

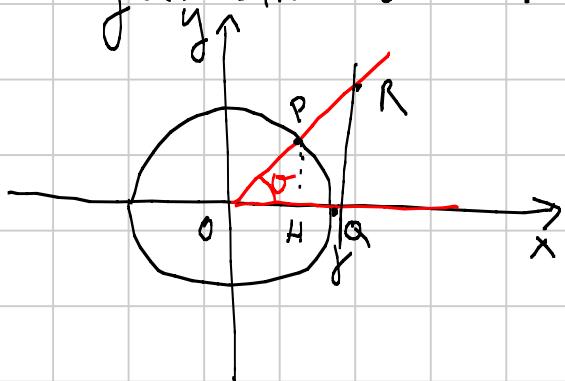
# G1 - basic - Senior '15 - Giacchino

Titolo nota

22/08/2015

Consideriamo la circonferenza

goniometrica: (f)



Sia  $\theta$  un angolo.

1) Misuriamo, a partire dal senso positivo delle  $x$  in senso antiorario, un angolo  $\theta$  (es.  $\theta = 30^\circ$ ) e traccia una semiretta

Sia  $P$  l'int. di questa con la cir. gom.

$$\begin{aligned} PH &\stackrel{\text{def}}{=} \sin \theta \\ OH &\stackrel{\text{def}}{=} \cos \theta \end{aligned} \quad \sim P(\cos \theta, \sin \theta)$$

C.1 Pitagora su  $\hat{P}OH$ :  $PH^2 + OH^2 = 1 \rightarrow \boxed{\sin^2 \theta + \cos^2 \theta = 1}$

Tg Tracca da  $Q$  la  $\perp$  all'asse  $x$ . Interseca la semiretta in  $R$ .

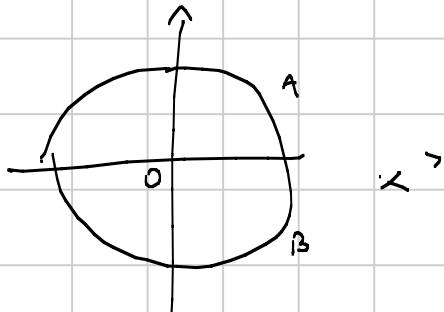
$$QR \stackrel{\text{def}}{=} \tan \theta$$

C.2  $\hat{P}OH \sim \hat{R}OQ \rightarrow \frac{PH}{OH} = \frac{QR}{OQ} \rightarrow \boxed{\frac{\sin \theta}{\cos \theta} = \tan \theta}$

$$\pi \rightsquigarrow 180^\circ$$

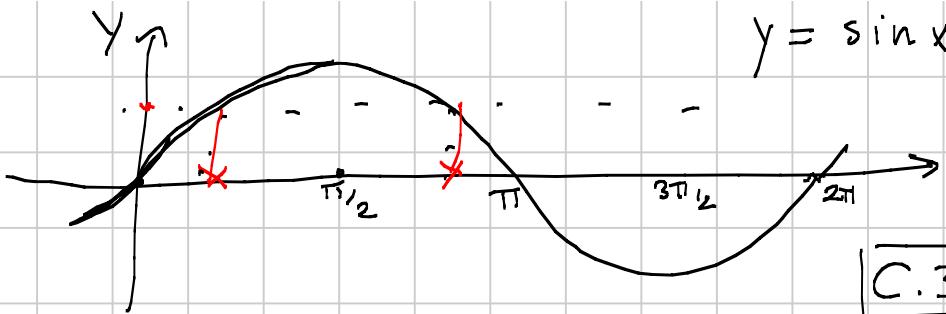
Inciso  $\pi : 180^\circ = \text{ang. in rad. : ang. in gradi}$

Per chiudere ...



