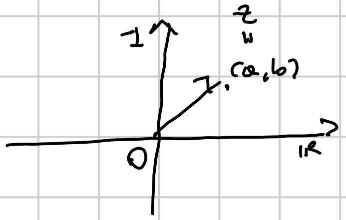
G2 BASIC - Vettori; Complessi; Cartesiane

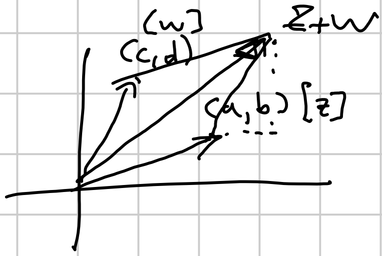
Titolo nota 03/09/2014

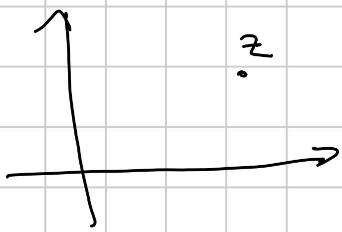
Vettori $\doteq$ Numeri Complessi $\doteq \mathbb{R}^2$

$a+bi=z$ $\qquad i^2=-1$ $\qquad \overrightarrow{OZ}$



$\underbrace{(a+bi)}_{z}+\underbrace{(c+di)}_{w}=(a+c)+(b+d)i$





$\alpha \in \mathbb{C}$

αz?

Rot orig α + Omotetia ragione mod α

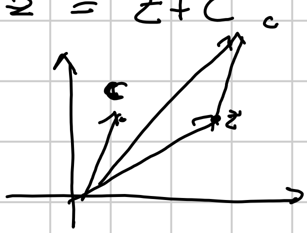
"SPIRAL SIMILARITY" ... Traslazione di un vettore c: $z'=z+c$

(1) Rotazione attorno all'origine di α

$z'=z\cdot e^{i\alpha}$

Rotazione attorno a c con angolo α?

$z'=(z-c)e^{i\alpha}+c$

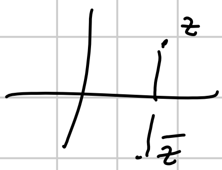


In comune

Punto medio a e b $\qquad \frac{a+b}{2}$ [Punto medio $\vec{A}$ e $\vec{B}$ $\frac{\vec{A}+\vec{B}}{2}$]

Oss. (0)

$\bar{z}$ = simmetrico wrt $\mathbb{R}$



$z\bar{z}=|z|^2$

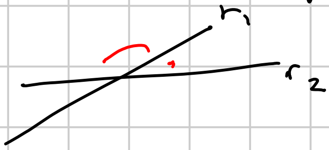
$\frac{z+\bar{z}}{2} = \operatorname{Re} z \qquad \frac{z-\bar{z}}{2i} = \operatorname{Im} z$

Oss. (1)

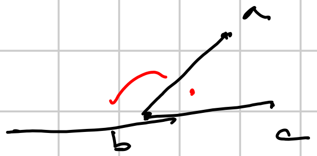
$\boxed{z\in\mathbb{R} \iff z=\bar{z}}$

$\boxed{z\in \operatorname{Im} \iff z=-\bar{z}}$

$\measuredangle(r_1,r_2)$ $\quad r_1$ in senso $\circlearrowleft$ per farla coincidere con r_2

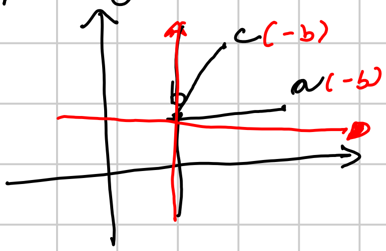


$\overset{\curvearrowleft}{\measuredangle}\, abc$ l'angolo con cui ruotare ab in senso ↺ per ottenere bc



Nota: In generale $\overset{\curvearrowleft}{\measuredangle}\, abc \neq \measuredangle\, abc$ ma può essere o lui o il suo supplementare

Eq. angolo



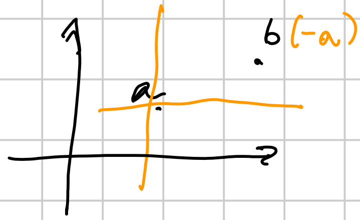
Una relazione con $\varphi = \overset{\curvearrowleft}{\measuredangle}\, abc$:

$$c-b = (a-b)\,e^{i\varphi}\,\frac{|c-b|}{|a-b|}$$

$$\boxed{\frac{c-b}{a-b} = e^{i\varphi}\,\frac{|c-b|}{|a-b|}}$$

— o —

$z = \lambda a \quad \lambda \in \mathbb{R}$



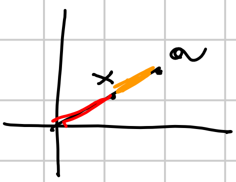
Retta per a e b ?

