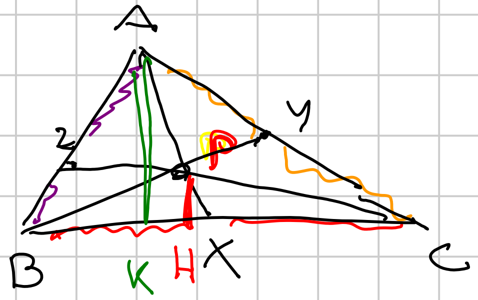
G3 BASIC - Sintetica

Teorema di Ceva.



AX, BY, CZ concorrono $\overset{?}{\Leftrightarrow}$ $\frac{BX}{XC}\cdot\frac{CY}{YA}\cdot\frac{AZ}{ZB}=1$

$\boxed{a:b=c:d}$ Hp

$\boxed{a-c:b-d=a:b}$ Th

$\frac{a-c}{b-d}=\frac{a}{b}$ $\qquad$ $\frac{a}{b}=\frac{c}{d}$

$\Updownarrow$

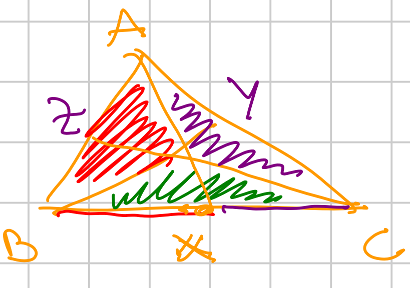
$\boxed{\cancel{ab}-bc=\cancel{ab}-ad}$ $\qquad$ $ad=bc$

$\Rightarrow$ (1) $\frac{BX}{XC}=\frac{[PBX]}{[PXC]}$

$\frac{BX}{XC}=\frac{[ABX]}{[AXC]}$

$\frac{[ABX]}{[AXC]}=\frac{[PBX]}{[PXC]} \Rightarrow \frac{[ABX]-[PBX]}{[AXC]-[PXC]}=\frac{[ABX]}{[AXC]} \Rightarrow$

$\Rightarrow \frac{[PAB]}{[PCA]}=\frac{[ABX]}{[AXC]}=\frac{BX}{XC}$



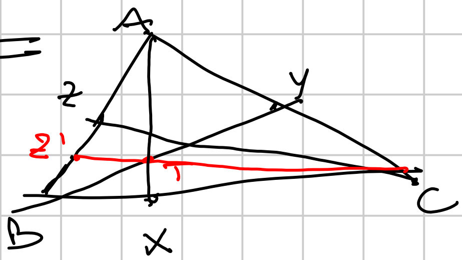
$\frac{BX}{XC}=\frac{\bullet}{\bullet}$

$\frac{CY}{YA}=\frac{\bullet}{\bullet}$

$\frac{AZ}{ZB}=\frac{\bullet}{\bullet}$

$\frac{BX}{XC}\cdot\frac{CY}{YA}\cdot\frac{AZ}{ZB}=1$

$\Leftarrow$

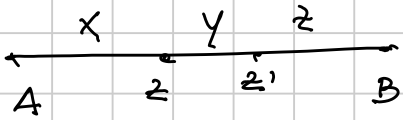


$\frac{BX}{XC}\frac{CY}{YA}\frac{AZ}{ZB}=1$ x hp

$T=AX\cap BY$

$\frac{BX}{XC}\frac{CY}{YA}\frac{AZ'}{Z'B}=1$ x ceva diretto

$$\frac{Az}{zB} = \frac{Az'}{z'B}$$



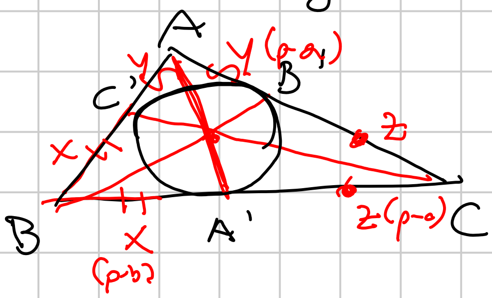
$$\frac{x}{y+z} = \frac{x+y}{z}$$

$\Updownarrow$

$$\cancel{xz} = xy + \cancel{xz} + y^2 + yz$$
$$xy + y^2 + yz = 0$$

$\Downarrow$

P.to di Gergonne



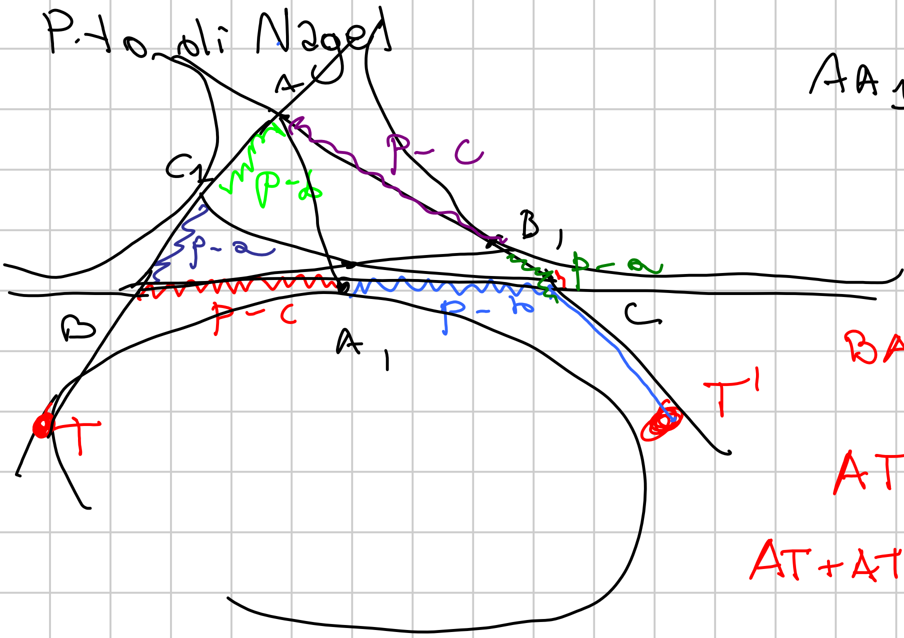
AA', BB', CC' concorrono

$$\begin{cases} x+z=a \\ y+z=b \\ x+y=c \end{cases} \Longrightarrow \begin{cases} y=\frac{b+c-a}{2} = p-a \\ x=\frac{c+a-b}{2} = p-b \\ z=\frac{a+b-c}{2} = p-c \end{cases}$$

$$x+y+z = \underbrace{\frac{a+b+c}{2}}_{p}$$

$$\frac{BA'}{A'C}\,\frac{CB'}{B'A}\,\frac{AC'}{C'B} = \frac{\cancel{p-b}}{\cancel{p-c}}\,\frac{\cancel{p-c}}{\cancel{p-a}}\,\frac{\cancel{p-a}}{\cancel{p-b}} = 1.$$