Angoli:  $r \sin \omega = -1 \tan$

Angoli:	0	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{\pi}{3}, \frac{1}{2}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{3}$	0
	0	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{\pi}{3}, \frac{1}{2}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{3}$	0
	$\frac{\pi}{2}$	$\pi$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{2\pi}{3}, \frac{1}{2}$	$\frac{\pi}{4}, \frac{1}{2}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{\pi}{2}$
	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{5\pi}{4}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{3\pi}{4}$
	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{2\pi}{3}, \frac{1}{2}$	$\frac{3\pi}{4}$	$\frac{4\pi}{3}$	$\frac{7\pi}{6}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{5\pi}{4}$
	$\frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{5\pi}{4}$	$\frac{6\pi}{5}$	$\frac{11\pi}{6}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{7\pi}{4}$
	$\frac{\pi}{12}$	$\frac{13\pi}{12}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{6}, \frac{1}{2}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{17\pi}{12}$	$\frac{13\pi}{6}, \frac{1}{2}$	$\frac{13\pi}{4}$
	$-\frac{\pi}{12}$	$-\frac{13\pi}{12}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$-\frac{7\pi}{6}, \frac{1}{2}$	$-\frac{11\pi}{8}$	$-\frac{13\pi}{8}$	$-\frac{17\pi}{12}$	$-\frac{13\pi}{6}, \frac{1}{2}$	$-\frac{13\pi}{4}$
	$-\frac{\pi}{6}$	$-\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$-\frac{4\pi}{3}, \frac{1}{2}$	$-\frac{3\pi}{4}$	$-\frac{4\pi}{3}$	$-\frac{5\pi}{6}$	$-\frac{4\pi}{3}, \frac{1}{2}$	$-\frac{5\pi}{4}$
	$-\frac{\pi}{3}$	$-\frac{5\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$-\frac{2\pi}{3}, \frac{1}{2}$	$-\frac{5\pi}{8}$	$-\frac{3\pi}{4}$	$-\frac{7\pi}{6}$	$-\frac{2\pi}{3}, \frac{1}{2}$	$-\frac{3\pi}{4}$
	$-\frac{\pi}{4}$	$-\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$-\frac{\pi}{2}, \frac{1}{2}$	$-\frac{3\pi}{8}$	$-\frac{\pi}{4}$	$-\frac{5\pi}{8}$	$-\frac{\pi}{2}, \frac{1}{2}$	$-\frac{\pi}{4}$
	$-\frac{\pi}{6}$	$-\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$-\frac{\pi}{3}, \frac{1}{2}$	$-\frac{\pi}{8}$	$-\frac{\pi}{6}$	$-\frac{3\pi}{8}$	$-\frac{\pi}{3}, \frac{1}{2}$	$-\frac{\pi}{6}$
	$-\frac{\pi}{12}$	$-\frac{13\pi}{12}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$-\frac{7\pi}{6}, \frac{1}{2}$	$-\frac{11\pi}{8}$	$-\frac{13\pi}{8}$	$-\frac{17\pi}{12}$	$-\frac{13\pi}{6}, \frac{1}{2}$	$-\frac{13\pi}{4}$
	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{17\pi}{12}$	$\frac{13\pi}{6}, \frac{1}{2}$	$\frac{13\pi}{4}$
	$\frac{11\pi}{8}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{2\pi}{3}, \frac{1}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{2\pi}{3}, \frac{1}{2}$	$\frac{3\pi}{4}$
	$\frac{13\pi}{8}$	$\frac{5\pi}{8}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{11\pi}{12}$	$\frac{7\pi}{8}$	$\frac{19\pi}{12}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{7\pi}{8}$
	$\frac{17\pi}{12}$	$\frac{11\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{6}, \frac{1}{2}$	$\frac{23\pi}{12}$	$\frac{13\pi}{8}$	$\frac{29\pi}{12}$	$\frac{7\pi}{6}, \frac{1}{2}$	$\frac{13\pi}{8}$
	$\frac{13\pi}{6}$	$\frac{13\pi}{4}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{3}, \frac{1}{2}$	$\frac{25\pi}{12}$	$\frac{17\pi}{8}$	$\frac{31\pi}{12}$	$\frac{13\pi}{3}, \frac{1}{2}$	$\frac{17\pi}{8}$
	$\frac{13\pi}{4}$	$\frac{13\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{27\pi}{12}$	$\frac{19\pi}{8}$	$\frac{33\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{19\pi}{8}$
	$\frac{13\pi}{2}$	$\pi$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\pi, \frac{1}{2}$	$\frac{29\pi}{12}$	$\frac{21\pi}{8}$	$\frac{35\pi}{12}$	$\pi, \frac{1}{2}$	$\frac{21\pi}{8}$
	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{31\pi}{12}$	$\frac{23\pi}{8}$	$\frac{37\pi}{12}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{23\pi}{8}$
