SENIOR 2012 Medium - G2

Titolo nota 05/09/2012

Metodi Algebrici

- coordinate
- vettori
- num complessi

1) num. complessi

$\mathbb{C} \quad a+ib \longleftrightarrow (a,b)$

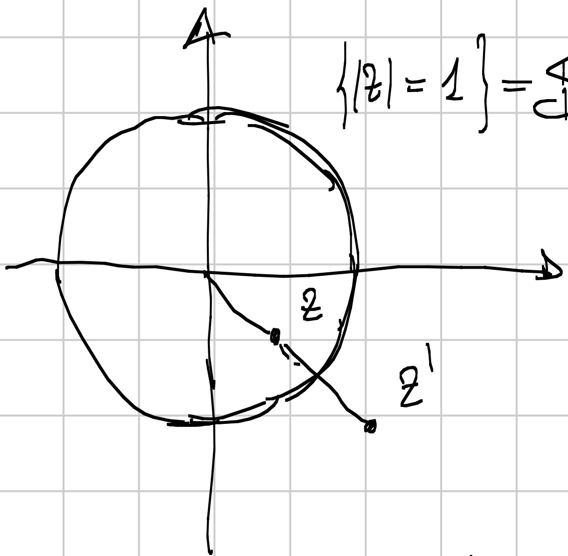
$z+w$ traslazione

αz omotetia $\alpha \in \mathbb{R} \setminus \{0\}$

λz rotazione $|\lambda| = 1$, $\lambda \in \mathbb{C}$

• Inversione rispetto alla cfr $\{|z|=1\}$

$\{|z|=1\} = S^1$



$$\begin{cases} \arg z \equiv \arg z' \quad (2\pi), \\ |z|\cdot|z'| = 1 \end{cases}$$

$$|z'| = \frac{1}{|z|} \Rightarrow z' = \frac{\lambda}{z} \quad |\lambda| = 1$$

$$z' = \frac{e^{i\theta}}{z}$$

$$\arg(z) = \arg(z') = \arg\left(\frac{e^{i\theta}}{z}\right) =$$

$$= \arg(e^{i\theta}) - \arg(z) = \theta - \arg(z)$$

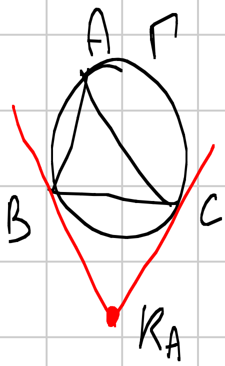
$\varphi = \arg(z)$ $\qquad \varphi \equiv \theta - \varphi \quad (2\pi)$

$$2\varphi \equiv \theta \quad (2\pi)$$

$$z' = \frac{e^{i\theta}}{z} = \frac{e^{2i\arg(z)}}{z} = \qquad e^{i\arg(z)} = \frac{z}{|z|}$$

$$= \frac{z/\bar z}{z} = \frac{1}{\bar z} \qquad e^{2i\arg z} = \frac{z^2}{|z|^2} = \frac{z^{\cancel{2}}}{\bar z \cancel{z}} = \frac{z}{\bar z}$$

Es: $O =$ circocentro $\Rightarrow h = a+b+c$



a, b, c

AK_A simmediana

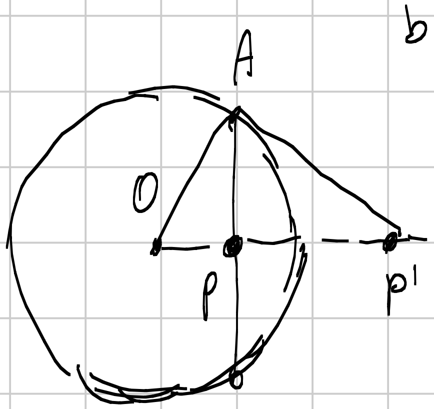
$K_A = ?$ $\qquad K_A =$ inverso risp. a Γ del pt medio di BC

