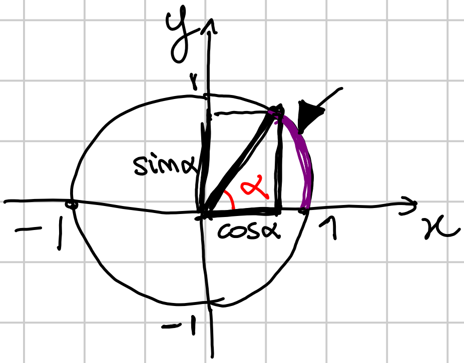
G1 basic

Giada

Titolo nota 02/09/2014

G3 Geometria sintetica
G2 Metodi analitici
G1 Trigonometria



Gradi $\longleftrightarrow$ Radianti

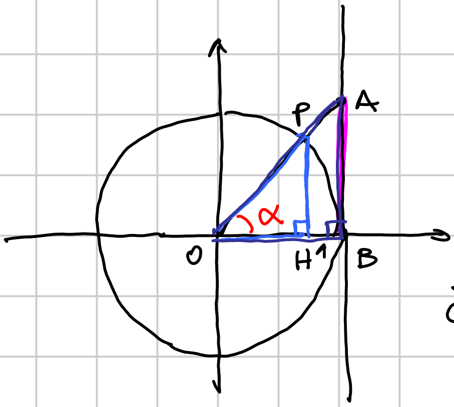
$360° \longleftrightarrow 2\pi$

$\sin, \cos \in [-1, 1]$

$\sin^2\alpha + \cos^2\alpha = 1$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\cot\alpha = \dfrac{1}{\tan\alpha} = \dfrac{\cos\alpha}{\sin\alpha}$



$\dfrac{PH}{OH} = \dfrac{AB}{BO}$

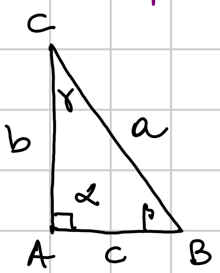
$\Downarrow$

$\dfrac{\sin\alpha}{\cos\alpha} = AB$

$\parallel$

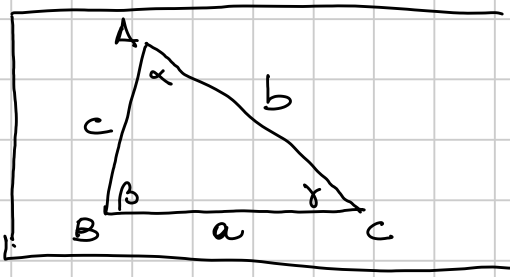
$tg\,\alpha$

Interpretazione geometrica



$b = a \sin\beta$

$c = a \sin\gamma = a\cos\beta$

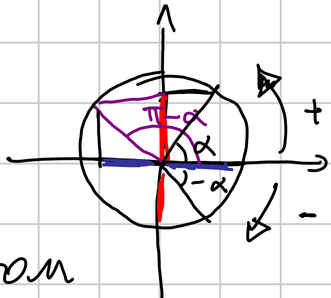


$\gamma = 180° - \alpha - \beta = 90° - \beta$

$\sin(90° - \beta) = \sin\gamma = \cos\beta$

$\tan\beta = \dfrac{b}{c}$

Simmetrie e periodicità



$\sin(90° - \alpha) = \cos\alpha$

$\sin(2\pi + \alpha) = \sin\alpha$ uguale cos, tan

$\sin(-\alpha) = -\sin\alpha$ funzione dispari $(f(-x) = -f(x))$

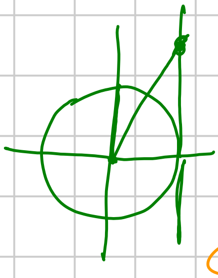
$\cos(-\alpha) = \cos\alpha$ funzione pari

