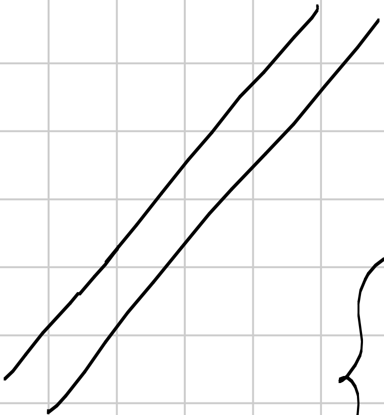
G2-MEDIUM [KFP]

Titolo nota 03/09/2014

Cosa cambia? Le rette parallele si incontrano



Piano Proiettivo

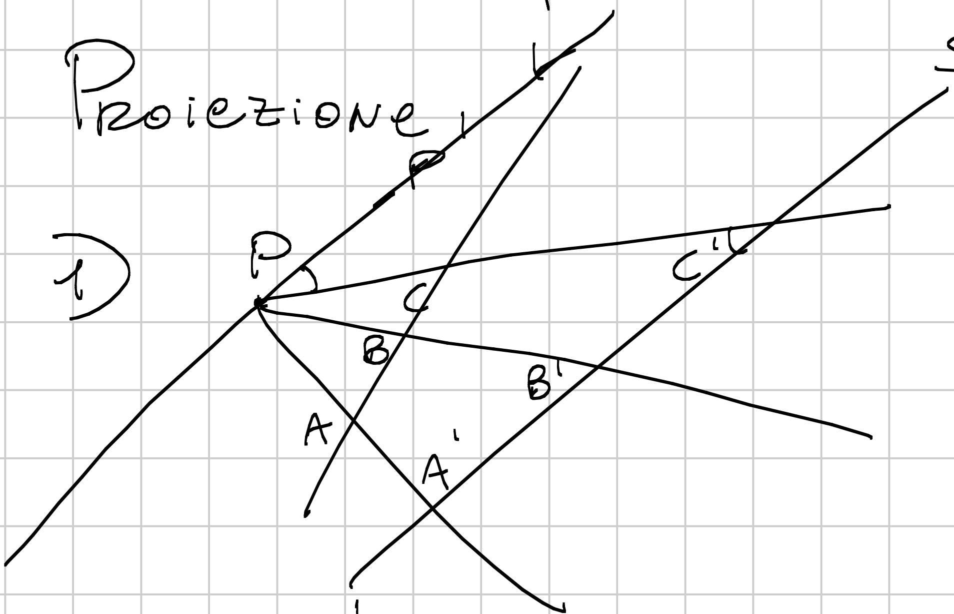
=

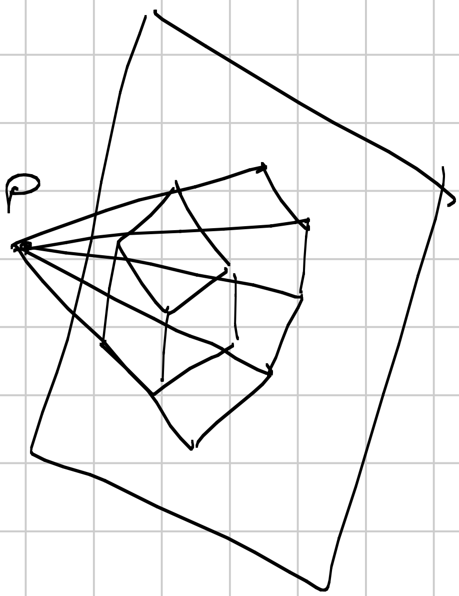
{Piano Euclideo} ∪ {Retta all'∞}

Ogni coppia di rette si incontra in uno e un solo punto.

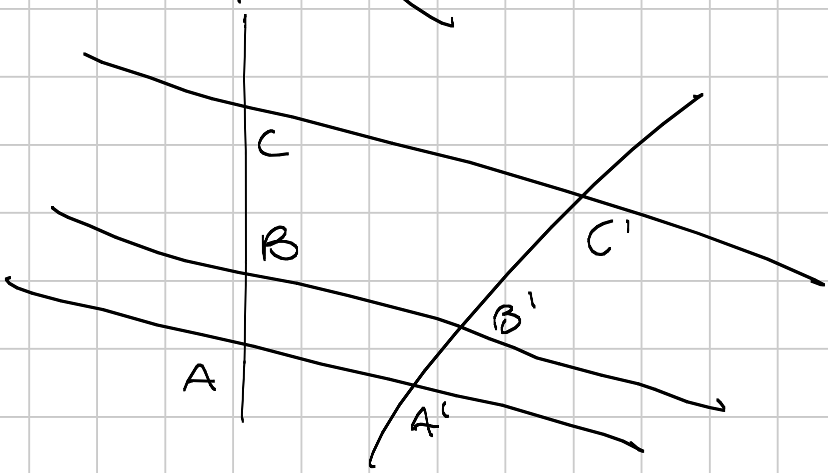
Proiezione

1)

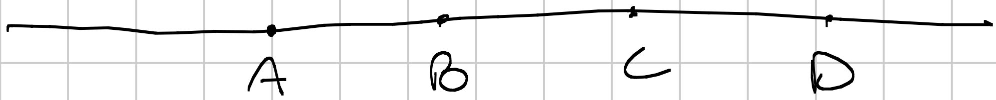




2)

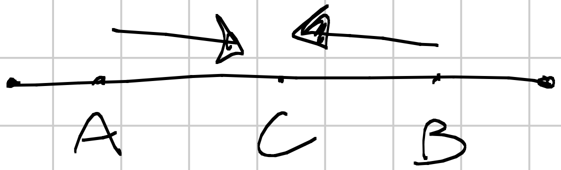


Birapporto



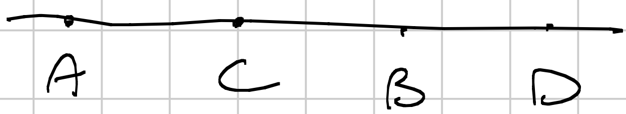
$$(A,B;C,D) = \frac{\frac{AC}{BC}}{\frac{AD}{BD}} = \frac{AC\cdot BD}{AD\cdot BC}$$

$\boxed{0 \qquad 1 \qquad +\infty}$ $\quad (A,C;B,D)$



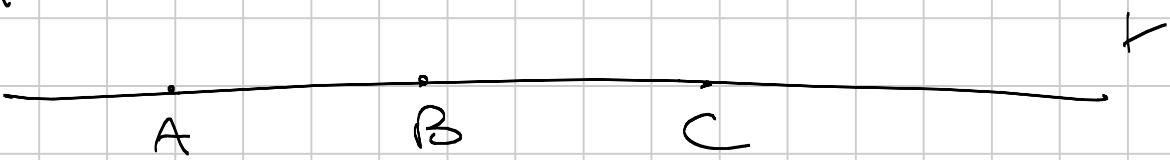
$$\frac{AC}{BC}$$

☆

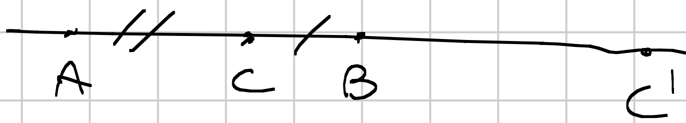


$(A,B;C,D)<0$

$$|(A,B;C,X)| = \lambda$$



$$\frac{A\cancel{C}\cdot BX}{\cancel{B}C\cdot AX}$$



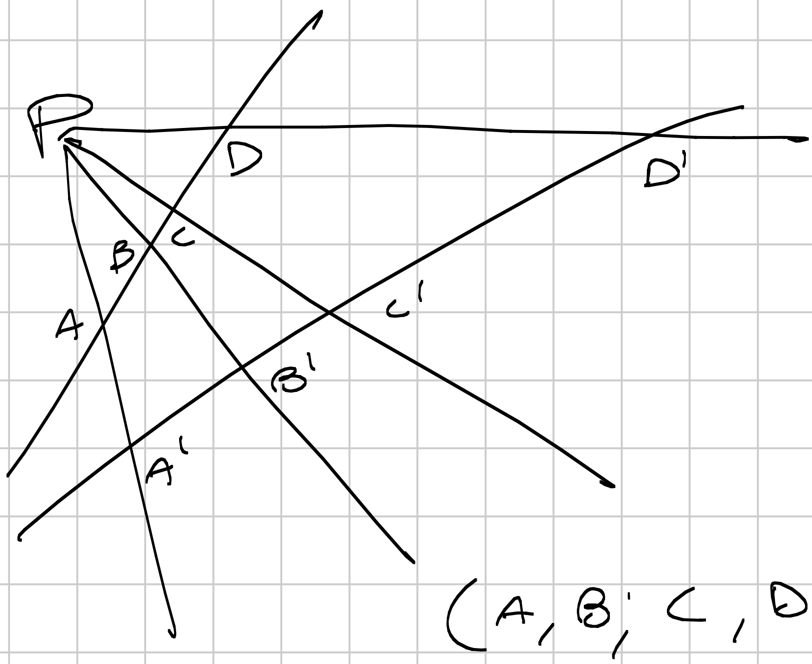
$$f(x) = (A, B; C, x)$$

$$\{\text{retta proiettiva}\} \to \{\text{reali} + \infty\}$$

$$(A, B; C, x) = \frac{AC}{BC}$$

$$(A, B; C, \infty) = \frac{AC \cdot \cancel{B\infty}}{BC \cdot \cancel{A\infty}}$$

