Algebra 2

ma_go

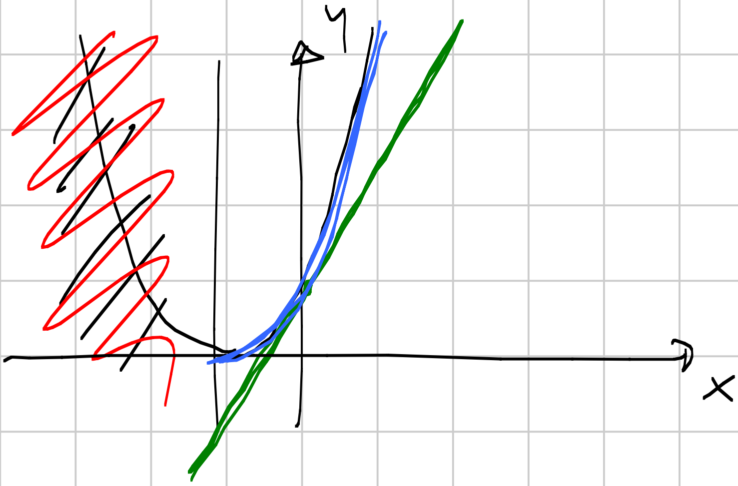
Titolo nota 24/08/2015

(0) Bernoulli: $\boxed{x \geq -1}$ reale,

$n \in \mathbb{N}$ intero ≥ 0

allora $(1+x)^n \geq 1+nx$.

es $2^n \geq 1+n$ $\forall n$ ....



(1) <u>Riarrangiamento</u>

$$\left.\begin{array}{l} a_1, a_2, \ldots, a_n \\ b_1, \ldots, b_n \end{array}\right\} \text{reali.}$$

$$a_1 \cdot b_1 + a_2 \cdot b_2 + \ldots + a_n \cdot b_n$$

$$a_1 \cdot b_2 + a_2 \cdot b_1 + \ldots + a_n \cdot b_n$$

σ perm. di $\{1, \ldots, n\}$

$$a_1 \cdot b_{\sigma(1)} + a_2 \cdot b_{\sigma(2)} + \ldots + a_n \cdot b_{\sigma(n)}$$

tra queste qual è la più grande?
quale la più piccola?

$a_1 = 1$, $a_2 = 2$
$b_1 = 3$, $b_2 = 4$

$\longrightarrow 1 \cdot 3 + 2 \cdot 4 = 11$

$\vee$

$\longrightarrow 1 \cdot 4 + 2 \cdot 3 = 10$

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ipotesi di comodo: $a_1 \leq a_2 \leq \dots \leq a_n$
$b_1 \leq b_2 \leq \dots \leq b_n$

<u>thm</u> (Riarrangiamento)

$$a_1 b_1 + \dots + a_n b_n \geq a_1 b_{\sigma(1)} + \dots + a_n b_{\sigma(n)} \geq$$
$$\geq a_1 b_n + a_2 b_{n-1} + \dots + a_n b_1$$

<u>dim</u> Le permutazioni sono prodotto di trasposizioni.

Cosa succede quando uno fa una trasposizione?

Immaginate di scambiare $i \leftrightarrow j$

σ, τ due permutaz. che diff. per una trasposizione.

$$a_1 \cancel{b_{\sigma(1)}} + \dots + a_n \cancel{b_{\sigma(n)}} \overset{?}{\gtrless} a_1 \cancel{b_{\tau(1)}} + \dots + a_n \cancel{b_{\tau(n)}}$$

$\leftarrow$ tutte $\quad \underbrace{a_i b_{\sigma(i)} + a_j b_{\sigma(j)} \gtrless a_i b_{\tau(i)} + a_j b_{\tau(j)}}_{= \text{il caso } n=2}$

$\tau(i) = \sigma(j)$, $\tau(j) = \sigma(i)$ $\quad (i<j)$

quello più grande sarà quello per cui $\sigma(i)<\sigma(j)$

[c'è nascosta un'induzione.] $\square$

es $\quad ab+bc+ca \leq a^2+b^2+c^2 \quad$ (a,b,c reali).

$(a,b,c) \quad (a_i)$

$(a,b,c) \quad (b_i)$

un altro accoppiam.

max fra tutti gli accoppiam.