$z = \lambda(b-a)+a \quad \lambda \in \mathbb{R}$

a, b, c allineati $\iff$ c $\in$ retta per a, b $\iff \exists\, \mu \in \mathbb{R}$ t.c. $c = \mu(b-a)+a \iff \frac{c-a}{b-a} \in \mathbb{R} \iff \boxed{\frac{c-a}{b-a} = \overline{\frac{c-a}{b-a}} = \frac{\bar c - \bar a}{\bar b - \bar a}}$

$\Longleftrightarrow$ a, b, c allineati

Domanda:



$\frac{|x|}{|a|} = \lambda$? $x = \lambda a$

ok.

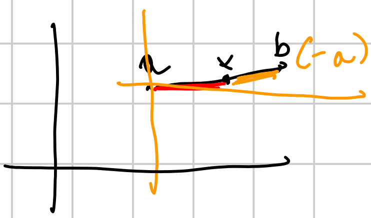
$\frac{|x|}{|x-a|} = \lambda$

$\frac{\bullet}{\bullet} = \lambda \Longrightarrow \frac{\bullet}{\bullet + \bullet} =$

$\frac{R+A}{R} = \frac{1}{\frac{R}{R+A}} = 1+\frac{A}{R} = 1+\frac{1}{\lambda}$

$\frac{R}{R+A} = \frac{\lambda}{1+\lambda}$

$$\boxed{x = \frac{\lambda}{\lambda+1}\, a}$$



$$\boxed{x = \frac{\lambda}{\lambda+1}(b-a)+a = \frac{a+b\lambda}{\lambda+1}}$$

$$\boxed{\vec{X} = \frac{\vec{A} + \vec{B}\lambda}{1+\lambda}}$$

Parallela a ab passante per c : $z = c + \lambda(b-a) \quad \lambda \in \mathbb{R}$

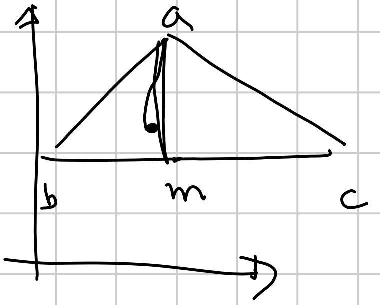
$$\frac{d-c}{b-a} = \frac{\bar{d}-\bar{c}}{\bar{b}-\bar{a}} \Longrightarrow ab \parallel cd$$

Perpendicolare ad ab passante per c: $z = c + i\lambda(b-a) \quad \lambda \in \mathbb{R}$

$$\frac{d-c}{b-a} = -\frac{\bar{d}-\bar{c}}{\bar{b}-\bar{a}} \Longleftrightarrow ab \perp cd$$

E gli angoli? $\measuredangle abc = \measuredangle xyz$

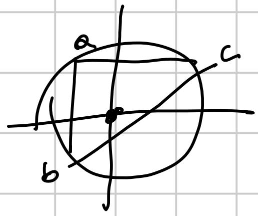
— o —



$$m = \frac{b+c}{2}$$

$$g = \frac{a+2m}{3} = \frac{a+b+c}{3}$$

~o~



$o \quad g \quad h \qquad \frac{og}{gh} = \frac{1}{2}$

$$g = \frac{o + \frac{1}{2}h}{1+\frac{1}{2}} = \frac{2o+h}{3}$$

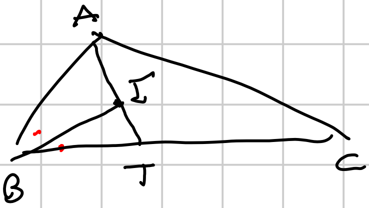
$$3g = 2o + h$$

$$\boxed{a+b+c = \cancel{2o}+h}$$

$h = a+b+c$

$\vec{O} \equiv$ origine $\left[\vec{H} = \vec{OA} + \vec{OB} + \vec{OC} \ \ldots \ \vec{G} = \frac{\vec{A}+\vec{B}+\vec{C}}{3} \right]$

