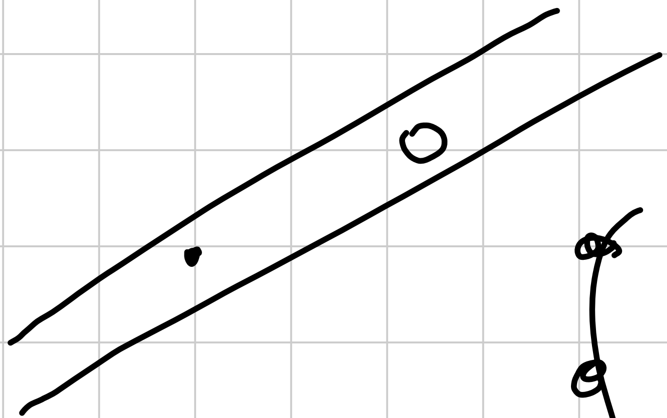
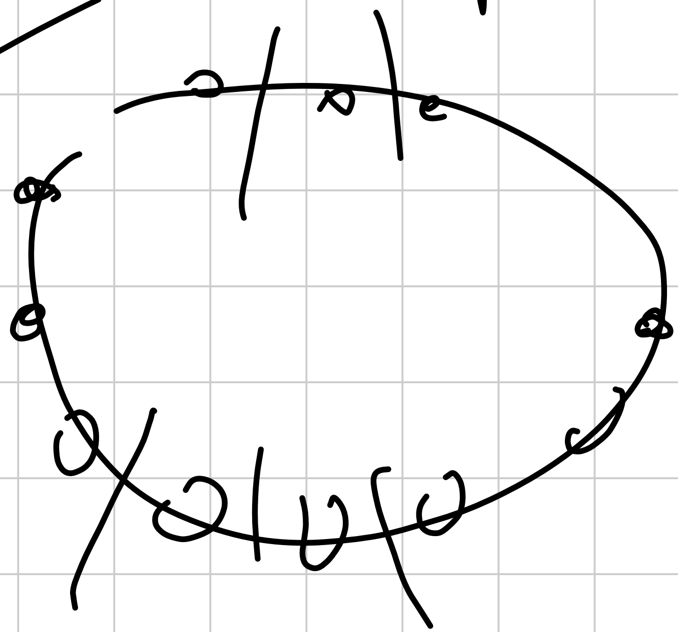
COMBI ADV. VENERDI

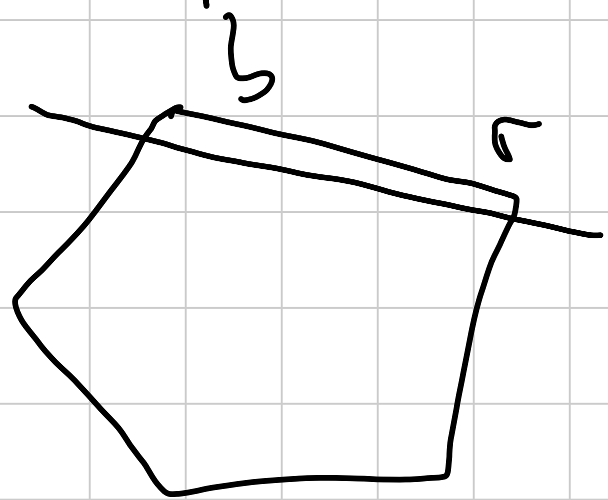
Titolo nota 06/09/2013

n red
n+1 blue

— $\boxed{n \text{ lines}}$ enough to separate same colour points.



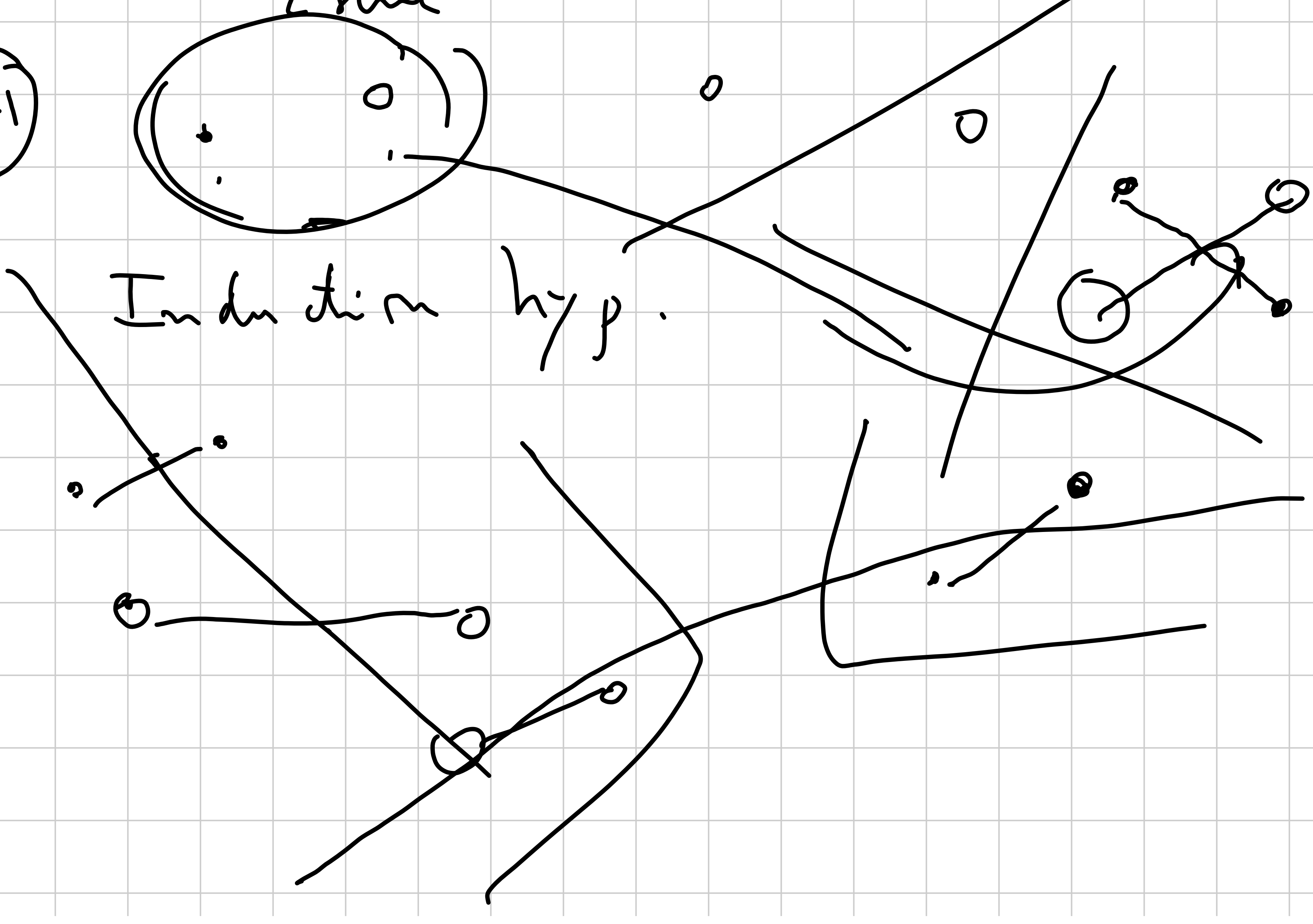




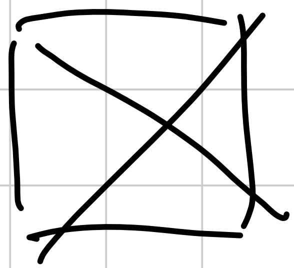
$n = 1$: 1 red
2 blue

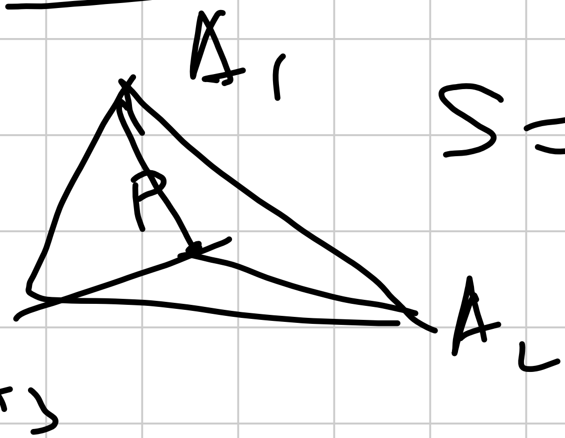
$n+1$

Induction hyp.

