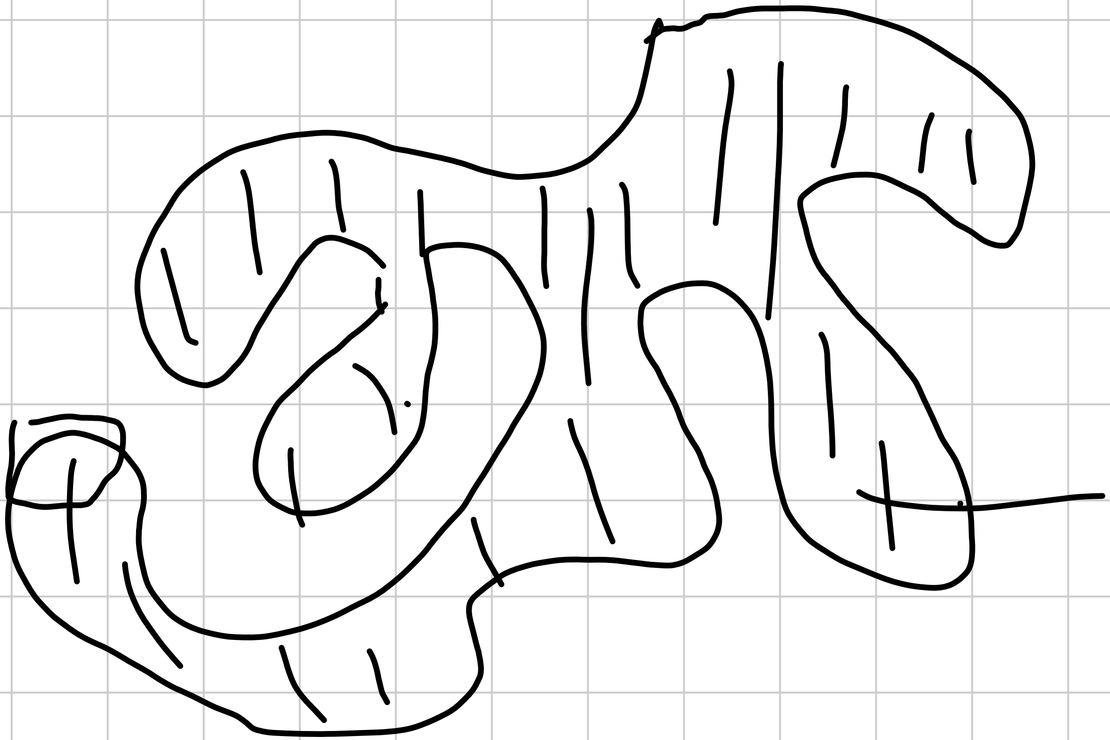
Titolo nota 03/09/2013

$2n+1$ interi distinti

$\Sigma_{n+1} > \Sigma_n \longrightarrow$

i) each $> n^2$

ii) find all configurations where one $= n^2+1$.



$\left(\underline{\text{Arc} \le 1}\right)$.

$P_1, P_2, \ldots, P_5$

$$1 \geq \mu\left(\bigcup_{i=1}^{5} P_i\right) \geq \underbrace{\sum_{i=1}^{5} \mu(P_i)}_{\frac{5}{2}} - \sum_{i<j} \mu(P_i \cap P_j)$$

$$\sum_{10} \mu(P_i \cap P_j) \geq \frac{3}{2} \longrightarrow \mu(P_i \cap P_j) \geq \frac{3}{20}$$

α_k = measure set of points which are covered by exactly k patches.

α_0, α_1

$$1 = \alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 \quad \Big| \; a$$

$$\frac{5}{2} = \sum_{i=1}^{5} \mu(P_i) = \alpha_1 + 2\alpha_2 + 3\alpha_3 + 4\alpha_4 + 5\alpha_5 \quad \Big| \; b$$

$$\overset{?}{=} \sum_{i=1}^{5} \mu(P_i \cap C_i) = \boxed{\alpha_2 + 3\alpha_3} + 6\alpha_4 + 10\alpha_5 \longleftarrow$$