"senza perdita di generalità" WLOG

$a \leq b \leq c$

$a \leq b \leq c$

per riarrangiam., abbiamo finito.

tentazione: $\quad ac + \cancel{b^2} + ca \leq a^2 + \cancel{b^2} + c^2$.

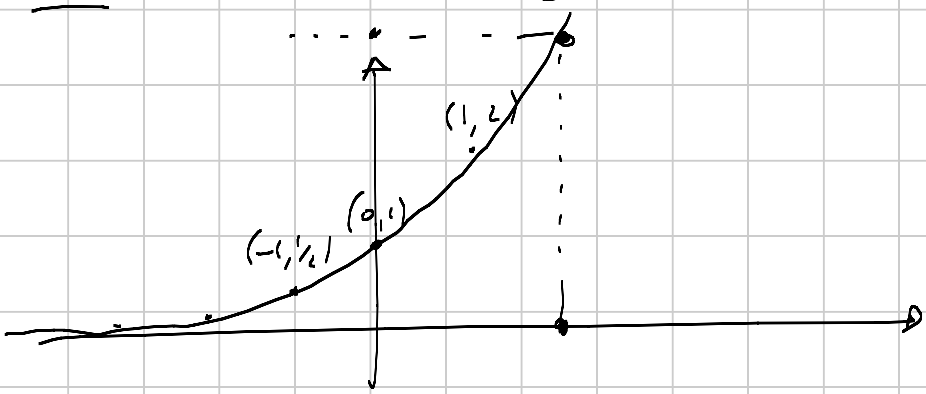
perché non è WLOG

$$\boxed{ac + b^2 + ac \overset{?}{\leq} ab + bc + ca}$$

non è simmetrica: NO WLOG!

es

$$a^b \cdot b^c \cdot c^a \le a^a b^b c^c \qquad a,b,c>0$$



$y = 2^x$

$37 = 2^x$

$x = \log_2 37$

$$37 \cdot 87 = 2^z \qquad z = \log_2 37 \cdot 87$$

$$2^{x+y} = 2^x \cdot 2^y$$

$$= x+y = \log_2 37 + \log_2 87$$

$$a^b b^c c^a \le a^a b^b c^c \iff \log a^b + \log b^c + \log c^a \le$$
$$\le \log a^a + \log b^b + \log c^c.$$

$$\log_2 37^{87} = 87 \cdot \log_2 37 \qquad (\text{es.})$$

$$b \cdot \log a + c \cdot \log b + a \cdot \log c \le a \cdot \log a + b \cdot \log b + c \cdot \log c.$$

accoppiati "non dritti" . $\le$ "accoppiati dritti"

RIARR.

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② Medie (basic)

$$QM \ge AM \ge GM$$

$x_1, \dots, x_n > 0$ reali

$$QM = \sqrt{\frac{x_1^2 + \dots + x_n^2}{n}}$$

$$AM = \frac{x_1 + \dots + x_n}{n}$$

$$GM = (x_1 \cdot x_2 \cdot \dots \cdot x_n)^{1/n}$$

1. $x_1 \le \dots \le x_n \rightsquigarrow \underset{\substack{a\\ \downarrow}}{\frac{x_1+x_n}{2}}, \underset{\substack{b\\ \downarrow}}{\frac{x_1+x_n}{2}}, x_2, \dots, x_{n-1}$

(stessa somma $\Rightarrow$ stessa AM)

$$QM(a, b, x_i)^2 = a^2 + b^2 + \underline{x_2^2 + \dots + x_{n-1}^2}$$

$$\updownarrow$$

$$QM(x_1, \dots, x_n)^2 = x_1^2 + x_n^2 + \underline{(x_2^2 \dots x_{n-1}^2)}$$

Capire se $a^2 + b^2 \le x_1^2 + x_n^2$

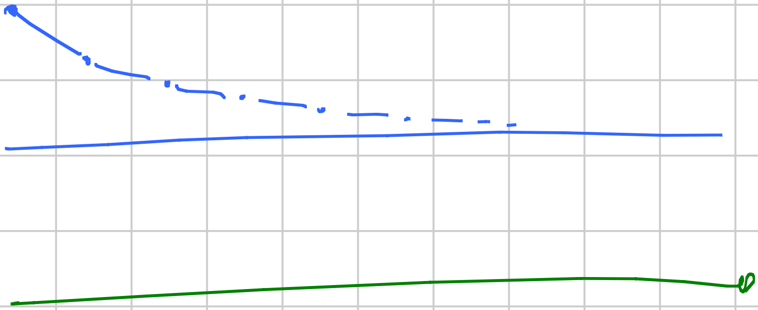
$$\overset{\shortparallel}{2\left(\frac{x_1+x_n}{2}\right)^2} < \frac{x_1^2 + x_n^2}{2}$$

