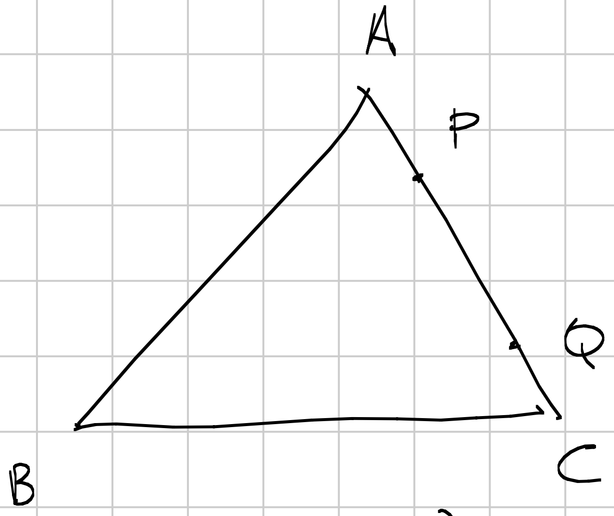
G1 - Advanced - Sam

Titolo nota 02/09/2014



$$AP + BC = AB + CQ$$

$$P = (p : 0 : 1-p)$$

$$Q = (q : 0 : 1-q)$$

$$0 \le p, q \le 1$$

$$A = (1 : 0 : 0)$$

$$A - P \left(= \overrightarrow{AP}\right) = (1-p, 0, p-1) = (x, y, z)$$

$$\mathrm{dist}(A,P)^2 = -a^2 yz - b^2 xz - c^2 xy$$

$$AP = \sqrt{-b^2(1-p)(p-1)} = b(1-p)$$

$$CQ = bq$$

$$\boxed{b(1-p) + a = c + bq}$$

$$R \text{ pt. med di } PQ = \left(\frac{p+q}{2} : 0 : \frac{2-p-q}{2}\right)$$

$$\boxed{\frac{p+q}{2-p-q} = \frac{a}{c}} \qquad q = \frac{b(1-p) + a - c}{b}$$

$$\frac{\cancel{bp}+b-\cancel{bp}+a-c}{2b-\cancel{bp}-b+\cancel{bp}-a+c}=\frac{a}{c}$$

$$bc+\cancel{ac}-c^2=ab-a^2+\cancel{ac}$$

$$a^2-c^2+b(c-a)=0$$

$$(a+c)(a-c)+b(c-a)=0$$

$$(a-c)(a+c-b)=0 \implies a=c.$$

Distanza tra due punti

$$P=(p_1:p_2:p_3) \qquad Q=(q_1:q_2:q_3)$$

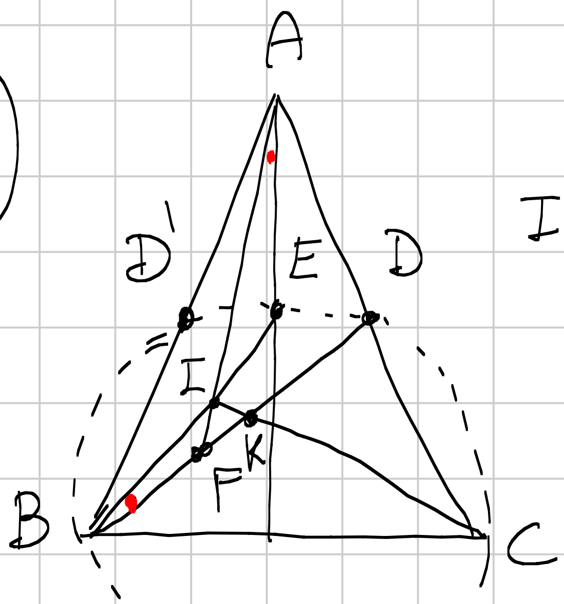
$$\rho=p_1+p_2+p_3 \qquad \sigma=q_1+q_2+q_3$$

$$PQ^2=-a^2\left(\frac{p_2}{\rho}-\frac{q_2}{\sigma}\right)\left(\frac{p_3}{\rho}-\frac{q_3}{\sigma}\right)-b^2(1)(3)$$

$$-c^2(1)(2)$$

$\rho=\sigma=1$ semplifica le cose.

