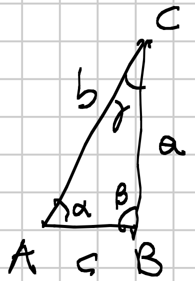
Geometria 1 Basic (LAB)

Titolo nota 02/09/2013

Geom: Sintetica G3, Analitica ... G2, Trigonometria G1



$\beta = 90°$

$\alpha + \gamma = 90° \Rightarrow \alpha, \gamma < 90°$ ACUTI

$$\cos\alpha = \frac{c}{b} = \frac{AB}{AC} = \frac{\text{cateto adiacente ad } \alpha}{\text{ipotenusa}}$$

$$\sin\alpha = \frac{a}{b} = \frac{CB}{AC} = \frac{\text{cat. opposto } \alpha}{\text{ipotenusa}}$$

$$\cos\gamma = \frac{a}{b} = \sin\alpha \qquad \cos(90° - \alpha) = \sin\alpha$$

$$\sin\gamma = \frac{c}{b} = \cos\alpha \qquad \sin(90° - \alpha) = \cos\alpha$$

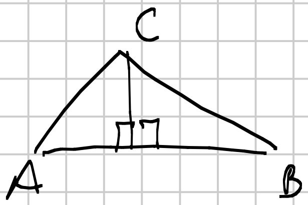
$$\operatorname{tg}\alpha = \tan\alpha = \frac{\text{cateto opposto}}{\text{cat. adiacente}} = \frac{a}{c} = \frac{a}{b}\cdot\frac{b}{c} = \frac{\sin\alpha}{\cos\alpha}$$

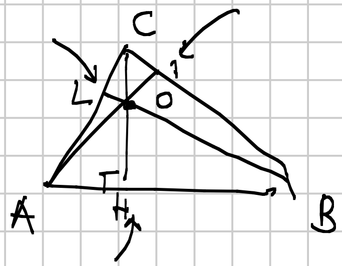
$$\operatorname{tg}\gamma = \frac{c}{a} = \frac{1}{\operatorname{tg}\alpha} =: \operatorname{cotg}\alpha \qquad \operatorname{tg}(90° - \alpha) = \operatorname{cotg}\alpha$$

$a = b \cdot \sin\alpha = b \cdot \cos\gamma$ $\qquad$ $c = b \cdot \cos\alpha = b \cdot \sin\gamma$

$$a^2 + c^2 = b^2 \longrightarrow \cancel{b^2}\cdot\sin^2\alpha + \cancel{b^2}\cdot\cos^2\alpha = \cancel{b^2}^{\,1}$$

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$





- Distanza di un vertice dall'ortocentro
- " " " lato " "

$AO = ?$ $\quad$ T_{AOH} retto in H

$$\frac{AH}{AO} = \cos(O\hat{A}H) = \cos(90° - \beta) = \sin\beta$$

$$OA = \frac{AH}{\sin\beta} \qquad AH = b \cdot \cos\alpha$$

$$= c - a \cdot \cos\beta$$

$$= b \cdot \frac{\cos\alpha}{\sin\beta}$$

$$= \frac{c - a \cdot \cos\beta}{\sin\beta}$$

$OI = ?$ $\quad$ 1. per differenze

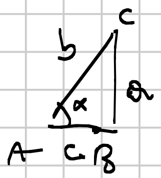
2. diretto $\qquad O\hat{C}B = 90° - \beta$

$$\frac{OI}{CI} = tg(O\hat{C}B) = cotg(\beta)$$

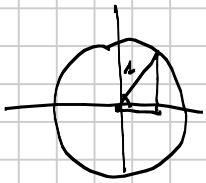
$$OI = (a - c \cdot \cos\beta) \cdot cotg\beta$$

$$CI = a - c \cdot \cos\beta$$

Torniamo indietro



$a = b \cdot \sin\alpha$ $\qquad$ caso interessante $b = 1$



Come misuriamo gli angol

gradi vs lunghezza arco sotteso

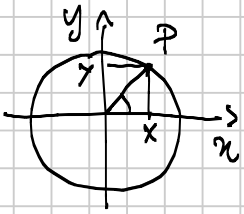
$$\alpha^{\circ} \longleftrightarrow \alpha^{rad}$$

$$\frac{\alpha^{\circ}}{360°} = \frac{\alpha^{rad}}{2\pi}$$

N.B. Stiamo considerando gli angoli con segno

↺ positivo $\qquad$ ↻ negativo

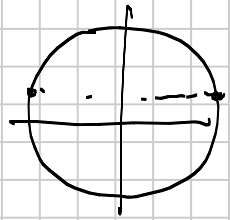
Come sono qu. $\sin\alpha$ e $\cos\alpha$