AM - QM su due: $\sqrt{\frac{x^2+y^2}{2}} \ge \left(\frac{x+y}{2}\right)^2$

$$2(x^2+y^2) \ge (x+y)^2$$

$$\cancel{2}x^2 + \cancel{2}y^2 \ge \cancel{x^2} + \cancel{y^2} + 2xy$$

$$(x-y)^2 = x^2 - 2xy + y^2 \ge 0$$



AM ferma

n. iterazioni

non funziona perché "andare all'infinito è pericolo".

dim "vera": sostituisco a $(x_1, x_n) \longrightarrow (\underbrace{A}_{AM(x_1, \dots, x_n)}, \overbrace{x_1+x_n-A}^{y})$

che succede?

$$AM(A, x_2, \dots, x_{n-1}, y) = A = AM(x_i)$$

$$QM(A, x_2, \dots, x_{n-1}, y)^2 = A^2 + y^2 + (x_2^2 + \dots + x_{n-1}^2)$$

$$QM(x_1 \dots x_n)^2 = x_1^2 + x_n^2 + (\quad '' \quad)$$

Confronto A^2+y^2 con $x_1^2+x_n^2$?

$$A^2 + (x_1+x_n-A)^2 = A^2 + x_1^2 + x_2^2 + 2x_1x_2 - 2A(x_1+x_n) + A^2$$

$$\vee\wedge$$

$$x_1^2 + x_n^2$$

$$\cancel{2}A^2 - \cancel{2}A(x_1+x_n) + \cancel{2}x_1x_n \gtrless 0$$

$$(A-x_1)(A-x_n) \leq 0$$

$X_1 = \min x_i$

$X_n = \max x_i$

$A = AM(x_i)$

$\downarrow$

≤ 0 , in più è

< 0 se

$X_1 \neq X_n$..

[nascosta un'induzione]

<u>es.</u> $ab + bc + ca \leq a^2 + b^2 + c^2$ (2)

$$\parallel$$

$$\frac{(a+b+c)^2 - (a^2+b^2+c^2)}{2} \leq a^2+b^2+c^2$$

$$\frac{(a+b+c)^2}{9} \leq \frac{3(a^2+b^2+c^2)}{9}$$

$$\parallel$$

$$AM(a,b,c)^2 \leq QM(a,b,c)^2$$

- $x + 2y + 3z \geq 6$ se $xy^2z^3 = 1$

$$\parallel$$

$$GM(x, y^2, z^3)^3$$

$$\parallel$$

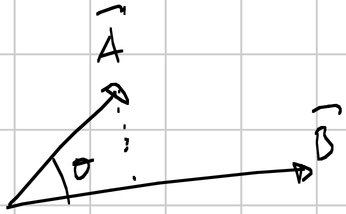
$AM(x,y,y,z,z,z) \qquad GM(x,y,y,z,z,z)^6$

$$\parallel$$

$$\frac{x+2y+3z}{6} \geq 1 \quad \checkmark$$

③ CAUCHY - SCHWARZ

<u>obiettivo</u> "$|\cos\theta| \leq 1$"



$$\vec{A}\cdot\vec{B} = \|\vec{A}\|\cdot\|\vec{B}\|\cdot\cos\theta.$$

$$|\cos\theta| \le 1 \implies$$

$$|\vec{A}\cdot\vec{B}| \le \|\vec{A}\|\cdot\|\vec{B}\|$$

$\vec{A} = (a_1, a_2)$

$\vec{B} = (b_1, b_2)$

$$\vec{A}\cdot\vec{B} = a_1 b_1 + a_2 b_2$$

$\vec{A} = (a_1, a_2, a_3)$

$\vec{B} = \ldots$

$$\vec{A}\cdot\vec{B} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$\vec{A} = (a_1, \ldots, a_n)$