2)



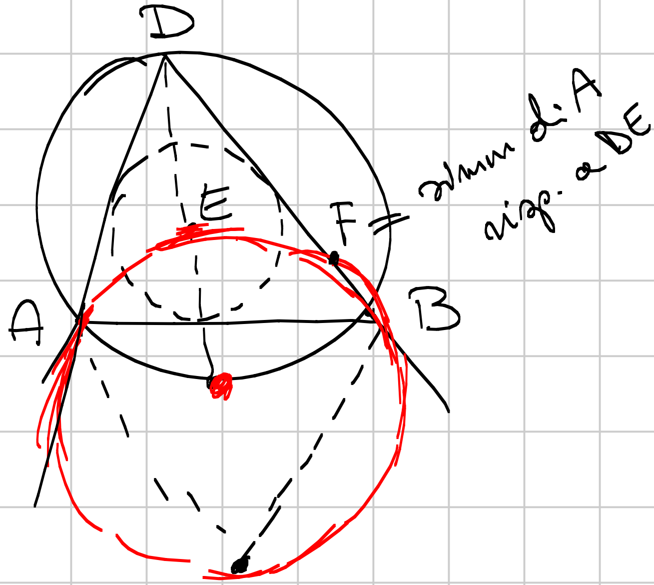
I incentro di $\overset{\triangle}{KAB}$

E pt. medio di $\overset{\frown}{DD'}$

$\implies BE$ bisett. di $\widehat{ABD}$

$\implies E$ incentro di $\overset{\triangle}{ABD}$

Il centro della cf. per A, E, B sta su DE



nel triangolo ABD

$a = BD \quad AD = b \quad AB = c = 2b$

$$E = (a : b : c)$$

$$F = (0 : b : a-b)$$

$$I = \left(a(a-b) : bc : c(a-b) \right)$$

$$C = (-1 : 0 : 2) \qquad K = \left(0 : \frac{b^2}{a^2} : 1 - \frac{b^2}{a^2} \right)$$

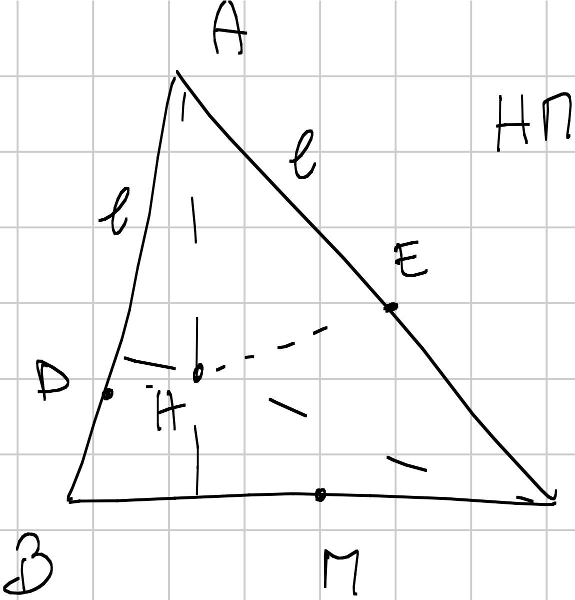
$$D = (0 : 0 : 1)$$

$$DK^2 = \frac{b^4}{a^2} \quad \longleftarrow \quad D - K = \left(0 : -\frac{b^2}{a^2} : \frac{b^2}{a^2} \right)$$

$$DK = \frac{b^2}{a} \qquad a \cdot DK = b^2$$

$$\boxed{DB \cdot DK = DA^2}$$

3)