$\sin(\pi - \alpha) = \sin\alpha$

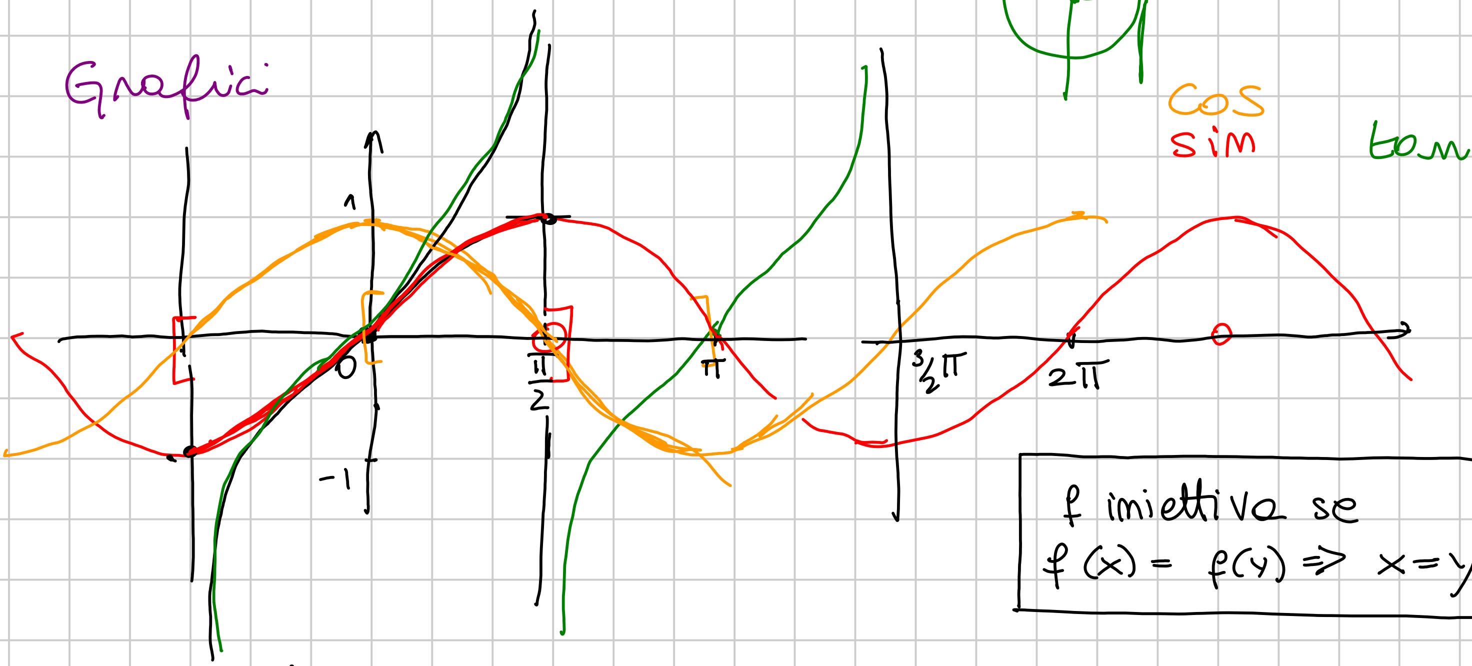
$\cos(\pi - \alpha) = -\cos\alpha$

Valori speciali

	sin	cos	tan
0	0	1	0
$(60°)\ \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$



Grafici



f iniettiva se
$f(x) = f(y) \Rightarrow x = y$

$\sin x = 1$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow$ sin iniettivo

$[0, \pi] \longrightarrow$ cos iniettivo

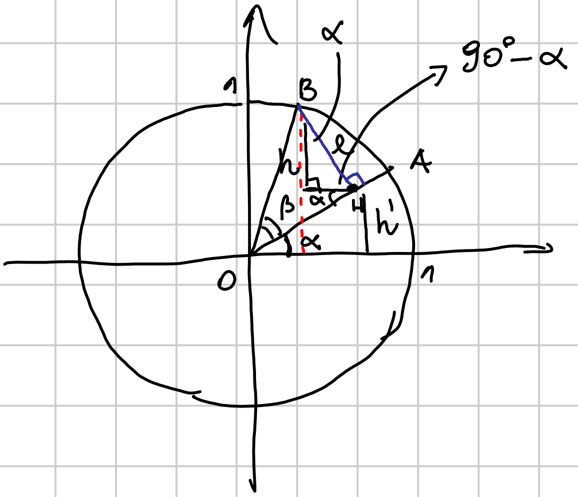
$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow$ tan biettiva (= iniettiva + suriettiva)

Funzioni inverse

arctan : $\mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
arcsin : $[-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
arccos : $[-1,1] \longrightarrow [0,\pi]$

Formule

Addizione, sottrazione, ...