pt. med. di $BC = \dfrac{b+c}{2} = m$

$$k_A = \frac{1}{\bar m} = \frac{2}{\bar b + \bar c} = $$

$$= \frac{2}{\frac{1}{b} + \frac{1}{c}} = \frac{2bc}{b+c}$$

$$\boxed{z \in S^1 \iff \frac{1}{z} = \bar z}$$

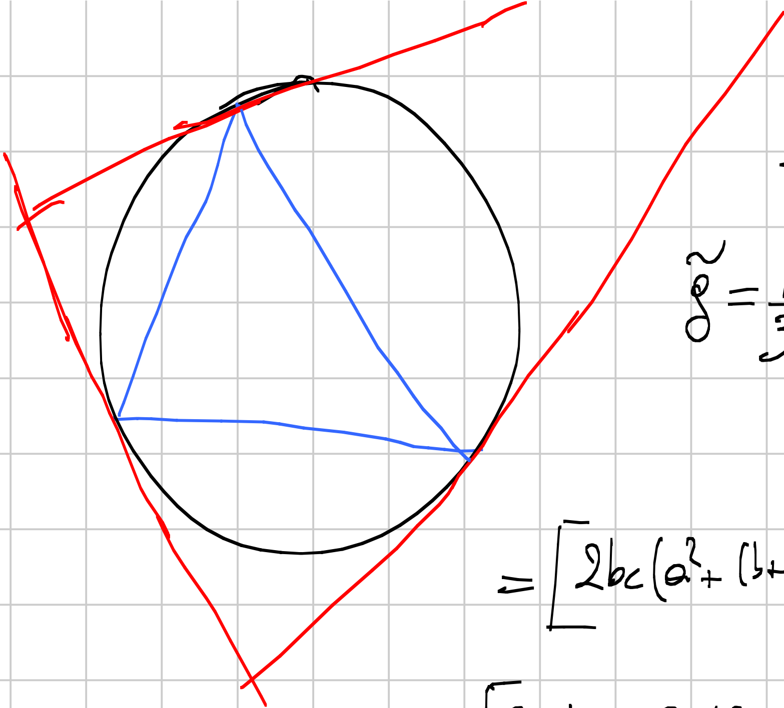


$OP \cdot OP' = R^2 \qquad OA = R$

$OP \cdot OP' = OA^2$ I Teo Euclide

$\Updownarrow$

$O\hat{A}P' = \frac{\pi}{2} \iff AP'$ tg alla cfr



$$\frac{2bc}{b+c} \qquad \frac{2ac}{a+c} \qquad \frac{2ab}{a+b}$$

$$\tilde{g} = \frac{1}{3}\Big[2bc(a+c)(a+b) + 2ac(a+b)(b+c) + 2ab(b+c)(a+c)\Big]\frac{1}{(\,)(\,)(\,)} =$$

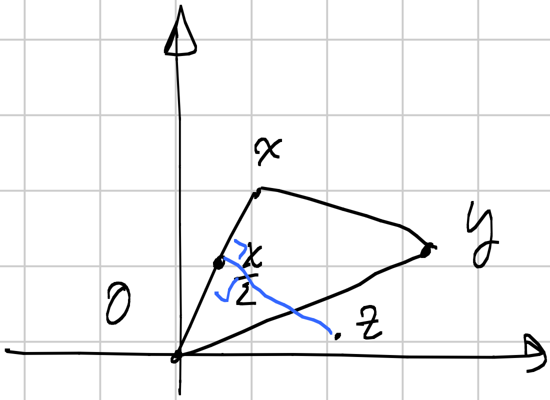
$$= \Big[2bc(a^2+(b+c)a+bc) + \dots + \dots\Big]\frac{1}{3(\,)(\,)(\,)} =$$

$$= \Big[2a^2bc + 2ab^2c + 2abc^2 + 2b^2c^2 + \dots + \dots\Big]\frac{1}{3(\,)} =$$

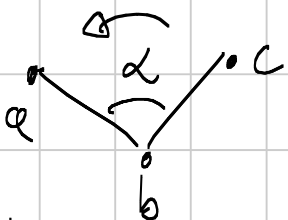
$$= \frac{\Big[6a^2bc + 6ab^2c + 6abc^2 + 2(b^2c^2 + c^2a^2 + a^2b^2)\Big]}{3(a^2+(b+c)a+bc)(b+c)}$$

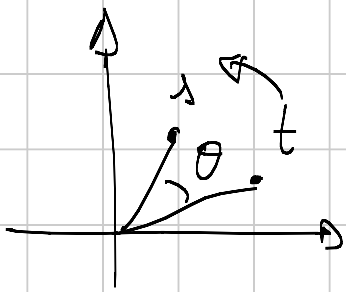
$$3(a^2b + a^2c + b^2a + c^2a + 2abc + b^2c + c^2b)$$

Es: Circocentro di $0, x, y$ $= z$



$$\begin{cases} \dfrac{z - \frac{x}{2}}{+\frac{x}{2}} = -\dfrac{\bar{z} - \frac{\bar{x}}{2}}{+\frac{\bar{x}}{2}} \\[2ex] \dfrac{z - \frac{y}{2}}{\frac{y}{2}} = -\dfrac{\bar{z} - \frac{\bar{y}}{2}}{\frac{\bar{y}}{2}} \end{cases}$$





$b \to 0$

$a \to a-b$

$c \to c-b$

$\theta = \arg\left(\frac{s}{t}\right)$

$\alpha = \arg\left(\frac{a-b}{c-b}\right)$

$\alpha = \pm\frac{\pi}{2} \iff \frac{a-b}{c-b} \in i\mathbb{R}$

$z \in i\mathbb{R} \iff \boxed{z = -\bar{z}}$

$$\begin{cases} \dfrac{2z}{x} - 1 = 1 - \dfrac{2\bar{z}}{\bar{x}} \\[2ex] \dfrac{2z}{y} - 1 = 1 - \dfrac{2\bar{z}}{\bar{y}} \end{cases} \qquad \begin{cases} 2z\,\dfrac{\bar{x}}{x} - \bar{x} = \bar{x} - 2\bar{z} \\[2ex] 2z\,\dfrac{\bar{y}}{y} - \bar{y} = \bar{y} - 2\bar{z} \end{cases}$$

$$\cancel{2}z\left(\frac{\bar{x}}{x} - \frac{\bar{y}}{y}\right) = \cancel{2}\bar{x} - \cancel{2}\bar{y}$$

$$z = \frac{\bar{x} - \bar{y}}{\dfrac{\bar{x}}{x} - \dfrac{\bar{y}}{y}} = \frac{xy(\bar{x} - \bar{y})}{y\bar{x} - x\bar{y}}$$

Es: ABC tri acutangolo $AB > BC$, $AC > BC$

O, H. Cfr circo a AHC interseca AB in A, M

Cfr circo a AHB interseca AC in A, N

$\Rightarrow$ Circocentro di MNH sta su OH.