$\vec{B} = (b_1, \ldots, b_n)$

$$\vec{A}\cdot\vec{B} \overset{\text{def}}{=} a_1 b_1 + \ldots + a_n b_n$$

$$\|\vec{A}\|^2 = \vec{A}\cdot\vec{A} = a_1^2 + \ldots + a_n^2$$

$$\|\vec{A}\|^2 = a_1^2 + a_2^2$$

$$\|\vec{A}\|^2 = \vec{A}\cdot\vec{A} = a_1^2 + a_2^2 + a_3^2$$

thm (CS) $\quad (\vec{A}\cdot\vec{B})^2 \le \|\vec{A}\|^2 \cdot \|\vec{B}\|^2$

$$\left(\sum_i a_i b_i\right)^2 \le \left(\sum a_i^2\right)\cdot\left(\sum b_i^2\right)$$

dim 1 $\quad \vec{C}_t$ vettore $\quad (t \in \mathbb{R})$

$$\vec{C}_t = \vec{A} + t\vec{B}$$

$$0 \leq \| \vec{C}_t \|^2 = \vec{C}_t \cdot \vec{C}_t = (\vec{A} + t\vec{B}) \cdot (\vec{A} + t\vec{B})$$

$$= \underbrace{\vec{A} \cdot \vec{A}}_{\|A\|^2} + 2t \underbrace{(\vec{A} \cdot \vec{B})}_{A \cdot B} + t^2 \underbrace{\vec{B} \cdot \vec{B}}_{\|B\|^2}$$

$$0 \leq \|A\|^2 + 2t(A \cdot B) + \|B\|^2 \cdot t^2 \qquad \forall t$$

$$\Downarrow$$

$$\Delta/4 \leq 0$$

$$(A \cdot B)^2 - \|A\|^2 \|B\|^2 \leq 0 .$$ win. ☺

$=$ in CS se e solo se $A \parallel B$ $\left(\begin{array}{l} A = tB \\ \text{oppure } B = tA \end{array} \right)$

$\square$

<u>dim</u> (2) (per casa) $x^2 \geq 0$

$$(a_i b_j - a_j b_i)^2 \geq 0 \quad \forall i, \forall j.$$

$$\Downarrow$$

$$\sum (a_i b_j - a_j \cdot b_i)^2 \geq 0 . \quad (= CS) \quad \square .$$

<u>es</u> $ab + bc + ca \leq \underbrace{a^2 + b^2 + c^2}_{\text{norma}^2 \text{ del vettore } (a,b,c)}$ (3)

$$\underset{\vec{A}}{(a,b,c)} \cdot \underset{\vec{B}}{(b,c,a)}$$

C.S. $(ab + bc + ca)^2 = (A \cdot B)^2 \leq \|A\|^2 \cdot \|B\|^2 = (a^2 + b^2 + c^2)^2$

<u>es</u> (Lemma di Titu) $a_i > 0,\ x_i > 0$

$$\frac{\left(\sum_i a_i\right)^2}{\sum_i x_i} \le \sum_i \frac{a_i^2}{x_i}$$

$$A \cdot B \le \|A\| \cdot \|B\|$$

$$\Rightarrow \underbrace{\left(\sum_i a_i\right)^2}_{A\cdot B} \le \underbrace{\left(\sum_i a_i^2/x_i\right)}_{\left\|\frac{a_1}{\sqrt{x_1}}, \dots, \frac{a_n}{\sqrt{x_n}}\right\|^2 \ (A)} \underbrace{\left(\sum_i x_i\right)}_{\left\|\sqrt{x_1}, \dots, \sqrt{x_n}\right\|^2 \ (B)}$$

Per CS, abbiamo finito. $\square$

④ JENSEN (& Convessità).