10

$$\begin{cases} a + 2b = 1 \\ a + 3b = 3 \end{cases} \longrightarrow \begin{cases} b = 2 \\ a = -3 \end{cases}$$

$$-3(\mathrm{I}) + 2(\mathrm{II}) : 2 = -3\alpha_0 - \alpha_1 + \boxed{\alpha_2 + 3\alpha_3} \; \boxed{+6\alpha_4 + 10\alpha_5} =$$

$+5\alpha_4 + 7\alpha_5$

$-\alpha_4 - 3\alpha_5$

$$= \underbrace{\sum_{i<j} \mu(P_i \cap P_j)} - \underbrace{\left(3\alpha_0 + \alpha_1 + \alpha_2 + 3\alpha_3\right)} \leq$$

$$\leq \sum_{i<j} \mu(P_i \cap P_j) - 1 \longrightarrow \exists\ \mu(P_i \cap P_j) \geq \underline{\frac{2}{15} - \frac{1}{5}}\,.$$

$\cancel{5}$ <u>6 patches</u>.

COVERINGS & PACKINGS.

DESIGN THEORY.

(v, b, k, r, λ) — BIB (D)

balanced incomplete block

v elements varieties $\leftarrow b$

subset = block. Each block has k elements

Each of the v belongs to r blocks.

Each pair —"— —"— λ —"—.

1. $v \cdot r = bk$

2. $\lambda \cdot \frac{v(v-1)}{\cancel{2}} = b\frac{k(k-1)}{\cancel{2}}$

|| Necessary condition

Fisher's Inequality: $b \geq v$

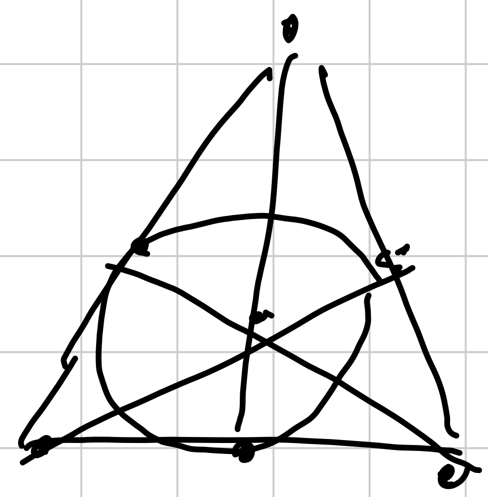
$b = v$ — BIB symmetric. (v, r, λ)

Ryser-Brooke-Chowla Theorem

$\underline{\lambda = 1}$. $\left[\begin{array}{l} v = q^2 + q + 1 \\ r = q + 1 \end{array}\right.$

$q = p^\alpha$, p prime.

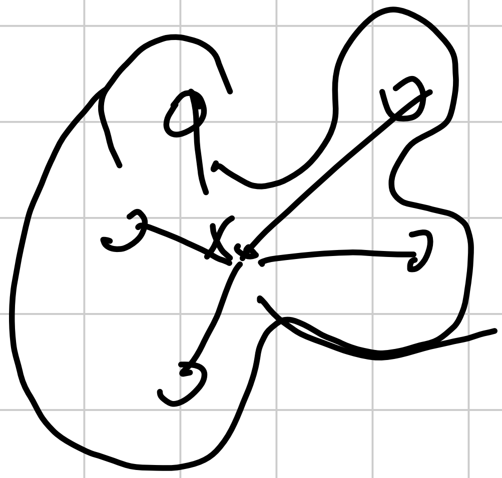
FANO



: 12k   3k + 6   N

A   B

$|N(x)| = \deg x = 3k+6$

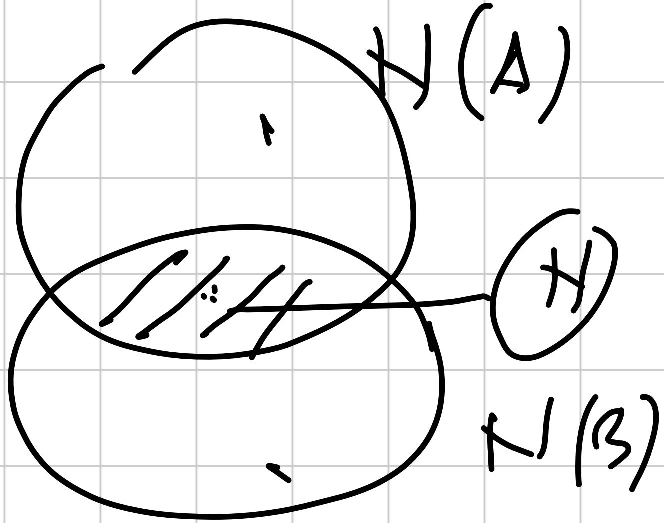


$3k+6$

$\|$

deg

B i B ?