Es: $x = \rho e^{i\theta}$ $\quad \bar{x} = \rho e^{-i\theta}$ $\quad \dfrac{\bar{x}}{x} = \dfrac{\cancel{\rho} e^{-i\theta}}{\rho e^{i\theta}} = e^{-2i\theta}$

$\dfrac{\bar{x}}{x} = 1 \iff 2\theta \equiv 0 \ (2\pi)$

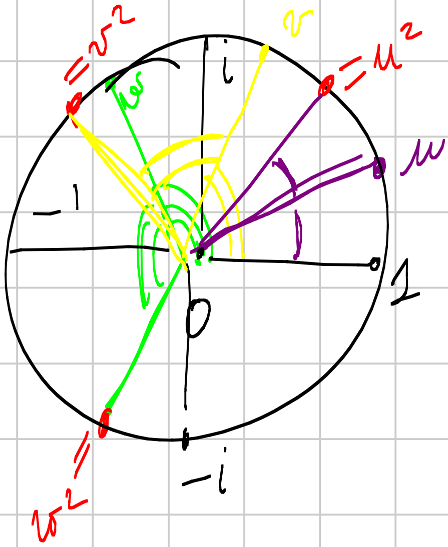
$\theta = 0 \quad \mathbb{R}_{>0}$

$\theta = \pi \quad \mathbb{R}_{<0}$

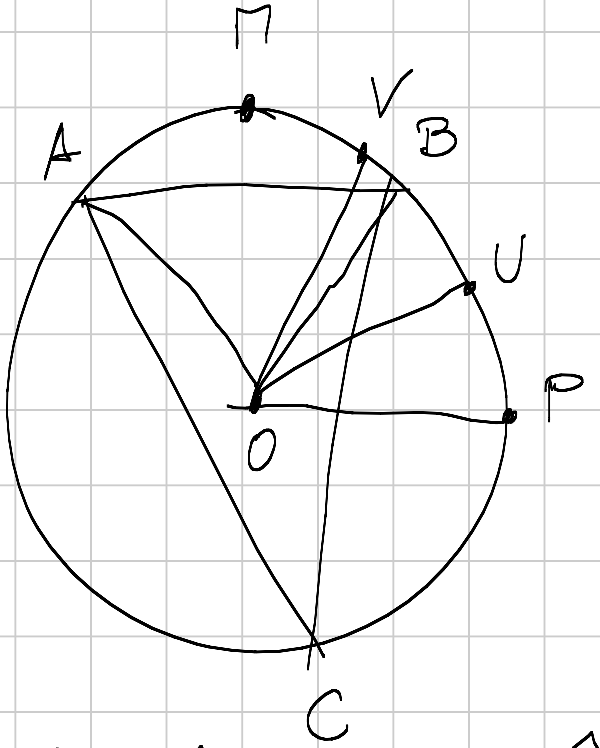
Oss sull'incentro: $\dfrac{|b-c|a + |c-a|b + |a-b|c}{|a-b| + |b-c| + |c-a|} = j$ incentro

A, B, C cfr. circo $= \Gamma = \{|z| = 1\}$

$\exists\, u, v, w \in \{|z| = 1\}$ tali che $A = u^2$ $B = v^2$ $C = w^2$



uv = pt. med dell'arco $\widehat{AB}$



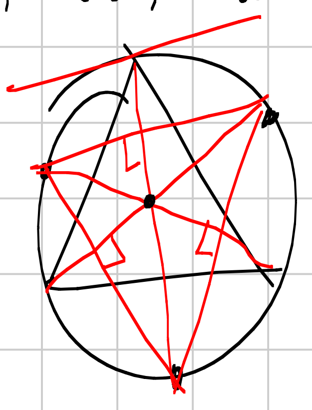
$$\widehat{VOP} = \frac{1}{2}\widehat{AOP}$$

$$\widehat{UOP} = \frac{1}{2}\widehat{BOP}$$

$$\frac{1}{2}\widehat{AOB} = \frac{1}{2}\widehat{AOP} - \frac{1}{2}\widehat{BOP}$$

$$\widehat{MOP} = \widehat{BOP} + \frac{1}{2}\widehat{AOB} = \frac{1}{2}\widehat{AOP} + \frac{1}{2}\widehat{BOP} = \widehat{VOP} + \widehat{UOP}$$

Di solito si scelgono u, v, w tali che
u^2, v^2, w^2 siano i vertici
$-uv, -vw, -wu$ siano i pt. medi degli archi.



$uv - wu \perp u^2 + vw$

$I = -uv - vw - uw$
$\uparrow$
ortocentro di $-uv, -vw, -uw$.

$h = u^2 + v^2 + w^2$ $\quad f = \dfrac{u^2 + v^2 + w^2}{2}$

$$IF = \frac{|u+v+w|^2}{2}$$

$$OI = |-uv - vw - uw| = |uvw| \left| \frac{1}{w} + \frac{1}{u} + \frac{1}{v} \right| =$$

$$= |uvw| \, |\bar{w} + \bar{u} + \bar{v}| = |uvw| \, |u+v+w| =$$

$$= |u+v+w|$$

$$IF = \frac{OI^2}{2} = \frac{R^2 - 2Rr}{2} = \frac{1-2r}{2} = \frac{1}{2} - r$$

2) Vettori: A, B, C

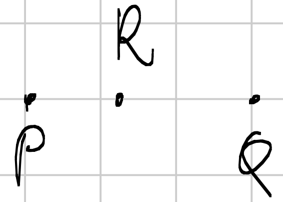
$$\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C}}{3} \quad \text{qualunque sia l'origine}$$

- se l'origine è O (il circocentro)

$$\Rightarrow \vec{H} = \vec{A} + \vec{B} + \vec{C} \qquad \vec{F} = \frac{\vec{A} + \vec{B} + \vec{C}}{2}$$

- con qualunque origine $\vec{I} = \dfrac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$

Step 0:



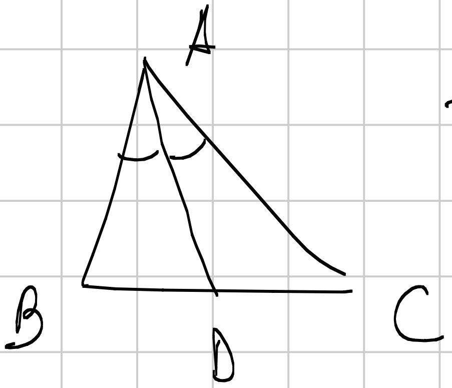
$$\frac{PR}{RQ} = \lambda \quad \text{rapporto con segno}$$

$$\vec{P} + k(\vec{Q} - \vec{P}) = \vec{S_k} \qquad \frac{\|\vec{P} - \vec{S_k}\|}{\|\vec{S_k} - \vec{Q}\|} = |\lambda|$$

$$\frac{\|k(\vec{Q} - \vec{P})\|}{\|(1-k)\vec{P} - (1-k)\vec{Q}\|} = \frac{|k|}{|1-k|} \cdot \frac{\cancel{\|\vec{Q} - \vec{P}\|}}{\cancel{\|\vec{P} - \vec{Q}\|}} = \left| \frac{k}{1-k} \right|$$

$$\lambda = \frac{k}{1-k} \qquad k(1+\lambda)=\lambda \qquad k=\frac{\lambda}{1+\lambda}$$

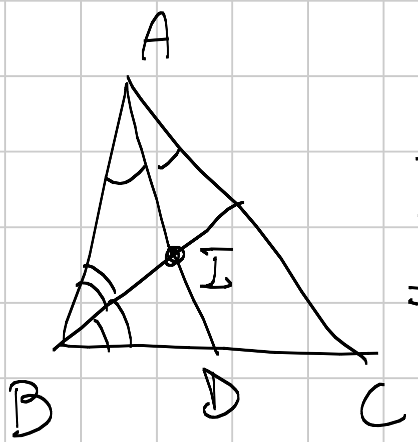
Step 1:



$$\frac{BD}{DC} = \frac{c}{b} = \lambda \quad \text{(Teo BISETT)}$$

$$\vec D = \vec B + \frac{\frac{c}{b}}{1+\frac{c}{b}}(\vec C - \vec B) = \vec B + \frac{c}{b+c}(\vec C - \vec B) = \frac{b}{b+c}\vec B + \frac{c}{b+c}\vec C$$

Step 2:



$$\frac{AI}{ID} = \frac{AB}{BD} = \frac{\cancel{c}}{\frac{\cancel{c}a}{b+c}} = \frac{b+c}{a}$$

$$\frac{DC}{BD} = \frac{b}{c}$$

$$\frac{b+c}{c} = \frac{DC}{BD}+1 = \frac{DC+BD}{BD} = \frac{BC}{BD} = \frac{a}{BD}$$

$$BD = \frac{ca}{b+c}$$

$$\vec I = \vec A + \frac{\frac{b+c}{a}}{\frac{b+c}{a}+1}(\vec D - \vec A) = \vec A + \frac{b+c}{b+c+a}(\vec D - \vec A) =$$

$$= \frac{a}{a+b+c}\vec A + \frac{b+c}{a+b+c}\vec D = \frac{a\vec A + b\vec B + c\vec C}{a+b+c}$$

$$\vec K = \frac{a^2\vec A + b^2\vec B + c^2\vec C}{a^2+b^2+c^2}$$

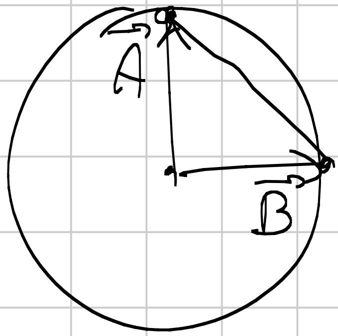
<u>x esercizio</u>

• $$IF^2 = \left\|\vec I - \vec F\right\|^2 = \left\|\frac{a\vec A + b\vec B + c\vec C}{a+b+c} - \frac{\vec A + \vec B + \vec C}{2}\right\|^2 =$$

Origine in O

$$= \left\|\frac{\vec A(b+c-a) + \vec B(a+c-b) + \vec C(a+b-c)}{2(a+b+c)}\right\|^2 =$$

$$= \frac{1}{4p^2}\left[\sum_{cyc}(b+c-a)^2\,\underbrace{\cancel{\vec{A}\cdot\vec{A}}}_{R^2} + \sum_{cyc} 2\vec{A}\cdot\vec{B}\,(b+c-a)(a+c-b)\right] =$$



$$2\vec{A}\cdot\vec{B} \qquad \underbrace{\|\vec{A}-\vec{B}\|^2}_{=\,c^2} = \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} - 2\vec{A}\cdot\vec{B} = 2R^2 - 2\vec{A}\cdot\vec{B}$$

$$2\vec{A}\cdot\vec{B} = 2R^2 - c^2$$

$$= \frac{1}{4p^2}\left[R^2\sum_{cyc}\left[b^2+c^2+a^2+2bc-2ab-2ca\right] + \sum_{cyc}(2R^2-c^2)(c^2-b^2-a^2+2ab)\right] =$$

$$= \frac{1}{4p^2}\Big[R^2\big[3a^2+3b^2+3c^2-2ab-2bc-2ca\;-2a^2-2b^2-2c^2+4ab+4bc+4ca\big]$$

$$-\big[c^4-c^2b^2-c^2a^2+2abc^2+b^4-b^2a^2-b^2c^2+2ab^2c+a^4-a^2b^2-a^2c^2+2a^2bc\big]\Big] =$$