$h + h' = ?$

$l = OB \cdot \sin\beta = \sin\beta$
$h = l \cos\alpha = \sin\beta \cos\alpha$
$h' = \underset{\cos\beta}{\underset{\shortparallel}{OH}} \cdot \sin\alpha = \sin\alpha \cos\beta$

$$\boxed{\sin(\alpha+\beta) = h + h' = \sin\alpha\cos\beta + \sin\beta\cos\alpha}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\operatorname{tg}(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$
$$= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{1 - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1 - \operatorname{tg}\alpha\operatorname{tg}\beta}$$

$\beta = -\gamma$ $\quad \sin(\alpha-\gamma) = \sin\alpha\cos\gamma - \sin\gamma\cos\alpha$

$\beta = \alpha$ $\quad \sin(2\alpha) = 2\sin\alpha\cos\alpha$
$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$
$= 2\cos^2\alpha - 1$
$\operatorname{tg}(2\alpha) = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$

$$\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}} \implies \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

Prostaferesi e Werner

× esercizio

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)$$

Parametriche in $tg\frac{\theta}{2}$

$$t = tg\frac{\theta}{2}$$

$$tg\,\theta = \frac{2t}{1-t^2}$$

$$\cos^2\frac{\theta}{2} = \frac{1}{\tan^2\frac{\theta}{2}+1} = \frac{1}{t^2+1}$$

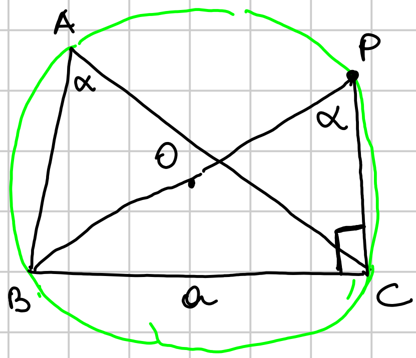
$$\frac{1}{\tan^2\frac{\theta}{2}+1} = \frac{1}{\frac{\sin^2}{\cos^2}+1}$$

$$\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = \frac{2}{t^2+1} - 1 = \frac{1-t^2}{1+t^2}$$

$$\sin\theta = tg\,\theta\cdot\cos\theta = \frac{2t}{1+t^2}$$

Geometria del triangolo

Teorema dei seni



$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R}$$

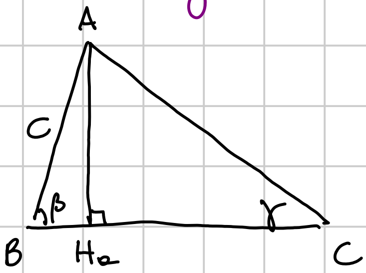
$$\Downarrow$$

$$\sin\gamma = \frac{c}{2R}$$

$$\frac{a}{\sin\alpha} = 2R$$

$$a = 2R\sin\alpha$$

Area trigonometria



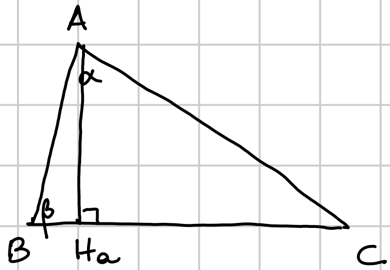
$[ABC]$, S_{ABC} ← area

$$S_{ABC} = \frac{BC \cdot AH_a}{2}$$

$BC = a \qquad AH_a = c\sin\beta = b\sin\gamma$

$$\boxed{S_{ABC} = \frac{1}{2} ab\sin\gamma} \qquad \boxed{S_{ABC} = \frac{abc}{4R}}$$