(v, r, λ)

$v = b = 12k$, $\quad r = 3k+5$, $\quad \lambda = t$

$x_1, x_2, \ldots, x_{12k}$

$H(x_1), \ldots, H(x_{12k})$

$$\lambda \frac{\cancel{v}(v-1)}{\cancel{2}} = \cancel{b} \frac{r(r-1)}{\cancel{2}}, \quad b = v$$

$$\boxed{\lambda = \frac{r(r-1)}{v-1}} \in \mathbb{N}^*$$

$$\therefore \frac{3(k+2)(3k+5)}{12k-1} \in \mathbb{N}^*$$

$12k-1 \mid 3k^2 + 11k + 10$

$12k^2 + 44k + 40 \;—\; 45k + 40$

$-k + k$

$180k + 160$

$-15 \quad +15 = 175 = 5^2 \cdot 7$

$12k-1 \in \{1, 5, 7, 25, 35, 175\}$

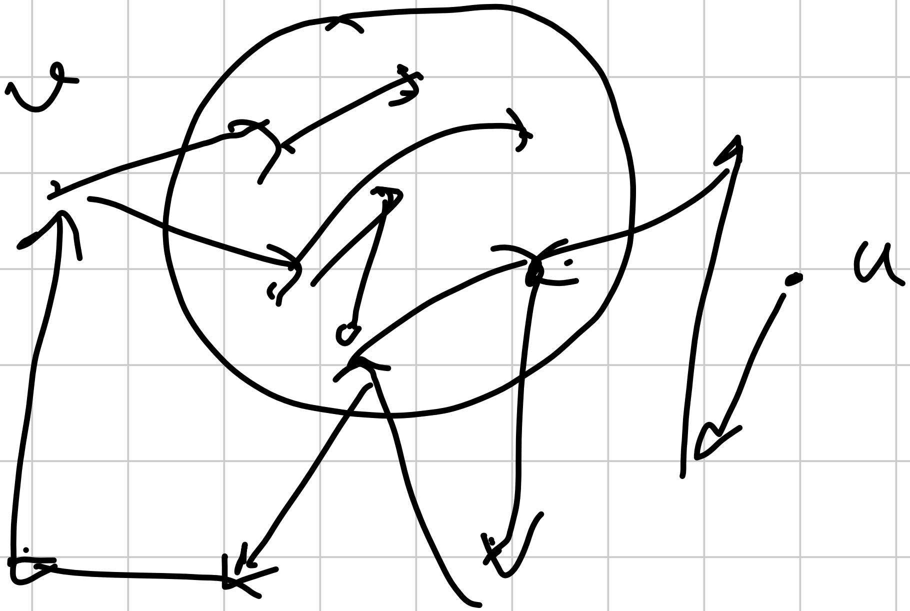
$12k \in \cancel{\{2, 6, 8, 26,} \boxed{36,} \cancel{176\}} \longrightarrow k = 3$