$$= \frac{1}{4p^2}\Big[R^2\big[a^2+b^2+c^2+2ab+2bc+2ca\big] - \underbrace{\big[a^4+b^4+c^4-2a^2b^2-2b^2c^2-2c^2a^2\big]}_{-16S^2} - 2abc(a+b+c)\Big] =$$

$$= \frac{1}{4p^2}\left[R^2p^2 + 16S^2 - 2abc\,p\right] = \frac{R^2}{4} + r^2 - \frac{abc}{2p} =$$

$$2\,\frac{abc}{2S}\cdot\frac{2S}{p} = \frac{2Rr}{2} \qquad = \frac{R^2}{4} + r^2 - \frac{2Rr}{2} =$$

$$= \left(\frac{R}{2} - r\right)^2$$

$\cdot)$ $OH^2 = \| \vec{A}+\vec{B}+\vec{C} \|^2 = 3R^2 + \sum_{cyc} 2\vec{A}\cdot\vec{B} = 3R^2 + \sum_{cyc} (2R^2 - c^2) =$

$= 9R^2 - (a^2+b^2+c^2)$

$\boxed{\overset{\Delta}{GH} \text{ è sempre ottusangolo}}$ $\times$ casa

Es: ABCD è un parallelogramma sse $AB^2+BC^2+CD^2+DA^2 = AC^2+BD^2$

$\Leftarrow)$

$(A-B)\cdot(A-B) + (B-C)\cdot(B-C) + (C-D)\cdot(C-D) + (D-A)\cdot(D-A) -$

$- (A-C)\cdot(A-C) - (B-D)\cdot(B-D) = 0$

$\Updownarrow$

$\cancel{|A|^2} + \cancel{|B|^2} - 2A\cdot B + |B|^2 + \cancel{|C|^2} - 2B\cdot C + |C|^2 + \cancel{|D|^2} - 2C\cdot D +$

$+ |D|^2 + |A|^2 - 2A\cdot D - \cancel{|A|^2} - \cancel{|C|^2} + 2A\cdot C - \cancel{|B|^2} - \cancel{|D|^2} + 2B\cdot D = 0$

$\Updownarrow$

$|A|^2 + |B|^2 - 2A\cdot B + |C|^2 + |D|^2 - 2C\cdot D + 2C\cdot(A-B) - 2D(A-B) = 0$

$\Updownarrow$

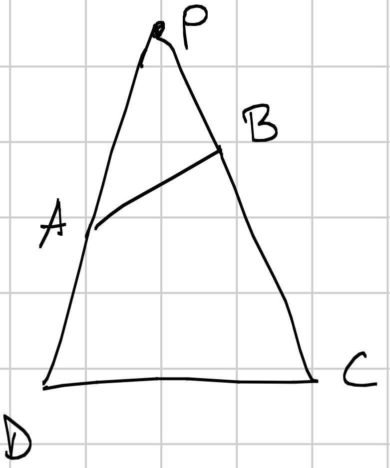
$(A-B)\cdot(A-B) + (C-D)\cdot(C-D) + 2(A-B)\cdot(C-D) = 0$

$\Updownarrow$

$(A-B+C-D, A-B+C-D) = 0 \iff A-B+C-D = 0$

$\iff A-B = D-C \iff AB \parallel CD$ e $\overline{AB} = \overline{CD}$.

Es:



H_1, H_2 ortocentri di $\hat{APB}$ e $\hat{CPD}$,
E_1, E_2 i centri delle cf. di Feuerbach di $\hat{APB}$ e $\hat{CPD}$.

Allora: (i) la perp da E_1 a CD
(ii) • la perp da E_2 a AB
• H_1H_2
concorrono.

(IMO-SL 2009/6)

Sol: Origine in P.

$$H_1-\cancel{O_1} = A+B+\cancel{P}-O_1-O_1-\cancel{O_1}$$

$$O_1 = \frac{A+B-H_1}{2}$$

$$O_2 = \frac{C+D-H_2}{2}$$

$$E_1 = \frac{O_1+H_1}{2} = \frac{A+B+H_1}{4} \qquad E_2 = \frac{O_2+H_2}{2} = \frac{C+D+H_2}{4}$$

X = intersezione di (i), (ii)

$$\begin{cases} 4(X-E_1)\cdot(C-D)=0 \\ 4(X-E_2)\cdot(A-B)=0 \end{cases}$$

$$(A+B)\cdot(C-D)+(C+D)(A-B)$$

$$A\cdot C-\cancel{A\cdot D}+\cancel{BC}-B\cdot D+$$
$$+CA-\cancel{BC}+\cancel{DA}-DB$$

$$\begin{cases} (4X-A-B-H_1)\cdot(C-D)=0 \\ (4X-D-C-H_2)\cdot(A-B)=0 \end{cases}$$

$$4X(A-B+C-D) = H_1(C-D)+H_2(A-B)+2A\cdot C-2B\cdot D$$

$(H_1-A)\cdot B=0$ P, B, C allineati $\Rightarrow (H_1-A)\cdot C=0$

$$H_1 \cdot C = A \cdot C \qquad H_1 \cdot D = B \cdot D$$
$$H_2 \cdot A = A \cdot C \qquad H_2 \cdot B = B \cdot D$$

$$H_1 (C - D) + H_2 (A - B) = 2 A \cdot C - 2 B \cdot D$$

$$X \cdot (A - B + C - D) = A C - B \cdot D$$

$$H_1 \cdot (A - B + C - D) = \cancel{H_1 \cdot (A - B)} + H_1 (C - D) = A C - B D$$