P.to di Nagel



AA_1, BB_1, CC_1 concorrono

p.to di Nagel ABC

$$BA_1 \overset{?}{=} p-c$$

$$AT = AT' \quad (1)$$

$$AT+AT' = AB + \underbrace{\underset{\parallel}{BT} + AC + \underset{\parallel}{CT'}}_{BC \,=\, BA_1 + CA_1} = 2p$$

$BT = BA_1$, $CT' = CA_1$

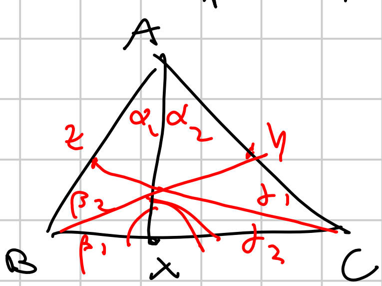
$$AT = AT' = p$$

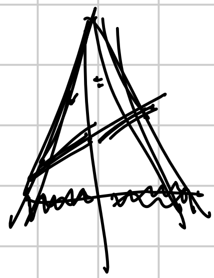
$$BA_1 = BT = AT - AB = p-c$$

$$\frac{BA_1}{A_1C}\,\frac{CB_1}{B_1A}\,\frac{AC_1}{C_1B} = \frac{\cancel{p-c}}{\cancel{p-b}}\,\frac{\cancel{p-a}}{\cancel{p-c}}\,\frac{\cancel{p-b}}{\cancel{p-a}} = 1. \quad \Downarrow$$

Ceva trigonometrico

$$\frac{BX}{XC} = \frac{\dfrac{AB\sin\alpha_1}{\cancel{\sin A\hat{X}B}}}{\dfrac{AC\sin\alpha_2}{\cancel{\sin A\hat{X}C}}} =$$





$= \frac{c \sin\alpha_1}{b \sin\alpha_2}$ (*)

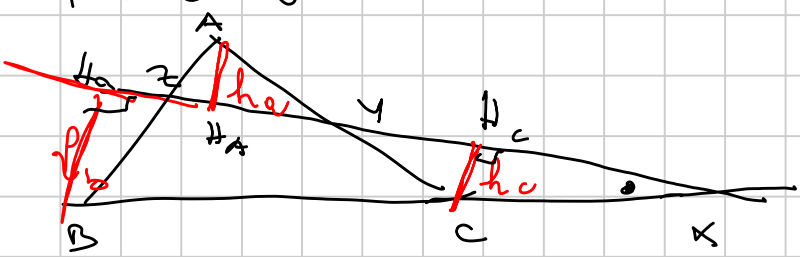
AX, BY, CZ concorrono $\Leftrightarrow$

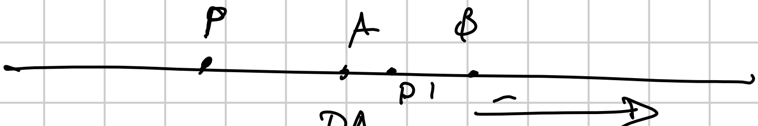
$$\frac{BX}{XC}\,\frac{CY}{YA}\,\frac{AZ}{ZB} = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{\cancel{c}\sin\alpha_1}{\cancel{b}\sin\alpha_2}\,\frac{\cancel{a}\sin\beta_1}{\cancel{c}\sin\beta_2}\,\frac{\cancel{b}}{\cancel{a}}\,\frac{\sin\gamma_1}{\sin\gamma_2} = 1 \Leftrightarrow$$

$$\Leftrightarrow \frac{\sin\alpha_1 \sin\beta_1 \sin\gamma_1}{\sin\alpha_2 \sin\beta_2 \sin\gamma_2} = 1$$

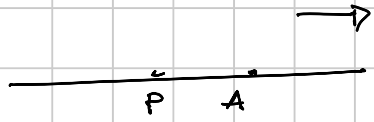
Menelao





$\frac{PA}{AB} > 0$

$\frac{P'A}{AB} < 0$



$PA > 0$
$AP < 0$

X, Y, Z allineati

$\Updownarrow$

$$\frac{BX}{XC}\,\frac{CY}{YA}\,\frac{AZ}{ZB} = -1$$

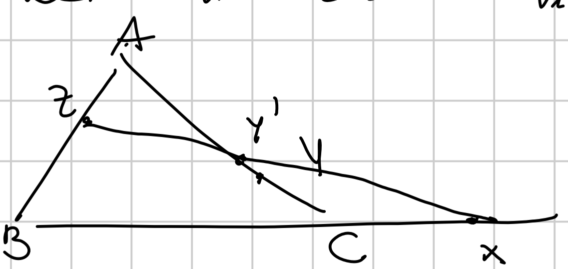
$\Longrightarrow$ $C\hat{X}H_c \sim B\hat{X}H_b \Rightarrow \frac{BX}{XC} = -\frac{BH_b}{CH_c} = -\frac{h_b}{h_c}$

$C\hat{Y}H_c \sim A\hat{Y}H_A \Rightarrow \frac{CY}{YA} = \frac{CH_c}{AH_A} = \frac{h_c}{h_a}$

$A\hat{Z}H_A \sim B\hat{Z}H_B \Rightarrow \frac{AZ}{ZB} = \frac{h_a}{h_b}$

$$\frac{BX}{XC}\cdot\frac{CY}{YA}\cdot\frac{AZ}{ZB} = -\frac{\cancel{h_b}}{\cancel{h_c}}\,\frac{\cancel{h_c}}{\cancel{h_a}}\,\frac{\cancel{h_a}}{\cancel{h_b}} = -1$$

$\Longleftarrow$

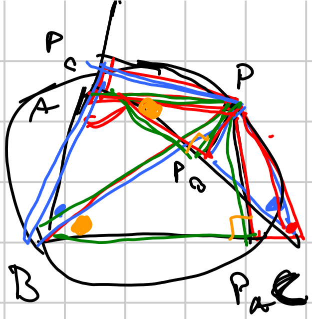


$\frac{BX}{XC}\,\frac{CY}{YA}\,\frac{AZ}{ZB} = -1$ x hp.

$\frac{BX}{XC}\,\frac{CY'}{Y'A}\,\frac{AZ}{ZB} = -1$ x Menelao nelle hp. del diretto

$\Downarrow$

$\frac{CY}{YA} = \frac{CY'}{Y'A}$ ASSURDO.