INVARIANZA PER PROIEZIONE



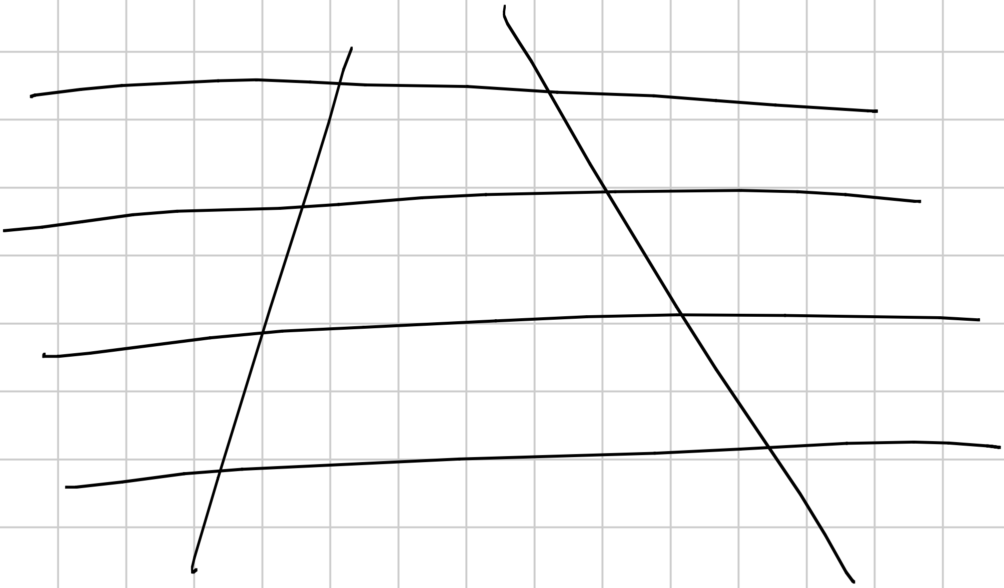
$$(A, B; C, D) = (A', B'; C', D')$$

$$AC = CP \frac{\sin \widehat{APC}}{\sin \widehat{PAC}} \qquad BC = CP \frac{\sin \widehat{BPC}}{\sin \widehat{PBC}}$$

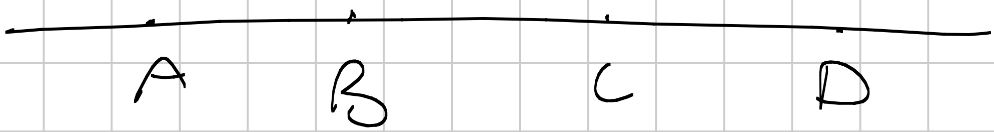
$$\frac{AC}{BC} = \frac{\sin \widehat{APC} \cdot \cancel{\sin \widehat{PBC}}}{\cancel{\sin \widehat{PAC}} \cdot \sin \widehat{BPC}}$$

$$\frac{AD}{BD} = \frac{\widehat{APD} \quad \cancel{\widehat{PBD}}}{\cancel{\widehat{PAD}} \cdot \widehat{BPD}}$$

$$= \frac{\sin \widehat{APC}}{\sin \widehat{BPC}} : \frac{\sin \widehat{APD}}{\sin \widehat{BPD}}$$



$$\boxed{(A, B; C, D) = K}$$

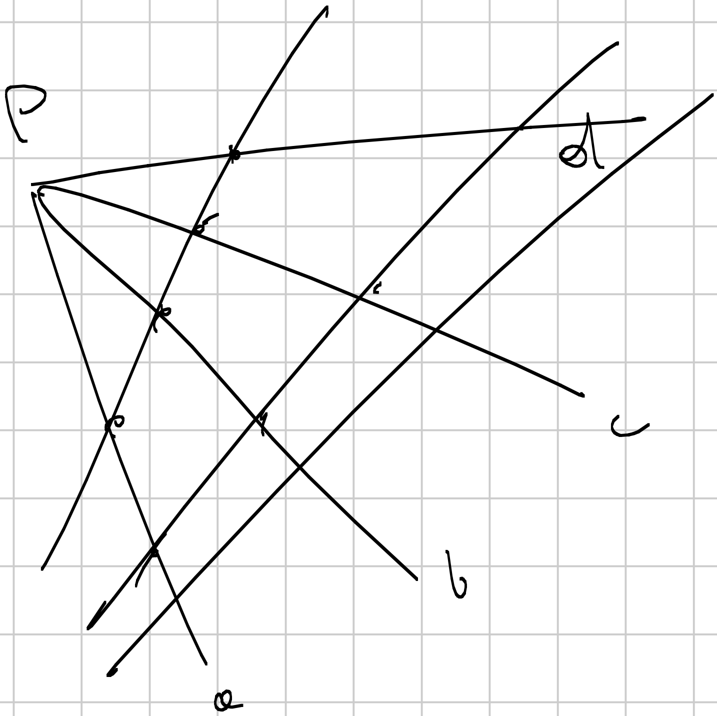


$$\left\{ k;\ \frac{1}{k};\ 1-k;\ \frac{k-1}{k};\ \frac{k}{k-1};\ \frac{1}{1-k} \right\}$$

$$(A, B; C, D) = k$$

$$(B, A; C, D) = \frac{1}{k}$$

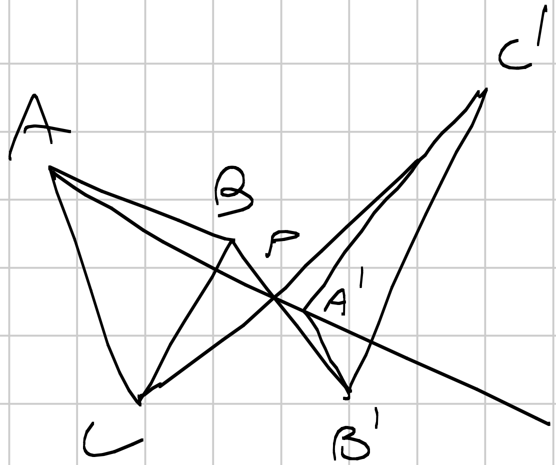
$$(A, B; D, C) = \frac{1}{k}$$



$$(a, b; c, d)$$

Il birapporto di 4 rette è il birapporto che si ottiene intersecando una retta qualunque con queste 4.

Teorema di Desargues



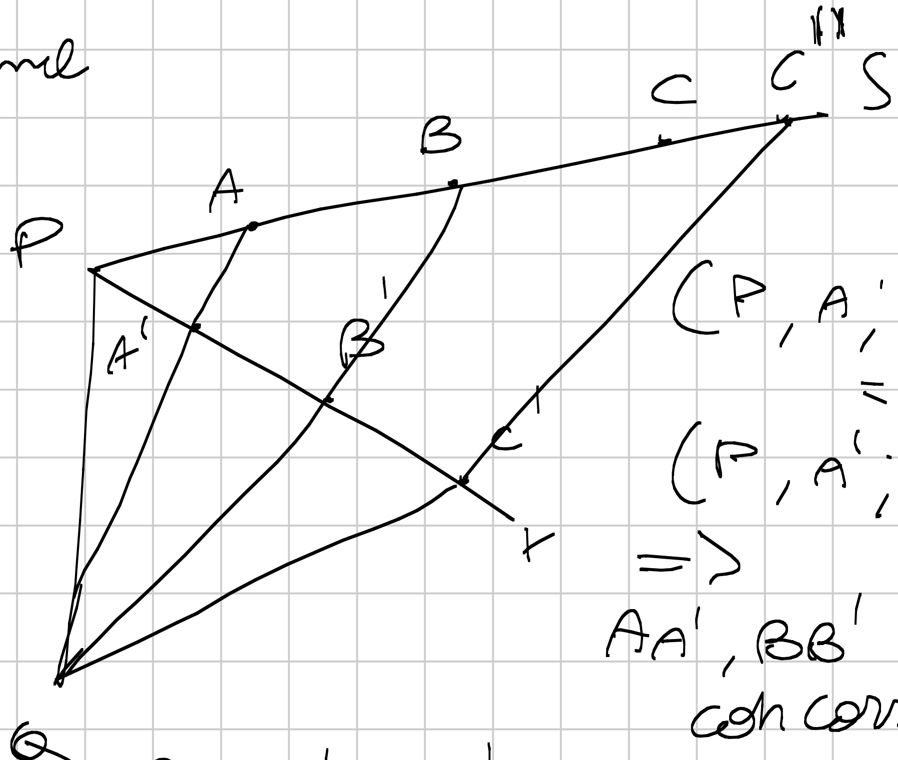
AA', BB', CC' concorrono in P

$$\Longleftrightarrow$$

$AB \cap A'B'$, $BC \cap B'C'$, $CA \cap C'A'$

sono allineati.