$$H_2 (A - B + C - D) = H_2 (A - B) = A C - B D$$

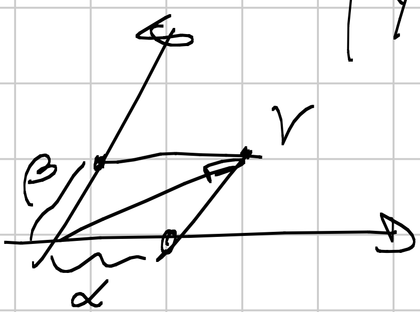
$$\begin{cases} (X - H_1) \cdot (A - B + C - D) = 0 \\ (X - H_2) \cdot (A - B + C - D) = 0. \end{cases} \qquad \begin{cases} v \cdot w = 0 \\ v \cdot u = 0 \end{cases} \Longleftarrow \begin{matrix} v = 0 \\ \text{oppure} \\ u \parallel w \end{matrix}$$

$$\begin{matrix} X H_1 \perp v \\ X H_2 \perp v \end{matrix} \Longrightarrow X, H_1, H_2 \text{ allineati poiché } A - B + C - D \neq 0$$

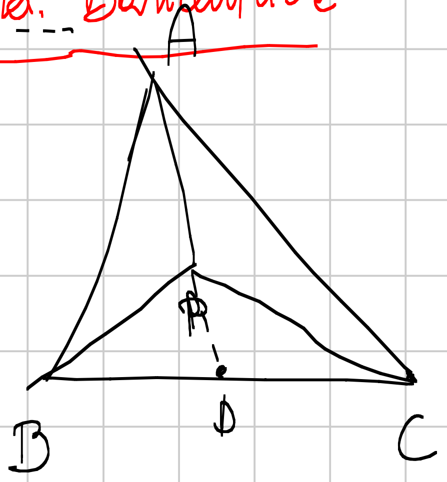
$$A - B = D - C$$
ma $AB \not\parallel CD$.

<u>Fatto</u>: Dati X, Y vettori <u>non</u> allineati

$$\begin{cases} X \cdot V = \alpha \\ Y \cdot V = \beta \end{cases}$$ ha <u>sempre</u> una e una sola soluzione



3) Coord. Baricentriche



$$\frac{[BPA]}{[CPA]} = \frac{[BDA]}{[CDA]} = \frac{BD}{DC}$$

$$\frac{x}{y} = \frac{z}{w} = \frac{z+x}{w+y}$$

Scrivere D

$$\frac{AP}{PD} = \frac{[APB]}{[PBD]} = \frac{[APB]+[APC]}{[PBC]}$$

Scrivere P

$$\frac{[APB]+[APC]}{[PBC]} = \frac{[APB]}{[PBD]} = \frac{[APC]}{[PDC]}$$

$$\boxed{\vec{P} = \frac{[PBC]\vec{A} + [PCA]\vec{B} + [PAB]\vec{C}}{[ABC]}}$$

$$\vec{P} = \alpha \vec{A} + \beta \vec{B} + \gamma \vec{C} \qquad \alpha+\beta+\gamma = 1$$

Fissato il Triangolo FIGO (Tri di riferimento)

posso scrivere solo α, β, γ

$$\vec{P} \longrightarrow [\alpha : \beta : \gamma]$$

Terne omogenee

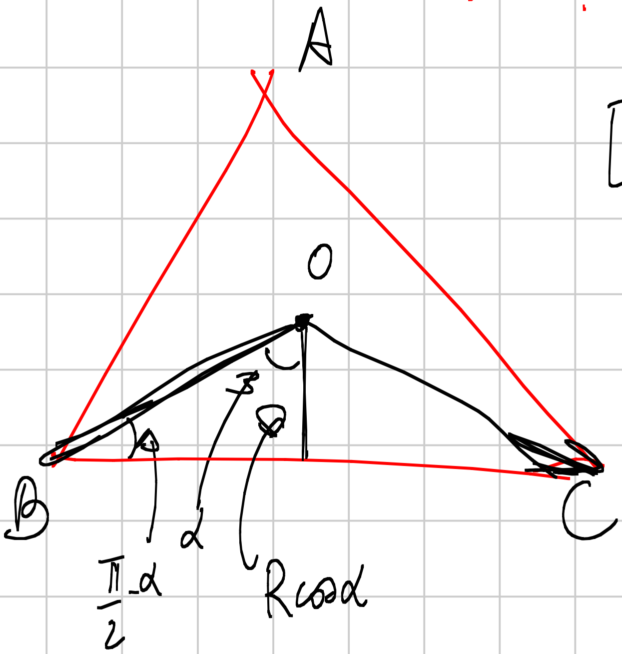
basta che $\alpha+\beta+\gamma \neq 0$

$[1:1:1] \longrightarrow \frac{A+B+C}{3}$ = baricentro.