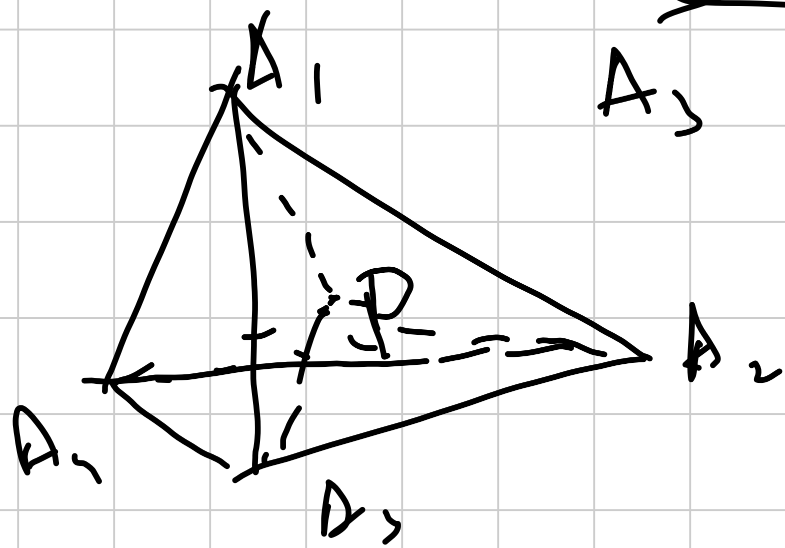


Find config. of 4 points plane s.t. 6 distances take <u>2 values</u>.





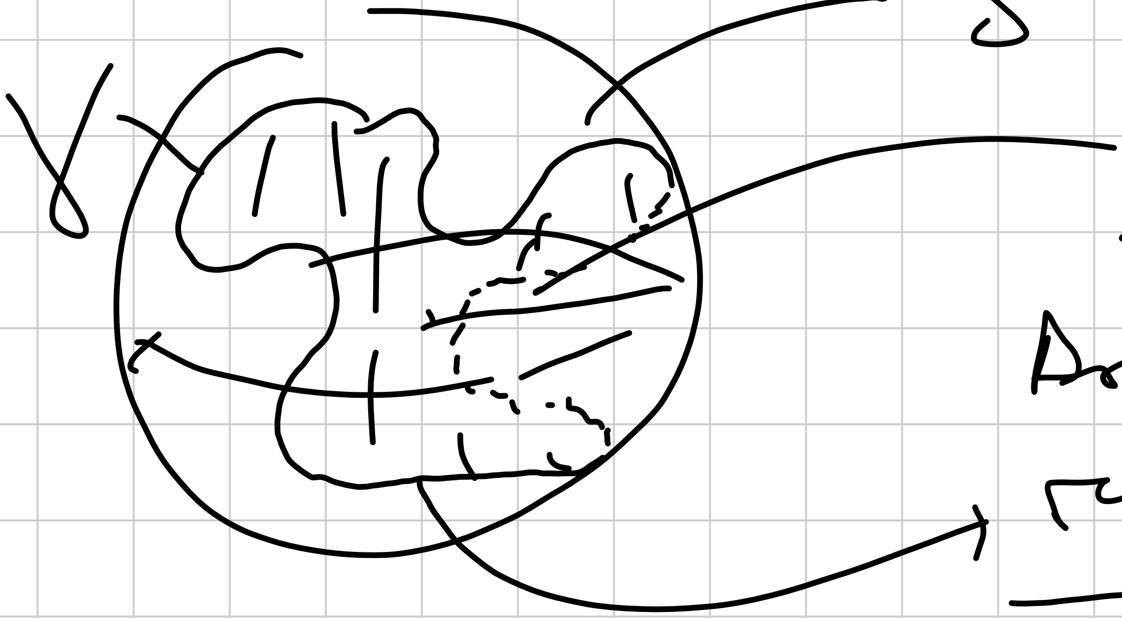
$$S = \sum_{i<j} \measuredangle A_i P A_j$$



3π $\quad$ 4π

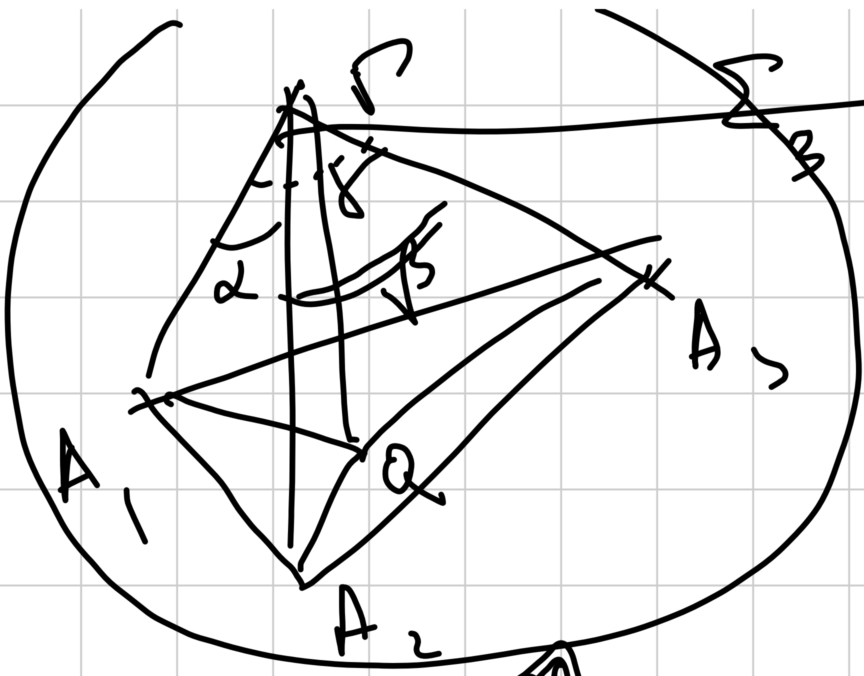
$m <$ $\quad$ $< M$

0 $\quad$ 6π



Areas these 2 regions are equal.