Incentro



$$\frac{BT}{TC} = \frac{c}{b} \qquad BT = \frac{ca}{b+c} \qquad TC = \frac{ba}{b+c}$$

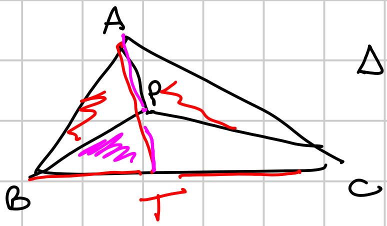
$$\vec{T} = \frac{\vec{B} + \frac{c}{b}\vec{C}}{1+\frac{c}{b}} = \frac{b\vec{B}+c\vec{C}}{b+c}$$

$$\frac{AI}{IT} = \frac{c}{\frac{ca}{b+c}} = \frac{b+c}{a}$$

$$\vec{I} = \frac{\vec{A} + \frac{b+c}{a}\vec{T}}{1+\frac{b+c}{a}} = \frac{a\vec{A}+b\vec{B}+c\vec{C}}{a+b+c}$$

— o —

$$\vec{P} = \frac{\alpha\vec{A}+\beta\vec{B}+\gamma\vec{C}}{\alpha+\beta+\gamma}$$

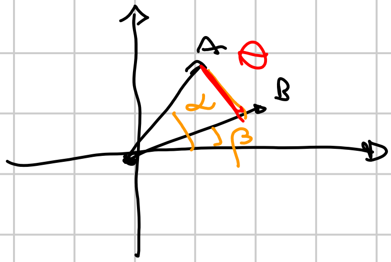


$\Delta PBC : \Delta PCA : \Delta PAB = \alpha : \beta : \gamma$

$$\left[\frac{\alpha}{\alpha+\beta+\gamma} : \frac{\beta}{\alpha+\beta+\gamma} : \frac{\gamma}{\alpha+\beta+\gamma}\right]$$

— o —

PRODOTTO SCALARE



Vettori $\in \mathbb{R}^2$
(0)

$a = |\overrightarrow{OA}|$
$b = |\overrightarrow{OB}|$

$A = (a_1, a_2)$
$B = (b_1, b_2)$

$\langle \vec{A}, \vec{B} \rangle = a_1 b_1 + a_2 b_2$
$= a\cos\alpha \, b\cos\beta + a\sin\alpha \, b\sin\beta =$

$\vec{A} \cdot \vec{B}$

$= ab(\cos\alpha\cos\beta + \sin\alpha\sin\beta) =$
$= ab\cos(\alpha - \beta) = ab\cos\theta =$
$= |\overrightarrow{OA}|\,|\overrightarrow{OB}|\cos\theta$

$a(b+c) \stackrel{?}{=} ab + ac$

<u>SIMM.</u> $\langle \vec{A}, \vec{B} \rangle = \langle \vec{B}, \vec{A} \rangle$

<u>~~BILINEARE~~</u> $\to \langle \vec{A}, \vec{B} + \vec{C} \rangle = \langle \vec{A}, \vec{B} \rangle + \langle \vec{A}, \vec{C} \rangle$ (1)
$\to \langle \vec{A} + \vec{B}, \vec{C} \rangle = \langle \vec{A}, \vec{C} \rangle + \langle \vec{B}, \vec{C} \rangle$ (2)

$\vec{B} = (b_1, b_2)$
$\vec{C} = (c_1, c_2)$
$\vec{B} + \vec{C} = (b_1 + c_1, b_2 + c_2)$

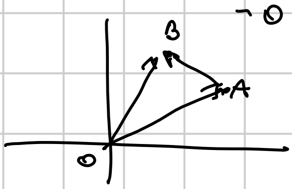
(1) LHS $= a_1(b_1 + c_1) + a_2(b_2 + c_2)$
RHS $= \underline{a_1 b_1 + a_2 b_2} + a_1 c_1 + a_2 c_2$