Teorema di Carnot (o del coseno)



$$\boxed{a^2 = b^2 + c^2 - 2bc\cos\alpha}$$

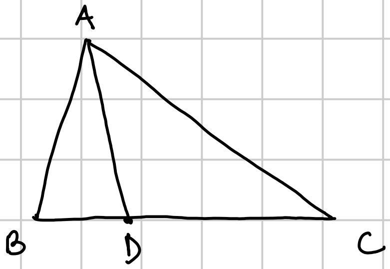
$$b^2 + c^2 - a^2 = (AH_a^2 + CH_a^2) + (AH_a^2 + BH_a^2) - (BH_a + CH_a)^2$$

$$= 2AH_a^2 + \cancel{BH_a^2} + \cancel{CH_a^2} - \cancel{BH_a^2} - \cancel{CH_a^2} - 2BH_a \cdot CH_a$$

$$= 2\underbrace{AH_a^2}_{\substack{= AH_a \cdot AH_a \\ = b\sin\gamma \cdot c\sin\beta}} - 2\underbrace{BH_a}_{= c\cos\beta} \cdot \underbrace{CH_a}_{= b\cos\gamma} = 2bc\underbrace{(\sin\beta\sin\gamma - \cos\beta\cos\gamma)}_{= -\cos(\beta+\gamma)}$$

$$= -2bc\cos(\beta+\gamma) = -2bc\cos(\pi - \alpha) = 2bc\cos\alpha$$

Teorema di Stewart

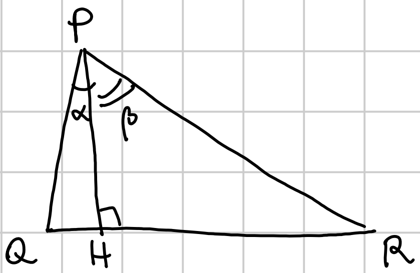


$$a(BD \cdot CD + AD^2) = b^2 \cdot BD + c^2 \cdot CD$$

× esercizio
[Carnot su $\overset{\triangle}{ABD}$ e $\overset{\triangle}{ACD}$]

Esempi

$\sin(\alpha+\beta)$



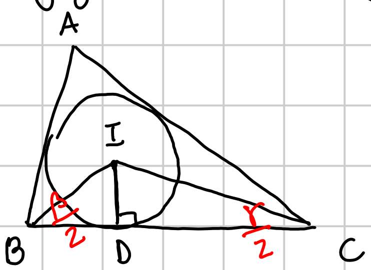
$$S_{PQR} = \frac{1}{2} PQ \cdot PR \cdot \sin(\alpha+\beta)$$
$$\parallel$$
$$S_{PQH} + S_{PHR}$$
$$\parallel$$
$$\frac{1}{2} PH \cdot QH + \frac{1}{2} PH \cdot HR$$
$$\parallel$$
$$\frac{1}{2} PQ \cdot \sin\alpha \, PR\cos\beta + \frac{1}{2} PR \sin\beta \, PQ\cos\alpha$$
$$\Downarrow$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

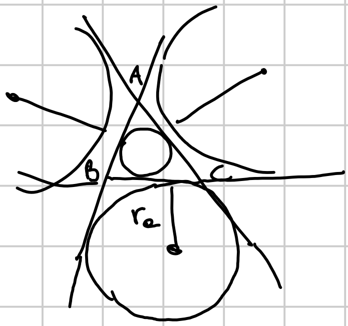
$PH = PQ \cdot \cos\alpha = PR \cdot \cos\beta$

$QH = PQ \cdot \sin\alpha$

$HR = PR \cdot \sin\beta$

Raggio inscritta





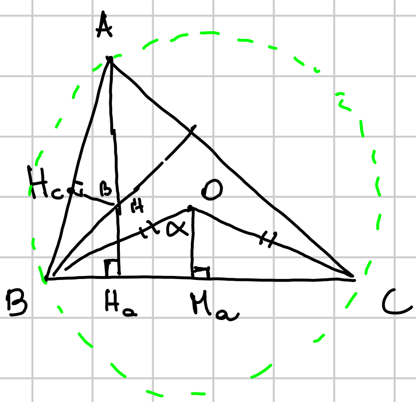
$r \leftarrow$ raggio cfr inscritta

$$\frac{ID}{BD} = \operatorname{tg}\frac{\beta}{2} \qquad \frac{ID}{CD} = \operatorname{tg}\frac{\gamma}{2}$$

$$\frac{BD}{r} = \cot\frac{\beta}{2} \qquad \frac{CD}{r} = \cot\frac{\gamma}{2}$$