$I = [a:b:c]$

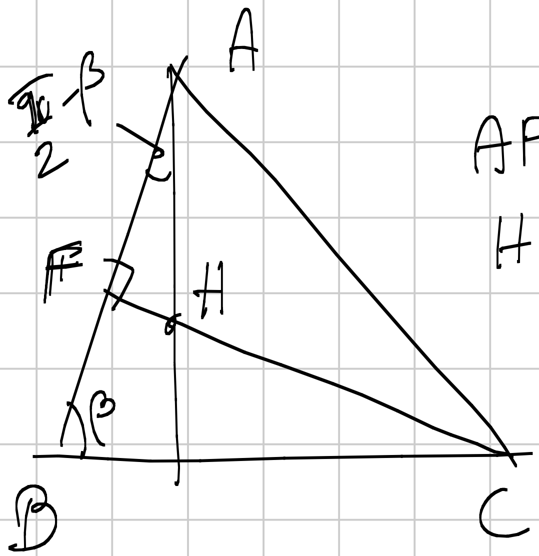
$O = ?$

$H = ?$



$[OBC] = R\cos\alpha \cdot a = aR\cos\alpha$

$O = [a\cos\alpha : b\cos\beta : c\cos\gamma]$



$AF = b\cos\alpha$

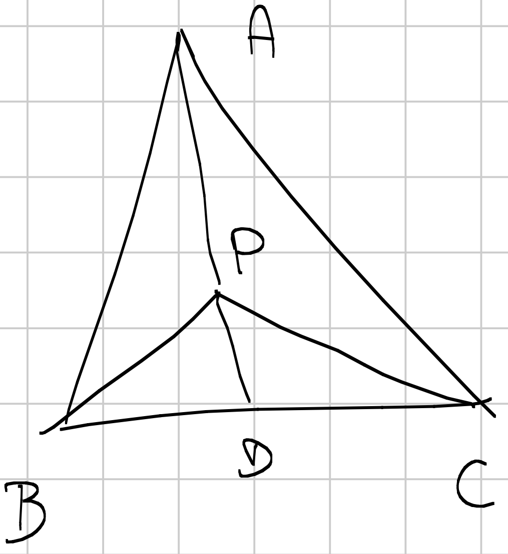
$HF = b\cos\alpha \cdot \operatorname{tg}\frac{\pi}{2}-\beta = b\cos\alpha \frac{1}{\operatorname{tg}\beta} =$

$= b\cos\alpha \frac{\cos\beta}{\sin\beta} = 2R\cos\alpha\cos\beta$

$H = [a\cos\beta\cos\gamma : b\cos\alpha\cos\gamma : c\cos\alpha\cos\beta] =$

$\underbrace{= \left[\frac{a}{\cos\alpha} : \frac{b}{\cos\beta} : \frac{c}{\cos\gamma}\right] = [\operatorname{tg}\alpha : \operatorname{tg}\beta : \operatorname{tg}\gamma]}_{\times \text{ Tri non rettangoli.}}$

Oss:



$$\frac{[PAB]}{[PCA]} = \frac{[DAB]}{[DCA]}$$

$P = [\alpha : \beta : \gamma]$

$\Downarrow$

$D = [0 : \beta : \gamma]$

$E = [\alpha : 0 : \gamma]$

$F = [\alpha : \beta : 0]$

Fatto: Ogni retta si scrive come

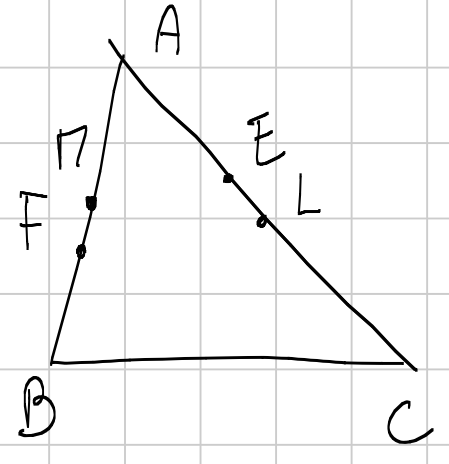
$$\{[x : y : z] \text{ t.c. } lx + my + nz = 0\} \longleftrightarrow [l : m : n]$$

Oss: se uno tra l, m, n è 0, la retta passa per un vertice.

Es: $x + y + z = 0$ $\quad (-x - y - z = 0, \ 2x + 2y + 2z = 0)$

NON ESISTE (si chiama RETTA all'∞)

Es:



BE, CF altezze

BL, CM bisettrici

$I \in EF \iff O \in LM$

$$O = [a\cos\alpha : b\cos\beta : c\cos\gamma]$$

$$H = [\operatorname{tg}\alpha : \operatorname{tg}\beta : \operatorname{tg}\gamma] \qquad E = [\operatorname{tg}\alpha : 0 : \operatorname{tg}\gamma]$$

$$F = [\operatorname{tg}\alpha : \operatorname{tg}\beta : 0]$$

$$I = [a : b : c] = [\sin\alpha : \sin\beta : \sin\gamma]$$

$$L = [a : 0 : c] \qquad M = [a : b : 0]$$

retta per EF

$$\begin{pmatrix} \operatorname{tg}\alpha \\ 0 \\ \operatorname{tg}\gamma \end{pmatrix} \overset{(\times)}{\wedge} \begin{pmatrix} \operatorname{tg}\alpha \\ \operatorname{tg}\beta \\ 0 \end{pmatrix} = \begin{pmatrix} -\operatorname{tg}\gamma\operatorname{tg}\beta \\ \operatorname{tg}\gamma\operatorname{tg}\alpha \\ \operatorname{tg}\alpha\operatorname{tg}\beta \end{pmatrix}$$

Determinante

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = aei + bfg + dhc - ceg - fha - bdi$$