<u>Oss.</u>
$OA \perp OB \iff \langle \overrightarrow{OA}, \overrightarrow{OB} \rangle = 0$

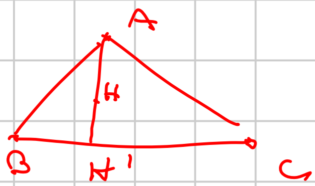
$\overrightarrow{AB} \cdot \overrightarrow{CD} = |\overrightarrow{AB}|\,|\overrightarrow{CD}|\cos\theta$ $\quad (AB \perp CD \iff \overrightarrow{AB} \cdot \overrightarrow{CD} = 0)$

$\| \;(\overrightarrow{OB} - \overrightarrow{OA})$

$\hookrightarrow (\overrightarrow{OB} - \overrightarrow{OA}) \cdot (\overrightarrow{OD} - \overrightarrow{OC}) \stackrel{\downarrow}{=} \overrightarrow{OB} \cdot \overrightarrow{OD} - \overrightarrow{OB} \cdot \overrightarrow{OC}$ ......
$- \overrightarrow{OA} \cdot \overrightarrow{OD} + \overrightarrow{OA} \cdot \overrightarrow{OC}$



Problemino



$\vec{H} = \vec{A} + \vec{B} + \vec{C}$
$\vec{H'} = ?$

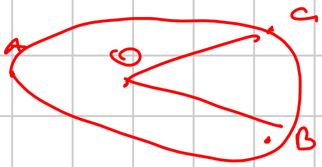
$H' \in BC \to \vec{H'} = \dfrac{\vec{B} + \lambda\vec{C}}{1+\lambda}$

$AH' \perp BC \iff \overrightarrow{AH'} \cdot \overrightarrow{BC} = 0$
$(\vec{H'} - \vec{A}) \cdot (\vec{C} - \vec{B}) = 0$

$\vec{H'} \cdot \vec{C} - \vec{H'} \cdot \vec{B} - \vec{A} \cdot \vec{C} + \vec{A} \cdot \vec{B} = 0$

$\left(\dfrac{\vec{B} + \lambda\vec{C}}{1+\lambda}\right) \cdot \vec{C} - \left(\dfrac{\vec{B} + \lambda\vec{C}}{1+\lambda}\right) \cdot \vec{B} - \vec{A} \cdot \vec{C} + \vec{A} \cdot \vec{B} = 0$

$$\frac{1}{1+\lambda}\underbrace{\vec B\cdot\vec C} + \frac{\lambda}{1+\lambda}\underbrace{\vec C\cdot\vec C} - \frac{1}{1+\lambda}\underbrace{\vec B\cdot\vec B} - \frac{\lambda}{1+b}\underbrace{\vec B\cdot\vec C} - \underbrace{\vec A\cdot\vec C} + \underbrace{\vec A\cdot\vec B} = 0$$



$$\vec C\cdot\vec C = OC^2 = R^2$$

$$\vec B\cdot\vec C = R^2\cdot\cos 2\alpha$$

$$\vec C\cdot\vec A = R^2\cdot\cos 2\beta$$

$$\vec A\cdot\vec B = R^2\cdot\cos 2\gamma$$

GI

$$GI^2 = \vec{GI}\cdot\vec{GI} =$$

$$= (\vec I - \vec G)(\vec I - \vec G) =$$

$$= \left(\frac{a\vec A + b\vec B + c\vec C}{a+b+c} - \frac{\vec A+\vec B+\vec C}{3}\right)\left(a \ldots\right) =$$

$$= \left(\frac{(2a-b-c)\vec A + (-a+2b-c)\vec B + (-a-b+2c)\vec C}{3(a+b+c)}\right)$$

$$(\ldots) =$$

$AH =$

$$\vec H = \vec A + \vec B + \vec C$$

$$\vec{A'} = \vec A$$

$$AH^2 = (\vec B + \vec C)\cdot(\vec B + \vec C) =$$

$$R^2 + 2R^2\cos 2\alpha + R^2 =$$

$$= 2R^2(1+\cos 2\alpha)$$

$$AH = \sqrt{2}\,R\sqrt{1+\cos 2\alpha} = 2R\cos\alpha \text{ [Controlla]}$$

$$\shortparallel$$

$$2\cos^2\alpha$$

$$x^2 + y^2 + \alpha x + \beta y + c = 0$$

$$C\left(-\frac{\alpha}{2}, -\frac{\beta}{2}\right)$$

$$r = \sqrt{\left(-\frac{\alpha}{2}\right)^2 + \left(-\frac{\beta}{2}\right)^2 - c}$$

$AX = XB$?