Length curve $\gamma \overset{?}{\geq} 2\pi$



$\Sigma_B \le 2\pi \rightsquigarrow P$

$=$

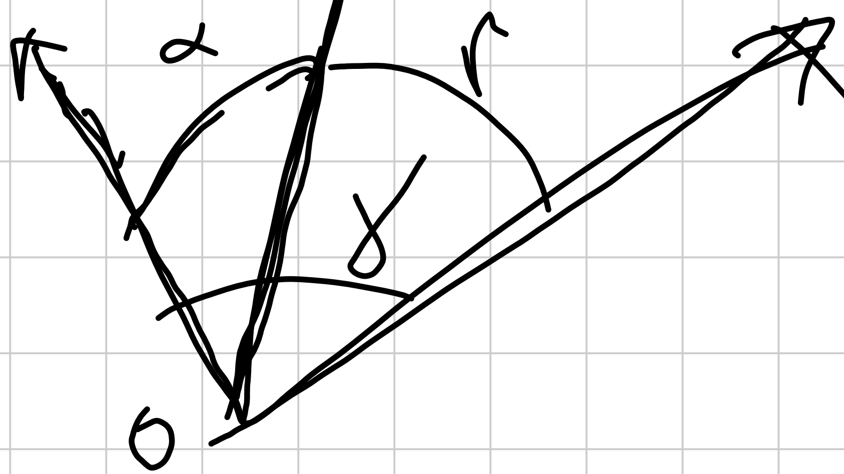
$P \in (A_1 A_2 A_3)$

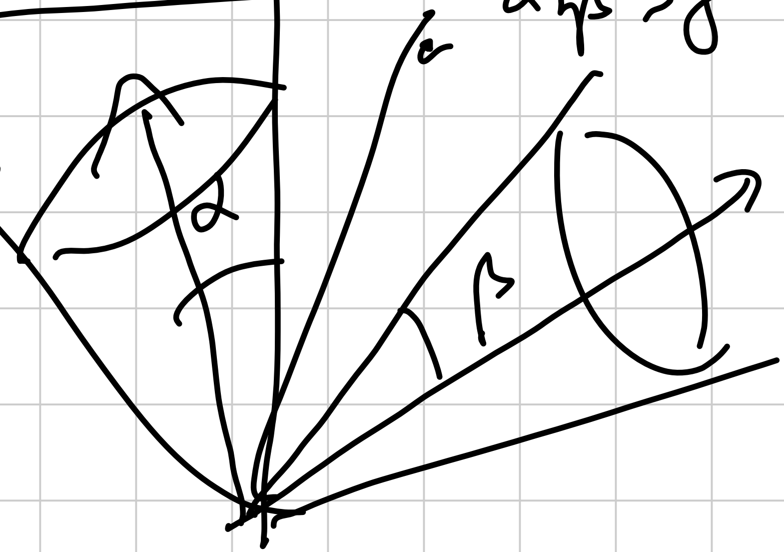
$\alpha + \beta + \gamma \le 2\pi$

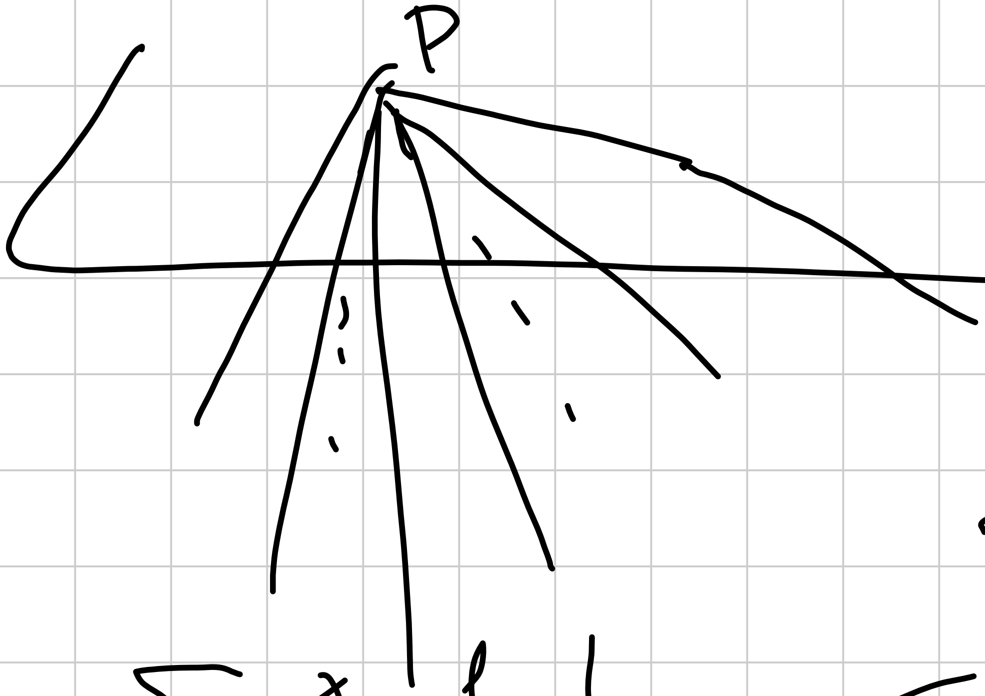
$\boxed{\begin{array}{l} \alpha+\beta > \gamma \\ \alpha+\gamma > \beta \\ \beta+\gamma > \alpha \end{array}}$

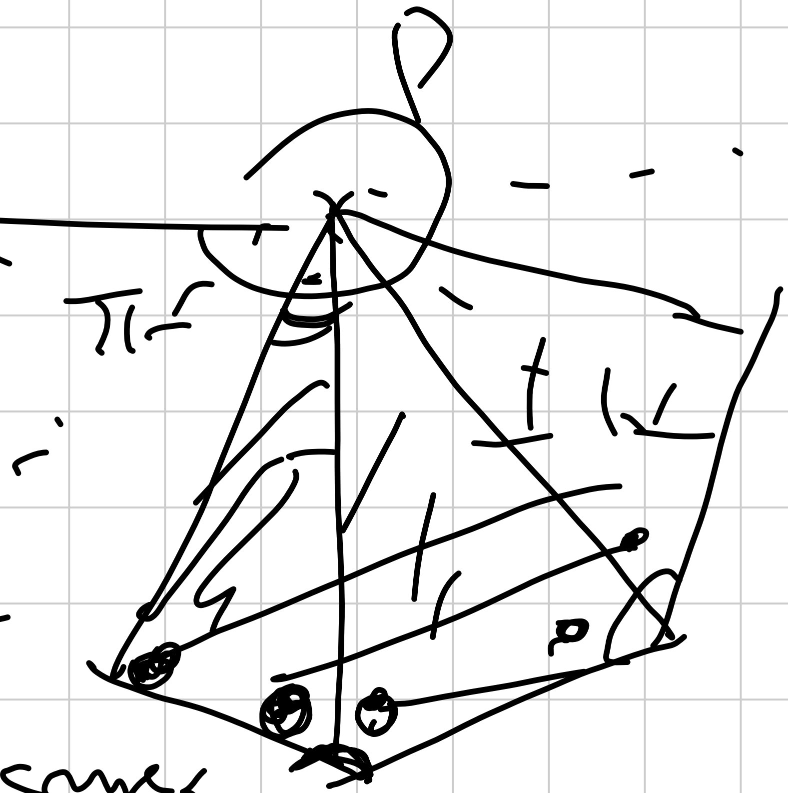
WLOG $\alpha \le \beta \le \gamma$

$\alpha + \beta > \gamma$









$\sum \measuredangle$ of base

For any vertex P of convex polyhedron

$\text{defect}(P) = 2\pi - \sum \alpha$ around $P > 0$

$\sum$ defect$_P$ $= 4\pi$.
P vertices.

$$\boxed{V + F = E + 2}$$

$\sum_3 (1,2,3) \le 2\pi$

$\sum_3 (1,2,4) \le 2\pi$

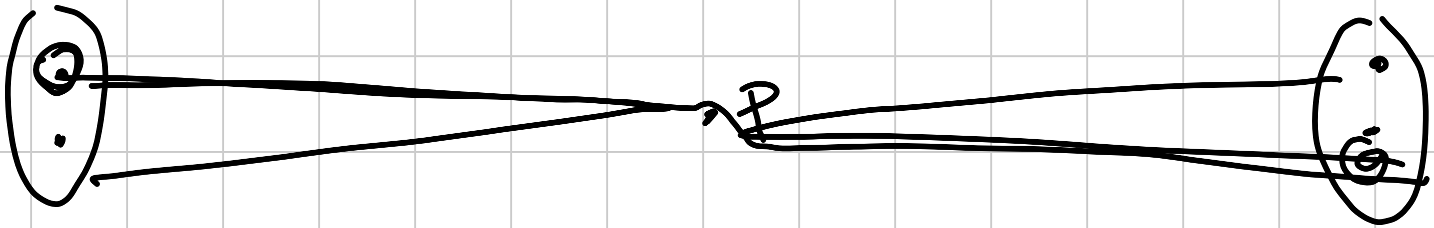
$\sum_3 (1,3,4) \le 2\pi$

$\sum_3 (2,3,4) \le 2\pi$

$\cancel{2S}$ $\le \cancel{8\pi}$ 4

4π good candidate

M



$\sum_4 = \widehat{A_1 A_2} + \widehat{A_2 A_3} + \widehat{A_3 A_4} + \widehat{A_4 A_1} \boxed{> 2\pi}$