Lemma



$$(P, A; B, C) = (P, A'; B', C') \Rightarrow$$

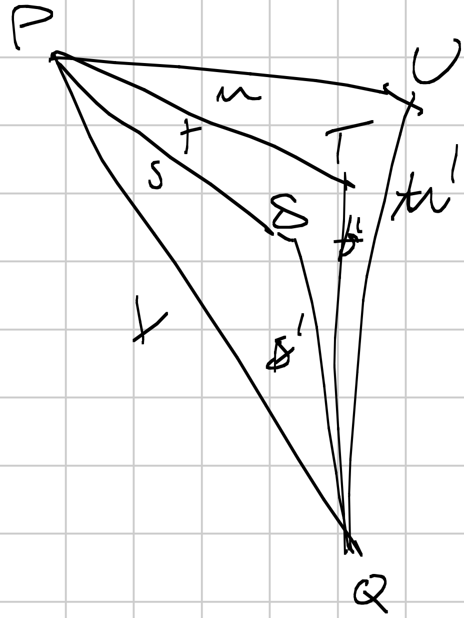
AA', BB', CC' concorrono

$Q = AA' \cap BB'$

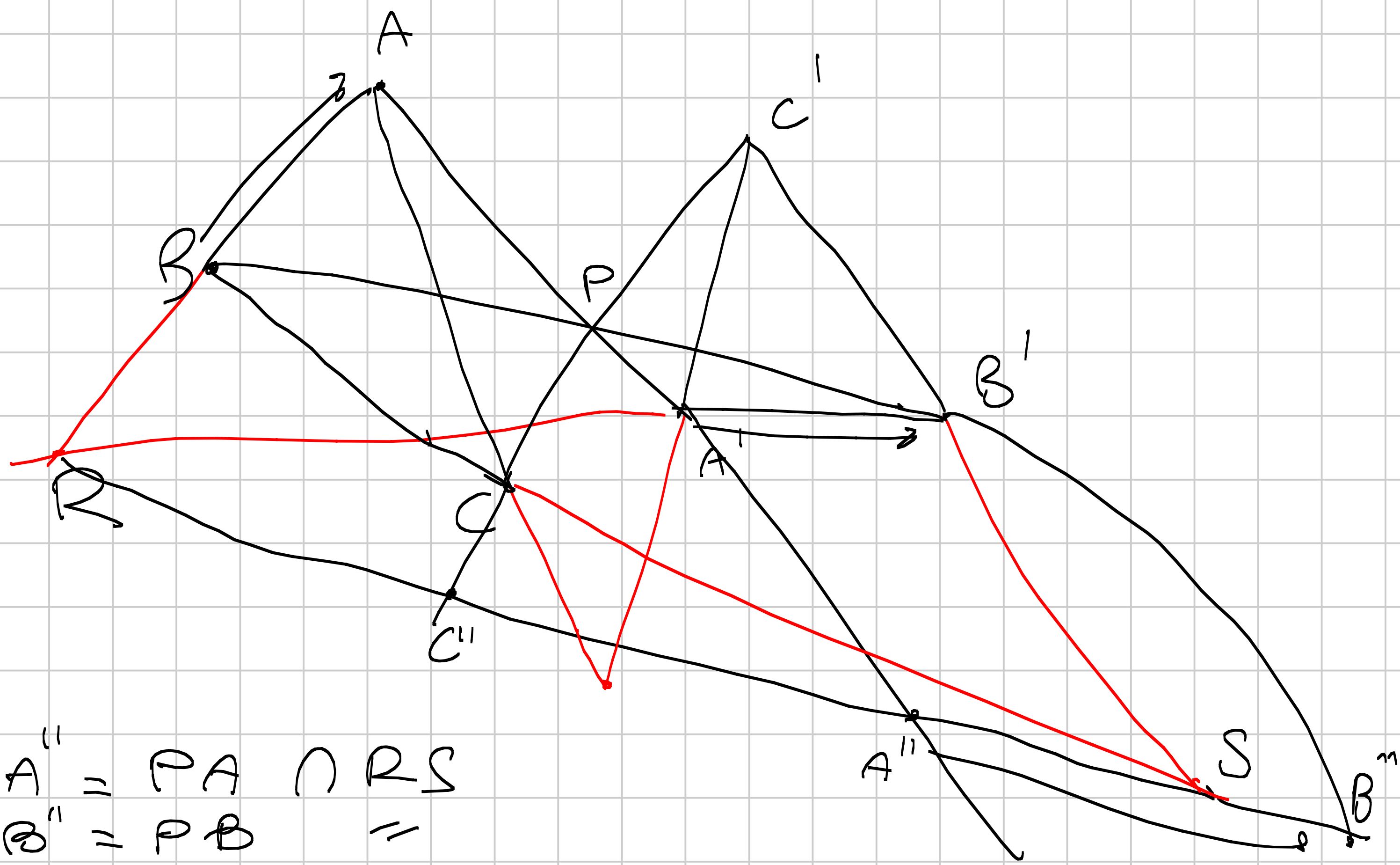
$$(P, A', B', C') = (P, A; B, C'')$$

$\boxed{C'' \equiv C}$

Lemme OK



$$(r, s, t, u) = (r, s', t', u')$$
$\Rightarrow$ S T U all.



$A'' = PA \cap RS$
$B'' = PB \quad$ "
$C'' = PC \quad$ "

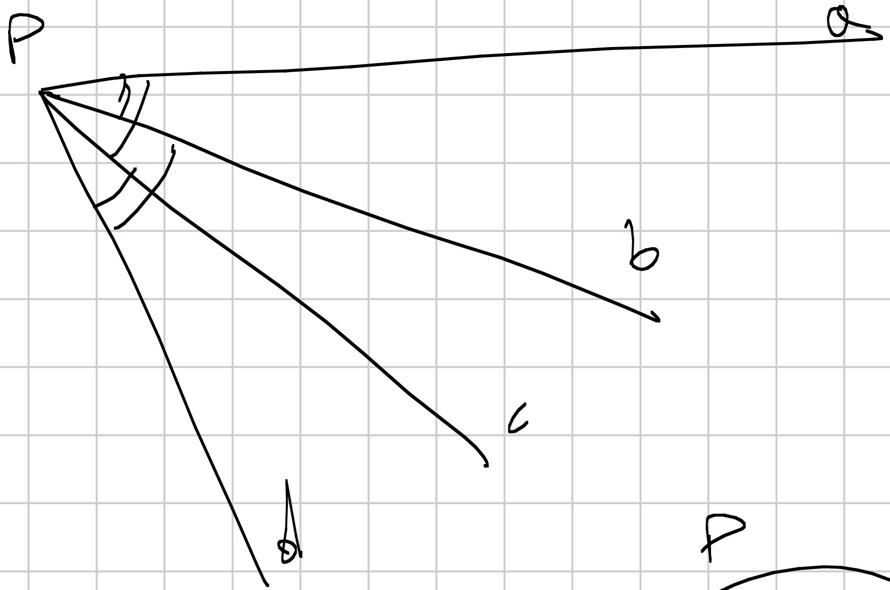
$$(P, B; B', B'') \overset{R}{\barwedge} (P, A; A', A'')$$

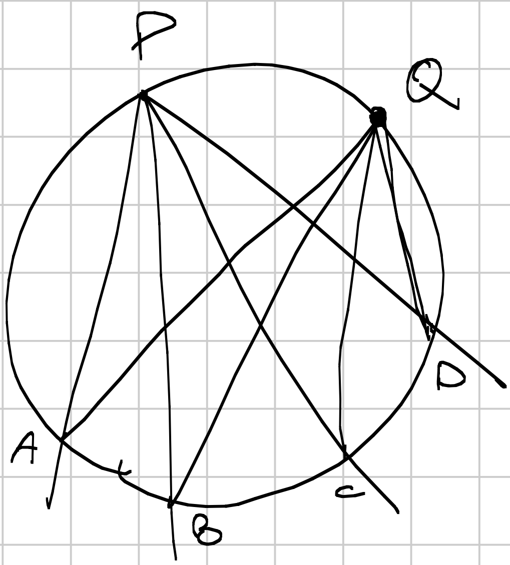
$$\| \left(\begin{matrix} (P, B; B', B'') \\ \hat{S}'' \\ (P, C; C', C'') \end{matrix} \right.$$

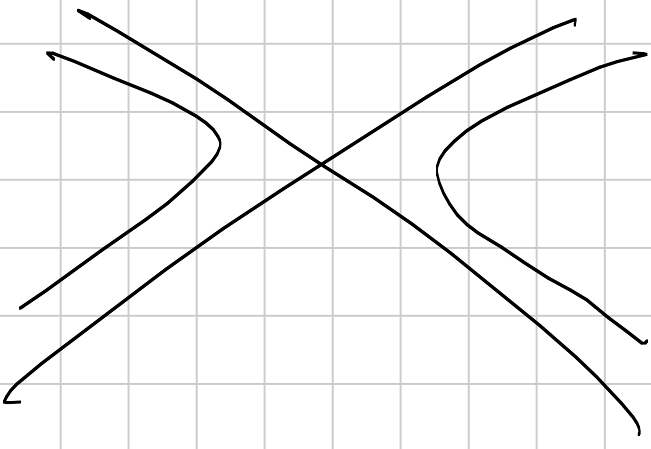
$A'' B'' = RS$

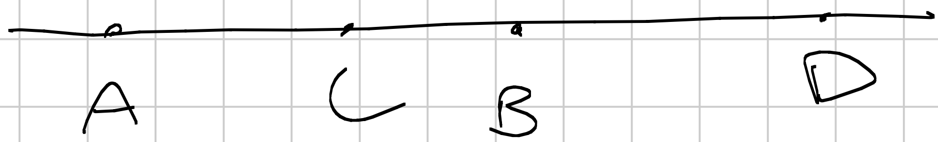
$RS, AC, A'C'$

Prima Freccia (ALTRA UGUALE)



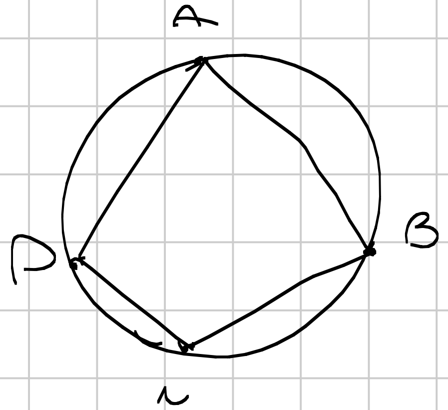






$$(A, B; C, D) = -1$$

Quando la quaterna è su una circonferenza, il quadrilatero si dice armonico.

