN \cap DF$ $\Rightarrow$ X, Y, A, I sono allineati.

$(BX, CY \perp AI)$.

1) Conti in baricentriche

$M = [0:1:1]$ $N = [1:0:1]$ $P = [1:1:0]$

$D = [0 : p-c : p-b]$ $E = [p-c : 0 : p-a]$

$MN: \; z - x - y$ $MP: \; y - x - z = 0$

$DE: \; z(p-c) - y(p-b) - x(p-a) = 0$

$AI: \; A = [1:0:0]$ $I = [a:b:c]$ $AI: \; cy - bz = 0$

$$\det\begin{pmatrix} -1 & 1 & -1 \\ -(p-a) & -(p-b) & (p-c) \\ 0 & c & -b \end{pmatrix} =$$

$$= \det\begin{pmatrix} 1 & -1 & 1 \\ p-a & p-b & c-p \\ 0 & c & -b \end{pmatrix} = -b\,(p-b+p-a) - c\,(c-p+a-p) =$$

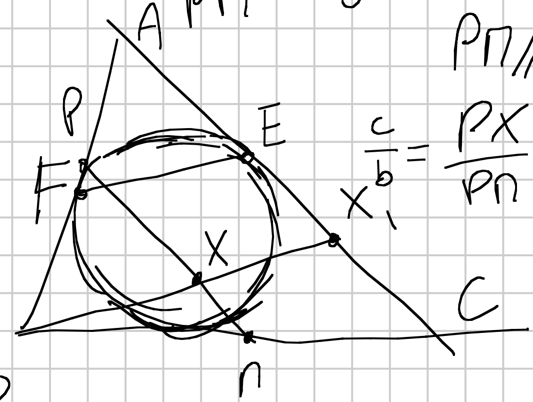
$$= +b^2 - 2bp + ab - c^2 - ac + 2cp =$$

$$= 2p(c-b) + (c+b)(b-c) + a(b-c) =$$

$$= (2p-a-b-c)(c-b) = 0.$$

2) Con gli conti (e $\widehat{DXM} \cong \widehat{DEC}$)

ricaviamo $\dfrac{PX}{PM} = \dfrac{c}{b}$



$PM \parallel AC$

$\dfrac{c}{b} = \dfrac{PX}{PM} = \dfrac{AX_1}{AC=b} \Rightarrow AX_1 = c$

$\Rightarrow B, X, X' \perp AI$

$\dfrac{BX}{XX_1} = \dfrac{BM}{MC} = 1 \Rightarrow X \in AI$

3) $IA \cap MP = X' \Rightarrow X'E \perp IC$ ($\Rightarrow X, D, E$ allineati)

4) $L = BI \cap EF \Rightarrow LB \perp LC$.

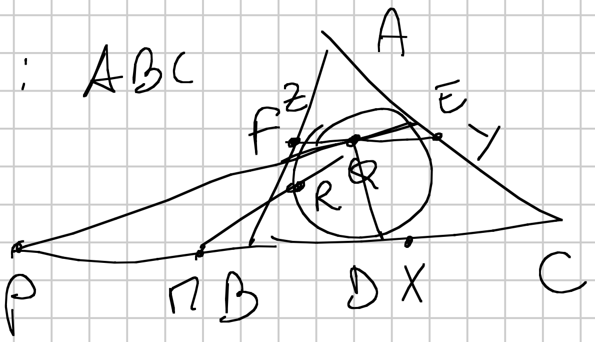
$L' = BI \cap ED \Rightarrow L'B \perp L'A$

<u>Oss</u>: $A' = AI \cap BC$ (A, X, A', Y) armonica

<u>Pb</u>: X, Y (circ. di diam XY) $\cap BC = \{K, L\}$

$\Rightarrow X, Y$ sono incentro ed excentro di AKL.

<u>Es 4:</u> ABC



$P = BC \cap EF$

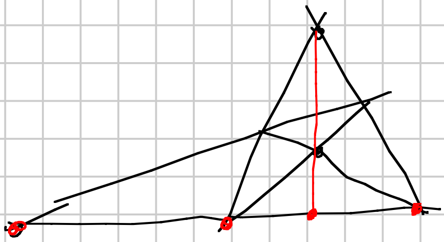
$Q = EF \cap$ lato tri mediale

QM mediana in $\hat{DQP}$

$R = QM \cap$ cerchio inscritto

$\Rightarrow B\hat{R}C = \frac{\pi}{2}$.

- Il prob. é equivalente a dim che QM é asse radTra cfr. inscritta e cfr di diam BC.



$(B, C; P, D) = -1$

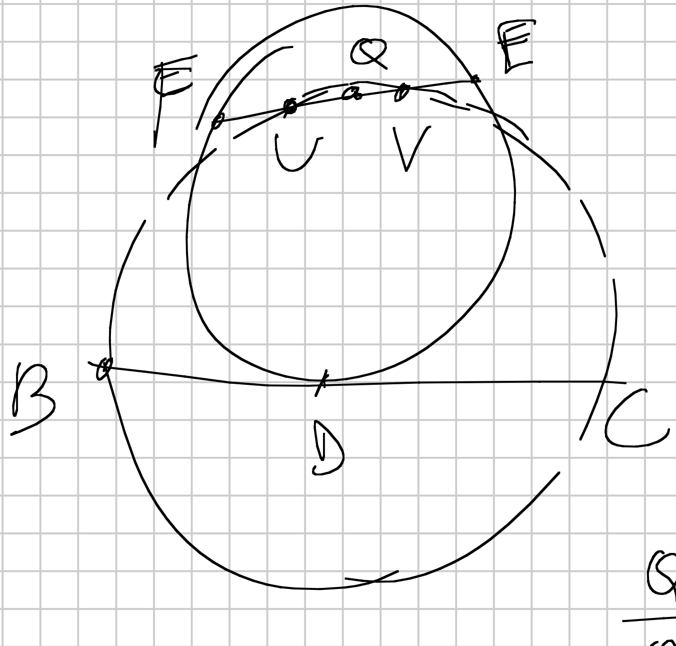
$\Downarrow$

$MD^2 = MB \cdot MC$

$QF \cdot QE$

Sappiamo che EF, BI, XY concorrono in U
EF, CI, XZ " in V

$B\hat{U}C = B\hat{V}C = \frac{\pi}{2}$. $\Rightarrow QU \cdot QV =$ pot di Q risp. alla cfr. di diam BC



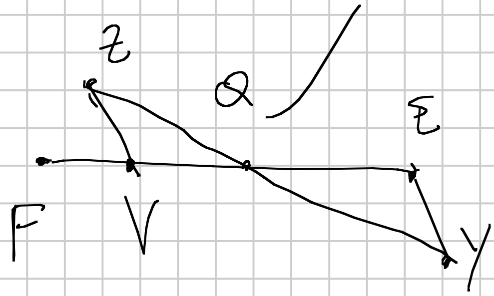
$U \in XY$

$ZF \subseteq AB$

$ZF \parallel UY$

$ZV \parallel YE$

$$\frac{QF}{QU} = \frac{QZ}{QY} = \frac{QV}{QE}$$



- Q ∩ n cp imm. = {R, S} ⟹ B, R, S, C ciclico.

  V, U come sopra ⟹ R, S, V, U ciclico

  B, U, V, C ciclico.