In modulo

LHS

$$\frac{BP_A}{P_AC}\cdot\frac{CP_B}{P_BA}\cdot\frac{AP_C}{P_CB} = 1$$

$$B\hat{P_A}P \sim A\hat{P_B}P$$

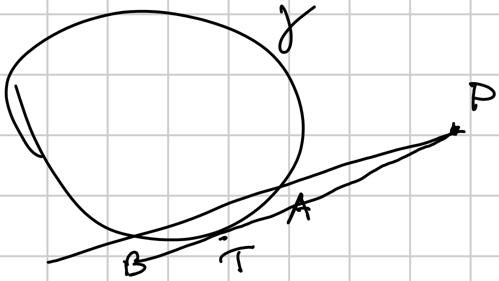
$$\frac{BP_A}{P_BA} = \frac{PB}{PA}$$

$$P\hat{P_B}C \sim P\hat{P_C}B \Rightarrow \frac{CP_B}{P_CB} = \frac{PC}{PB}$$

$$A\hat{P}P_C \sim P\hat{P_A}C \Rightarrow \frac{AP_C}{P_AC} = \frac{PA}{PC}$$

$$LHS = \frac{\cancel{PB}}{\cancel{PA}}\,\frac{\cancel{PC}}{\cancel{PB}}\,\frac{\cancel{PA}}{\cancel{PC}} = 1$$

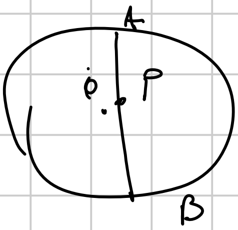
Est.



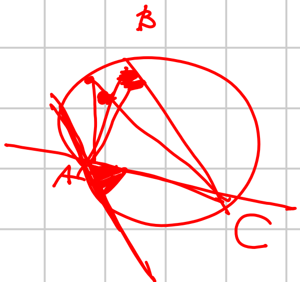
$Pow_\gamma P > 0$

$PA\cdot PB = cost$ al variare della retta

Int



$Pow_\gamma P < 0$

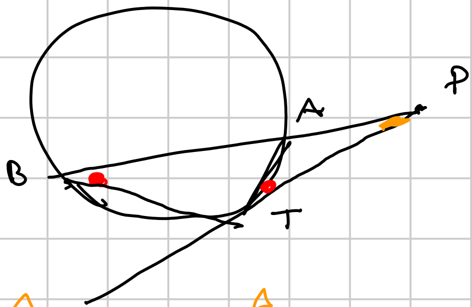


Rossi =

Oss.

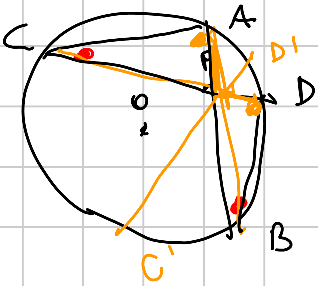
$PT^2 = PA\cdot PB$

Est.



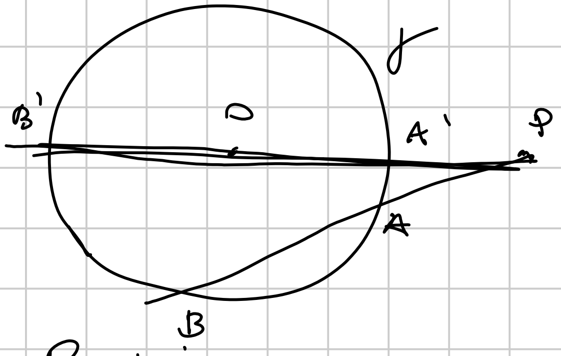
$$P\hat{T}A \sim P\hat{B}T \Rightarrow \frac{PT}{PA} = \frac{PB}{PT} \Rightarrow PT^2 = PA\cdot PB$$

Int.



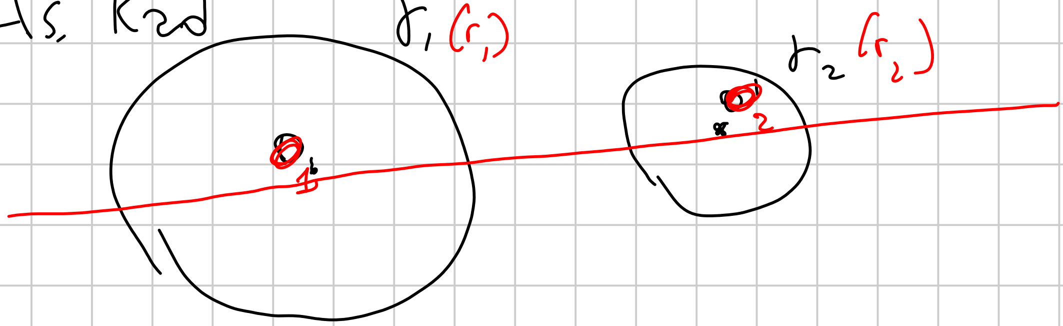
$$P\hat{A}C \sim P\hat{D}B \Rightarrow \frac{PA}{PC} = \frac{PD}{PB} \Rightarrow$$

$$\Rightarrow PA\cdot PB = PC\cdot PD$$



$$PA\cdot PB = Pow_\gamma P = PA'\cdot PB' =$$
$$= (PO - \overset{r}{OA'})(PO + \overset{r}{OB'}) =$$
$$= PO^2 - r^2$$

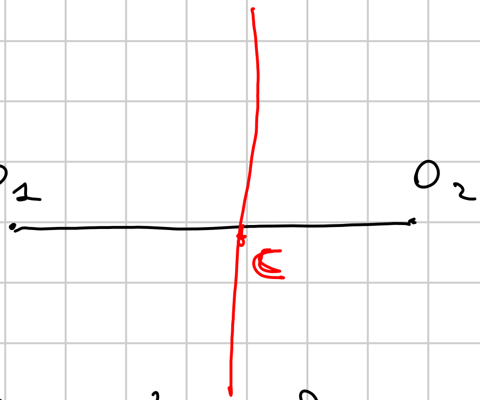
As. Rad



P s.t. $Pow_{\gamma_1} P = Pow_{\gamma_2} P$

$$PO_1^2 - r_1^2 = PO_2^2 - r_2^2$$

$$\boxed{PO_1^2 - PO_2^2 = \underbrace{r_1^2 - r_2^2}_{=k}} \quad (*)$$

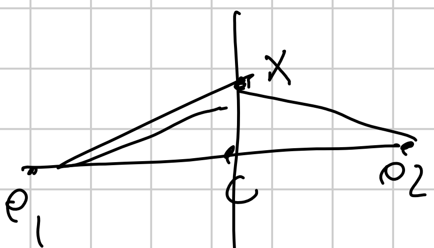


Chi è il luogo dei P t.c. $PO_1^2 - PO_2^2 = k$?