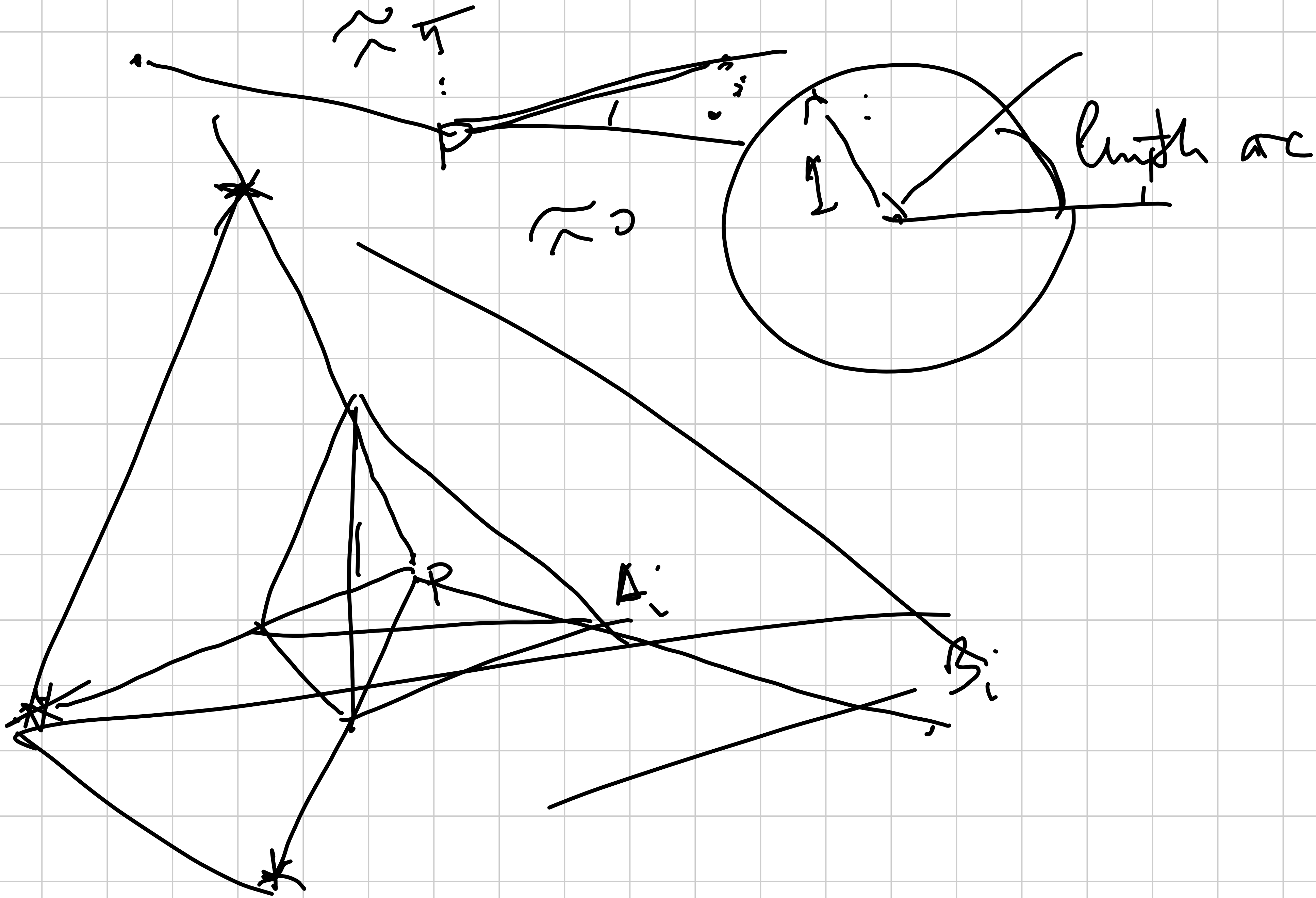
$(x, y, z, t) = (y, z, t, x) =$

similar: ...

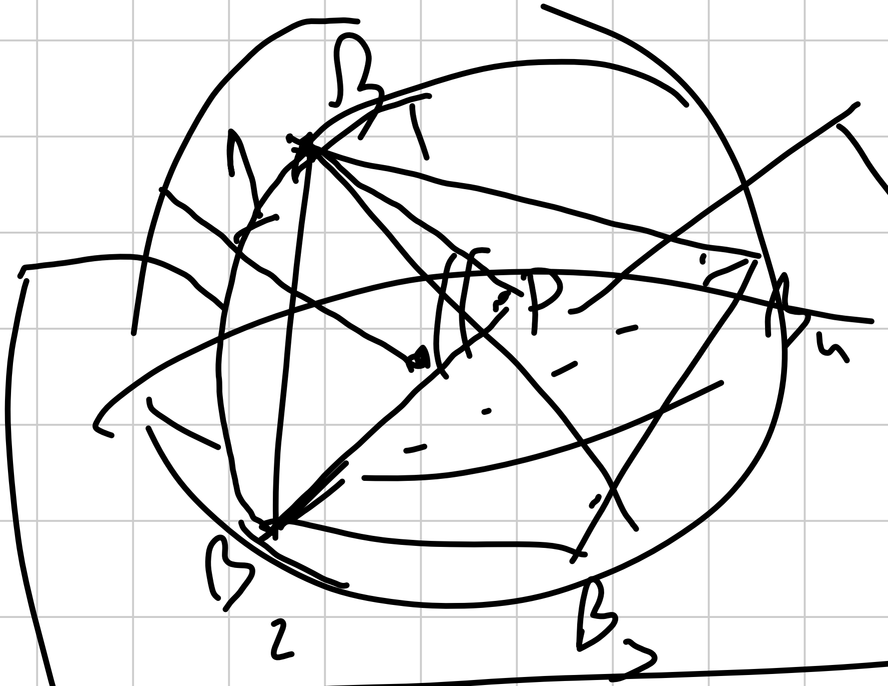
$S > 3\pi$

$\parallel$

m



P
Δ_i
B_i
r
1
length αr



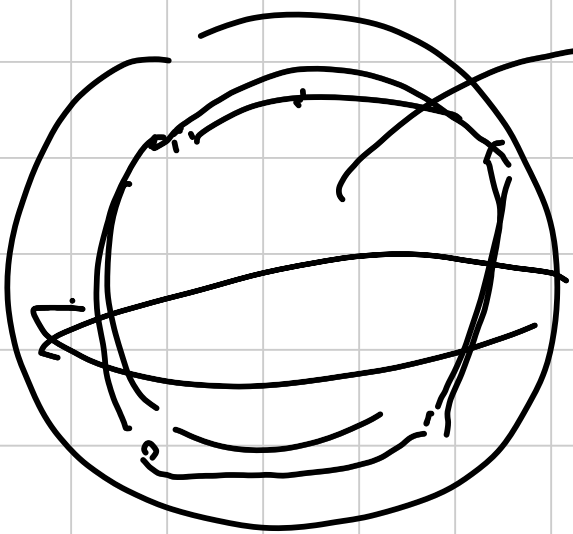
$\equiv$ center of sphere.

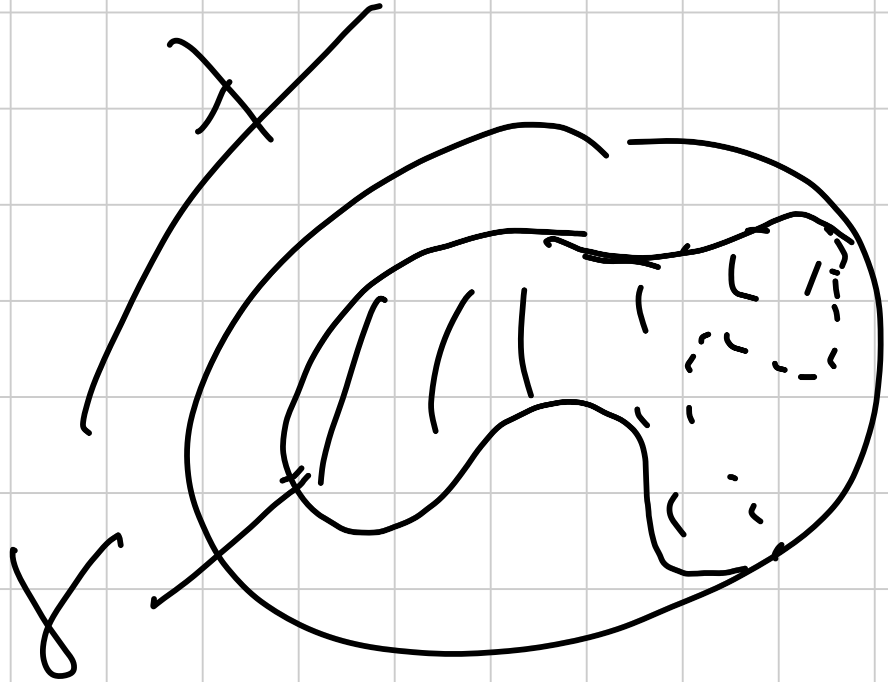
interior $B_1B_2B_3B_4$.