$x = \cos\alpha$
$y = \sin\alpha$

$360° \longleftrightarrow 2\pi$
$180° \longleftrightarrow \pi$
$90° \longleftrightarrow \pi/2$

Simmetrie & periodicità



$\sin(\alpha + 2\pi) = \sin\alpha$

$\cos(\alpha + 2\pi) = \cos\alpha$

$\sin(\pi - \alpha) = \sin\alpha$

$\cos(\pi - \alpha) = -\cos\alpha = \cos(\pi + \alpha)$

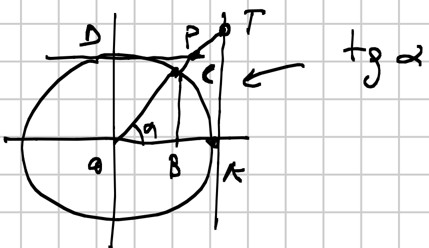
$\sin(\pi + \alpha) = -\sin\alpha = \sin(-\alpha)$

$\cos(-\alpha) = \cos\alpha$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$ $\qquad$ $\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin\alpha$

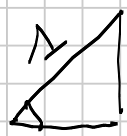
$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$ $\qquad$ $\sin\left(\frac{\pi}{2} + \alpha\right) = \cos\alpha$

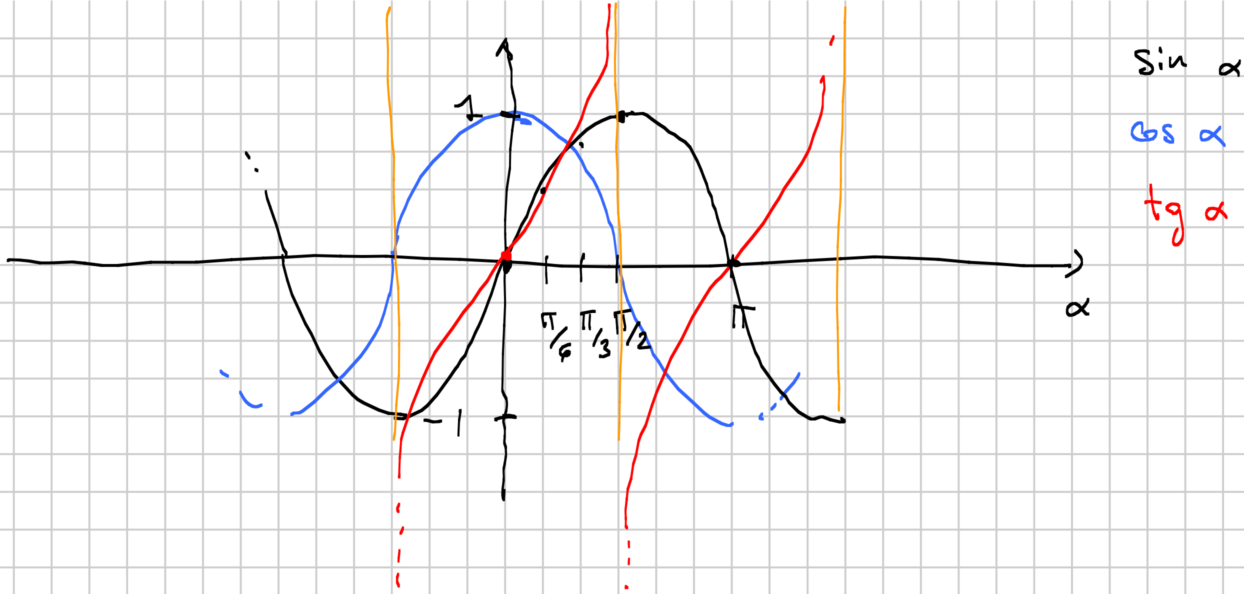
Tangente e cotangente



$$tg\,\alpha = TA = \frac{TA}{OA} = \frac{PB}{BO} = \frac{\sin\alpha}{\cos\alpha}$$

		sin	cos	tg
	0	0	1	0
	$\pi/2$	1	0	?
	π	0	-1	0
	$\frac{3}{2}\pi$	-1	0	?
(45°)	$\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
(60°)	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
(30°)	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\frac{\sqrt{3}}{3}$