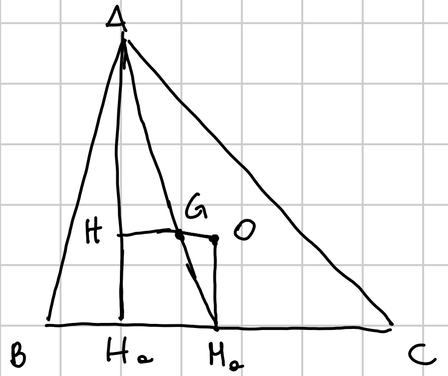
$$\frac{a}{r} = \frac{BD+CD}{r} = \cot\frac{\beta}{2} + \cot\frac{\gamma}{2}$$
$$\Downarrow$$
$$r = \frac{a}{\cot\frac{\beta}{2} + \cot\frac{\gamma}{2}}$$

Ortocentro e circocentro



$$OM_a = BO \cdot \cos\alpha = R\cos\alpha$$

$$AH = \frac{AH_c}{\sin\beta} = \frac{b\cos\alpha}{\sin\beta} = 2R\cos\alpha$$

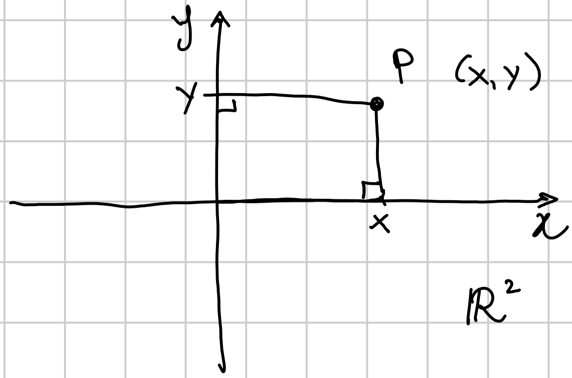


$\underline{AG = 2\,GH_a}$

$\frac{AH}{OM_a} = 2 \qquad \frac{AG}{GH_a} = 2$

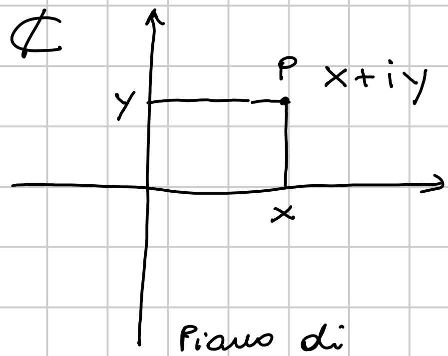
$\Longrightarrow$ O, G, H allineati

Complessi



Forma cartesiana

$\rightsquigarrow$ $\mathbb{C}$



Piano di Gauss

$i^2 = -1$

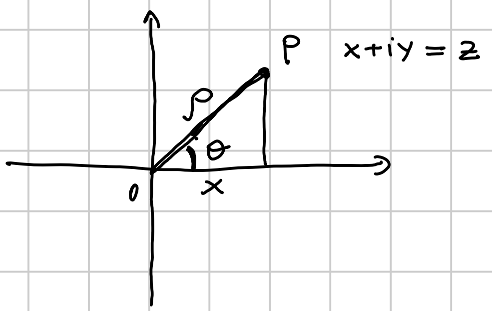
$i \Rightarrow$ unità immaginaria

$$\mathbb{C} = \{x + iy \;:\; x, y \in \mathbb{R}\}$$

$z = x + iy$

$x = \mathrm{Re}(z)$ parte reale

$y = \mathrm{Im}(z)$ parte immaginaria



$$z = x + iy = \rho\cos\theta + i\rho\sin\theta$$
$$= \rho(\cos\theta + i\sin\theta)$$

$\hookrightarrow$ forma polare

$$e^{i\theta} := \cos\theta + i\sin\theta \;\leftarrow$$
$$\Downarrow$$
$$z = \rho e^{i\theta}$$