$\frac{AX}{XB} = \lambda \qquad \lambda > 0$

$\Updownarrow$ ...

$A = (a_1, a_2)$

$B = (b_1, b_2)$

$X = (x, y)$

$AX^2 = \lambda^2 XB^2$

$(x-a_1)^2 + (y-a_2)^2 = \lambda^2\left[(x-b_1)^2 + (y-b_2)^2\right]$

$(1-\lambda^2)x^2 + (1-\lambda^2)y^2 +$

$+ 2(\lambda^2 b_1 - a_1)x + 2(\lambda^2 b_2 - a_2)y \dots = 0$

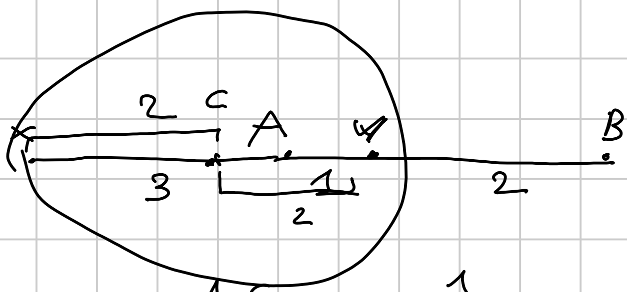
$$x^2 + y^2 \underbrace{-2\frac{a_1-\lambda^2 b_1}{1-\lambda^2}}_{\alpha_1} x \underbrace{-2\frac{a_2-\lambda^2 b_2}{1-\lambda^2}}_{\beta_1} y + \dots = 0$$

$$C\left(\frac{a_1-\lambda^2 b_1}{1-\lambda^2}, \frac{a_2-\lambda^2 b_2}{1-\lambda^2}\right) =$$

$$= \frac{\vec{A} - \lambda^2 \vec{B}}{1-\lambda^2}$$

$C \in AB$

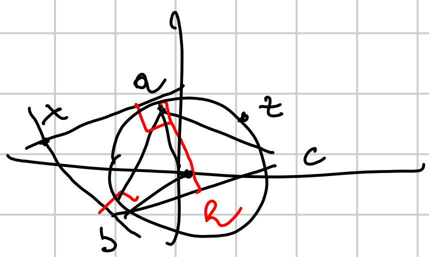
$\left|\frac{AC}{CB}\right| = \cancel{\phi}\ \lambda^4$



3

$\frac{AX}{XB} = \frac{1}{2}$

$\frac{AC}{CB} = \frac{1}{4}$



$|z| = 1 \Rightarrow |z|^2 = 1 \Rightarrow z\bar{z} = 1$

$\Rightarrow \bar{z} = \frac{1}{z}$

$\bar{a} = \frac{1}{a}$ e analog

$x = ?$ $\qquad xa \perp oa$ (1)

$xb \perp ob$

(1) $\frac{a - \cancel{0}}{a - x} = -\frac{\bar{a} - \cancel{0}}{\bar{a} - \bar{x}}$

$$\frac{a}{a-x} = -\frac{\frac{1}{a}}{\frac{1}{a}-\bar{x}} = -\frac{\frac{1}{a}}{\frac{1-a\bar{x}}{a}} = \frac{1}{a\bar{x}-1}$$

$$a(a\bar{x}-1) = a-x$$

$$a^2\bar{x} - a = a - x$$

$$a^2\bar{x} = 2a - x$$

$$\bar{x} = \frac{2a-x}{a^2}$$

(2) $\bar{x} = \frac{2b-x}{b^2}$

(1) = (2) $\Rightarrow$ $\frac{2a-x}{a^2} = \frac{2b-x}{b^2}$

$$b^2(2a-x) = a^2(2b-x)$$

$$2ab^2 - 2a^2b = x(b^2-a^2)$$

$$2ab(b-a) = x(b+a)(b-a)$$

$$x = \frac{2ab}{a+b}$$

$$h = \frac{1}{2}\left(a+b+c-\frac{bc}{a}\right) \quad \ldots$$

11-12 Veloce Correzione | correzione? (18)

13 - 15 - 16 - 19 Complessi

22 (2, 3) - 23 Vettori | pagg. - 3 seg.