Chiamo la retta $\perp$ ad O_1O_2 passante per C dove

$$\boxed{CO_1^2 - CO_2^2 = k}$$

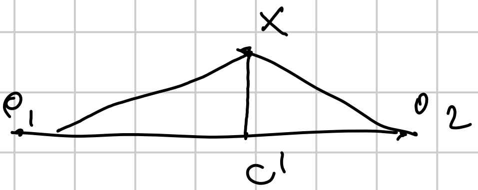
$$\begin{cases} CO_1 + CO_2 = O_1O_2 \\ CO_1^2 - CO_2^2 = k \end{cases}$$



$$XO_1^2 - XO_2^2 \overset{?}{=} (XC^2 + O_1C^2) - (XC^2 + O_2C^2) = O_1C^2 - O_2C^2 = k$$

X soddisfa $(*)$

$$XO_1^2 - XO_2^2 = k \quad (*_1)$$

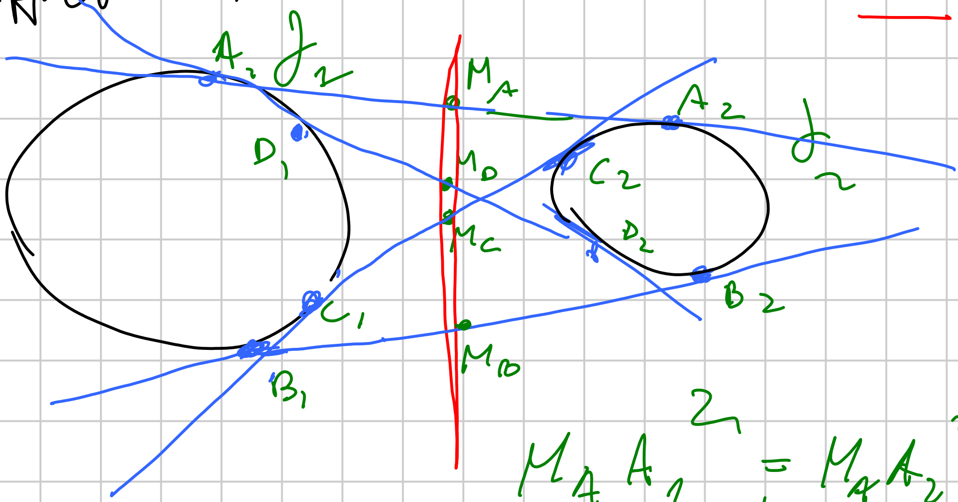


C'

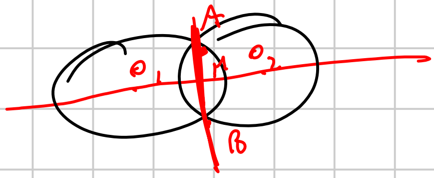
$$\boxed{O_1C'^2 - O_2C'^2} = (XO_1^2 - XC'^2) - (XO_2^2 - XC'^2) = XO_1^2 - XO_2^2 = k$$

Soddisfa $O_1C'^2 - O_2C'^2 = k$

Applicazione



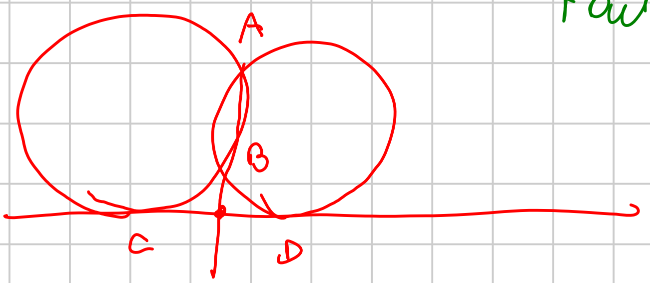
Oss.



$$M_A A_1^2 \underset{\text{t.p.}}{=} M_A A_2^2$$

$$\Updownarrow \qquad \Updownarrow$$

$$Pow_{\gamma_1} M_A = Pow_{\gamma_2} M_A$$



$AB \cap CD \rightarrow$ punto medio di $\underline{CD}$.

— o —

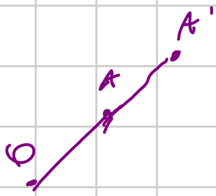
OMOTETIA

A'

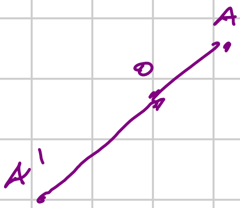
O, A, A' allineati

$$\frac{OA'}{OA} = \lambda$$

$\lambda > 0$

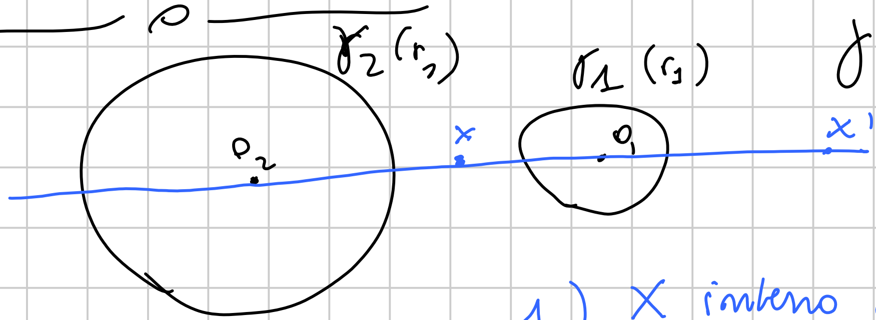


$\lambda < 0$



— o —

$\gamma_2 (r_2)$ $\gamma_1 (r_1)$ $\gamma_1 \longrightarrow \gamma_2$



$X \in O_1 O_2$

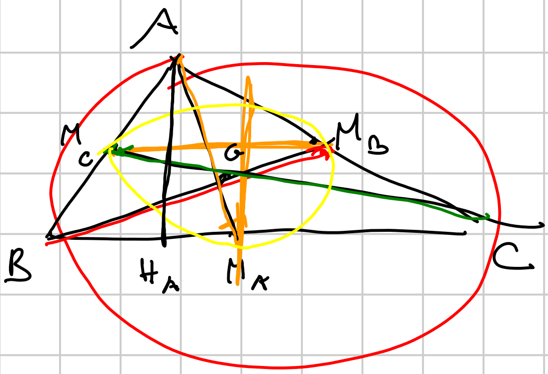
1) X interno ad $O_1 O_2$

X t.c. $\dfrac{XO_2}{XO_1} = -\dfrac{r_2}{r_1}$

$$\left[\frac{\vec{A} + \lambda \vec{B}}{1+\lambda}\right]$$

2) X' esterno ad $O_1 O_2$

X' t.c. $\frac{X'O_2}{X'O_1} = \frac{r_2}{r_1}$



Calc G $\lambda = -\frac{1}{2}$

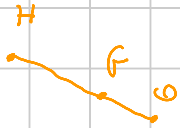
$BC \longrightarrow M_B M_C$

$Alt_A \longrightarrow \perp$ a $M_B M_C$ passante per M_A

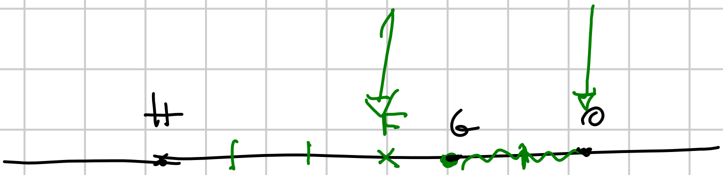
$\perp$ a BC passante per M_A

Asse di BC

$H(ABC) \longrightarrow O(ABC)$



H, G, O allineati $\frac{HG}{GO} = 2$ retta di Eulero



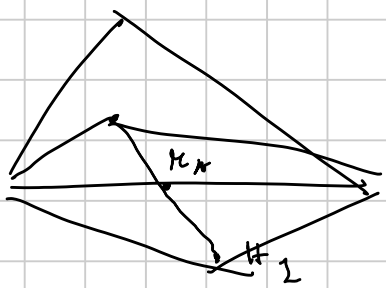
Un centro di omotetia che manda $\odot M_A M_B M_C$ in $\odot ABC$