$\theta = \mathrm{Arg}(z)$ argomento

$\rho = |z|$ modulo

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases} \longleftrightarrow \begin{cases} \rho = \sqrt{x^2 + y^2} \\ \theta = \arctan\frac{y}{x} \end{cases}$$

$$\rho^2(\sin^2\theta + \cos^2\theta) = x^2 + y^2$$

$$\Downarrow$$

$$\rho = \sqrt{x^2+y^2}$$

$$\operatorname{tg}\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x} \implies \theta = \operatorname{arctg}\frac{y}{x}$$

Operazioni

$$z = a+ib = \rho_1 e^{i\theta_1}$$
$$\omega = c+id = \rho_2 e^{i\theta_2}$$

- SOMMA

$$z+\omega = (a+ib)+(c+id) = (a+c)+i(b+d)$$

- PRODOTTO

$$z\cdot w = (a+ib)(c+id) = ac + iad + ibc - bd$$
$$= (ac-bd) + i(ad+bc)$$

$$\boxed{\begin{aligned} a &= \rho_1 \cos\theta_1 \\ b &= \rho_1 \sin\theta_1 \\ c &= \rho_2 \cos\theta_2 \\ d &= \rho_2 \sin\theta_2 \end{aligned}}$$

$$= \rho_1\rho_2(\cos\theta_2\cos\theta_1 - \sin\theta_1\sin\theta_2) + i\rho_1\rho_2(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)$$

$$= \rho_1\rho_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$
$$= \rho_1\rho_2 e^{i(\theta_1+\theta_2)}$$

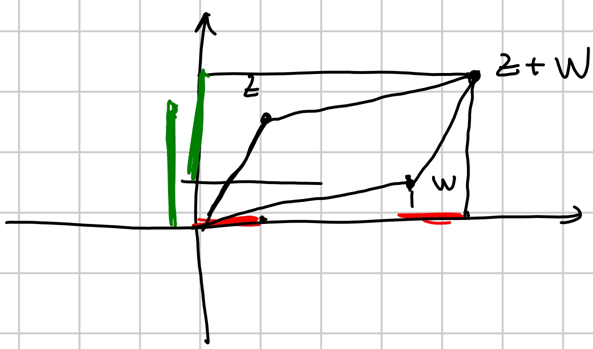
$$\rho_1 e^{i\theta_1}\cdot\rho_2 e^{i\theta_2} = \rho_1\rho_2 e^{i(\theta_1+\theta_2)}$$

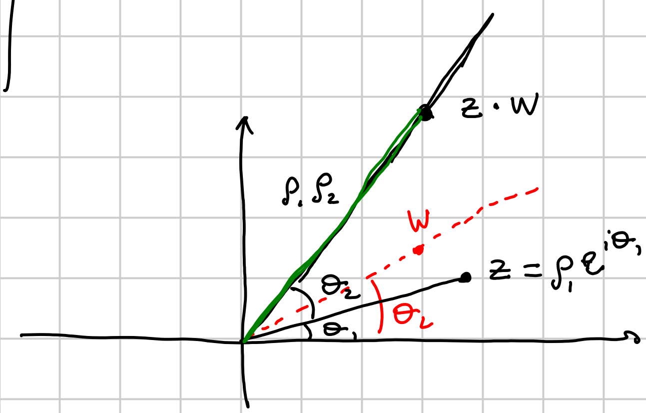
$$\boxed{e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}}$$

$$\Downarrow$$

$$\boxed{e^{i\theta_1}\cdot e^{i\theta_2} = e^{i(\theta_1+\theta_2)}}$$

$$a^x\cdot a^y = a^{x+y}$$





Se $|w| = \rho_2 = 1$, moltiplicare per w, equivale a ruotare di un angolo $\theta_2 = \arg(w)$