$\boxed{\text{length arc} = \text{measure } \angle B_1PB_2}$.

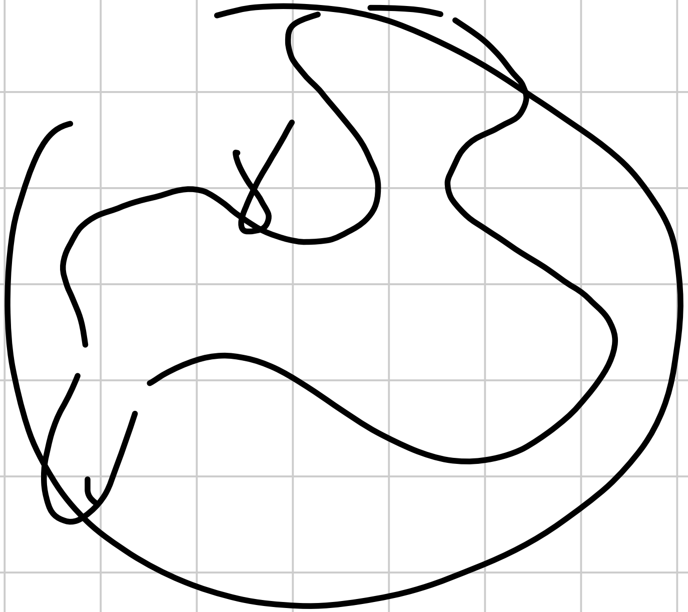
$$\sum_4 \text{angles} = \frac{\sum_4 \text{lengths arcs}}{\text{length}} \geq 2\pi \ ?$$

why





Thm If length $\lambda < 2\pi$ $\exists$ $\gamma \subset$ hemisphere.



γ

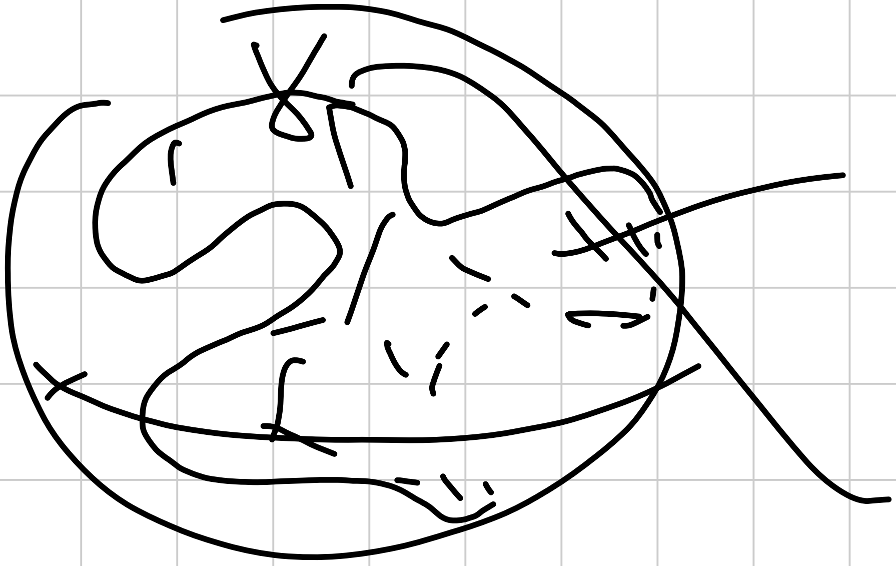
$0 \in \operatorname{conv}(\gamma)$

$\downarrow$ $\exists\, x_1, x_2, \ldots x_n$

$0 \in x_1 x_2 x_3 \cancel{x_4}$.

Carathéodory Theorem. Radon's

Helly's.



$1/2$ area sphere

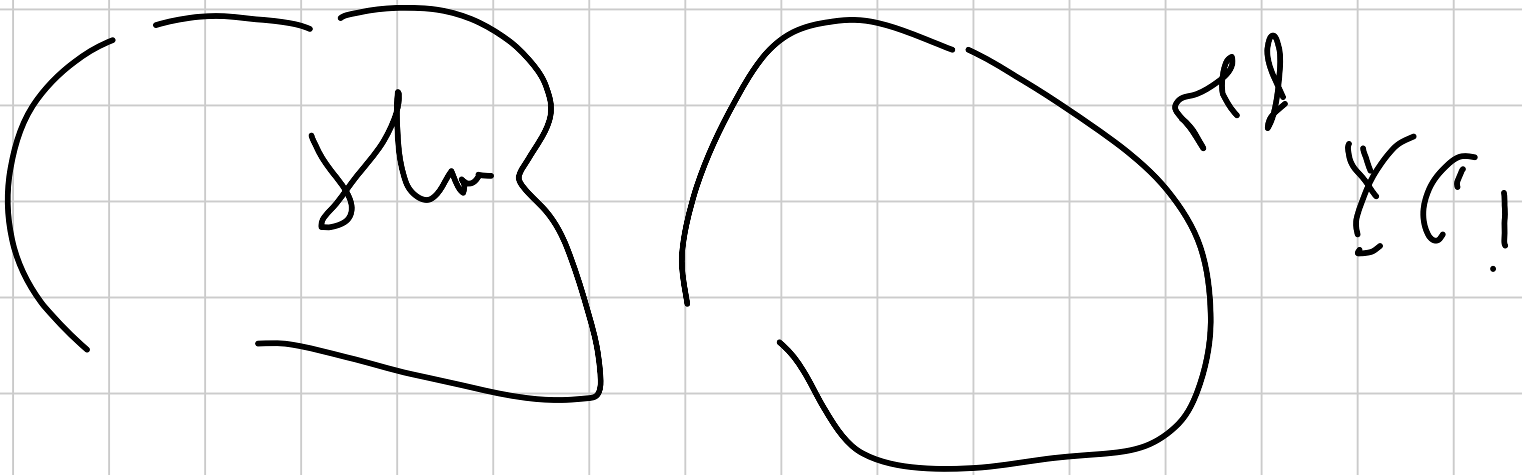
$\lambda \geq 2\pi$.

$\varphi : S^2 \longrightarrow S^2$

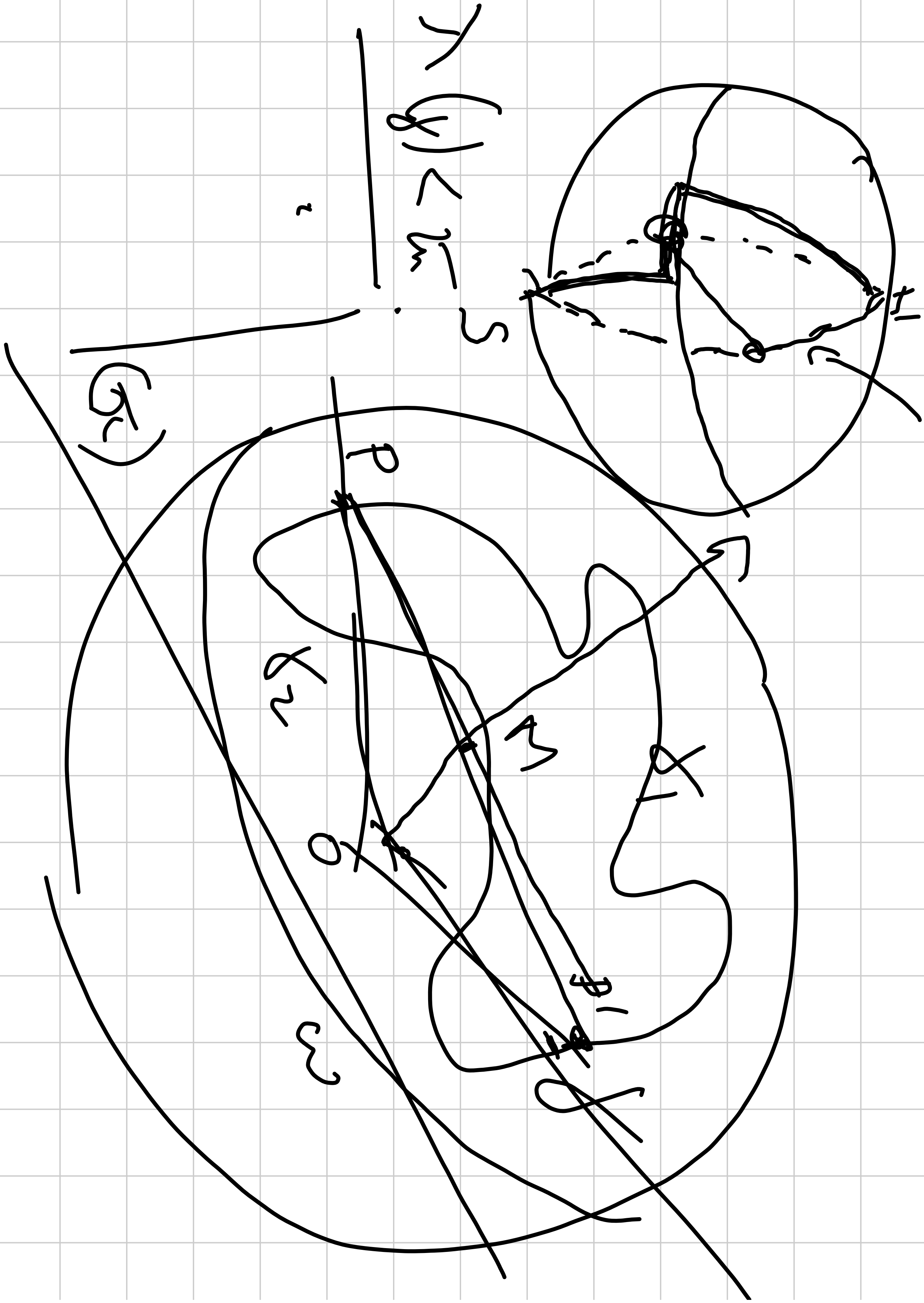
$P \longmapsto$ antipodal point.

