G1 - Advanced - BARICENTRICHE & co.

Titolo nota 02/09/2013

$$\mathbb{R}^n = \{ (x_1, \dots, x_n) \mid x_1, \dots, x_n \in \mathbb{R} \}$$

$$(x_1, \dots, x_n) + (y_1, \dots, y_n) = (x_1 + y_1, \dots, x_n + y_n)$$

$$\lambda \cdot (x_1, \dots, x_n) = (\lambda x_1, \dots, \lambda x_n) \qquad \lambda \in \mathbb{R}$$

<u>Oss</u>: $\mathbb{K}$ somma, prodotto, ogni el. ha un opposto e ogni el $\neq 0$ ha un inverso [e le due

operazioni di $+, \cdot$ sono commutative e associative] ($\mathbb{K}$ si chiama <u>CAMPO</u>)

$$\mathbb{K}^n = \{ (x_1, \dots, x_n) \mid x_1, \dots, x_n \in \mathbb{K} \}$$

<u>Es</u>: $\mathbb{C}$, $\mathbb{Q}$, $\mathbb{Q}[\sqrt{2}] = \{ a + b\sqrt{2} \mid a, b \in \mathbb{Q} \}$,

$\mathbb{F}_p = \{$ le classi di resto mod $p \}$ p primo.

<u>Oss</u>: $\mathbb{Q}[\sqrt{2}] \cong \mathbb{Q}^2 = \{ (x,y) \mid x, y \in \mathbb{Q} \}$

$\mathbb{Q}[\sqrt{2}] = (\mathbb{Q}[\sqrt{2}])^1$

$a + \sqrt{2}b \longleftrightarrow (a,b)$

$$F : \mathbb{Q}[\sqrt{2}] \longrightarrow \mathbb{Q}^2$$

$$F\left((a+\sqrt{2}b) + (c+\sqrt{2}d)\right) = F\left((a+c) + \sqrt{2}(b+d)\right) =$$

$$= (a+c,\ b+d) = F(a+\sqrt{2}b) + F(c+\sqrt{2}d)$$

$q \in \mathbb{Q}$

$$F\left(q(a+\sqrt{2}b)\right) = (qa,\ qb) = q \cdot F(a+\sqrt{2}b)$$

$$F\left((a+\sqrt{2}b)(c+\sqrt{2}d)\right) = F\left(ac+2bd+\sqrt{2}(bc+ad)\right) =$$

$$= (ac+2bd,\ bc+ad)$$

$$\begin{tikzcd} \mathbb{Q}[\sqrt{2}]^5 \arrow[r] & \mathbb{Q}^{10} \\ {} \arrow[u] & \end{tikzcd}$$

$$F : \mathbb{K}^n \longrightarrow \mathbb{K}^m$$

1) $F\big((x_1, \dots, x_n) + (y_1, \dots, y_n)\big) = F(x_1, \dots, x_n) + F(y_1, \dots, y_n)$

2) $F\big(\lambda(x_1, \dots, x_n)\big) = \lambda F(x_1, \dots, x_n)$

F lineare

__Es__: $f: \mathbb{R} \longrightarrow \mathbb{R}$ lineare $\iff f(x) = ax$

$g(x) = ax + b$

$g(\lambda x) = a\lambda x + b \neq \lambda g(x) = \lambda a x + \lambda b$

__In generale__: $F: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ lineare

$$F(x_1, x_2, x_3) = (a_{11}x_1 + a_{12}x_2 + a_{13}x_3,\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3)$$

con $a_{ij} \in \mathbb{R}$

$$F\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} =$$

$$= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \end{pmatrix}$$

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e_j = n-upla di zeri e uni formata da tutti 0 e un 1 in pos. j-esima

$$\mathbb{Q}^5 \quad \underset{e_1}{\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}} \quad \underset{e_2}{\begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}} \quad \underset{e_3}{\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix}} \quad \underset{e_4}{\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}} \quad \underset{e_5}{\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}}$$

$$\mathbb{Q}^5 \ni x = q_1 e_1 + q_2 e_2 + q_3 e_3 + q_4 e_4 + q_5 e_5$$

<u>scrittura unica</u>

<u>Base</u> = ins. di vettori $v_1, \dots, v_k$ t.c. ogni altro v si scrive

$$v = \lambda_1 v_1 + \dots + \lambda_k v_k$$

in maniera unica!