Pag. 3

Es. 5 Prostaferesi e Werner

Es. 1,2,3

* Es. 4

Es. 6 Stewart

Es. 7 Erone

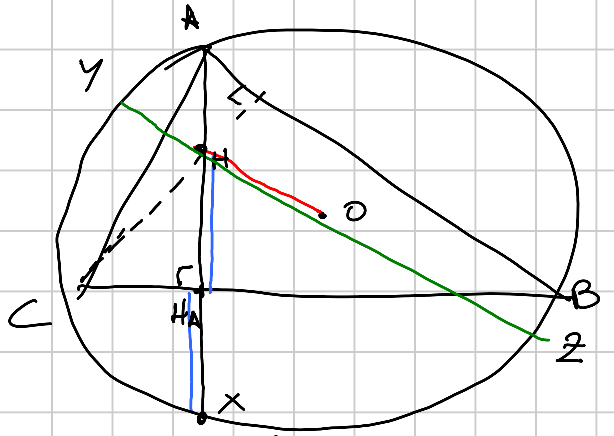
Es. 8 formule tg

Pag. 32

Es. 1, 4, 8, 9, 10

Correzione esercizi

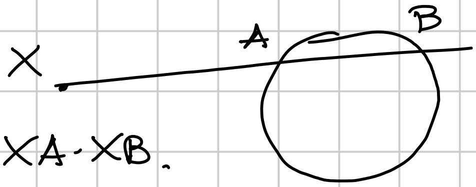
Es 4



$BH_A = c \cos\beta$

$OH^2 = 9R^2 - (a^2+b^2+c^2)$

Potenza rispetto ad una circonferenza + qualcosa sulla simmetrica di H rispetto ai lati.



$XA \cdot XB$.

$\text{Pow}\,H = HY \cdot HZ = (R-OH)(R+OH) = R^2 - OH^2$

$\overset{\shortparallel}{\text{Pow}}\,H = AH \cdot HX = 2\,AH \cdot HH_A = 2\,AH\,(AH_A - AH)$

$AH_A = b \sin\gamma = 2R \sin\beta \sin\gamma$ $\qquad \boxed{\dfrac{b}{\sin\beta} = 2R}$

$AH = 2R\cos\alpha$

$R^2 - OH^2 = 2(2R\cos\alpha)\big(2R(\sin\beta\sin\gamma - \cos\alpha)\big) = 8R^2(\cos\alpha\sin\beta\sin\gamma - \cos^2\alpha)$

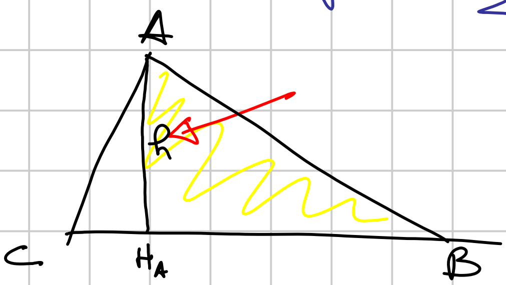
$= 8R^2(\cos\alpha\sin\beta\sin\gamma - 1 + \sin^2\alpha) = -8R^2 +$

$8R^2\left(\cos\alpha\,\dfrac{b}{2R}\,\dfrac{c}{2R} + \dfrac{a^2}{4R^2}\right) = -8R^2 + \underset{b^2+c^2-a^2}{\underset{\shortparallel}{2bc\cos\alpha}} + 2a^2$

$= -8R^2 + a^2 + b^2 + c^2$

$A^2 = p(p-a)(p-b)(p-c)$ con $p = \frac{a+b+c}{2}$.

$A^2 = \frac{a^2 h^2}{4}$



$h = c \cdot \sin\beta \Rightarrow$

$h^2 = c^2 \sin^2\beta = c^2(1-\cos^2\beta) = c^2 - \cancel{c^2}\left(\frac{a^2+c^2-b^2}{2a\cancel{c}}\right)^2$

$A^2 = \frac{a^2\left(c^2 - \left(\frac{a^2+c^2-b^2}{2a}\right)^2\right)}{4} = \frac{a^2c^2 - \frac{(a^2+c^2-b^2)^2}{4}}{4}$