$(\lambda = \mu = 6)$

$(36, 15, 6)$ BIB

__Digraph__:

$C_v = \{$ cities w that can be reached from v $\}$

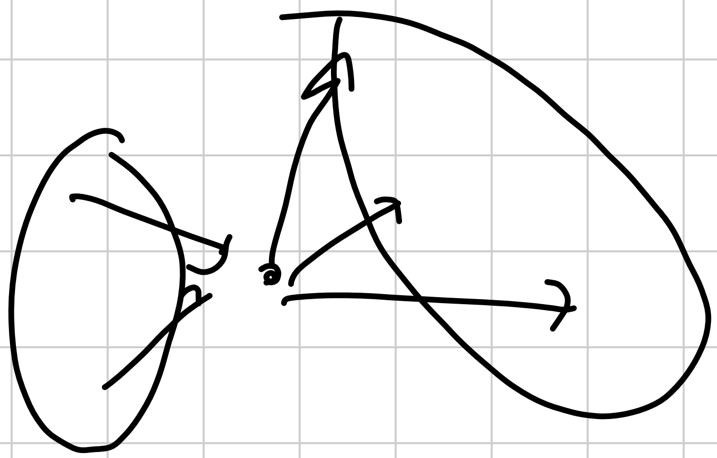


$C_u \not\supseteq C_v$

$\square$

~~S~~ T : from any city in T leave ≥ 3 buses in T

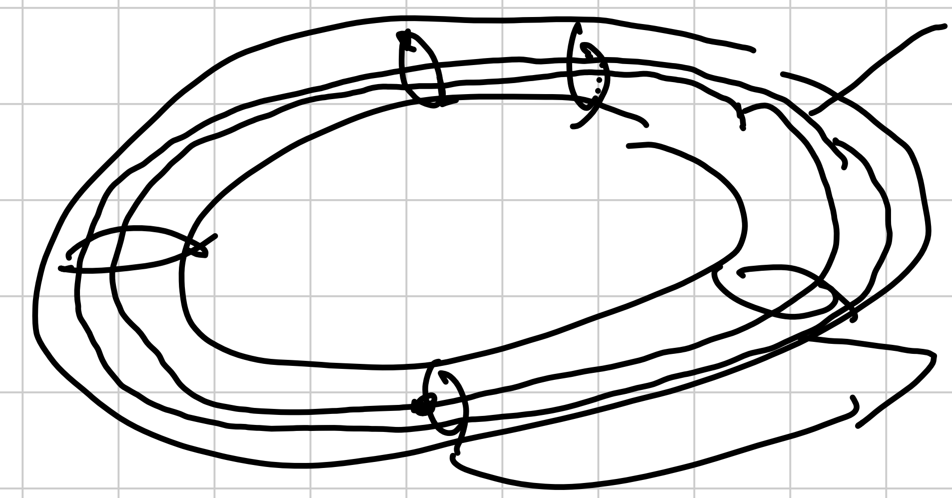
$\boxed{V}$: from any city in V $\to$ any other city in V.



$S \setminus T$

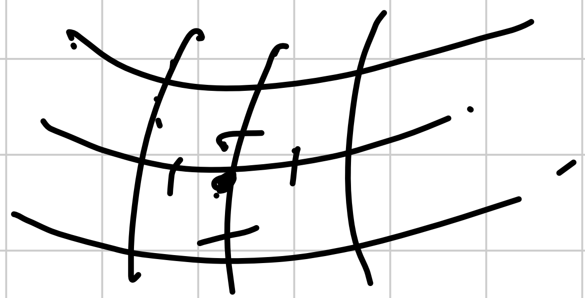
$\deg^+(v)$ out deg.

$\deg^-(v)$ in deg.