$HM \perp$ asse rad delle cfr. circo a $\overset{\triangle}{ABC}$ e $\overset{\triangle}{ADE}$

$$H = \left(\frac{1}{b^2+c^2-a^2} : \frac{1}{c^2+a^2-b^2} : \frac{1}{a^2+b^2-c^2} \right)$$

$$S_A = \frac{b^2+c^2-a^2}{2}$$

$$H = \left(\frac{1}{S_A} : \frac{1}{S_B} : \frac{1}{S_C} \right) \qquad S_A S_B + S_B S_C + S_C S_A = S^2$$

$$D = (c-\ell : \ell : 0) \qquad E = (b-\ell : 0 : \ell)$$

$$\det \begin{vmatrix} c-\ell & \ell & 0 \\ b-\ell & 0 & \ell \\ \frac{1}{S_A} & \frac{1}{S_B} & \frac{1}{S_C} \end{vmatrix} = 0$$

$$\ell = \frac{S_A (c S_B + b S_C)}{S_A S_B + S_B S_C + S_C S_A} =$$

$$= \frac{S_A \left(c(a^2+c^2-b^2) + b(a^2+b^2-c^2) \right)}{2 S^2} =$$

$$= \frac{S_A}{2S^2} (c+b)(a-b+c)(a+b-c) =$$

$$= \frac{(b^2+c^2-a^2)(b+c)}{(a+b+c)(b+c-a)}$$

cp per $A, B, C = \{a^2 yz + b^2 xz + c^2 xy = 0\}$

$$-a^2 yz - b^2 xz - c^2 xy + (x+y+z)\cdot(ux + vy + wz) = 0$$

$A: u = 0$

$D: -c^2(c-e)e + c\cdot(ve) = 0$

$E: -b^2(b-e)e + b(ew) = 0$

$v = c(c-e)$

$w = b(b-e)$

Asse rad tra circoscritta e generica è $ux + vy + wz = 0$

$$c(c-e)y + b(b-e)z = 0$$

Strada 1:

$X = (0 : b(b-e) : -c(c-e))$

$A = (1 : 0 : 0)$

$XA \perp HM$ conti con formula tipo prod. scalare

Strada 2: punti all'∞.

I pt. $(f:g:h)$ $(f':g':h')$ su $x+y+z=0$

rappresentano rette $\perp$ se e solo se

$$S_A ff' + S_B gg' + S_C hh' = 0$$

1) Troviamo il pt. all'∞ di

$$c(c-\ell)y + b(b-\ell)z = 0$$

in funzione di ℓ

2) troviamo il pt all'∞ di $H\Pi$

3) Imponiamo la perp.$^{\text{ne}}$

4) Ricaviamo ℓ.

$$\begin{cases} px+qy+rz=0 \\ x+y+z=0 \end{cases} \qquad \det \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ p & q & r \end{vmatrix} =$$

$$= i(r-q) + j(p-r) + k(q-p)$$

$$\boxed{(r-q : p-r : q-p)}$$

Notazione di Conway

$S = 2$ area di ABC

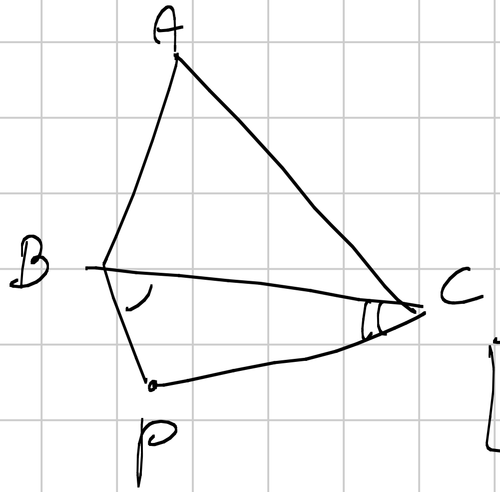
$S_\varphi = S \cdot \cot\varphi \qquad S_A = \dfrac{b^2+c^2-a^2}{2}$ etc.