12 - 13 (ultimi 2 punti) - 15 - 19

$$\langle \bar{A}, \bar{B} \rangle = R^2 \cos 2\gamma = R^2(2\cos^2\gamma - 1) =$$

$$=$$

$$R = \frac{abc}{4A}$$

$$A^2 = \frac{-a^4-b^4-c^4+2a^2b^2+2b^2c^2+2a^2c^2}{16} =$$

$$R^2 = \frac{a^2b^2c^2}{16A^2} = \frac{a^2b^2c^2}{--}$$

$$2\cos^2\gamma - 1 = 2\left(\frac{b^2+a^2-c^2}{2ab}\right)^2 - 1 =$$

$$= \frac{b^4+a^4+c^4+\cancel{2a^2b^2} - 2c^2b^2 - 2a^2c^2}{2a^2b^2}$$

$$\overline{AM}^2 = \vec{AM}\cdot\vec{AM} = \left(\frac{\vec{B}+\vec{C}-2\vec{A}}{2}\right)\cdot\left(\frac{\vec{B}+\vec{C}-2\vec{A}}{2}\right) =$$

$$LHS = \frac{1}{4}\left(\vec{B}\cdot\vec{B} + \vec{C}\cdot\vec{C} + 4\vec{A}\cdot\vec{A} + 2\vec{B}\cdot\vec{C} - 4\vec{A}\cdot\vec{B} - 4\vec{A}\cdot\vec{C}\right)$$

$$RHS \overset{?}{=} \alpha|\vec{AB}|^2 + \beta|\vec{AC}|^2 + \gamma|\vec{BC}|^2$$

$$\alpha(\vec{B}-\vec{A})\cdot(\vec{B}-\vec{A}) + \beta(\vec{C}-\vec{A})\cdot(\vec{C}-\vec{A}) + \gamma(\vec{C}-\vec{B})\cdot(\vec{C}-\vec{B})$$

$$= (\alpha+\gamma)\vec{B}\cdot\vec{B} + (\beta+\gamma)\vec{C}\cdot\vec{C} + (\alpha+\beta)\vec{A}\cdot\vec{A} - 2\alpha\vec{A}\cdot\vec{B} - 2\beta\vec{A}\cdot\vec{C}$$
$$-2\gamma\vec{B}\cdot\vec{C}$$

$$-2\gamma = \frac{1}{2} \qquad \gamma = -\frac{1}{4}$$

$$-2\alpha = -1 \Rightarrow \alpha = +\frac{1}{2}$$

$$\beta = +\frac{1}{2}$$

$$= \boxed{\frac{1}{2}\underbrace{|\vec{AB}|^2}_{c^2} + \frac{1}{2}\underbrace{|\vec{AC}|^2}_{b^2} - \frac{1}{4}\underbrace{|\vec{BC}|^2}_{a^2}}$$

$$\Sigma m^2 = \sum_{cyc}\frac{1}{2}c^2 + \frac{1}{2}b^2 - \frac{1}{4}a^2 = \boxed{\frac{3}{4}(a^2+b^2+c^2)}$$

12 $A(x_1,y_1)$ $\quad X(x,y)$

$B(x_2,y_2)$ $\qquad \vec{XA}\cdot\vec{XB} + \vec{XB}\cdot\vec{XC} + \vec{XA}\cdot\vec{XC} = 0$

$C(x_3,y_3)$

$$\vec{XA}\cdot\vec{XB} = (\vec{OA}-\vec{OX})\cdot(\vec{OB}-\vec{OX}) =$$

$$= x_1x_2 + y_1y_2 - (xx_1 + \boxed{yy_1}) - (xx_2 + yy_2) + x^2 + y^2$$

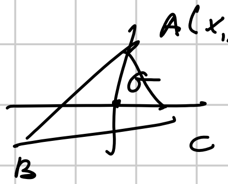
$$\vec{XB}\cdot\vec{XC} = x_2x_3 + y_2y_3 - (xx_2 + yy_2) - (xx_3 + yy_3) + x^2 + y^2$$

$$\vec{XC}\cdot\vec{XA} = x_1x_3 + y_1y_3 - (xx_1 + \boxed{yy_1}) - (xx_3 + yy_3) + x^2 + y^2$$