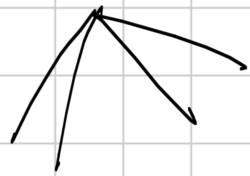


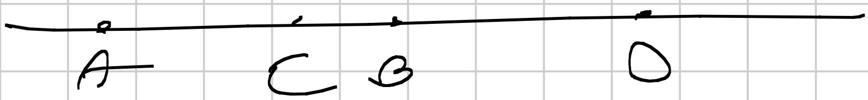
ABCD armonico

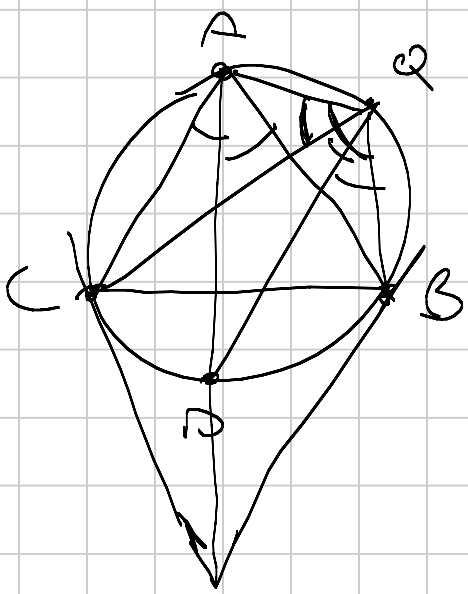
$\iff$

$$AB \cdot CD = AD \cdot BC$$

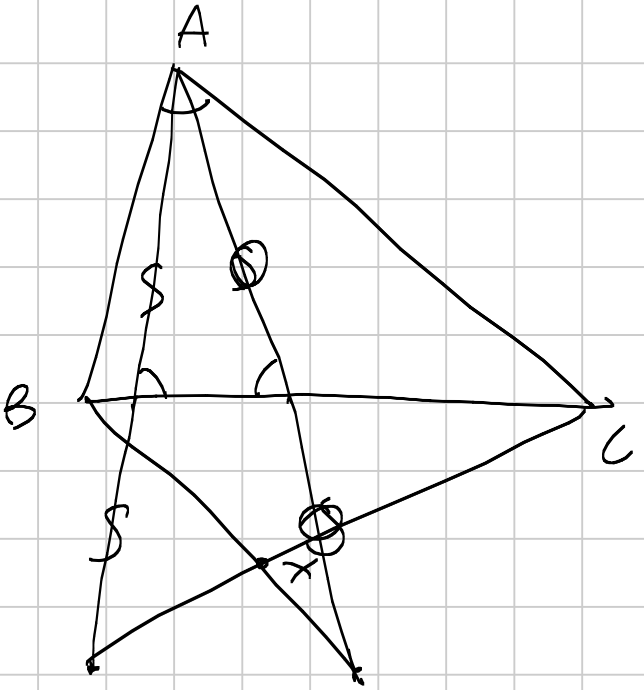
$$(A, C; B, D) = \frac{AB \cdot CD}{CB \cdot AD} = -1$$

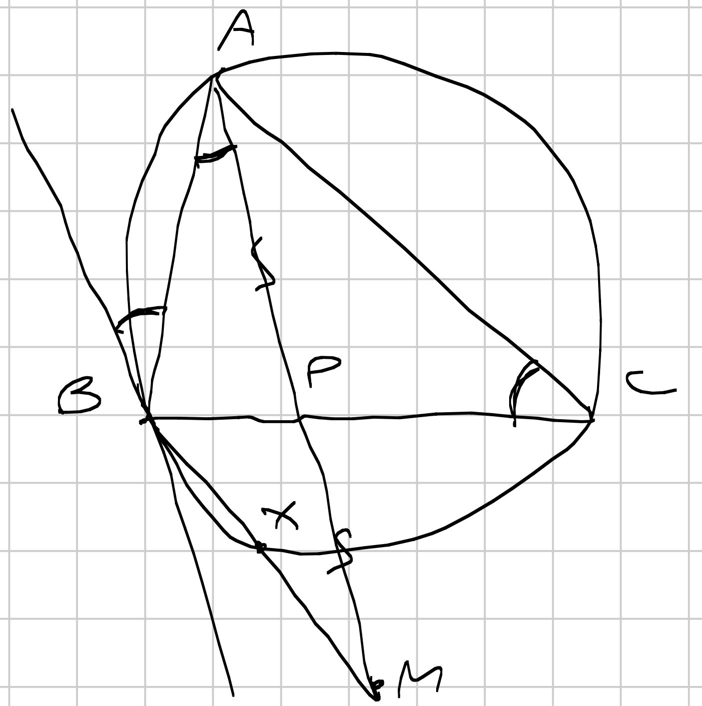






IMO-4 2014





$$(A, M; P, \infty) = \frac{AP \cdot \cancel{M\infty}}{MP \cdot \cancel{A\infty}} = -1$$

$$\hat{B} \downarrow$$

$$(A, X; C, B) = -1$$

BM posso per sim ∩ cieco CN

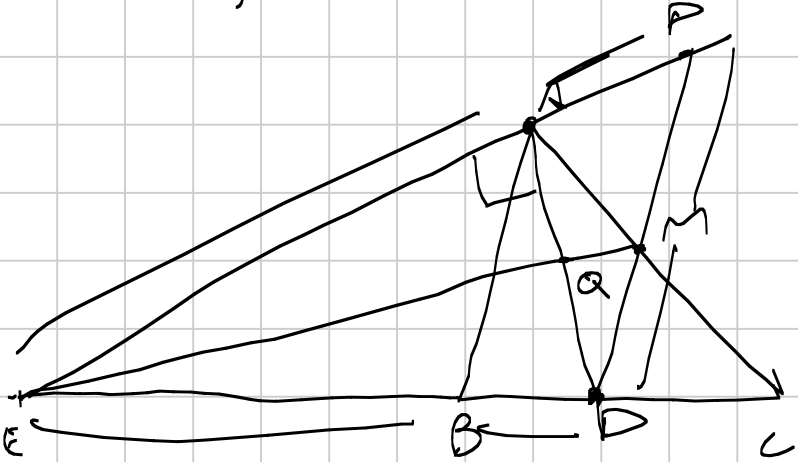
FINE

11

$(A\ \ M\ \ B\ \ \infty) = -1$

e simm.

TST Turchia 2008 - 1



PQ PASSA PER UN PUNTO FISSATO AL VARIARE DI P.

Teor. di Menelao su $\widehat{PED}$ & AC

$$\frac{PA}{AE}\cdot\boxed{\frac{EC}{CD}}\cdot\frac{DM}{MP}=1$$

$(C, B; D, E) = -1$

$$\boxed{\frac{DB}{DC}=\frac{AB}{AC}} \qquad \frac{AB}{AC}=\frac{AB}{AC} \qquad \boxed{\frac{EB}{EC}=\frac{AB}{AC}}$$

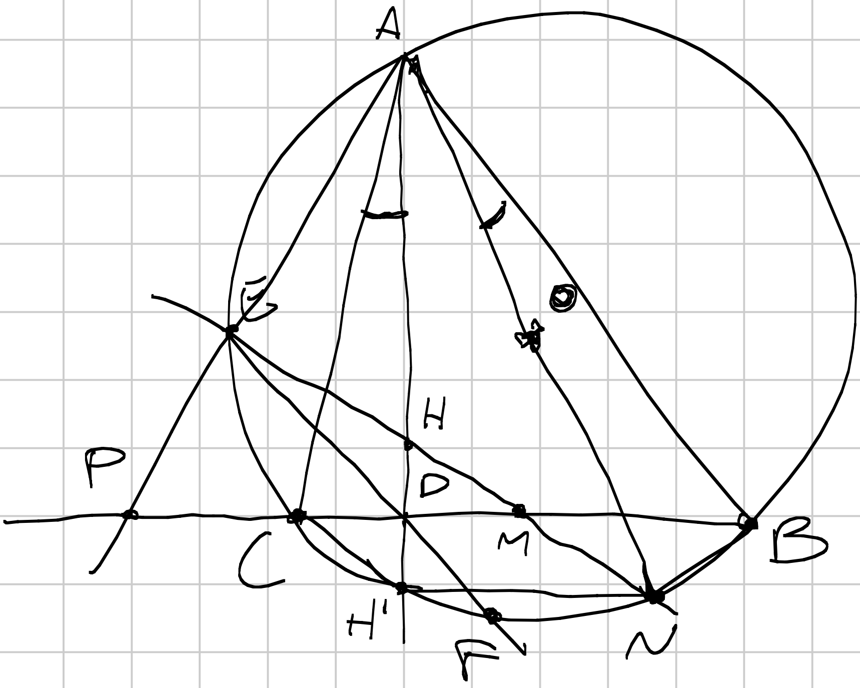
$$\frac{EC}{EB}=\frac{AC}{AB}$$

$$\frac{CD}{CE}=\frac{BD}{BE}$$

$$\boxed{\frac{PA}{AE}\cdot\frac{BE}{BD}\cdot\frac{DM}{MP}=1}$$

P, Q, B allineati

Cevi



$$\frac{BF}{FC}=\frac{AB}{AC}$$

$$\boxed{H'N \parallel BC}$$

$$(NH', NM; NB, NC) = -1$$

$$\infty \qquad M \qquad B \qquad C$$

$$\downarrow \hat{N}$$

$$(H', E; B, C)$$

$$\downarrow \hat{A}$$

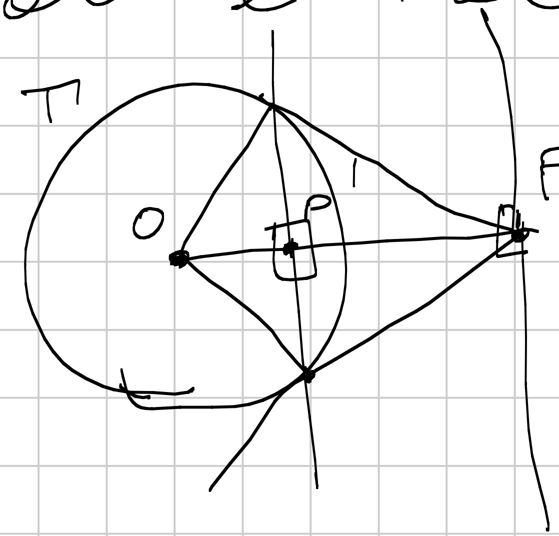
$$(D, P; B, C)$$

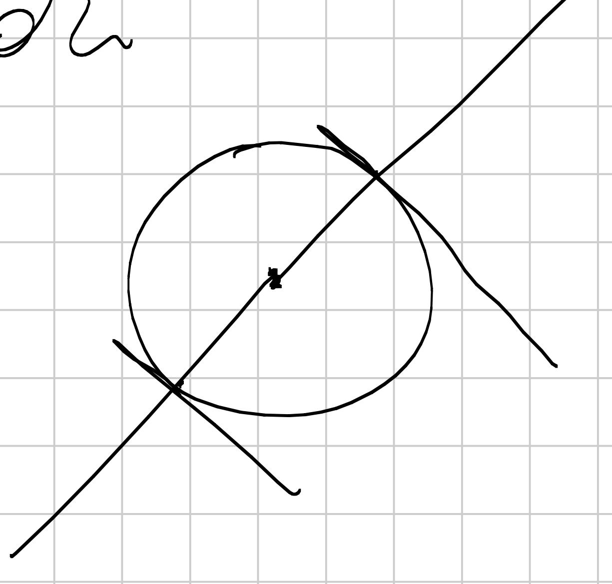
$$\downarrow \hat{E}$$

$$(F, A; B, C) = -1$$

Fine!