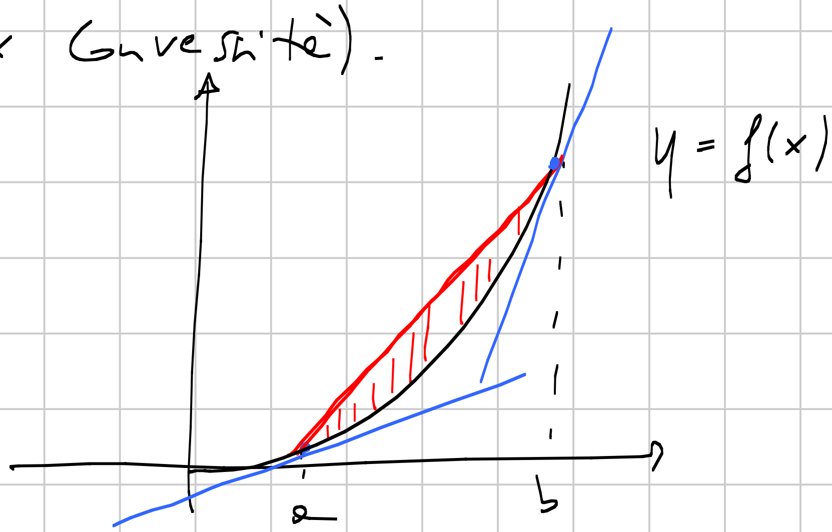
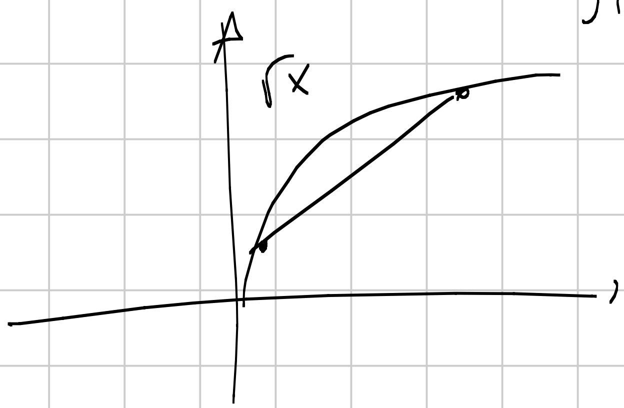
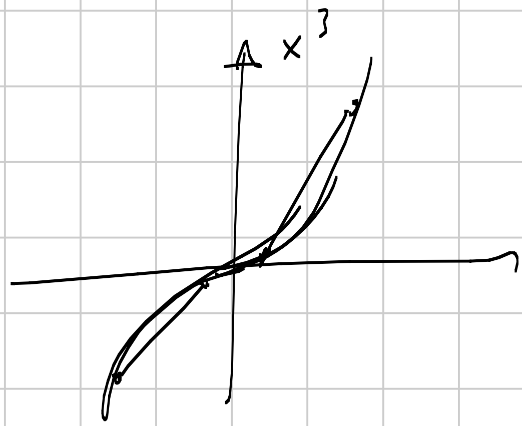
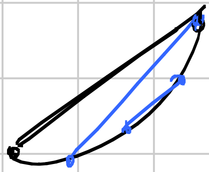


grafico di f tra a e b
sta <u>sotto</u> (il grafico del) segmento.

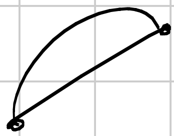




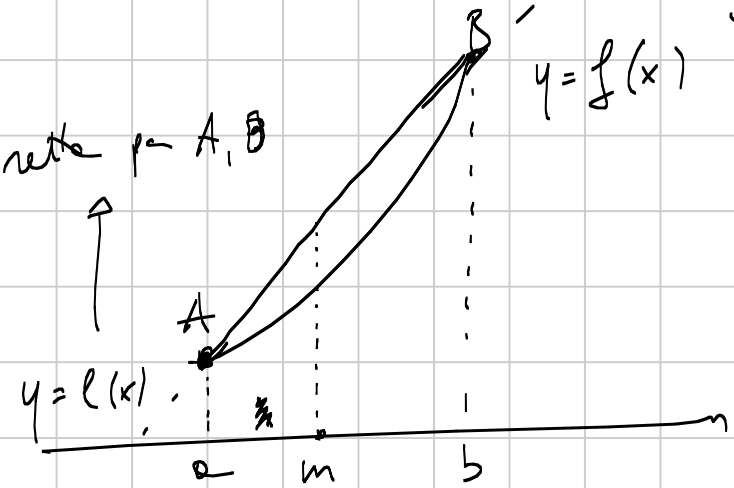
una funzione che ha la proprietà per tutti i punti a, b in $[0,37]$ ✓ si chiama <u>convessa</u> in $[0,37]$



se ha la proprietà opposta si dice <u>concava</u>.



Cosa vuol dire che "il grafico sta sotto il segmento"?



$$f(m) \leq \ell(m)$$

comb. convessa di a, b.

$$m = \lambda a + \mu b \quad \text{con } \lambda + \mu = 1, \ \lambda, \mu \geq 0$$

$$\text{"} \lambda = \frac{m-a}{b-a}, \quad \mu = \frac{b-m}{b-a} \text{"}.$$

$$f(\lambda a + \mu b) \leq \ell(\lambda a + \mu b) \stackrel{es}{=} \lambda f(a) + \mu f(b)$$

<u>thm</u> (Jensen) f è convessa, $\lambda_1 + \dots + \lambda_n = 1$, $\lambda_i \geq 0$

$$f(\lambda_1 x_1 + \dots + \lambda_n x_n) \leq \lambda_1 f(x_1) + \dots + \lambda_n f(x_n).$$

dim (es.).