$$\begin{cases} S_B + S_C = a^2 \text{ e cicliche} \\ S_A S_B + S_B S_C + S_C S_A = S^2 \end{cases} \qquad S_\varphi \cdot S_\psi = S_{\varphi\psi}$$

$$O = (a^2 S_A : b^2 S_B : c^2 S_C) = (S_A(S_B+S_C) : \dots : \dots)$$

$$N = (S^2 + S_B S_C : \dots : \dots)$$

Formula di Conway



$\widehat{CBP} = \varphi \quad \widehat{BCP} = \psi$

$$(-a^2 : S_C + S_\psi : S_B + S_\varphi)$$

$\varphi > 0$ se $\widehat{CBP}$ e $\widehat{CBA}$ hanno orientazione diversa

Rette parallele

$(\cdot)$ $x+y+z=0$ è la retta all'∞. (ℓ_∞)

due rette sono parallele se hanno la stessa intersezione con ℓ_∞

Es: ABC AL bisettrice $\qquad L = (0 : b : c)$

la perp. a BC per L

$H = \left(\frac{1}{S_A} : \frac{1}{S_B} : \frac{1}{S_C}\right)$

$A = (1 : 0 : 0)$

$$\{ S_B y - S_C z = 0 \} = AH$$

pt all'∞ (l'intersez. con ℓ_∞) di AH $\quad px+qy+rz=0$

$$(-S_C - S_B : S_C : S_B) = H_\infty \qquad (r-q : p-r : q-p)$$

$\Rightarrow$ la perp. a BC per L è la retta per L e H_∞

$$\det \begin{vmatrix} x & y & z \\ 0 & b & c \\ \underbrace{-S_C - S_B}_{-a^2} : & S_C : & S_B \end{vmatrix} =$$

$$= x(bS_B - cS_C) + y a^2 c - z a^2 b$$

Intercette

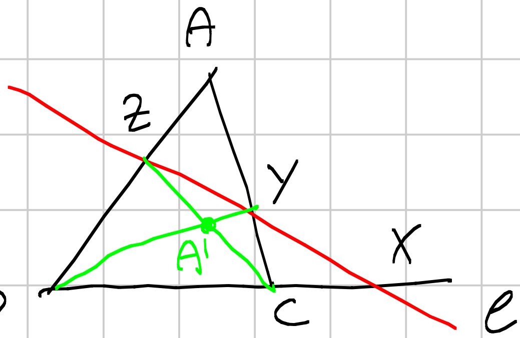
ℓ che interseca i lati in

$$(0 : v : -w) \,,\; (-u : 0 : w) \,,\; (u : -v : 0)$$

(devono avere questa forma per Menelao)

è della forma $\frac{x}{u} + \frac{y}{v} + \frac{z}{w} = 0$

$P = (u : v : w)$ TRIPOLO di ℓ rispetto ad ABC



$A' = BY \cap CZ$

$B' = AX \cap CZ$

$C' = AX \cap BY$

P = dove concorrono AA', BB', CC'

Triangolo pedale

$P = (u : v : w)$

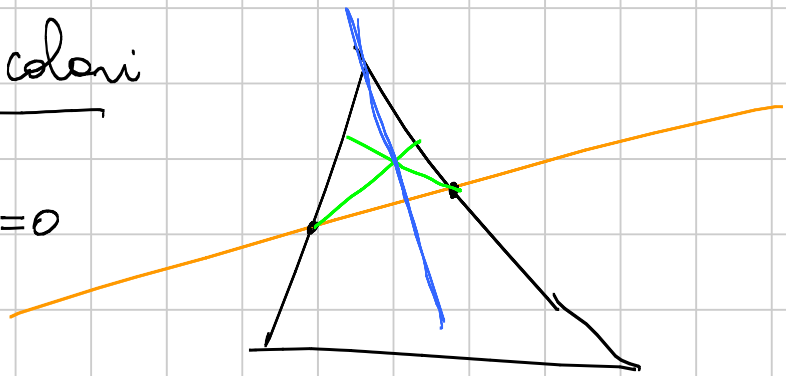
la perp da P a BC $\qquad \det \begin{vmatrix} -a^2 & S_C & S_B \\ u & v & w \\ x & y & z \end{vmatrix} = 0$