$\varphi^2 = id$.

γ coloured <u>red</u>. $\varphi(\gamma)$ <u>blue</u>



$\gamma \cap \varphi(\gamma) \neq \emptyset$ ($\geq$ <u>2 points</u>)



$\gamma_1 \cap \omega \neq \phi$. x

$\downarrow$ sym. wrt $O\Pi$ $\longmapsto$ $\gamma_1' \neq \gamma_2$

$\lambda(\gamma_1') = \lambda(\gamma_1) = 1/2\, \lambda(\gamma)$

$\gamma_1' \cap \omega \neq \phi$ $\quad x'$ are opposite (antipodes)

$\gamma_1 \cup \gamma_1' = \gamma'$ — $\lambda(\gamma') = \lambda(\gamma) < 2\pi$

containing a pair of

PZ

$1 < p < q \qquad (p,q) = 1$

$p \quad q$
$7 \quad 11$

$\{1, 2, 3, \ldots, p+q\}$

pick as many elements, so that $|x-y| \neq p, q$.

$$a-p \longrightarrow a \longleftarrow a+q$$

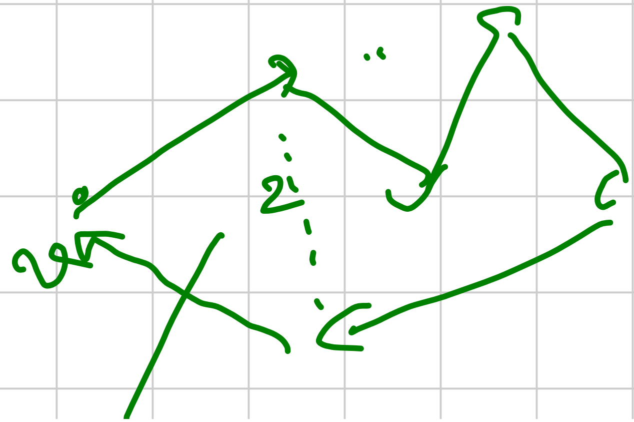
$1 \le a \le p$ $\qquad$ $a \rightarrow a+p$, $\quad a \leftarrow a+q$

$p < a \le q$ $\qquad$ $a-p \rightarrow a \longrightarrow a+p$

$q < a \le p+q$ $\qquad$ $a-p \rightarrow a$, $\quad a \rightarrow a-q$

$G\left(V = \{1, 2, \dots, p+q\},\ \text{red/blue arrows}\right)$

1-reg. digraph :



w

$$\boxed{\alpha \xi = \beta p}$$

$p \mid \alpha$

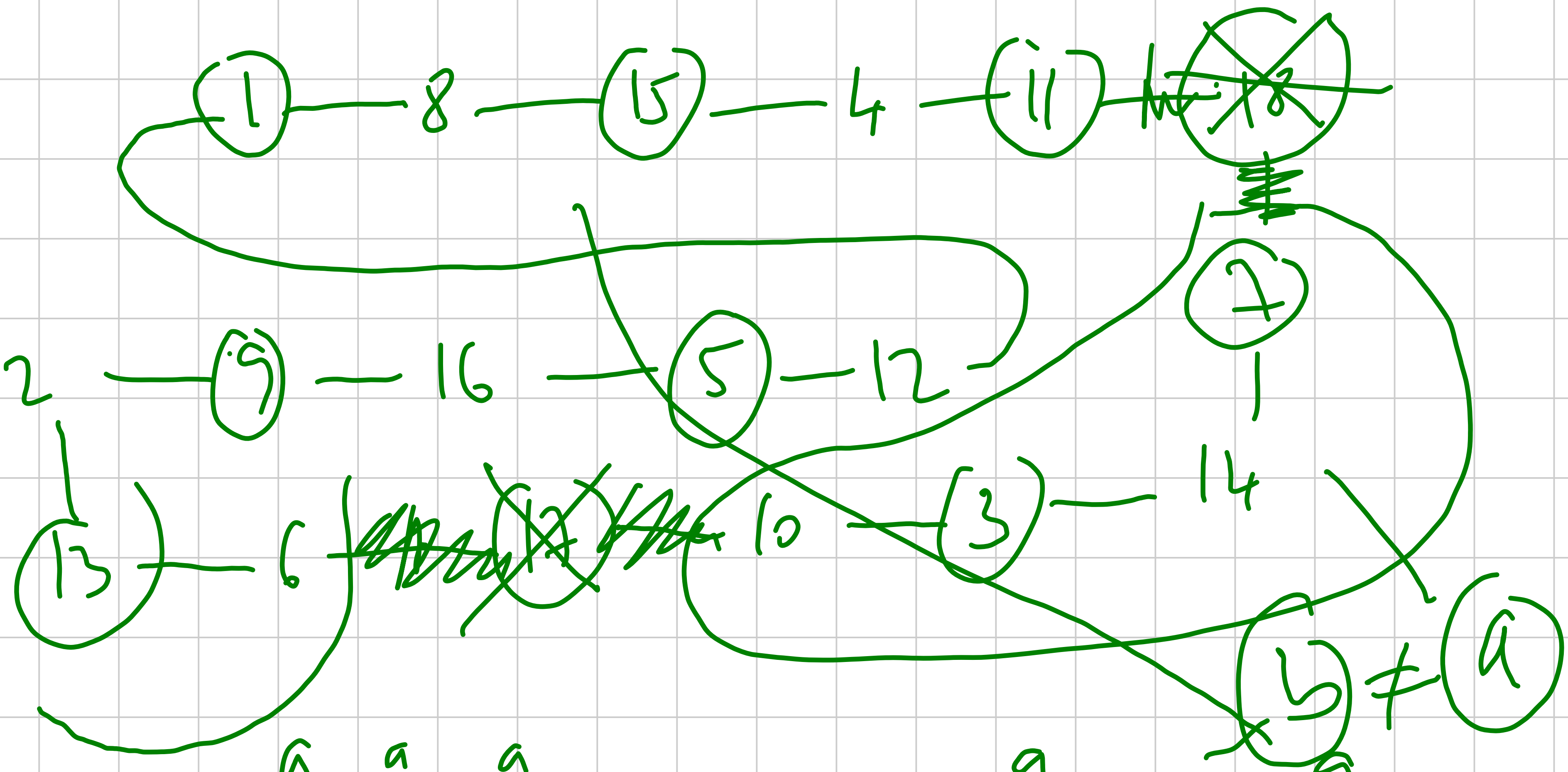
$\xi \mid \beta$

α red arrows

β blue arrows.