Omotetia di centro G e rapporto -2 manda $\odot M_A M_B M_C$ in $\odot ABC$

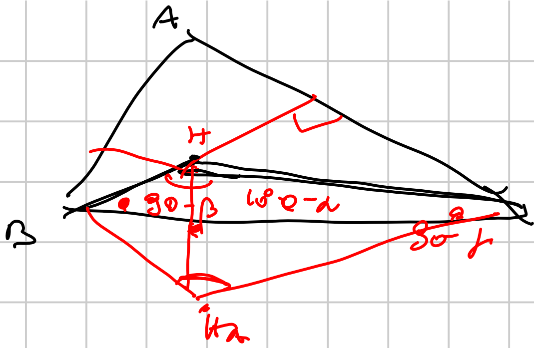
F O

Oss. 1 Il centro di $\odot M_A M_B M_C$ (F) è il punto medio di OH

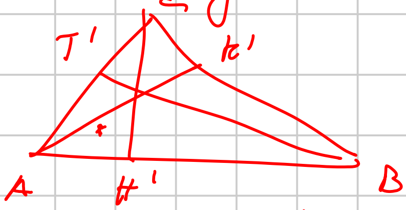
Oss. 2 H è l'altro di centro di omotetia fra $\odot M_A M_B M_C$ e $\odot ABC$



H_1 simmetrico di H wrt M_A ∈ $\odot ABC$



Oss: = facile



$H', K', T' \in \odot M_A M_B M_C$

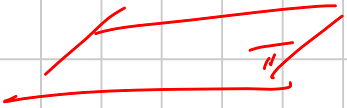
Centri di omotetia   Circoscritta - Inscritta
Inscritta Feuerbach (1)

INVERSIONE CIRCOLARE

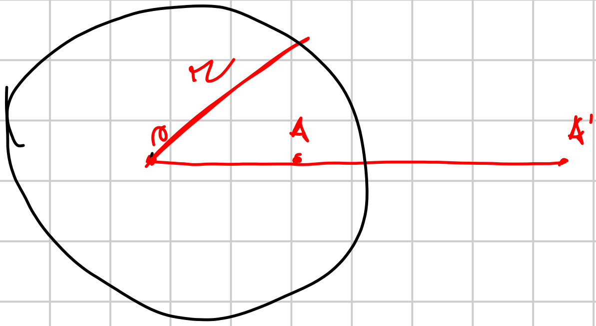
O, r

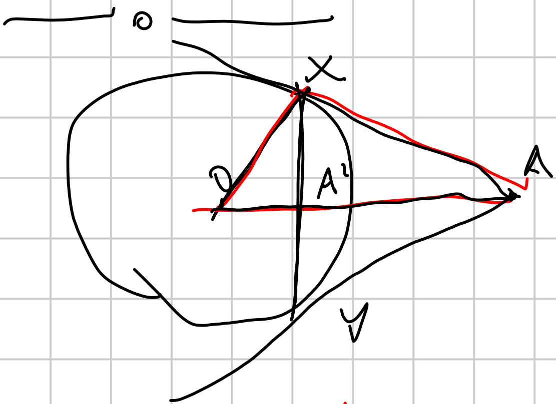
$A \longrightarrow A'$

1) O, A, A' allineati
2) A, A' dalla stessa parte di O
3) $OA \cdot OA' = r^2$



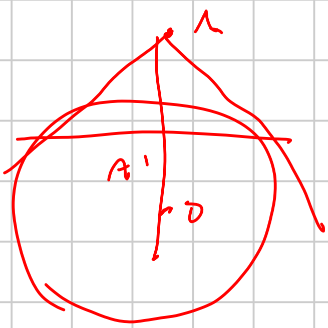
$OA \cdot OA' = r^2$

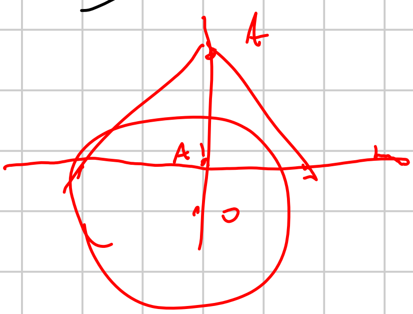


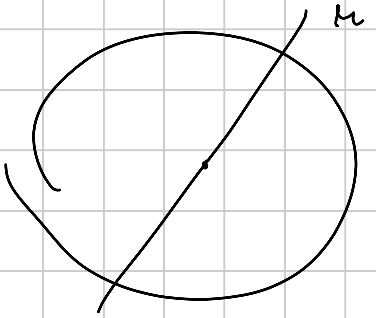


$A' = XY \cap OA$

$OA' \cdot OA = OX^2 = r^2$

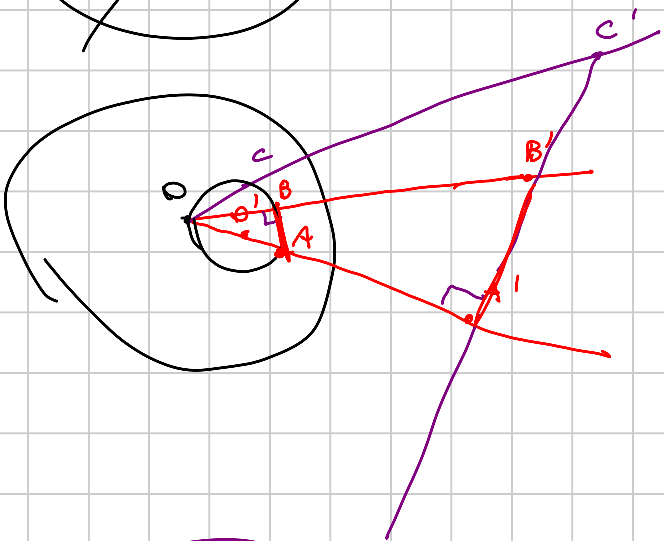




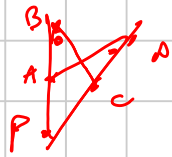


1) r per l'origine $\longrightarrow$ se stessa

2)