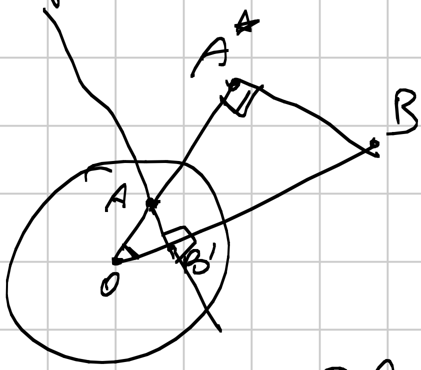
Poli e Polari





Teorema di La Hire

$A \in pol(B) \iff B \in pol(A)$

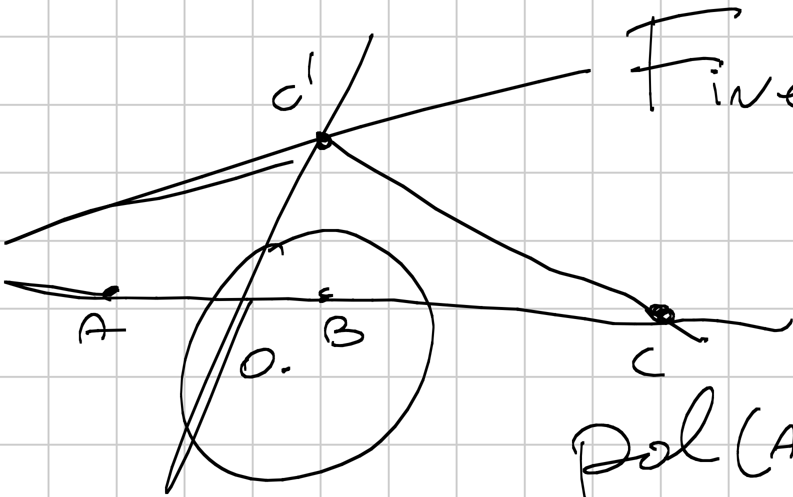


$OB'A$

OA^*B

$OA \cdot OA^* = OB' \cdot OB$

Fine

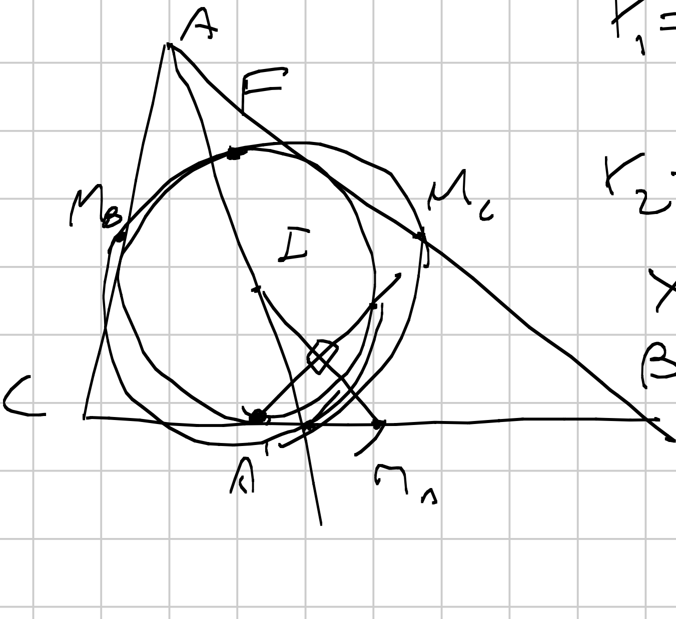


$C' = pol(A) \cap pol(B)$

$pol(A) \cap pol(B) = pol(AB)$

3 punti all $\iff$ polari concorrono

WC-2010 6



$r_1 =$ sim di BC rispetto ad AI

$r_2 = \perp$ da A_1 a $M_A I$

$X_A = r_1 \cap r_2$

B X_B X_C

Ts: X_A, X_B, X_C allineati

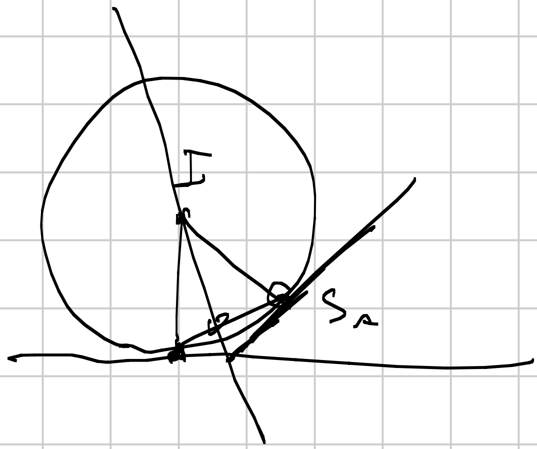
Tesi + forte: allineati su una retta tang. all' inscritta.

$\iff$ pol(X_A) ...

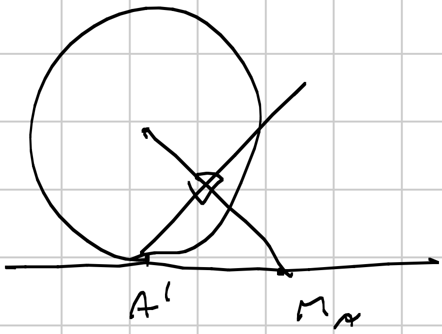
concorrono sull' inscritta

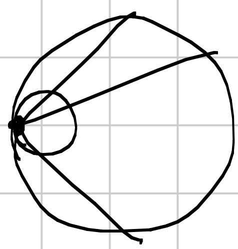
$X_A = r_1 \cap r_2$

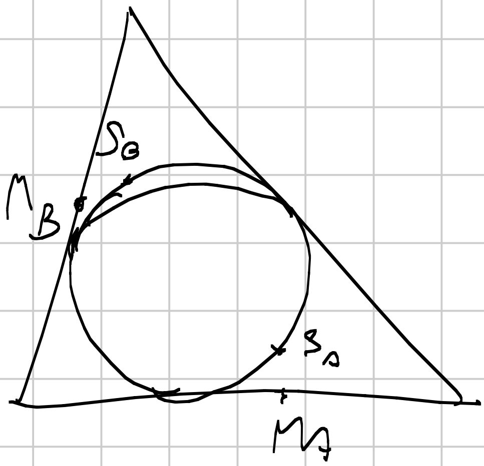
$\text{pol}(X_A) = \overline{\text{pol}(r_1)\ \text{pol}(r_2)}$



$M_A S_A$ concorron sull INSCR.





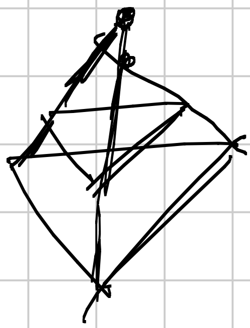


$\widehat{M_A M_B M_C}$

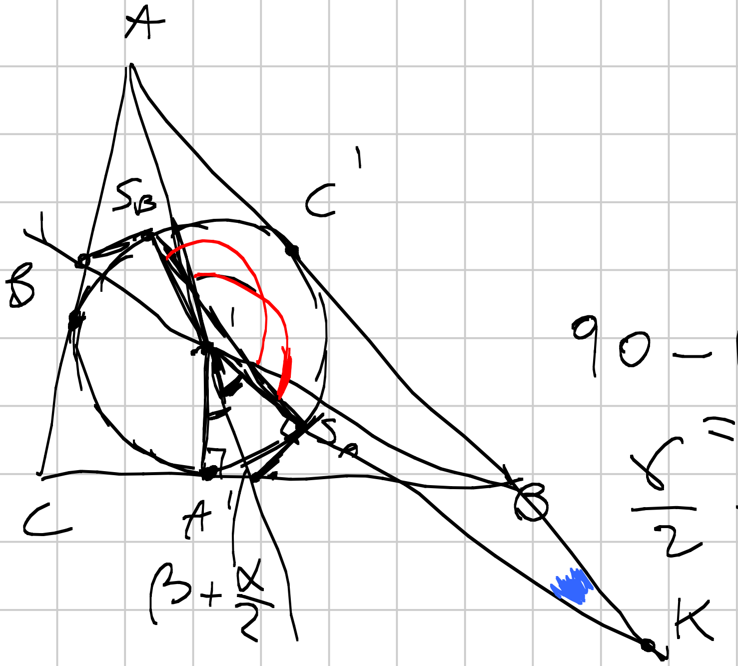
$\widehat{S_A S_B S_C}$

$\widehat{ABC}$ $\quad$ $\widehat{A_1 B_1 C_1}$

$AA_1 \quad BB_1 \quad CC_1$



$$\boxed{S_A S_B \parallel M_A M_B \parallel AB}$$



$$90 - \beta - \frac{\alpha}{2} = \frac{\gamma}{2} - \frac{\beta}{2}$$

$$\widehat{KIA} = + + \frac{\alpha}{2} = \pi$$

$$\pi - \frac{\gamma}{2} + \frac{\beta}{2}$$

$$+$$

$$\cancel{\pi} - \frac{\gamma}{2} + \frac{\alpha}{2}$$

$$+$$

$$\cancel{\pi} + \frac{\beta}{2} + \frac{\alpha}{2}$$

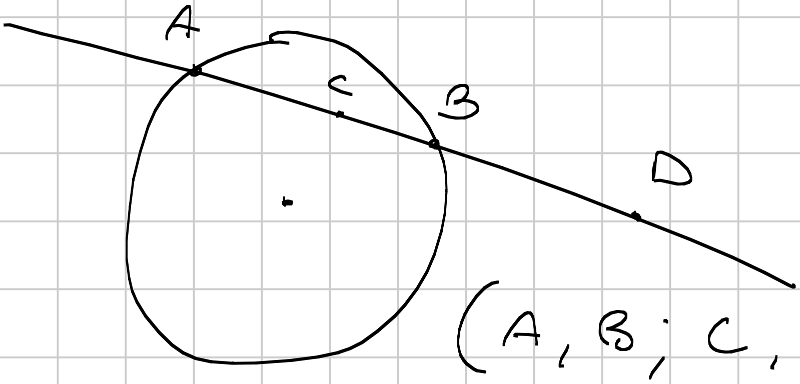
$$\pi - \gamma + \alpha + \beta$$

[illegible]