(1)

$$x^2 + y^2 \underbrace{- \frac{2x(x_1+x_2+x_3)}{3}}_{\alpha} \underbrace{- \frac{2y(y_1+y_2+y_3)}{3}}_{\beta} + \frac{x_1x_2+x_2x_3+x_1x_3}{3} + \frac{y_1y_2+y_2y_3+y_1y_3}{3} = 0$$

$$\left(-\frac{\alpha}{2}, -\frac{\beta}{2}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$



$\boxed{\text{Origine nel baricentro}}$

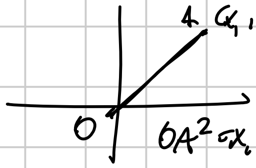
$$x_1+x_2+x_3 = 0$$
$$y_1+y_2+y_3 = 0$$

$(0,0) \equiv$ baricentro

$$x^2+y^2$$

$$\underset{\substack{\shortparallel \\ 0}}{(x_1+x_2+x_3)^2} = x_1^2+x_2^2+x_3^2+2(x_1x_2+x_2x_3+x_1x_3)$$

$$x_1x_2+x_2x_3+x_1x_3 = -\frac{x_1^2+x_2^2+x_3^2}{2}$$
$$y_1y_2+y_2y_3+y_1y_3 = -\frac{y_1^2+y_2^2+y_3^2}{2}$$



$$x^2+y^2 \underbrace{-\frac{(x_1^2+y_1^2)}{6} - \frac{(x_2^2+y_2^2)}{6} - \frac{(x_3^2+y_3^2)}{6}} = 0$$

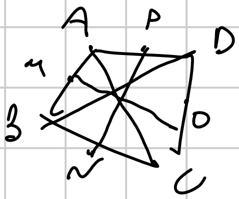
$$R^2 = \frac{GA^2}{6}+\frac{GB^2}{6}+\frac{GC^2}{6} = \frac{\left(\frac{2}{3}AM\right)^2}{6} + \dots =$$
$$= \frac{2}{27}\sum AM^2 = \frac{\cancel{2}}{\cancel{27}_9}\,\frac{\cancel{3}}{\cancel{4}_2}(a^2+b^2+c^2) = \frac{a^2+b^2+c^2}{18}$$

<u>19</u>

$AC \perp BD$

$\Updownarrow$

$MO = NP$



$$M = \frac{A+B}{2} \qquad O = \frac{C+D}{2}$$
$$N = \frac{B+C}{2} \qquad P = \frac{A+D}{2}$$

$MO = NP \iff$

$$\iff MO^2 = NP^2 \iff \vec{MO}\cdot\vec{MO} = \vec{NP}\cdot\vec{NP} \iff$$
$$\iff (\vec{O}-\vec{M})(\vec{O}-\vec{M}) = (\vec{P}-\vec{N})(\vec{P}-\vec{N}) \iff$$
$$\iff \left(\frac{C+D-A-B}{2}\right)\left(\frac{C+D-A-B}{2}\right) = \left(\frac{A+D-B-C}{2}\right)\left(\frac{A+D-B-C}{2}\right)$$

$$\cancel{C\cdot C} + \cancel{D\cdot D} + \cancel{A\cdot A} + \cancel{B\cdot B} + 2C\cdot D - \cancel{2A\cdot C} - 2B\cdot C - 2A\cdot D - \cancel{2B\cdot D} + 2A\cdot B$$
$$\cancel{C\cdot C} + \cancel{D\cdot D} + \cancel{A\cdot A} + \cancel{B\cdot B} - 2D\cdot C - \cancel{2A\cdot C} + 2B\cdot C + 2A\cdot D - \cancel{2B\cdot D} - 2A\cdot B$$
$$C\cdot D - B\cdot C - A\cdot D + A\cdot B = 0 \iff$$

$$\iff C(D-B) - A(D-B) = 0$$

$(C-A)(D-B) = 0$

$\Rightarrow \vec{AC}\cdot\vec{BD} = 0 \Rightarrow AC \perp BD$

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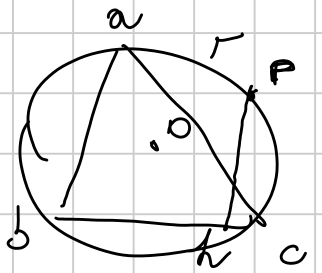