__Fatto__: due basi dello stesso sp. vett. hanno la stessa cardinalità.

__Def__: $v_1, \dots, v_k$ sono linearmente dipendenti se $\exists\ \lambda_1, \dots, \lambda_k$ non tutti nulli t.c.

$$\lambda_1 v_1 + \dots + \lambda_k v_k = 0.$$

__Es__: $\exists\ a(x), b(x), c(x), d(x)$ t.c.

$$1 + xy + x^2y^2 = a(x)c(y) + b(x)d(y)\ ?$$

$y = 0 \qquad 1 = a(x)c(0) + b(x)d(0)$

$y = 1 \qquad 1 + x + x^2 = c(1)a(x) + d(1)b(x)$

$y = -1 \qquad 1 - x + x^2 = c(-1)a(x) + d(-1)b(x)$

$$1 = \lambda a(x) + \mu b(x) \qquad \lambda, \mu, \ell, m, L, \Pi \in \mathbb{R}$$

$$x = \ell a(x) + m b(x)$$

$$x^2 = L a(x) + \Pi b(x)$$

$$\{p(x) \in \mathbb{R}[x],\ \deg p \leq 2\} \cong \mathbb{R}^3$$

<u>Fatto</u>: in $\mathbb{K}^n$ ogni base ha n elem.

Se ho $m < n$ vettori $\Rightarrow$ non posso scrivere <u>tutti</u> gli altri in funzione di loro.

Se ho $m > n$ vettori $\Rightarrow$ sono lin. dip.

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<u>Es</u>: n abitanti m società sportive.
Ogni società ha num. dispari di membri.
2 ogni società hanno in comune un num. pari di membri. $\Longrightarrow m \leq n$

1. numeriamo gli abitanti da $1, \dots, n$
2. società $\longleftrightarrow$ $\left(\quad \underset{\substack{\uparrow \\ 0 \text{ se } j \notin \text{società}}}{j} \quad \right) \in \mathbb{F}_2^n$

1 se $j \in$ scelti

$s \in \mathbb{F}^n \qquad s \cdot s = 1 \qquad$ (prodotto scalare $x \cdot y = \sum x_i y_i$)

$s, s' \in \mathbb{F}^n$

$s \neq s' \qquad s \cdot s' = 0$

$s^1, \dots, s^m \qquad s^i \cdot s^i = 1$

$s^i \cdot s^j = 0$

Se $m > n$, allora $\exists\, c_1, \dots, c_m \in \mathbb{F}_2$ t.c. $\to$ non tutti nulli

$$c_1 \cdot s^1 + \dots + c_m s^m = 0$$

$$0 = s^{j} \cdot \left(c_1 s^1 + \dots + c_m s^m\right) = c_j \qquad \forall j$$

$\Rightarrow$ essendo $\Rightarrow$ $m \leq n$.

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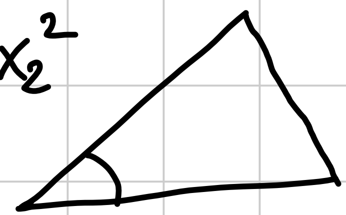
$(x_1, y_1) \in \mathbb{R}^2$ $\qquad \|(x_1, y_1)\| = \sqrt{x_1^2 + y_1^2}$

$(x_1, y_2) \in$ $\qquad \|(x_2, y_2)\| = \sqrt{x_2^2 + y_2^2}$

$$\frac{1}{2}\left((x_1 - x_2)^2 + (y_1 - y_2)^2 - x_1^2 - x_2^2 - y_1^2 - \cancel{y^2} \right)$$



$$= -x_1 x_2 - y_1 y_2 = -\|\bullet\| \cdot \|\bullet\| \cdot \cos \widehat{\ }$$