$\alpha + \beta = \lambda =$ length cycle $\leq p + \xi$.

$$\left.\begin{array}{l} p \leq \alpha \\ \xi \leq \beta \end{array}\right\} \quad p + \xi \leq \alpha + \beta$$



$a_1, a_2, a_3, \ldots, a_m, \ldots, a_{p+q}.$

$a_i = a_{i+p}$ for all i $(\ldots)$

$a_j = a_{j+q}$ $\ldots, j$ $(\ldots)$

For $1 < p, q$, $(p, q) = 1$

i) There is no sequence of $p+q-1$ terms (or longer), periodic of both p and q, s.t. $\neq$ constant.

ii) For $p+q-2$ terms there exists and a sequence. (using 2 values for terms).

$\boxed{\text{Wilf–Fine Theorem}}$.

$x_1, x_2, \ldots \ldots, x_n$ -- numbers.

- sum of any consecutive 7 > 0 ?
- sum of any ——— 11 < 0

Cannot $n = p + q - 1$ (or larger)

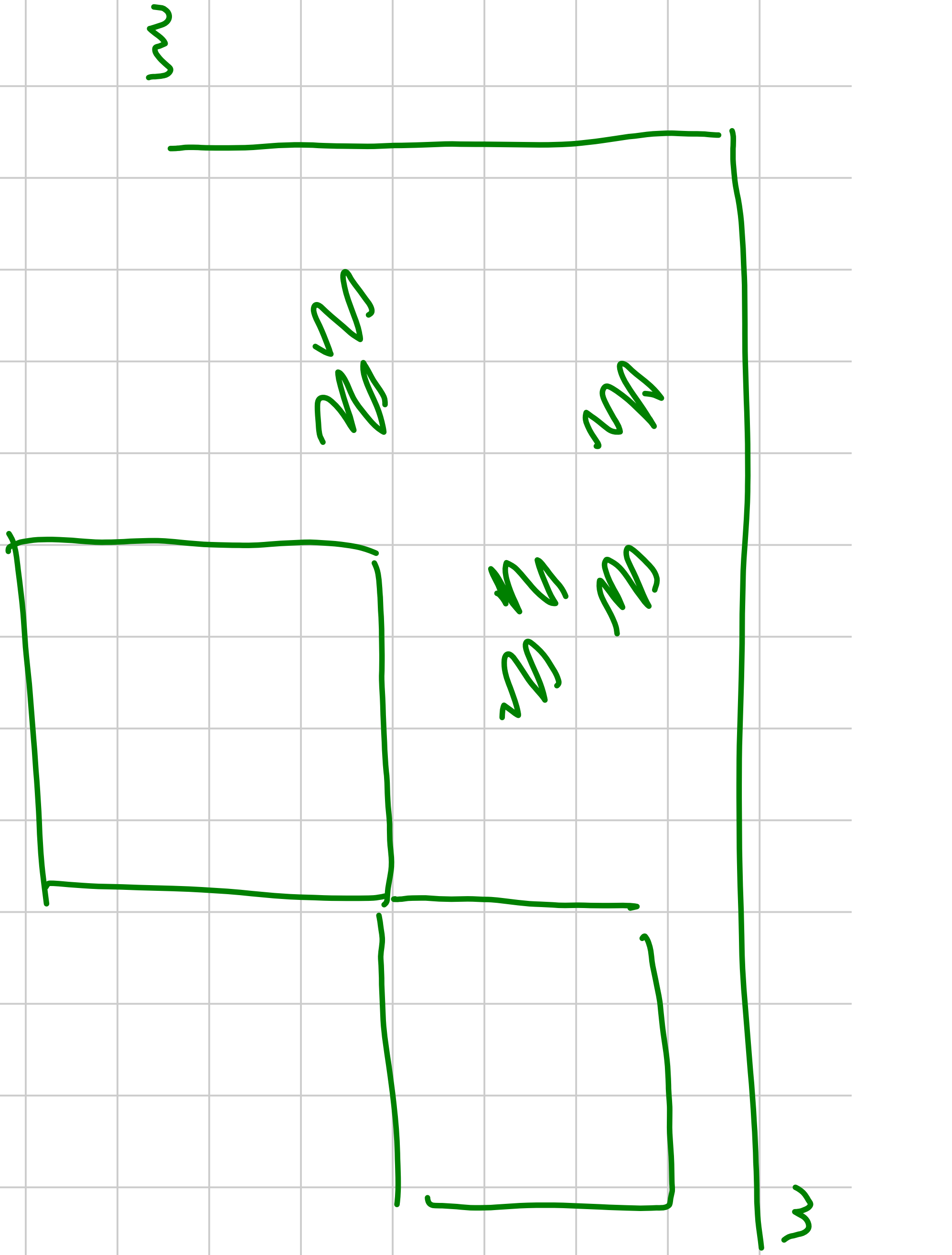
But can $\underline{n = p + q - 2}$.

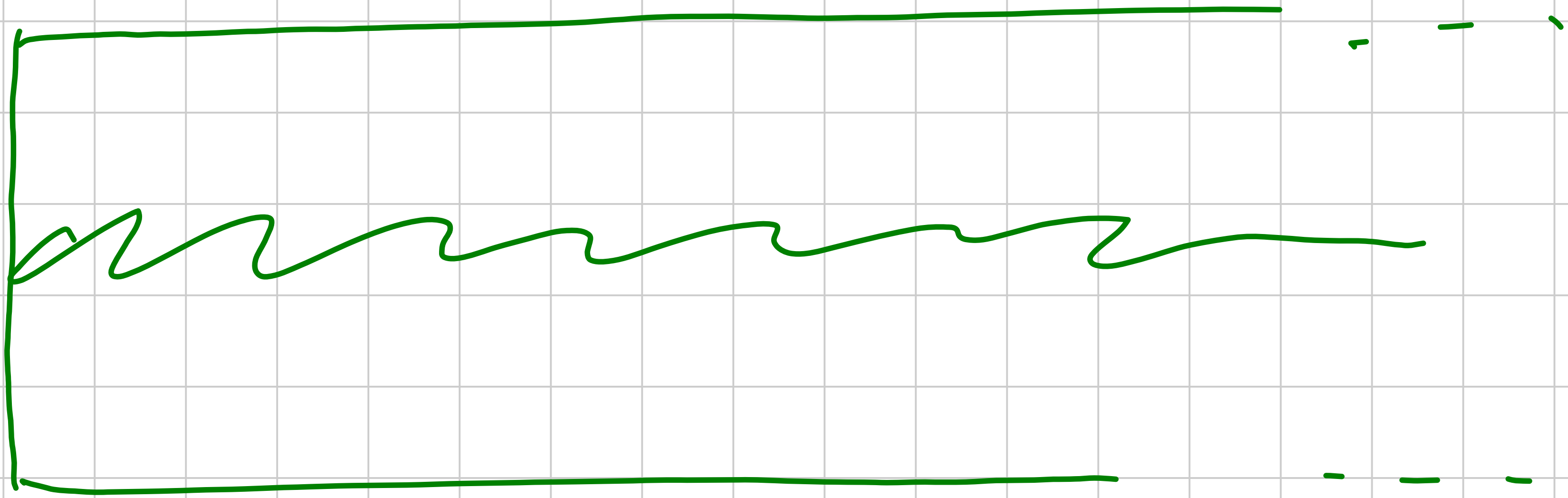
$\cancel{a^{\#} = x}$

$\cancel{b^{\#} = y}$

Some elem. $= a$

rest $= b$

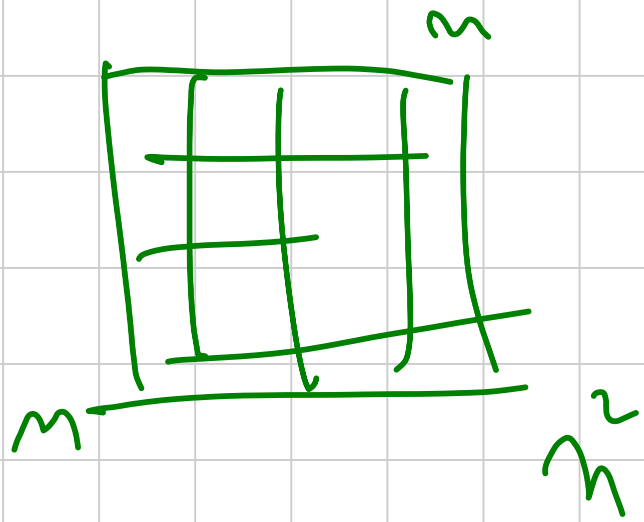




$3 + 4 - 2 = 5$ Any $p \in p$ S.