"sposta il problema" verso la convessità.

es $\boxed{\bullet\ x^{\cancel{n}\,\alpha} \text{ è convessa } \forall \alpha \geq 1,\ \forall x \geq 0}$

- $1/x^n$ è convessa per $x > 0$ $\forall n > 0$.
- a^x sono convessi $\forall x$, $\forall a > 0$
- x^n è concava $\forall n$ dispari, $\forall x \leq 0$
- $\log x$ è concava $\forall x > 0$
- x^α è concava per $x > 0$ e $\alpha \leq 1$ (reale)

es Medie generalizzate.

$x_1, \dots, x_n > 0$, $p > 0$

$$\text{Media p-esima} = \left(\frac{x_1^p + \dots + x_n^p}{n}\right)^{1/p}$$

thm Media p-esima $\leq$ Media q-esima se $p < q$.

dim Mi riconduco a AM $\leq$ Media q-esima con $q > 1$ (es. per voi).

$q > 1$, $f(x) = x^q$ è convessa.

per Jensen: $f(\lambda_1 x_1 + \dots + \lambda_n x_n) \leq \lambda_1 f(x_1) + \dots + \lambda_n f(x_n)$

$$\lambda_1 x_1^2 + \dots + \lambda_n x_n^2$$

scelgo $\lambda_1 = \dots = \lambda_n = \frac{1}{n}$

$$(AM)^2 \le \frac{x_1^2 + \dots + x_n^2}{n} = (M_2)^2 \qquad \square$$

(5) Nesbitt.

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2} \qquad \forall a,b,c \geq 0.$$

dim intelligente (CS) Lemma di Titu.

$$\sum_i \frac{a_i^2}{x_i} \qquad \sum x_i \quad \left(\sum a_i\right)^2$$

$$\frac{a}{b+c} = \frac{a^2}{ab+ac}$$

$a^2 \to a_i$ $\qquad ab+ac$: x_i

$$\frac{\left(\sum_i a_i\right)^2}{\sum_i x_i} \le \sum_i \frac{a_i^2}{x_i} = LHS$$

$$= \frac{(a+b+c)^2}{ab+ac+ba+bc+ca+cb} = \frac{(a+b+c)^2}{2(ab+bc+ca)}$$

$$LHS \geq \frac{(a+b+c)^2}{\cancel{2}(ab+bc+ca)} \overset{?}{\geq} \frac{3}{\cancel{2}} \qquad a^2+b^2+c^2+2(ab+bc+ca) \geq 3(ab+bc+ca) \text{ vero!}$$

$\square$

dim (2) Jensen.

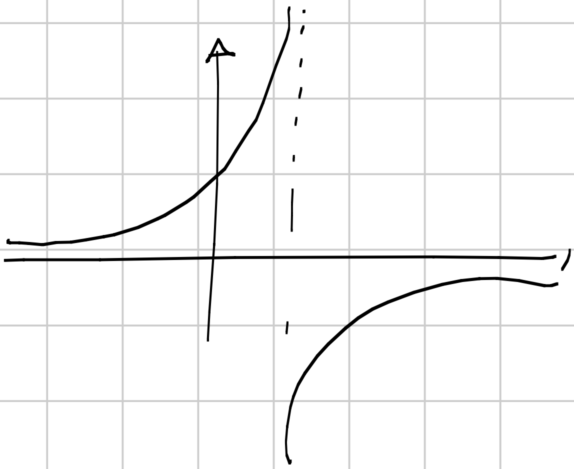
Posso supporre che $a+b+c=1$ (di omogeneità / normalizz.)

$$\frac{a}{b+c} = \frac{a}{1-a} = f(a)$$

$$f(a)+f(b)+f(c) \geq \frac{3}{2}$$

$$\frac{x}{1-x} = \frac{x-1}{1-x} + \frac{1}{1-x} = -1 + \frac{1}{1-x}$$

per la convessità, non conta



è convessa per $x<1$

ok!