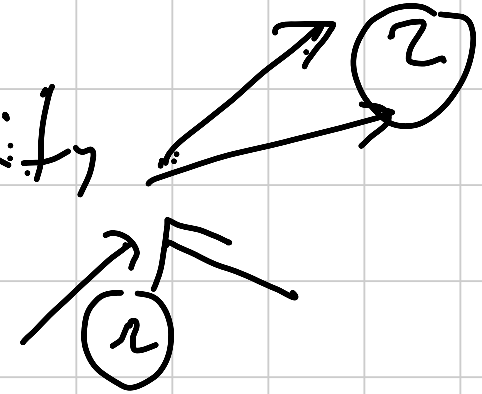


n blocks

m streets



For each city



2-reg. digraph ← 4-regular graph

=

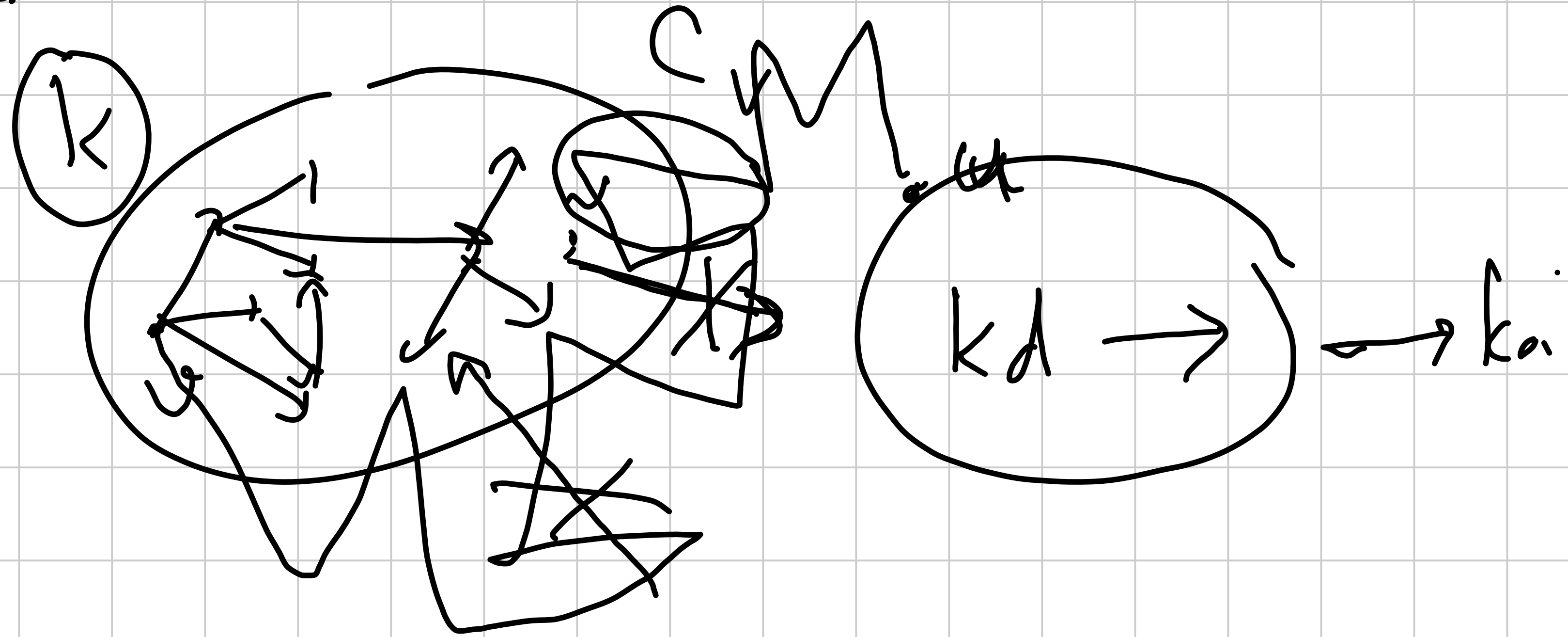
2d

4-reg —=—

$\deg^+(x) = \deg^-(x) = d$.

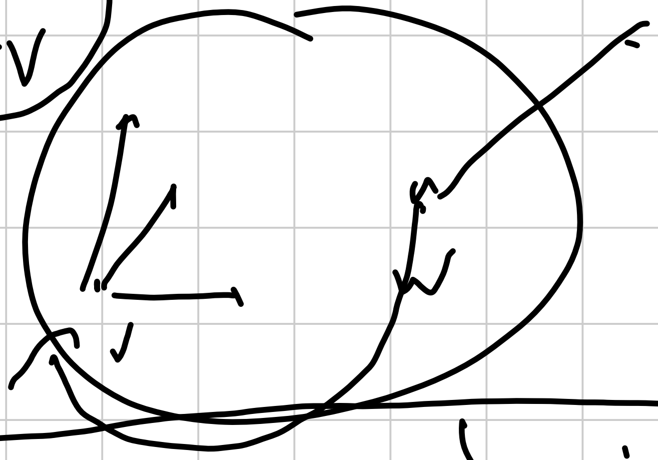
Thm. Any weakly connected d-reg. digraph is also $\boxed{\text{strongly connected}}$.

$\bigcup C_v = V$ Assume. Fix v



T V v $|C_v| =$ minimal.