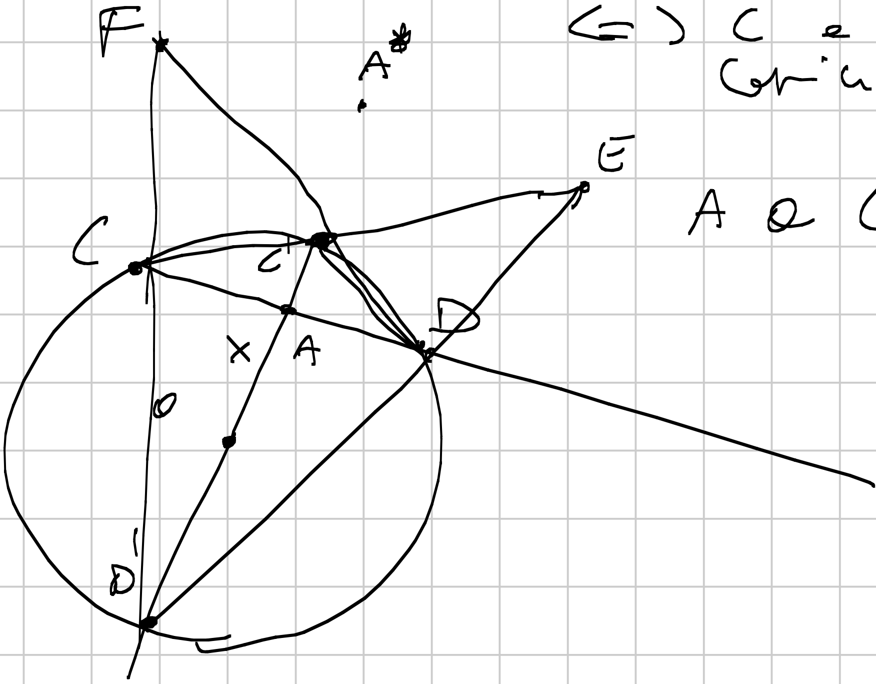
$$\frac{\pi - 2\alpha - 2\beta}{2}$$

$$\frac{\pi}{2} - \alpha - \beta$$

Lemma della Polare



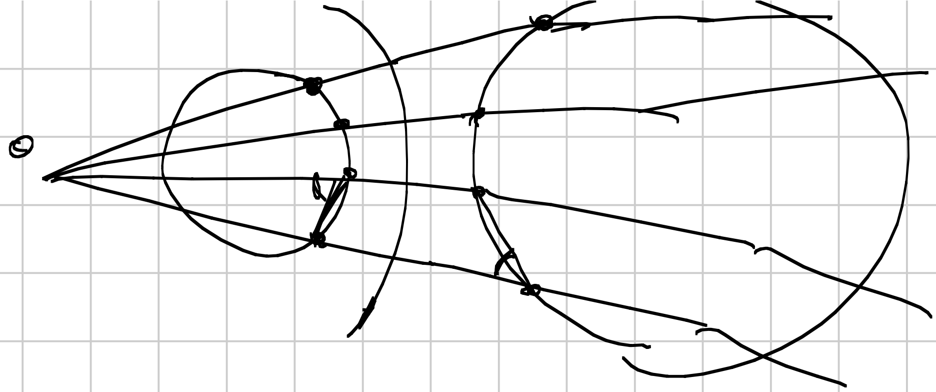
$(A, B; C, D) \iff C$ e D coniugati



A a CASO

$$(C, D; A, A^*) = -1$$

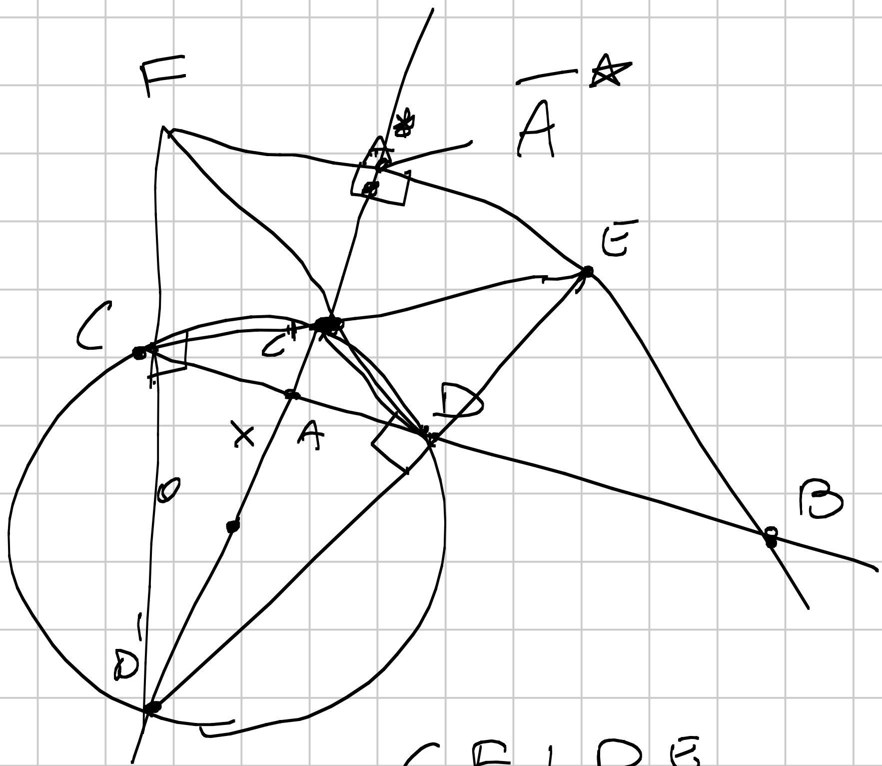
$$OA^* = \frac{R^2}{OA} \qquad C'D' = 2R$$



$$(C_1, D_1; A, A^*) = k = -1$$

$$=$$

$$(C_1, D_1; A^*, A) = \frac{1}{k}$$



$C_1F \perp D_1E$

$B = EF \cap CD$

$$(C_1, D_1, A, \bar{A}^*) \overset{E}{=} (C, D; A\, B)$$

$$\overset{F}{=}$$

$$= (D_1, C_1, A, \bar{A}^*)$$

$(C_1, D_1, A, \bar{A}^*) = -1$

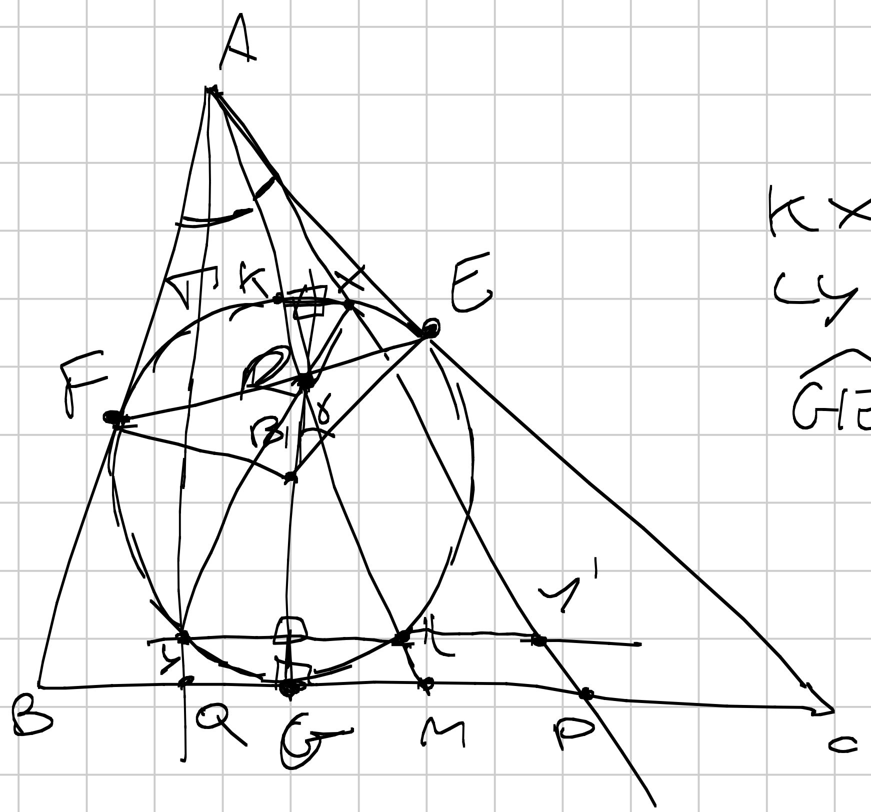
$(C_1, D_1, A, A^*) = -1$

$\bar{A}^* = A^*$

$(C, D; A, B) = -1$

Fine

G6 – 2005



$KX \parallel BC$

$LY \parallel BC$

$\widehat{GE} = \pi - \gamma$

$BQ = CP \quad (BM = MC)$

$QM = MP$

$\boxed{YL = LY'}$

$\boxed{\frac{LY}{KX}} = \frac{RL}{RK}$

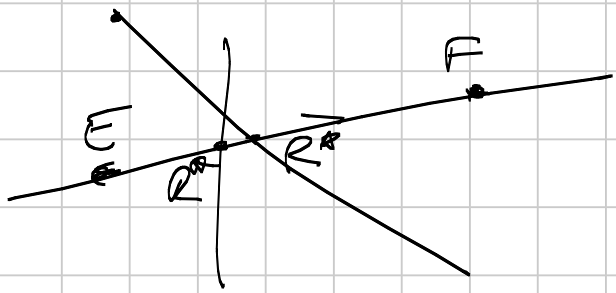
$\boxed{\frac{LY_1}{KX}} = \frac{AL}{AK}$