$p \in \Gamma$

$h = ?$

$h \in bc \longleftrightarrow \frac{h-c}{b-c} = \frac{\bar h - \bar c}{\bar b - \bar c}$ (1)

$ph \perp bc \longleftrightarrow \frac{c-b}{h-p} = -\frac{\bar c - \bar b}{\bar h - \bar p}$ (2)

$$-\frac{(h-c)}{b-c} = \frac{\bar h - \frac{1}{c}}{\frac{1}{b}-\frac{1}{c}} = \frac{\frac{c\bar h - 1}{\cancel{c}}}{\frac{c-b}{b\cancel{c}}} \to c-h = bc\bar h - b$$

$$\boxed{\bar h = \frac{b+c-h}{bc}}$$

allineamento con il lato

$$(2) \quad \frac{\cancel{c-b}}{h-p} = -\frac{\frac{1}{c}-\frac{1}{b}}{\frac{b+c-h}{bc}-\frac{1}{p}} = \frac{\frac{\cancel{b-c}}{bc}}{\frac{p(b+c-h)-bc}{bcp}} \Rightarrow$$

$$\frac{1}{h-p} = \frac{p}{p(b+c-h)-bc}$$

MODULO L'ERRORE

~~h-p~~

$p(b+c) - hp - bc = ph - p^2$

$p(b+c) - bc + p^2 = 2ph$

$$h = \frac{1}{2}\left[(b+c) - \frac{bc}{p} + p\right] = \frac{1}{2}\left[p+b+c-\frac{bc}{p}\right]$$

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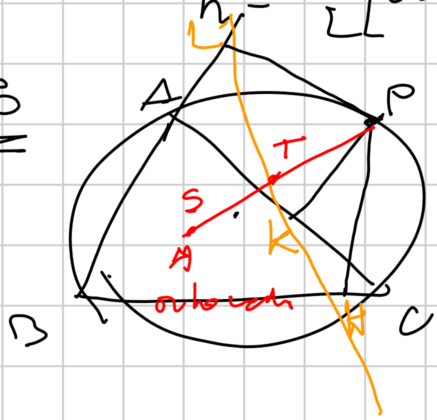

Centrato in O

⊙ABC ha raggio 1

$h = \frac{1}{2}\left(p+b+c-\frac{bc}{p}\right)$

$k = \frac{1}{2}\left(p+a+c-\frac{ac}{p}\right)$

$l = \frac{1}{2}\left(p+a+b-\frac{ab}{p}\right)$

$$\frac{h-k}{l-k} \overset{!}{=} \frac{\bar h - \bar k}{\bar l - \bar k}$$

$$LHS = \frac{\cancel{\frac{1}{2}}\left(b-a-\frac{bc}{p}+\frac{ac}{p}\right)}{\cancel{\frac{1}{2}}\left(b-c-\frac{ab}{p}+\frac{ac}{p}\right)} = \frac{\frac{p(b-a)-c(b-a)}{\cancel{p}}}{\frac{p(b-c)-a(b-c)}{\cancel{p}}} = \frac{(p-c)(b-a)}{(p-a)(b-c)}$$

$$\overline{LHS} = \frac{(\bar p-\bar c)(\bar b-\bar a)}{(\bar p-\bar a)(\bar b-\bar c)} = \frac{\left(\frac{1}{p}-\frac{1}{c}\right)\left(\frac{1}{b}-\frac{1}{a}\right)}{\left(\frac{1}{p}-\frac{1}{a}\right)\left(\frac{1}{b}-\frac{1}{c}\right)} = \frac{\frac{c-p}{\cancel{pc}}\,\frac{a-b}{\cancel{ba}}}{\frac{a-p}{\cancel{pa}}\,\frac{c-b}{\cancel{bc}}} = \frac{(c-p)(a-b)}{(a-p)(c-b)} =$$

ES. $\Rightarrow$

PH è bisecata dalla retta di simson di P (← ovvio in sintetico)

↑ ortocentro

FINE