$\mathbb{R}^{10}$ $\quad V = \{ (x, y, 0, \dots, 0) \mid x, y \in \mathbb{R} \}$

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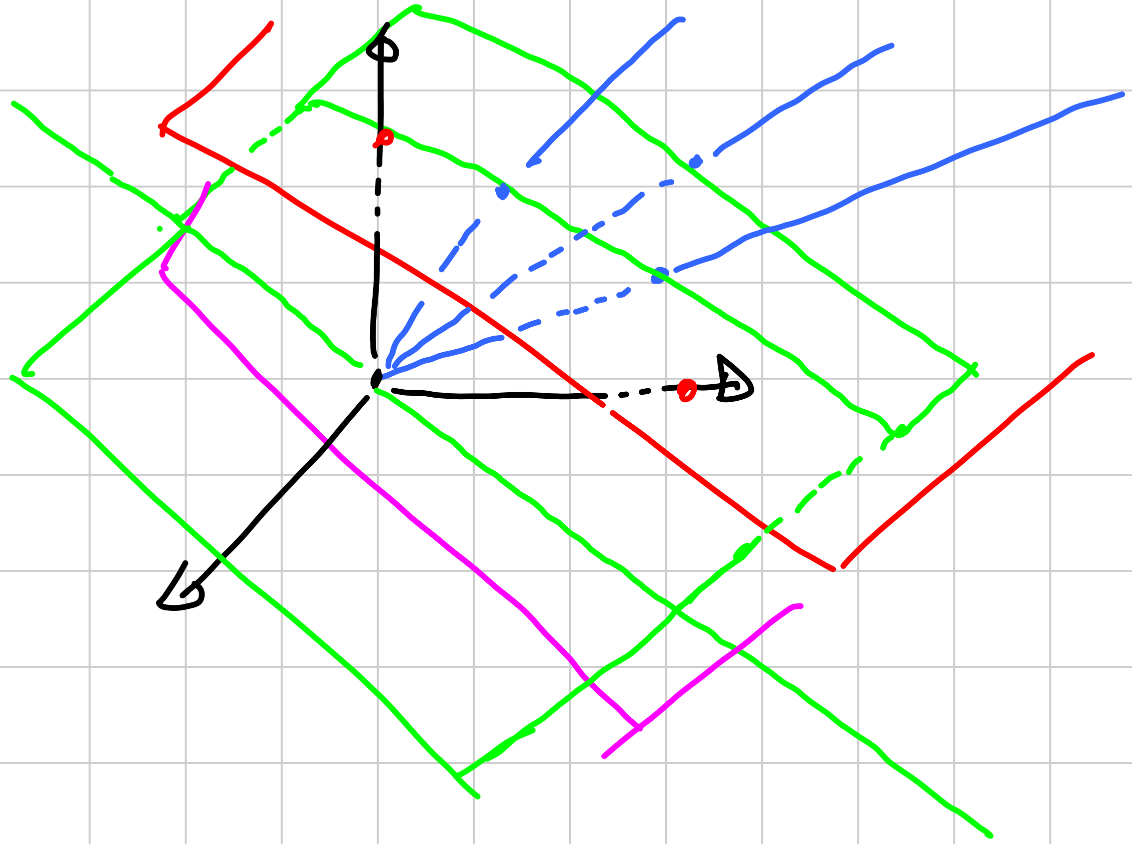
Coord. omogenee

$[x, y, z] = \{ (kx, ky, kz) \in \mathbb{R}^3, \ k \in \mathbb{R}^* \}$

Terne omogenee

$(x, y, z) \neq (0, 0, 0)$

$\{ [x, y, z], \ (x, y, z) \in \mathbb{R}^3 \setminus \{0\} \}$

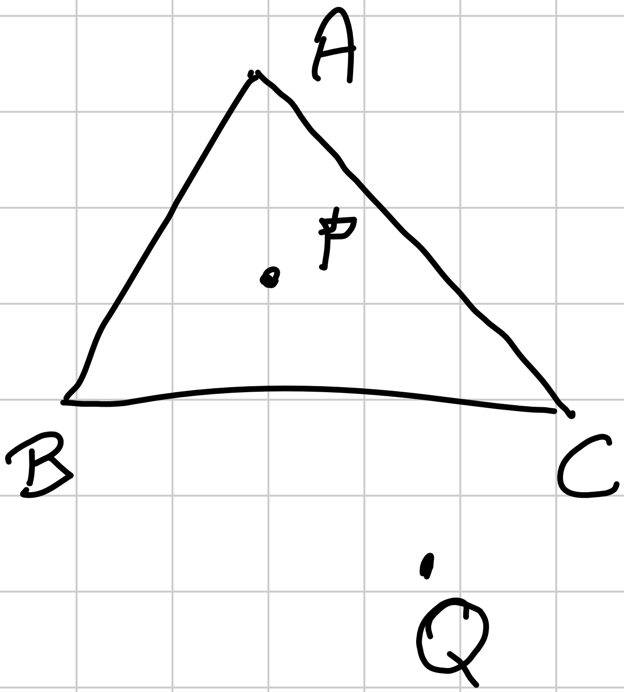


Coord. baricentriche

ABC triangolo

$$P \longrightarrow \big[[PBC], [APC], [ABP]\big]$$

$[DEF]$ = area orientata.



$[PBC]$

$[QBC] < 0$

$$\frac{x\vec{A} + y\vec{B} + z\vec{C}}{x+y+z} = \vec{P}$$

$$[x, y, z] = [[PBC], \ldots, \ldots]$$