$$\frac{RL}{RK} = \frac{AL}{AK} \iff (A,R;K,L) = -1$$

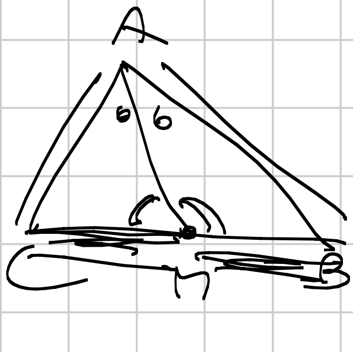
$$\iff E, R, F$$ allineati.

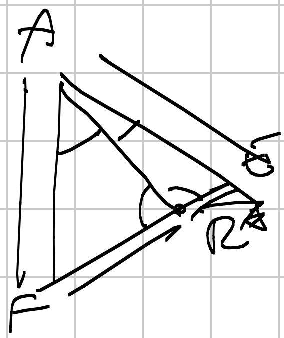
R^* $\quad$ $\bar{R}^*$

$R^* \to GI \cap EF$ $\qquad$ $\bar{R}^* \to AM \cap EF$



$$\boxed{\frac{ER^*}{R^*F} = \frac{E\bar{R}^*}{\bar{R}^*R}}$$

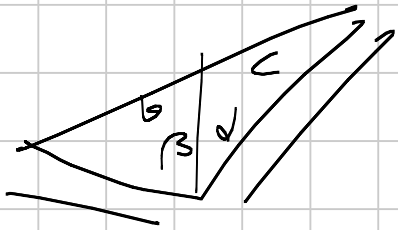




$$\frac{\sin \widehat{FA\bar{R}^*}}{\sin \widehat{\bar{R}^*AE}} = \frac{b}{c}$$

$$\frac{AF}{\sin \bar{R}^*} = \frac{F\bar{R}}{\sin \widehat{FA\bar{R}}} \qquad \frac{AE}{\bar{R}^*} = \frac{\bar{R}^*R}{\ }$$

$$\frac{FR^*}{R^*E} = \frac{b}{c}$$



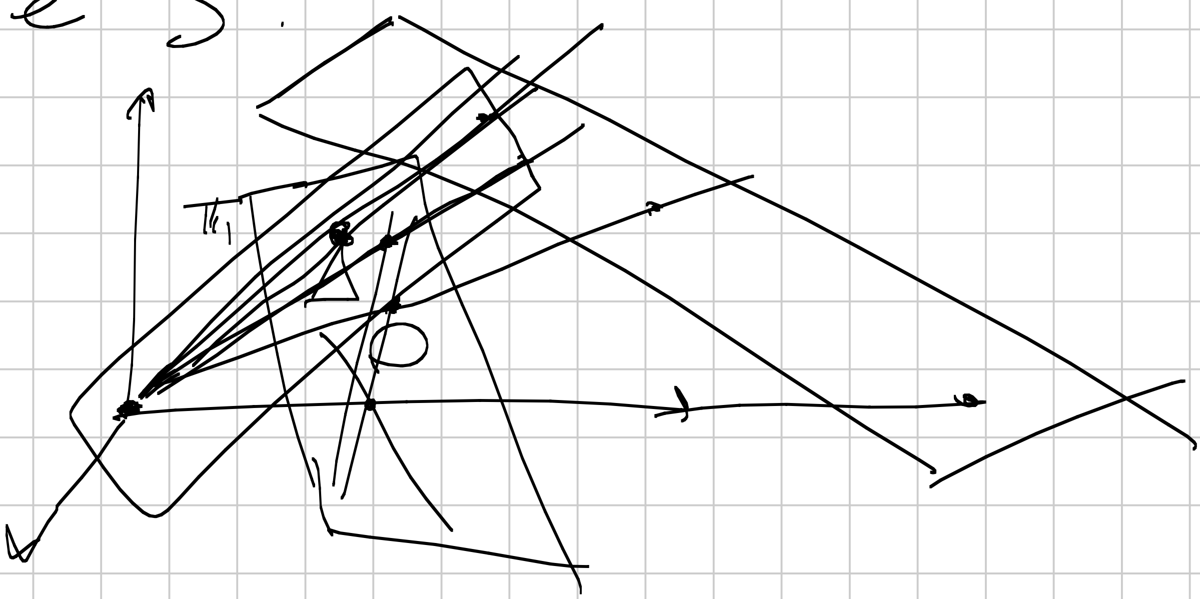
$$\frac{FR^*}{R^*E} = \frac{b}{c} = \frac{F\bar{R}^*}{\bar{R}^*E}$$

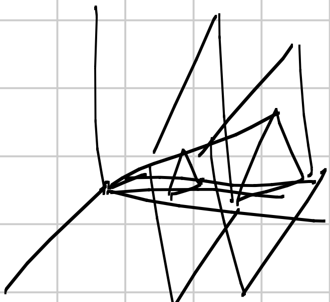
$$\bar{R}^* \equiv R^*$$

$\Rightarrow$ EF, GI, AM concorrono.

FINE

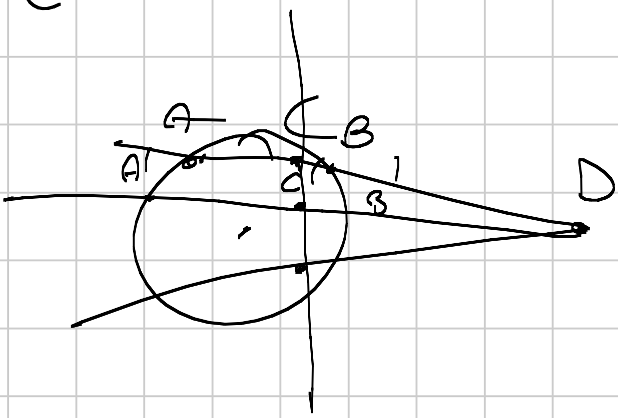
Parte 3:





♡ SALA ♡

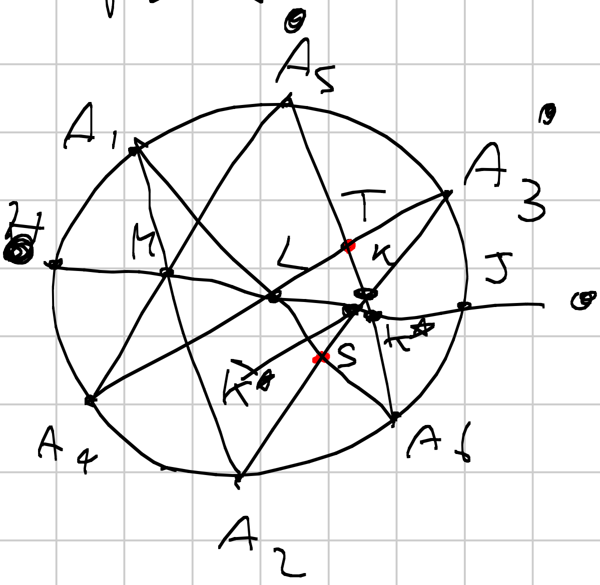
La Proiettività manda coniche in coniche



Teorema di Pascal

$A_1A_2 \cap A_4A_5$
$A_2A_3 \cap A_5A_6$
$A_3A_4 \cap A_6A_1$

Sono allineati



$$(H, M; L, J) \overset{\hat{A}_4}{=} (H, A_5; A_3, J)$$

$$\| \qquad \overset{\hat{T}}{=} (H, K^*; L; J)$$

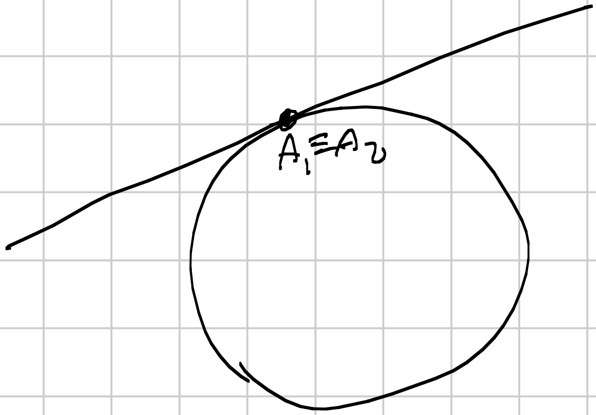
$$(H, M; L, J) \overset{\hat{A}_1}{=} (H, A_2; A_6, J)$$