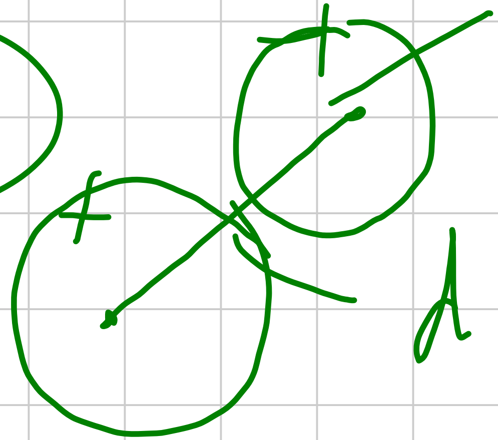
(p, q). $n \in S \times S$.

$pq - 2$ now
Any $n7$, − Column 5

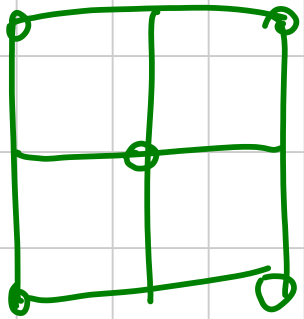


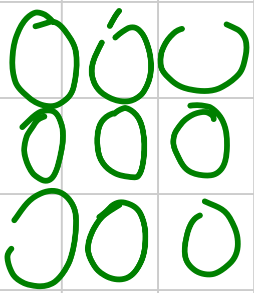
same points

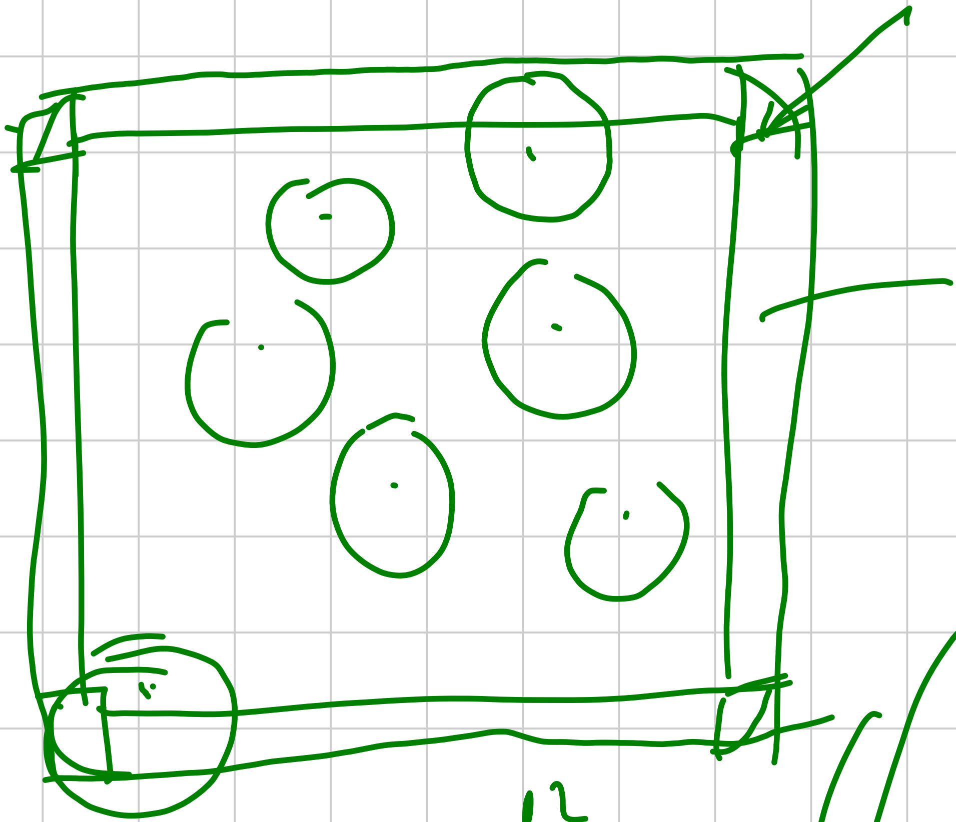
$$\frac{d(x,y) \le \sqrt{2}}{n^2+1}$$



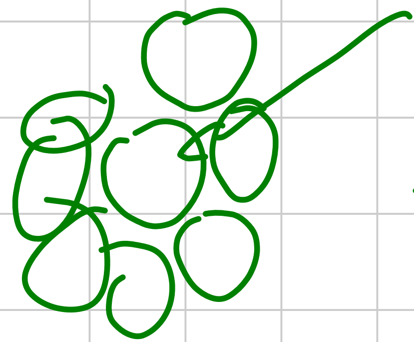
$n=2$







width $= \frac{1}{2}$

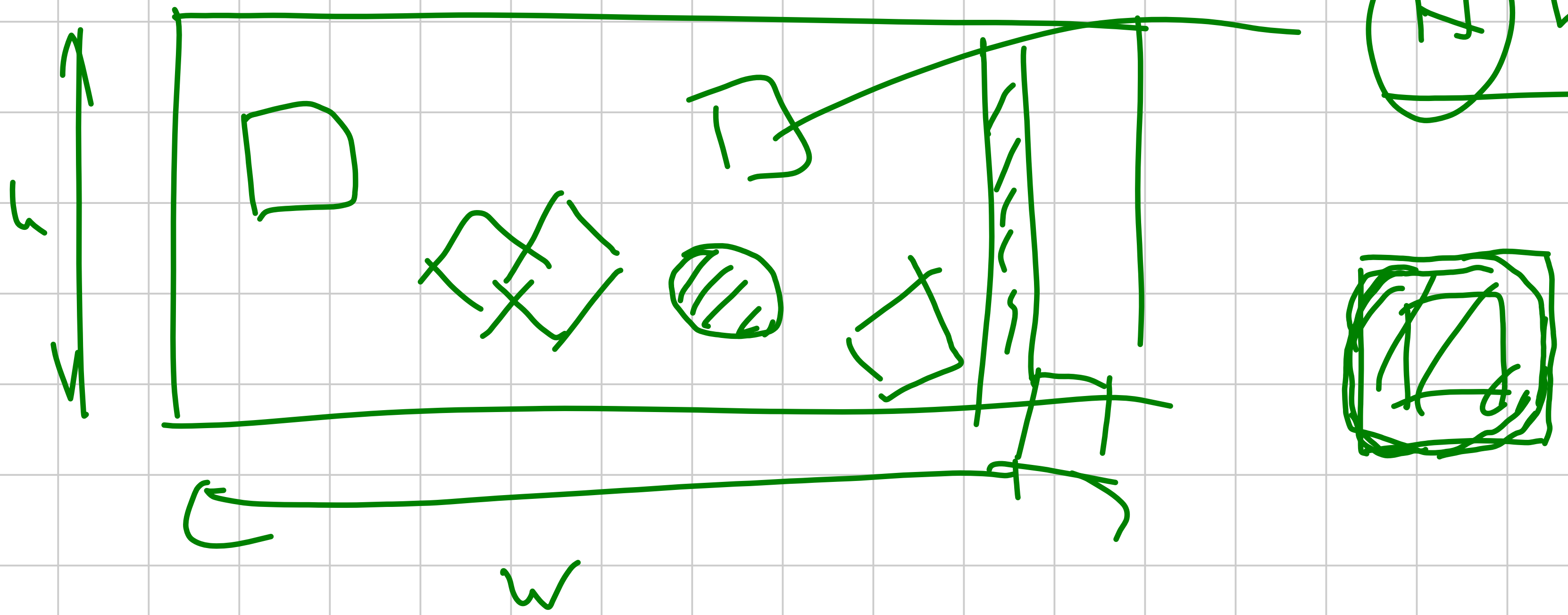


n^2+1

$(N-\cancel{1})\,\pi\frac{d^2}{4} \leq \left(n^2 + 4n\frac{d}{2} + \cancel{\pi\frac{d^2}{4}}\right)$

Density $\simeq 0.92\ldots = \beta$

Formula offers for N the
true asymptotic value



H holes.