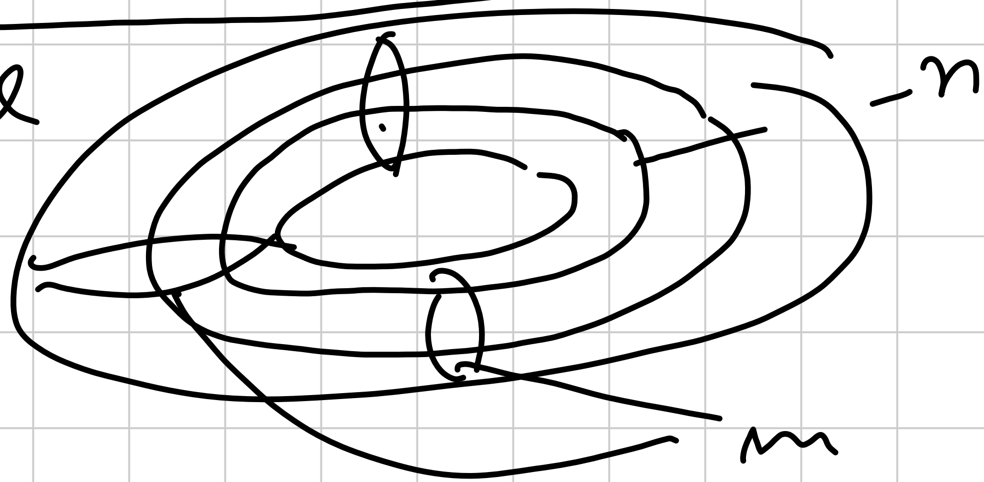
C_v



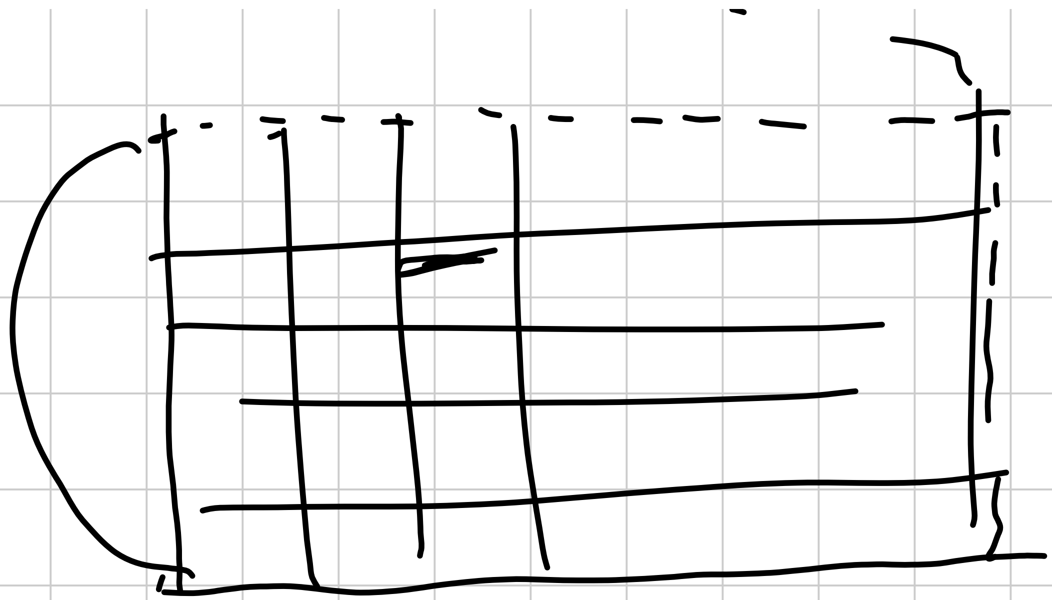
$C_w \subsetneq C_v \Rightarrow =$

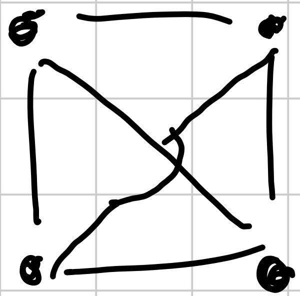
toroidal lattice

$\mathbb{Z}_m \times \mathbb{Z}_n$

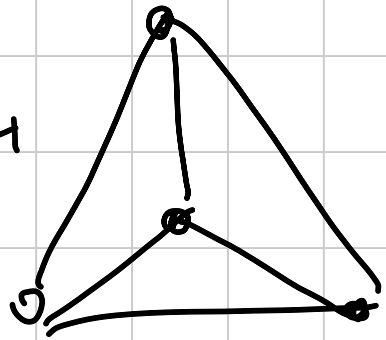


$\exists$ even length cycle..





K_4



$\underline{K_5}$ — never <u>succeed</u>

$\underline{K_{3,3}}$

Kuratowski