Q: È vero che $\sin x = \sin y \Rightarrow x = y$? NO

Restringere $\pi/2$, $\frac{3}{2}\pi$

$\left[\frac{\pi}{2} + 2k\pi, \frac{3}{2}\pi + 2k\pi\right]$ è monotona decresc

$\Rightarrow$ iniettiva :)

$\left[-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right]$ è monot. cresc.

$\Rightarrow$ iniettiva

Stessa cosa ma int. diversi per il coseno

Se le voglio entrambe $\longrightarrow \left[0 + \frac{k\pi}{2}, \frac{\pi}{2} + \frac{k\pi}{2}\right]$

La tangente? In $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ è monot. cres

$\Rightarrow$ iniettiva

ma è anche suriettiva

$\Downarrow$ invertibile

Funzioni inverse

$\operatorname{atan} x = \arctan x = \operatorname{arctg} x$ "arco la cui tangente è x"

$\mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

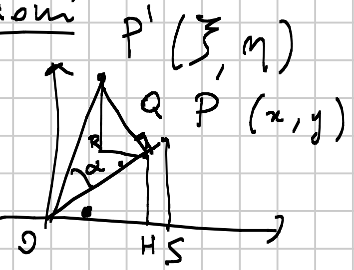
$\arcsin x = \operatorname{asin} x$

$[-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\arccos x = \operatorname{acos} x$

$[-1, 1] \longrightarrow [0, \pi]$.

Rotazioni



Vogliamo ξ, η in termini di x, y, α

$$\rho^2 = x^2 + y^2 = \xi^2 + \eta^2$$

ρ

$$OQ = OP \cdot \cos\alpha$$

$$\frac{QH}{OQ} = \frac{PS}{OP} = \frac{y}{\rho}$$

$$\eta = P'R + QH \qquad \boxed{QH = y \cdot \cos\alpha}$$

$$O\hat{P}S = O\hat{Q}H = 90° - \bullet$$

$$90° = O\hat{Q}P' = O\hat{Q}R + R\hat{Q}P = \bullet + R\hat{Q}P \quad \Rightarrow O\hat{P}S = R\hat{Q}P$$

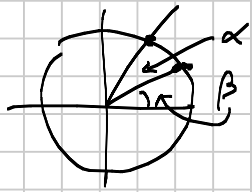
$$\frac{P'R}{P'Q} = \frac{OS}{OP} = \frac{x}{\rho} \qquad P'Q = \rho \cdot \sin\alpha$$

$$P'R = x \cdot \sin\alpha$$

$$(EX) \quad \begin{cases} \eta = x \cdot \sin\alpha + y \cdot \cos\alpha \\ \xi = x \cdot \cos\alpha - y \cdot \sin\alpha \end{cases}$$

Se $P \in B(0,1)$ $\quad x = \cos\beta \quad y = \sin\beta$

$$\xi = \cos(\alpha+\beta) = \cos\beta \cdot \cos\alpha - \sin\beta \cdot \sin\alpha$$



$$\eta = \sin(\alpha+\beta) = \cos\beta \cdot \sin\alpha + \sin\beta \cos\alpha$$

"EX" $\quad \cos(\alpha-\beta) = ?$

$\sin(\alpha-\beta) = ?$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

"EX" $\quad \cos\left(\frac{\alpha}{2}\right)$

$\sin\left(\frac{\alpha}{2}\right)$

Tangenti

$$tg(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dots = \frac{tg\alpha + tg\beta}{1 - tg\alpha\, tg\beta}$$

$$tg(2\alpha) =$$

$$tg\left(\frac{\alpha}{2}\right) = \dots = \frac{\sin\alpha}{1+\cos\alpha}$$

modo geom.