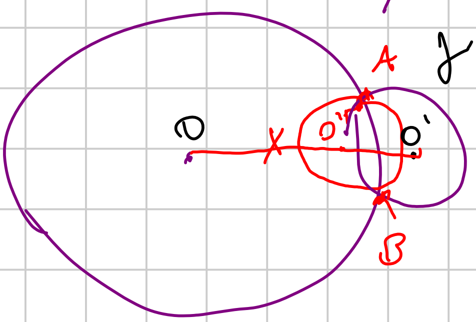


$OB \cdot OB' = r^2 = OA \cdot OA'$



$PA \cdot PB = PC \cdot PD$

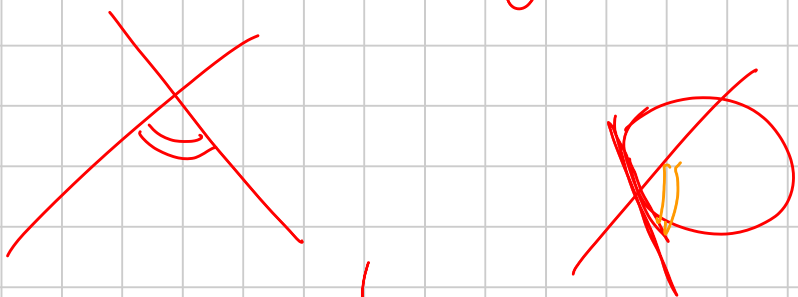
retta a cui $O \notin$ $\longleftrightarrow$ circonferenza per l'origine

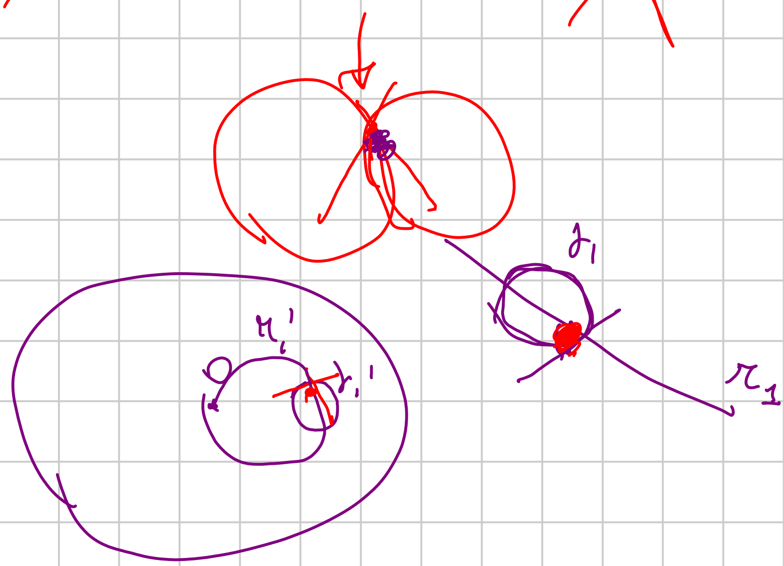


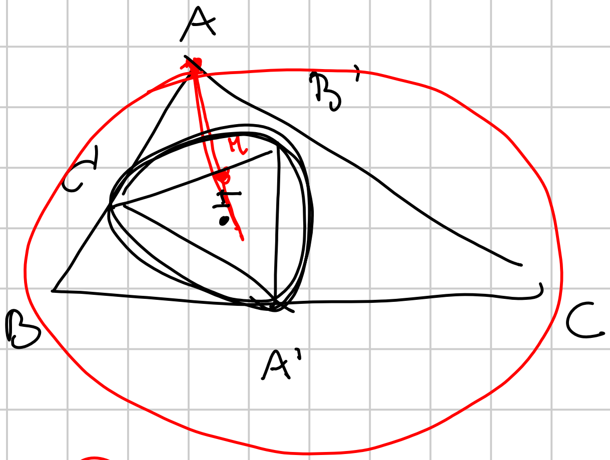
O, O', O'', il cerchio della sfera trasformata sono allineati

O $\downarrow$ centro di inversione

Inversione conserva gli angoli







$I, O, H(A'B'C')$ allineati
(ABC)

$A \to M_{B'C'}$
$B \to M_{A'C'}$
$C \to M_{A'B'}$

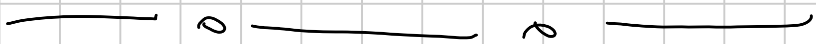
$\odot\, ABC \longrightarrow$ Feuerbach di $A'B'C'$
$O \qquad F(A'B'C')$

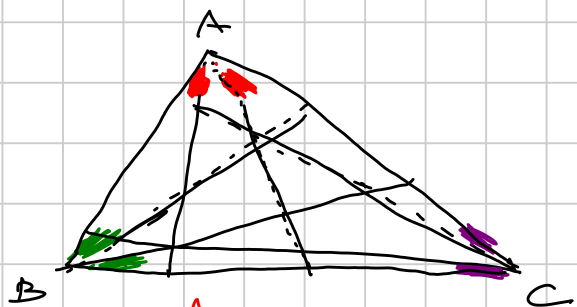
$I, \boxed{O}, F(A'B'C')$ allineati

$F(A'B'C')$, $O(A'B'C')$, $\boxed{H(A'B'C')}$ allineati x prima
$ I$

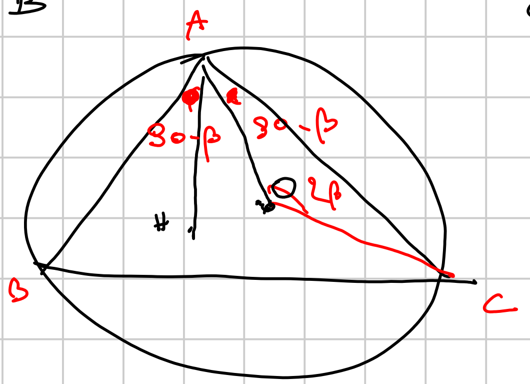
$\downarrow$

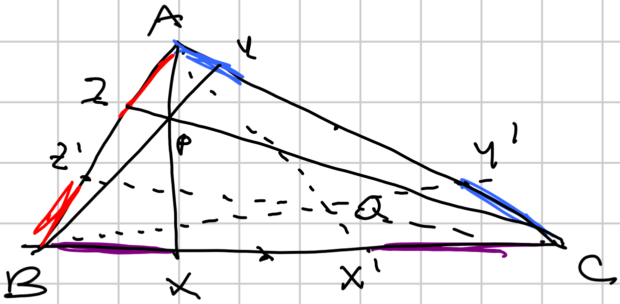
sulla retta OI $O, I, H(A'B'C')$ sono allineati