$$-(S_B v - S_C w)x + (S_B u + a^2 w)y - (S_C u + a^2 v)z = 0$$

$$P_a = (0 : S_C u + a^2 v : S_B u + a^2 w)$$

↳ somma delle coord $= a^2(u+v+w)$

Rette perpendicolari



$\ell : px + qy + rz = 0$

$(f : g : h)$

$(r-q : p-r : q-p)$

$$(f' : g' : h') = (S_B g - S_C h : S_C h - S_A f : S_A f - S_B g)$$

punto all'∞ delle rette $\perp$ a ℓ.

$\Rightarrow (f : g : h)$ e $(f' : g' : h')$ su $x+y+z=0$ "sono $\perp$" se

$$S_A f f' + S_B g g' + S_C h h' = 0$$

Es: tg alla cf. circo in A è $c^2 y + b^2 z = 0$

Tri tg. ha lati $\quad c^2 y + b^2 z = 0 \quad c^2 x + a^2 z = 0 \quad b^2 x + a^2 y = 0$

$\Rightarrow$ ho verifici

$C'(a^2 : b^2 : -c^2)$, $A'(-a^2 : b^2 : c^2)$, $B'(a^2 : -b^2 : c^2)$

$\Rightarrow$ CC', AA', BB' concorrono in $K = (a^2 : b^2 : c^2)$

Coniugato isogonale $(x : y : z) \to \left(\frac{a^2}{x} : \frac{b^2}{y} : \frac{c^2}{z}\right)$

Coniug. isog di ℓ_∞ : $a^2yz + b^2xz + c^2xy = 0$

Eq. generica di una cf : $-a^2yz - b^2xz - c^2xy +$

$+ (x+y+z)(ux + vy + cz) = 0$

Es: tg alla cf circo in A

$$a^2yz + b^2xz + c^2xy = 0$$

$$\cancel{a^2\frac{y_0z + z_0y}{2}} + b^2\frac{x_0z + \cancel{z_0x}}{2} + c^2\frac{x_0y + \cancel{y_0x}}{2} = 0$$

$$b^2z + c^2y = 0$$

Polare di I : $a^2bz + a^2cy + b^2az + b^2cx + c^2ay + c^2bx = 0$

$$x(b^2c + c^2b) + y(a^2c + c^2a) + z(a^2b + b^2a) = 0$$

Ultime cose: P, Q, R, S con coord. normalizzate

$$P-Q=(x_1,y_1,z_1) \qquad R-S=(x_2,y_2,z_2)$$

1) $PQ \perp RS$

$$\Updownarrow$$

$$0=a^2(z_1y_2+y_1z_2)+b^2(x_1z_2+z_1x_2)+c^2(y_1x_2+x_1y_2)$$

2) $PQ^2=-a^2y_1z_1-b^2x_1z_1-c^2x_1y_1$

3) $[PQR]=[ABC]\cdot \det\begin{pmatrix} P \\ Q \\ R\end{pmatrix}$

4*) Se $\overrightarrow{PQ}=\alpha_1\overrightarrow{AO}+\beta_1\overrightarrow{BO}+\gamma_1\overrightarrow{CO}$
$\overrightarrow{RS}=\alpha_2\overrightarrow{AO}+\beta_2\overrightarrow{BO}+\gamma_2\overrightarrow{CO}$

con $\alpha_i+\beta_i+\gamma_i=0 \quad i=1,2$

$$PQ \perp RS \Leftrightarrow a^2(\beta_1\gamma_2+\beta_2\gamma_1)+ b^2(\alpha_1\gamma_2+\alpha_2\gamma_1)+ c^2(\alpha_1\beta_2+\alpha_2\beta_1)$$