Es 8

$(\alpha+\beta+\gamma = 180°)$

$tg(\alpha) + tg(\beta) + tg(\gamma) = tg\,\alpha\; tg\,\beta\; tg\,\gamma$

$tg(180° - x) = -tg(x)$

$\gamma = 180° - \alpha - \beta.$

$tg\,\gamma = tg(180 - \alpha - \beta) = -tg(\alpha+\beta)$

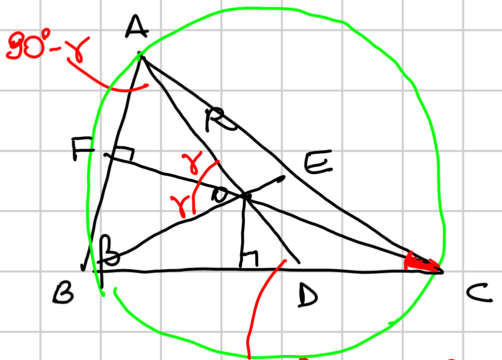
$= \boxed{-\frac{tg(\alpha)+tg(\beta)}{1 - tg(\alpha)tg(\beta)}}$

$a + b - \frac{a+b}{1-ab} = -ab\frac{a+b}{1-ab} \Leftrightarrow \cancel{a+b} - a^2b - ab^2 - \cancel{a+b}$ ok

$= -ab(a+b)$

$tg\,\gamma = \boxed{\frac{tg\,\alpha + tg\,\beta}{tg\,\alpha\, tg\,\beta - 1}}$

Es. 8 pag. 32



$180° - \beta - (90° - \gamma) = 90° - \beta + \gamma$

$\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{2}{AO}$

$\frac{AD}{\sin\beta} = \frac{AB}{\sin(A\hat{D}B)} = \frac{c}{\sin(90° - \beta + \gamma)}$

$= \frac{c}{\cos(\beta - \gamma)}$

$$AD = \frac{c \sin\beta}{\cos(\beta-\gamma)} = \frac{2R \sin\beta \sin\gamma}{\cos(\beta-\gamma)} \qquad \frac{c}{\sin\gamma} = 2R$$

$$BE = 2R\,\frac{\sin\alpha \sin\gamma}{\cos(\alpha-\gamma)}$$

$$\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{2}{R}$$

$$\|$$

$$\frac{\cos(\beta-\gamma)}{2\cancel{R}\sin\beta\sin\gamma} + \frac{\cos(\gamma-\alpha)}{2\cancel{R}\sin\alpha\sin\gamma} + \frac{\cos(\alpha-\beta)}{2\cancel{R}\sin\alpha\sin\beta} = \frac{2}{\cancel{R}}$$

$$\sin\alpha\cos(\beta-\gamma) + \sin\beta\cos(\gamma-\alpha) + \sin\gamma\cos(\alpha-\beta) = 4\sin\alpha\sin\beta\sin\gamma$$

$$\|$$

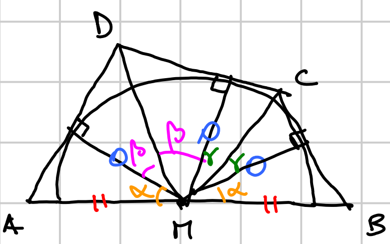
$$\sin\alpha(\cos\beta\cos\gamma + \sin\beta\sin\gamma) + \sin\beta(\qquad) + \sin\gamma(\cos\ldots)$$

$$\sin\alpha\sin\beta\sin\gamma = \sin\alpha\cos\beta\cos\gamma + \sin\beta\cos\alpha\cos\gamma + \sin\gamma\cos\alpha\cos\beta$$

$$\Updownarrow$$

$$\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma = \operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma$$

Es. 9 p. 32



$$AB^2 = 4\,BC \cdot AD$$

$$AB = \frac{2R}{\cos\alpha} \qquad BC = R(\operatorname{tg}\alpha + \operatorname{tg}\gamma)$$

$$AD = R(\operatorname{tg}\alpha + \operatorname{tg}\beta)$$

$$\frac{1}{\cos^2\alpha} = (\operatorname{tg}\alpha + \operatorname{tg}\gamma)(\operatorname{tg}\alpha + \operatorname{tg}\beta)$$

$$\|$$

$$\operatorname{tg}^2\alpha + 1$$

$$\alpha + \beta + \gamma = 90°$$