Formule parametriche $\qquad t = tg\left(\frac{\alpha}{2}\right)$

$$\sin\alpha = \frac{2t}{1+t^2} \qquad \cos\alpha = \frac{1-t^2}{1+t^2}$$

Proprietà interessante

$$0 < \alpha, \beta, \gamma < \pi$$

$$\alpha+\beta+\gamma = \pi \iff tg\frac{\alpha}{2}\, tg\frac{\beta}{2} + tg\frac{\beta}{2}\, tg\frac{\gamma}{2} + tg\frac{\alpha}{2}\cdot tg\frac{\gamma}{2} = 1 \quad (*)$$

[$\Rightarrow$]

$$tg\frac{\gamma}{2} = tg\frac{\pi-\alpha-\beta}{2} = tg\left(\frac{\pi}{2} - \frac{\alpha+\beta}{2}\right)$$

$$= \frac{1}{tg\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)} = \frac{1 - tg\frac{\alpha}{2}\, tg\beta/2}{tg\frac{\alpha}{2} + tg\,\beta/2}$$

che si ottiene da $(*)$ raccogliendo

[$\Leftarrow$]

$$tg\left(\frac{\pi}{2} - \frac{\alpha+\beta}{2}\right) = \frac{1 - tg\frac{\alpha}{2}\, tg\,\beta/2}{tg\frac{\alpha}{2} + tg\,\beta/2} = tg\frac{\gamma}{2}$$

$$\Downarrow$$

tg è iniettiva in $0, \frac{\pi}{2}$ & $\gamma \in (0,\pi) \Rightarrow \frac{\gamma}{2} \in (0, \pi/2)$

$$\Rightarrow \gamma/2 = \frac{\pi}{2} - \frac{\alpha}{2} - \frac{\beta}{2} \Rightarrow \text{tesi} \qquad \blacksquare$$

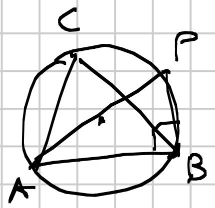
Ex: $\quad \alpha+\beta+\gamma = \pi \Rightarrow tg\alpha + tg\beta + tg\gamma = tg\alpha\cdot tg\beta\cdot tg\gamma$

$$\Rightarrow \sin\alpha + \sin\beta + \sin\gamma = 4\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}$$

Teo dei Seni

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$$

raggio circ. circoscritta



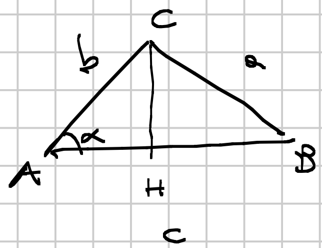
$$A\hat{P}B = \gamma$$

$$c = 2R \cdot \sin\gamma$$

Teorema di Carnot (o dei coseni o Pitagora general.)

Dati b, c lati e angolo α tra essi

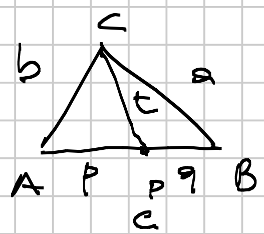
$$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$$



Dim

$$\begin{aligned} a^2 &= BH^2 + CH^2 \\ &= BH^2 + b^2\sin^2\alpha \\ &= (c - b\cdot\cos\alpha)^2 + b^2\sin^2\alpha \\ &= c^2 - 2bc\cos\alpha \underbrace{+ b^2\cos^2\alpha + b^2\sin^2\alpha}_{= b^2} \end{aligned}$$

Teo Stewart



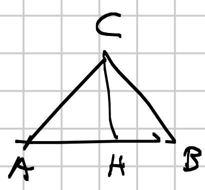
$$c\cdot(t^2 + p\cdot q) = b^2 q + a^2 p$$

Dim (EX) hint Carnot ..