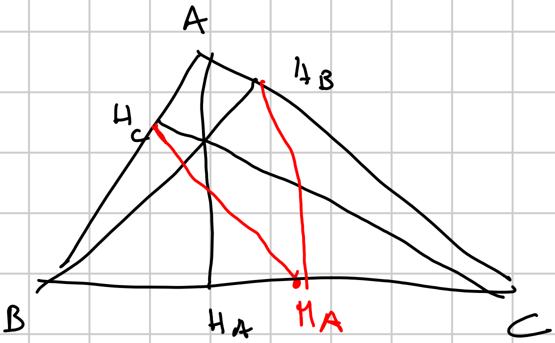
FIGURE

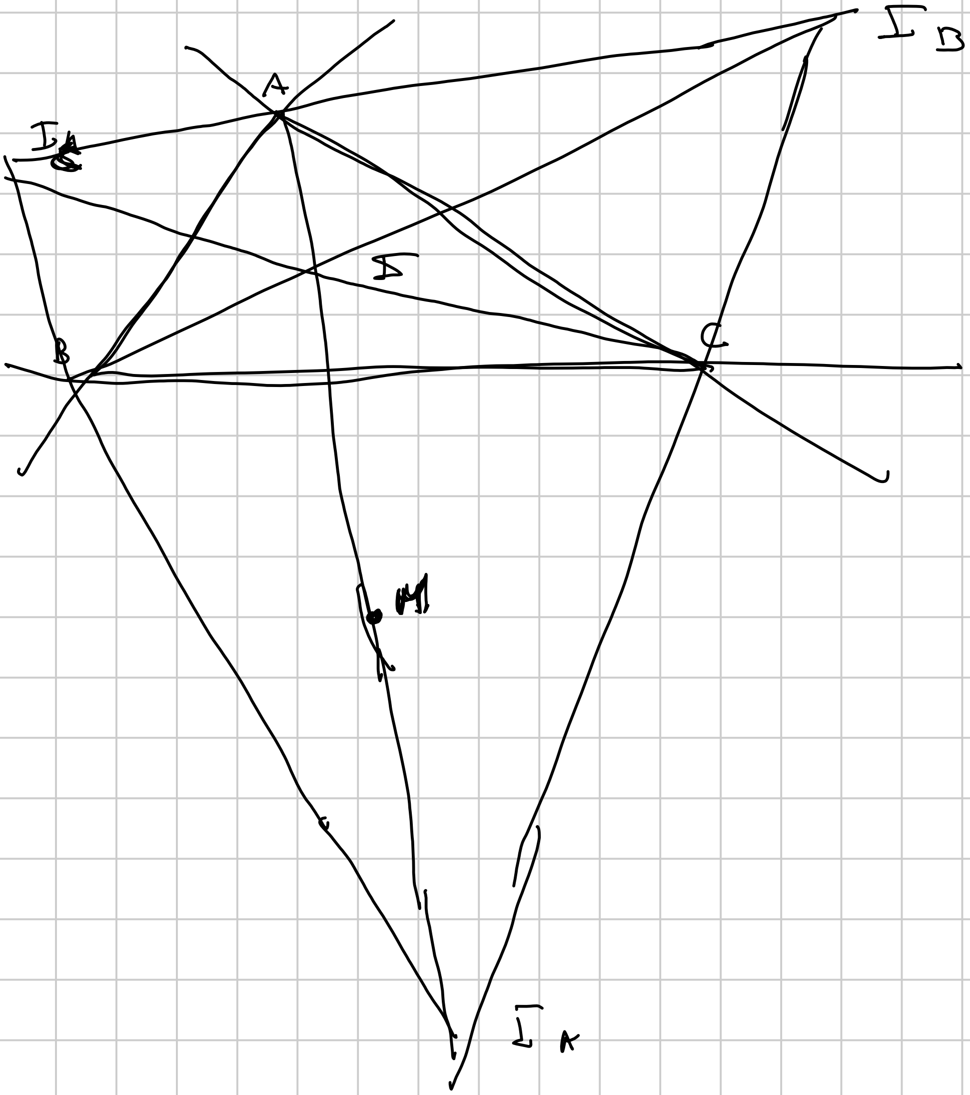




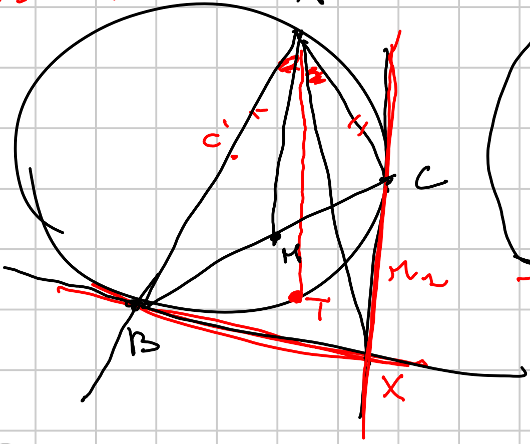
PROBLEMI

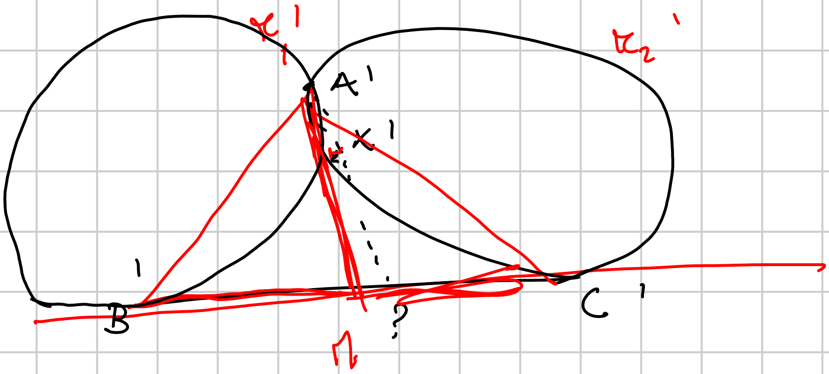
VELOCI





Lemma Simmediana





· Inversione centro A e $r=\sqrt{AB\cdot AC}$ + Simmetria rispetto alla bisettrice di $\hat{A}$,