$$f(a)+f(b)+f(c) = \left(\frac{1}{3} f(a) + \frac{1}{3} f(b) + \frac{1}{3} f(c)\right)\cdot 3$$

$$\geq 3\cdot f\left(\frac{a+b+c}{3}\right) = f\left(\frac{1}{3}\right)\cdot 3 = \frac{\frac{1}{3}}{1-\frac{1}{3}}\cdot 3 = 3/2.$$

dim 3

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$$

WLOG $a \leq b \leq c$

$a+b \leq a+c \leq b+c$

$\vee\!\!/$ RIARR

$$\frac{1}{b+c} \le \frac{1}{a+c} \le \frac{1}{a+b}$$

$$\frac{a}{a+b} + \frac{b}{b+c} + \frac{c}{c+a}$$

$$\frac{b}{a+b} \qquad \frac{c}{b+c} \qquad \frac{a}{a+c}$$

$$\frac{a}{a+c} \qquad \frac{b}{a+b} \qquad \frac{c}{b+c}$$

RR : $LHS \ge \frac{a}{a+b} + \frac{b}{b+c} + \frac{c}{c+a}$

RA : $LHS \ge \frac{b}{a+b} + \frac{c}{b+c} + \frac{a}{c+a}$

RV : $2LHS \ge 1 + 1 + 1 = 3$

- trovare il minimo di

$\rightarrow$ $x + 2y + 3z$ al variare di $x, y, z > 0$ t.c. $x^3 \cdot y^2 \cdot z = 7$

- $\rightarrow$ $\min \left(\sqrt{x+y+z} - \sqrt{x-1} - \sqrt{y-1} - \sqrt{z-1} \right)$

con $x, y, z \ge 1$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$

- dim che $\left(\frac{1}{n} \sum_i a_i b_{n+1-i} \right) \le \left(\frac{\sum_i a_i}{n} \right) \left(\frac{\sum_i b_i}{n} \right) \le$

$\le \left(\frac{1}{n} \sum_i a_i b_i \right)$

se $a_1 \le \dots \le a_n$
$b_1 \le \dots \le b_n$

+ es. 86, 88 pp. 15-16

+ es 5, 6, 9 p. 24

Cominciamo a correggere?

2) $\min \sqrt{x+y+z} - \left(\sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1}\right)$

con i vincoli $x,y,z \geq 1$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$.

claim $\min = 0$.

$$\sqrt{x+y+z} \geq \sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1}$$

$$\cancel{x}+\cancel{y}+\cancel{z} \geq \cancel{x}+\cancel{y}+\cancel{z} - 3 + 2\left(\sqrt{(x-1)(y-1)} + \sqrt{(y-1)(z-1)} + \sqrt{\ }\right)$$

$$\sum_{cyc} 2\sqrt{(x-1)(y-1)} \overset{?}{\leq} 3$$

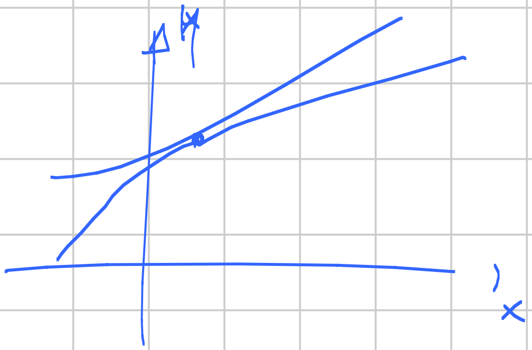
$$\hookrightarrow \overset{AM-GM}{\leq} \sum_{cyc} (x-1)+(y-1) \overset{?}{\leq} 3$$

$= x=y=z=\frac{3}{2}$

$\max\ 2(x+y+z-3)$ col vincolo $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 2$

$$A(x) \leq B(x)$$

$A(x) = B(x)$ solo se $x = \pi$



- $xy + yz + zx = 2xyz$

$\longrightarrow \quad xy + yz + zx \geq 3(xyz)^{2/3}$

$2xyz \geq 3(xyz)^{2/3} \Rightarrow xyz \geq \left(\frac{3}{2}\right)^3$

$x + y + z \geq \frac{9}{2}$

QM-AM: $\sqrt{x-1}, \sqrt{y-1}, \sqrt{z-1}$ $\quad \geq 9/2$

$$\frac{\sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1}}{3} \leq \sqrt{\frac{x+y+z-3}{3}}$$

nel modo infido: $\quad \min = 0$

$$\underbrace{\sqrt{x+y+z}}_{\text{prodotto di due norme}} \geq \underbrace{\sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1}}_{\text{prodotto scalare}}$$

$$\sqrt{x+y+z} = \overset{A}{\|(\sqrt{x}, \sqrt{y}, \sqrt{z})\|} \rightsquigarrow B = \left(\frac{\sqrt{x-1}}{\sqrt{x}}, \sqrt{\frac{y-1}{y}}, \sqrt{\frac{z-1}{z}}\right)$$

ok.