Prod. vettore tra $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ e $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\det\begin{pmatrix} i & j & k \\ a & b & c \\ x & y & z \end{pmatrix} = i(bz - cy) + j(xc - az) + k(ay - bx)$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} bz - cy \\ xc - az \\ ay - bx \end{pmatrix}$$

$$\{-x\operatorname{tg}\gamma\operatorname{tg}\beta + y\operatorname{tg}\alpha\operatorname{tg}\gamma + z\operatorname{tg}\alpha\operatorname{tg}\beta = 0\} \to EF$$

$$\begin{pmatrix} a \\ 0 \\ c \end{pmatrix} \times \begin{pmatrix} a \\ b \\ 0 \end{pmatrix} \qquad \det \begin{pmatrix} i & j & k \\ a & 0 & c \\ a & b & 0 \end{pmatrix} = i(-bc) + j(ac) + k(ab)$$

$$\left\{ -bcx + acy + abz = 0 \right\} \text{ retta per } L\Pi$$

$$\left\{ -\frac{x}{\sin\alpha} + \frac{y}{\sin\beta} + \frac{z}{\sin\gamma} = 0 \right\} \qquad O = [\sin\alpha\cos\alpha : \sin\beta\cos\beta : \sin\gamma\cos\gamma]$$

$$-\cos\alpha + \cos\beta + \cos\gamma = 0 \Leftrightarrow O \in L\Pi.$$

$$\left\{ -\frac{x}{\operatorname{tg}\alpha} + \frac{y}{\operatorname{tg}\beta} + \frac{z}{\operatorname{tg}\gamma} = 0 \right\} \text{ retta per } EF$$

$$I = [\sin\alpha ; \sin\beta ; \sin\gamma]$$

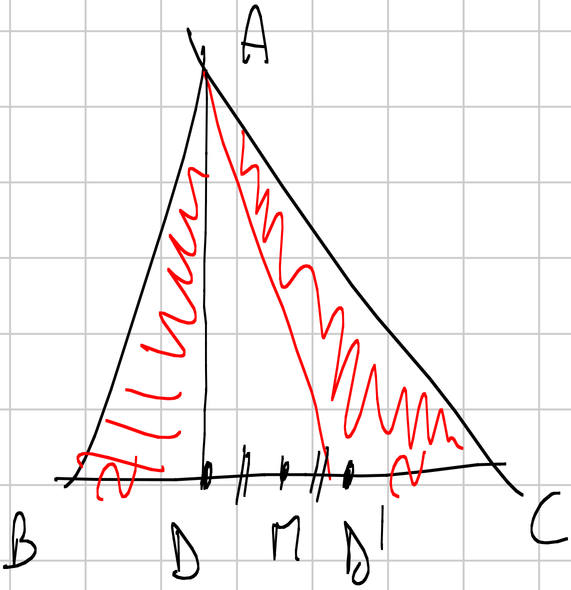
$$-\frac{\sin\alpha}{\operatorname{tg}\alpha} + \frac{\sin\beta}{\operatorname{tg}\beta} + \frac{\sin\gamma}{\operatorname{tg}\gamma} = 0$$

$$-\cos\alpha + \cos\beta + \cos\gamma = 0 \Leftrightarrow I \in EF$$

$$I \in EF \Leftrightarrow \cos\beta + \cos\gamma = \cos\alpha \Leftrightarrow O \in L\Pi. \quad \square$$

<u>Fatto!</u>

$$\begin{cases} lx + my + nz = 0 \\ \lambda x + \mu y + \nu z = 0 \end{cases} \qquad \begin{pmatrix} l \\ m \\ n \end{pmatrix} \times \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} = \begin{pmatrix} m\nu - \mu n \\ n\lambda - l\nu \\ l\mu - m\lambda \end{pmatrix}$$



$D = [0 : q : r]$ $\quad$ $D' = [0 : r : q]$

$D = [0 : q : r]$

$E = [p : 0 : r]$ $\longrightarrow$ piedi delle ceviane

$F = [p : q : 0]$ $\quad$ di $P = [p : q : r]$

$$\Downarrow$$

D', E', F' simm. nei pt. medi dei lati

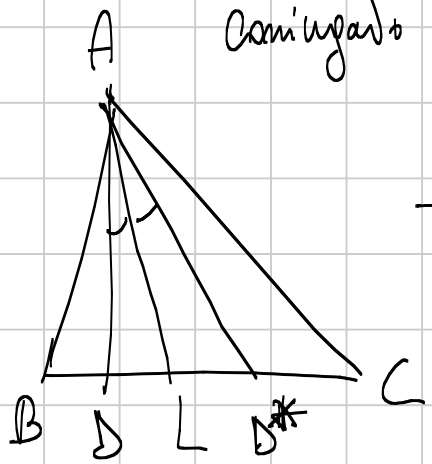
$D' = [0 : r : q] = \left[0 : \frac{1}{q} : \frac{1}{r}\right]$

$E' = [r : 0 : p] = \left[\frac{1}{p} : 0 : \frac{1}{r}\right]$

$F' = [q : p : 0] = \left[\frac{1}{p} : \frac{1}{q} : 0\right]$

$\Rightarrow P' = \left[\frac{1}{p} : \frac{1}{q} : \frac{1}{r}\right]$

coniugato isotomico



$$\frac{d(D,AB)}{d(D,AC)} = \frac{d(D^*,AC)}{d(D^*,AB)}$$

$$\frac{[D^*CA]}{[D^*AB]} = \frac{d(D^*,AC)\cdot b}{d(D^*,AB)\cdot c} = \frac{d(D,AB)}{d(D,AC)}\cdot\frac{b}{c} =$$

$D = [0 : q : r]$

$$= \frac{[DAB]/c}{[DCA]/b}\cdot\frac{b}{c} = \frac{[DAB]}{[DCA]}\cdot\frac{b^2}{c^2}$$

$$D^* = [0 : rb^2 : qc^2] = \left[0 ; \frac{b^2}{q} ; \frac{c^2}{r}\right]$$

$P^* = \left[\frac{a^2}{p} : \frac{b^2}{q} : \frac{c^2}{r}\right]$ se $P = [p : q : r]$

$AD^* \cap BE^* \cap CF^*$

coniugato ISOGONALE

$(G = K^*, \ H = O^*)$