Mediana (cor.)

$$t^2 = \frac{a^2 + b^2}{2} - \frac{c^2}{4}$$

Area



$$\begin{aligned} S_{ABC} &= \frac{AB \cdot CH}{2} \\ &= \frac{1}{2} c \cdot b \cdot \sin\alpha \\ &\overset{(\text{Seni})}{=} \frac{a\cdot b\cdot c}{4R} \end{aligned}$$

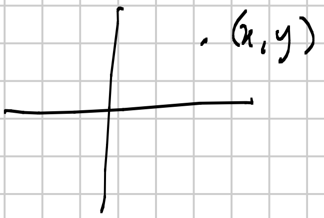
Erone $\quad p = \dfrac{a+b+c}{2} \qquad S = \sqrt{p(p-a)(p-b)(p-c)}$

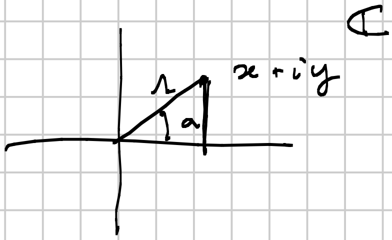
Dim $\quad \cos\gamma = \dfrac{a^2+b^2-c^2}{2a\cdot b}$

$$\sin\gamma = \dots$$

$$S = \frac{1}{2}\, a\cdot b\cdot \sin\gamma = \dots$$

Complessi





i t.c. $i^2 = -1$

$$(a+ib) + (c+id) = (a+c) + i(b+d)$$

$z \in \mathbb{C}$ $\qquad \mathrm{Re}(z) = \Re(z) = a$

$\shortparallel$

$a+ib$ $\qquad \mathrm{Im}(z) = \Im(z) = b$

$$(a+ib)\cdot(c+id) = ac + iad + ibc + i^2bd$$
$$= (ac-bd) + i(ad+bc)$$

$$|x+iy| = \sqrt{x^2+y^2}$$

$$\alpha = \arctan\left(\frac{y}{x}\right)$$

se $x=0$ problem..

$$|z|\cdot(\cos\alpha + i\sin\alpha)$$

$$(\cos\alpha + i\sin\alpha)(x+iy) =$$

$$= x\cos\alpha - y\sin\alpha + i(x\sin\alpha + y\cos\alpha)$$

$$(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) =$$
$$= \cos(\alpha+\beta) + i\sin(\alpha+\beta)$$

$$e^{i\alpha} := \cos\alpha + i\sin\alpha$$

$$e^{i\alpha} \cdot e^{i\beta} = e^{i(\alpha+\beta)}$$

$$\boxed{z = |z| e^{i\alpha}}$$

$(x+iy)^n = ? = (EX)$ de Moivre

$n \in \mathbb{N}$

Esercizi

- $\sin\alpha \cdot \cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$
- $5\cos x + 2\sin x = 1$ risolvere (trovare x)
- Pag 29 esercizi 4 e 9

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

- $5X + 2Y = 1$

$t = tg\frac{x}{2}$

$$5\,\frac{1-t^2}{1+t^2} + 2\,\frac{2t}{1+t^2} = 1$$

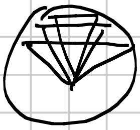
$$5 - 5t^2 + 4t - 1 - t^2 = 0$$

$$6t^2 - 4t - 4 = 0$$

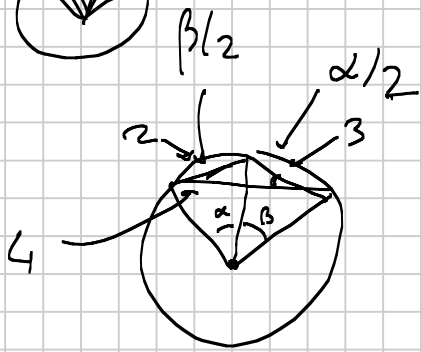
$$tg\frac{x}{2} = t = \frac{1 \pm \sqrt{7}}{3}$$

$$\frac{x}{2} = \operatorname{atan}\left(\frac{1 \pm \sqrt{7}}{3}\right) + k\pi$$

corde lunghe 2, 3, 4



$\alpha \quad \beta \quad \alpha+\beta$



Dobbiamo calcolare $\cos\alpha$

Carnot

$$2^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cdot \cos\frac{\alpha}{2}$$

$$\cos\frac{\alpha}{2} = \frac{7}{8}$$

+ formula duplicazione ..

$$\cos\alpha = \frac{17}{32}$$