C.S. $\quad A \cdot B = RHS \leq \|A\| \cdot \|B\|$

$\|B\|^2 = 1 - \frac{1}{x} + 1 - \frac{1}{y} + 1 - \frac{1}{z} = 1.$

5

$x_i y_i^{1/2}$

$x_1\sqrt{y_1} + \dots + x_n\sqrt{y_n} = 9n$ $\quad \longrightarrow$ stima QM(x_i)

$y' \longrightarrow y_1 + \dots + y_n = 8n$

$$QM^2(x_i) = \frac{x_1^2 + \dots + x_n^2}{n} \longleftarrow x^2$$

$$B = (\sqrt{y_1}, \dots, \sqrt{y_n}) \longrightarrow \|B\|^2 = 8n$$

$$A = (x_1, \dots, x_n) \longrightarrow \|A\|^2 = QM(x_i)^2 \cdot n$$

vincolo: $A \cdot B$

C.S. $(A \cdot B)^2 \leq \|B\|^2 \cdot \|A\|^2$

$$\overset{\shortparallel}{81 \cancel{n^2}} \leq 8 \cancel{n} \cdot \cancel{n} \cdot QM^2 \Longrightarrow$$

$$QM(x_i) \geq \frac{9}{2\sqrt{2}}$$

Cosa serve per avere $=$?

$=$ in CS $\Longleftrightarrow$ $A \parallel B$.

$\Longleftrightarrow$ $x_i = \lambda\sqrt{y_i}$ $\forall i$ (per un certo λ)

$$QM(x_i)^2 = \frac{\sum \lambda^2 y_i}{n} = \lambda^2 \frac{\sum y_i}{n} = \lambda^2 8.$$

vincolo: $\sum x_i \sqrt{y_i} = 9n$

$$\sum \lambda y_i = \lambda \cdot 8n \Longrightarrow \lambda = \frac{9}{8}$$

6. trovare la migliore C t.c. $(x-y)(2y-x) \le Cxy$

vincoli $0 \le y \le x \le 2y$ reali.

$2 \ge \frac{x}{y} \ge 1$
$\quad\quad\;\; \| $
$\quad\quad\;\; t$

Possiamo supporre $x, y \neq 0$.

$$\frac{(x-y)(2y-x)}{xy} \le C .$$

$$\min C = \max_{x,y,\dots} \left(\frac{(x-y)(2y-x)}{xy} \right).$$

$$\frac{(x-y)(2y-x)}{xy} = \Big(1 - \underbrace{\frac{y}{x}}_{1/t}\Big)\Big(2 - \underbrace{\frac{x}{y}}_{t}\Big) =$$

$$= (2 - t)\left(1 - \frac{1}{t}\right) = \qquad 1 \le t \le 2$$

$$= 2 - \frac{2}{t} - t + 1 = 3 - \frac{2}{t} - t$$

$$\max(\text{———}) = 3 - \min\left(\frac{2}{t} + t\right)$$

$$\sqrt{\frac{2}{t} \cdot t} = \sqrt{2} \overset{AMGM}{\underset{=}{\le}} \frac{\frac{2}{t} + t}{2} \Rightarrow$$

$$\min\{\quad\} \underset{=}{\ge} 2\sqrt{2} \Rightarrow$$

$$\max\{\cdot \text{—}\} = 3 - 2\sqrt{2} \Rightarrow C = 3 - 2\sqrt{2}$$

$=$ $\Leftarrow$ $\frac{2}{t} = t \Rightarrow t = \sqrt{2}$ ✓

$$(c-3)xy + x^2 + 2y^2 \geq 0 \quad \rightsquigarrow \quad (c-3)t + t^2 + 2 \geq 0$$

$$(x - y\sqrt{2})^2 \geq 0$$

$$2y^2 + x^2 - 2\sqrt{2}\,xy$$