$\hat{S} = (H, \bar{K}^{*}_{R} ; L, S)$

$\boxed{\bar{K}^{*} \equiv K^{*}}$

$\bar{K}^{*} \equiv K^{*}$

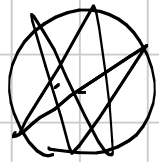
Fine

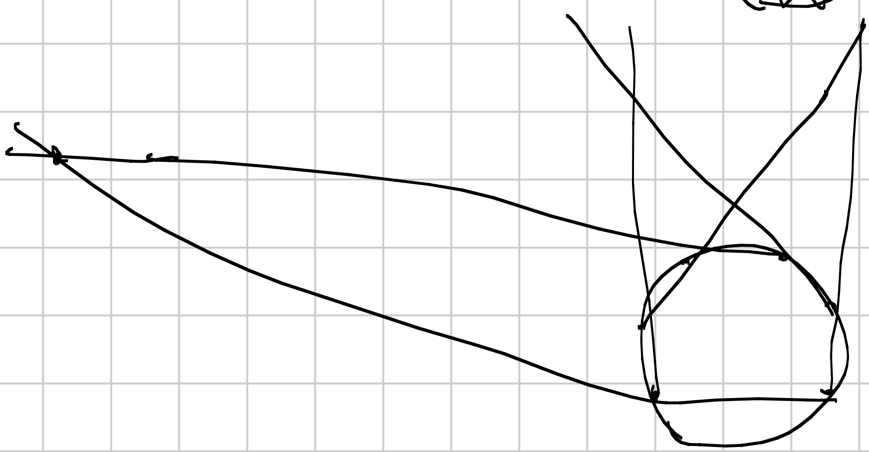


$A_1A_2 \cap A_4A_5$

120 modi di ordinare i punti

$\Rightarrow$ 120 allineati



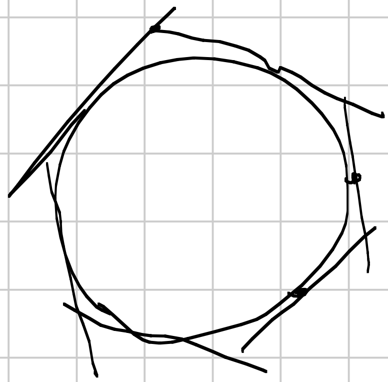


Teorema di Pappo

≡ Pascal, con coppie di rette.

∃ una proiettività che manda una circonferenza in una coppia di rette qualunque.

Teorema di BRIANCHON

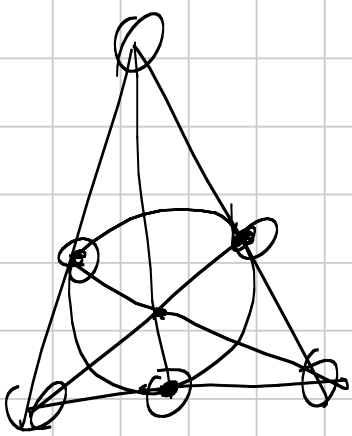


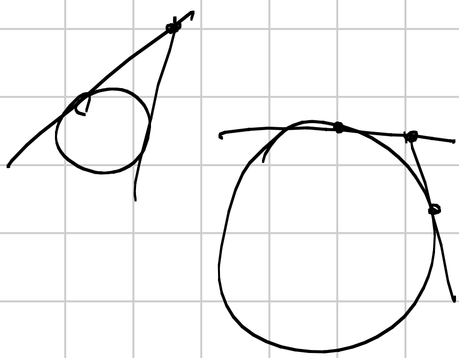
$r_1 \quad r_2 \quad r_3 \quad r_4 \quad r_5 \quad r_6$

$\overset{A_1}{r_1} \cap \overset{A_2}{r_2}$	$r_4 \cap r_5$	R_1
$r_2 \cap r_3$	$r_5 \cap r_6$	R_2
$r_3 \cap r_4$	$r_6 \cap r_1$	R_3

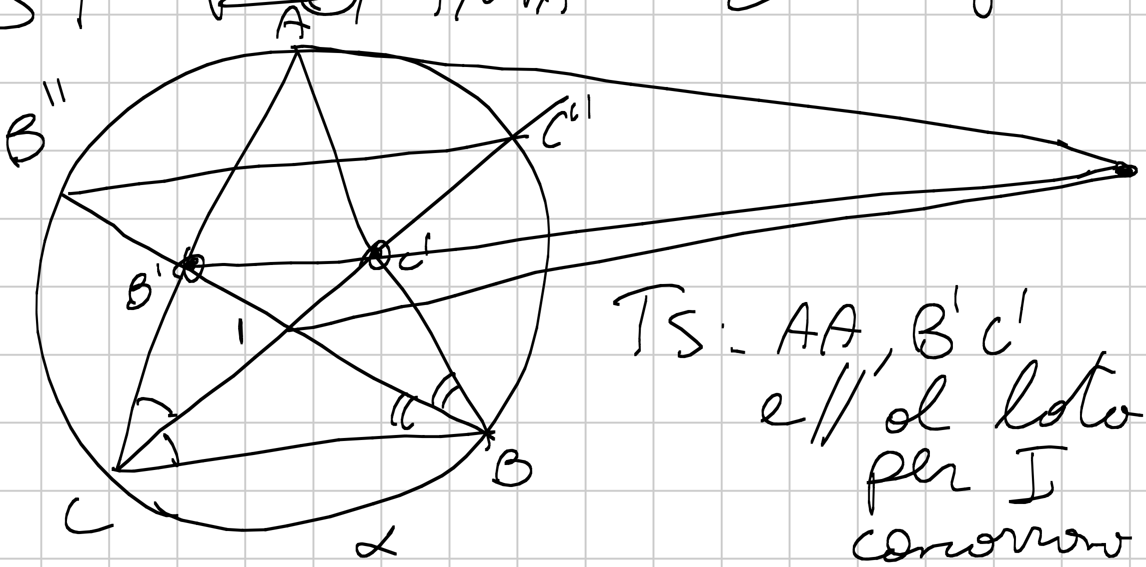
$R_1 \; R_2 \; R_3$ concorrono

Dim: Pascal + polari di ogni cosa.

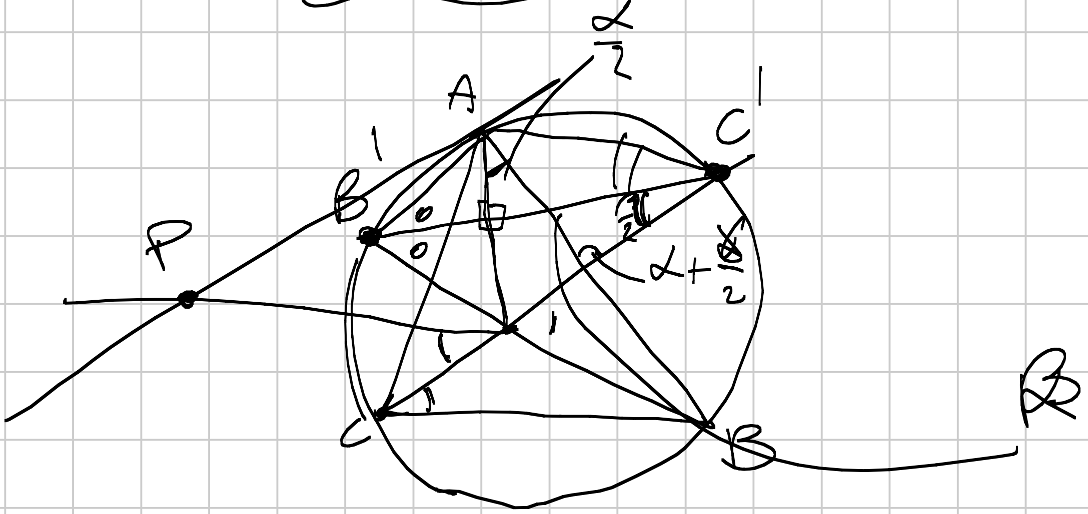




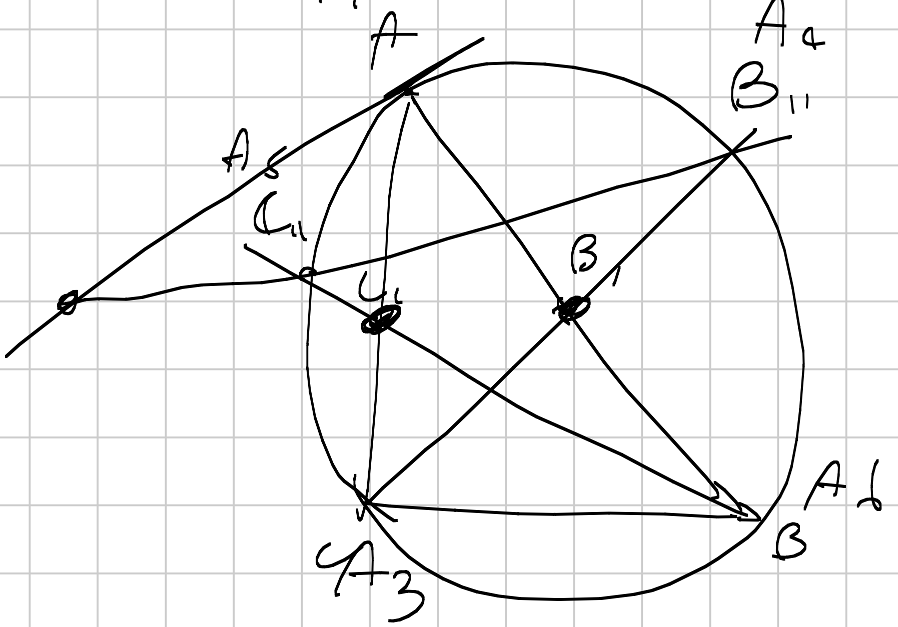
TST Romania 2008



Ts: AA, $B'C'$ e // al lato BC per I concorrono



$\widehat{PAU} = \beta \quad \widehat{CAI} = \frac{\alpha}{2} \qquad \boxed{\beta+\frac{\alpha}{2}}+\frac{\gamma}{2}$



$A_1A_2 \cap A_4A_5$

$A_2A_3 \cap A_5A_6$

$A_3A_4 \cap A_6A_1$

Fine.