· $B \to C' \longrightarrow C$

$AC'\cdot \cancel{AB} = \cancel{AB}\cdot AC$

$AC' = AC$

$C \to B$

$\omega_{ABC} \to BC$

$\mu_1 \to$ Cfr per A, B tangente a BC

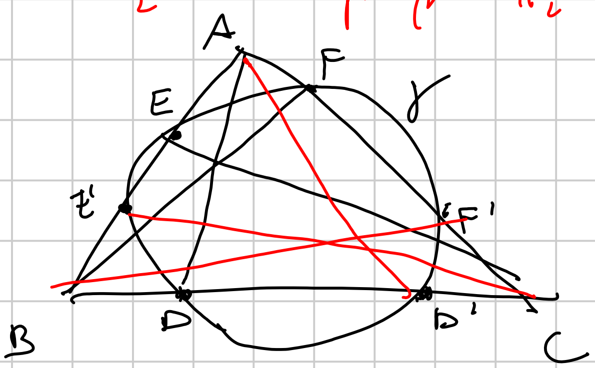
$\mu_2 \to$ Cfr per A, C tangente a BC

$X \to X' = \mu_1' \cap \mu_2'$

$X \in$ simmetrica di AM rispetto a $\hat{A}$

$\Updownarrow$

$X' \in AM$



$$\frac{BD}{DC}\cdot\frac{CF}{FA}\cdot\frac{AE}{EB}=1$$

$$\frac{BD'}{D'C}\cdot\frac{CF'}{F'A}\cdot\frac{AE'}{E'B}=1$$

$$\frac{BE'}{BD'}\cdot\frac{CD'}{CF'}\cdot\frac{AF'}{AE'}=1$$

(1) $BE'\cdot BE = BD'\cdot BD$

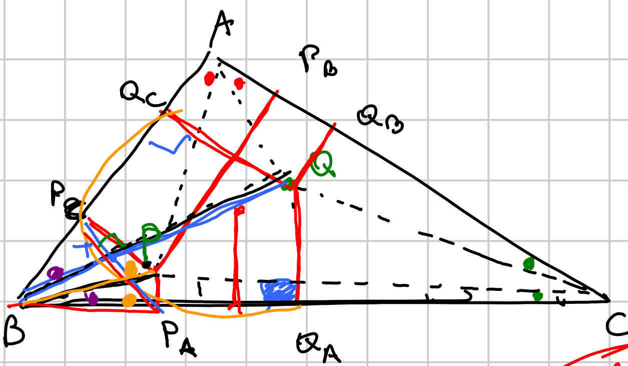
(2) $CD'\cdot CD = CF'\cdot CF$

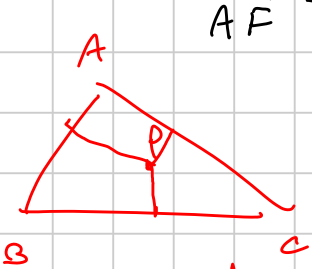
(3) $AE\cdot AE' = AF\cdot AF'$

(1) $\dfrac{BD}{BE} = \dfrac{BE'}{BD'}$

(2) $\dfrac{CF}{CD} = \dfrac{CD'}{CF'}$

(3) $\dfrac{AE}{AF} = \dfrac{AF'}{AE'}$





$\widehat{TP_AB}$ //≅

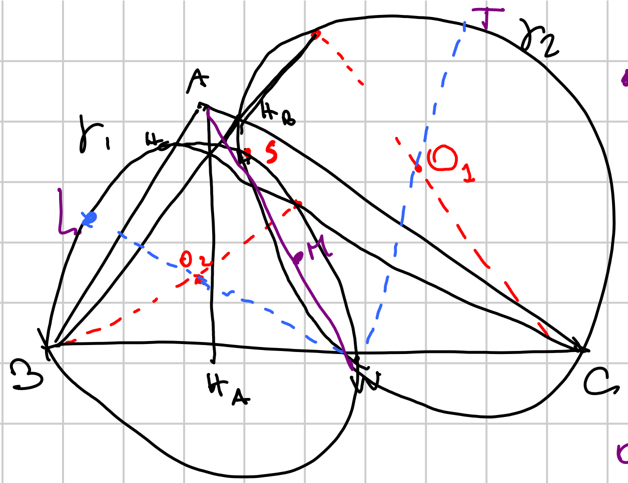
$P P_A B P_C$ ciclico

$\widehat{P_CP_AB} \cong \widehat{P_CPB} \cong \cong \frac{\pi}{2} - \widehat{PBP_C}$

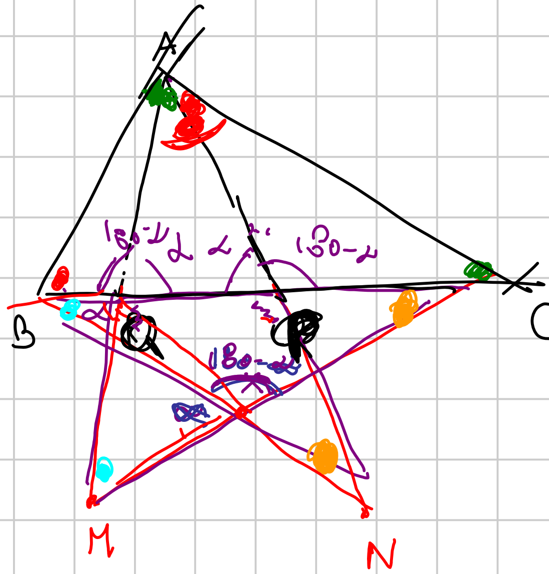
$\cong \frac{\pi}{2} - \widehat{QBQ_A} \cong \frac{\pi}{2} - \widehat{TBP_A}$

$\Rightarrow \widehat{BTP_A} \cong \frac{\pi}{2}$

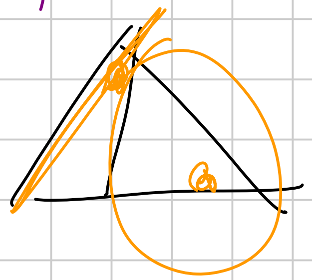
$BP_A\cdot BQ_A = BT\cdot BQ = BP_C\cdot BQ_C$



- Fatto 1: $A \in SW$
  SW è l'asse radicale di γ_1 e γ_2
  e $\underbrace{AH_C \cdot AB}_{Pow_{\gamma_1} A} = AH_B \cdot AC = Pow_{\gamma_2} A \Rightarrow A \in SW$
- H, O_1, O_2 allineati (fatto noto)
  $\Rightarrow S\,L\,T$ allineati (omotetia di centro W e rapporto 2)
  quindi $H \in LT$
- $H \in SL$ poiché HSH_AW è ciclico e dunque $H\hat{S}W = 90°$
  Essendo poi $L\hat{S}W = 90°$, $H \in SL$.
- Dunque H, S, L, T allineati e in particolare $H \in LT$.



$\underline{1°}$



$AB^2 = BP \cdot BC$

$AC^2 = CQ \cdot BC$

$Q\hat{A}C \sim B\hat{A}P \quad (1)$

$Q\hat{C}M \sim B\hat{P}N$

$$\frac{CQ}{QM} = \frac{CQ}{QA} \overset{(1)}{=} \frac{AP}{BP} = \frac{PN}{BP}$$

$$\frac{CQ}{QM} = \frac{PN}{BP}$$

$BQMX$ ciclico poiché i celesti sono uguali