	$\frac{11\pi}{8}$	$\frac{5\pi}{8}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{33\pi}{12}$	$\frac{25\pi}{8}$	$\frac{39\pi}{12}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{25\pi}{8}$
	$\frac{13\pi}{8}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{35\pi}{12}$	$\frac{27\pi}{8}$	$\frac{41\pi}{12}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{27\pi}{8}$
	$\frac{17\pi}{12}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{37\pi}{12}$	$\frac{29\pi}{8}$	$\frac{43\pi}{12}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{29\pi}{8}$
	$\frac{13\pi}{6}$	$\frac{13\pi}{4}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{39\pi}{12}$	$\frac{31\pi}{8}$	$\frac{45\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{31\pi}{8}$
	$\frac{13\pi}{4}$	$\frac{13\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{41\pi}{12}$	$\frac{33\pi}{8}$	$\frac{47\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{33\pi}{8}$
	$\frac{13\pi}{2}$	$\pi$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\pi, \frac{1}{2}$	$\frac{43\pi}{12}$	$\frac{35\pi}{8}$	$\frac{49\pi}{12}$	$\pi, \frac{1}{2}$	$\frac{35\pi}{8}$
	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{45\pi}{12}$	$\frac{37\pi}{8}$	$\frac{51\pi}{12}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{37\pi}{8}$
	$\frac{11\pi}{8}$	$\frac{5\pi}{8}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{47\pi}{12}$	$\frac{39\pi}{8}$	$\frac{53\pi}{12}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{39\pi}{8}$
	$\frac{13\pi}{8}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{49\pi}{12}$	$\frac{41\pi}{8}$	$\frac{55\pi}{12}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{41\pi}{8}$
	$\frac{17\pi}{12}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{51\pi}{12}$	$\frac{43\pi}{8}$	$\frac{57\pi}{12}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{43\pi}{8}$
	$\frac{13\pi}{6}$	$\frac{13\pi}{4}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{53\pi}{12}$	$\frac{45\pi}{8}$	$\frac{59\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{45\pi}{8}$
	$\frac{13\pi}{4}$	$\frac{13\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{55\pi}{12}$	$\frac{47\pi}{8}$	$\frac{61\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{47\pi}{8}$
	$\frac{13\pi}{2}$	$\pi$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\pi, \frac{1}{2}$	$\frac{57\pi}{12}$	$\frac{49\pi}{8}$	$\frac{63\pi}{12}$	$\pi, \frac{1}{2}$	$\frac{49\pi}{8}$
	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{59\pi}{12}$	$\frac{51\pi}{8}$	$\frac{65\pi}{12}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{51\pi}{8}$
	$\frac{11\pi}{8}$	$\frac{5\pi}{8}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{61\pi}{12}$	$\frac{53\pi}{8}$	$\frac{67\pi}{12}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{53\pi}{8}$
	$\frac{13\pi}{8}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{63\pi}{12}$	$\frac{55\pi}{8}$	$\frac{69\pi}{12}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{55\pi}{8}$
	$\frac{17\pi}{12}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{65\pi}{12}$	$\frac{57\pi}{8}$	$\frac{71\pi}{12}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{57\pi}{8}$
	$\frac{13\pi}{6}$	$\frac{13\pi}{4}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{67\pi}{12}$	$\frac{59\pi}{8}$	$\frac{73\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{59\pi}{8}$
	$\frac{13\pi}{4}$	$\frac{13\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{69\pi}{12}$	$\frac{61\pi}{8}$	$\frac{75\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{61\pi}{8}$
	$\frac{13\pi}{2}$	$\pi$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\pi, \frac{1}{2}$	$\frac{71\pi}{12}$	$\frac{63\pi}{8}$	$\frac{77\pi}{12}$	$\pi, \frac{1}{2}$	$\frac{63\pi}{8}$
	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{73\pi}{12}$	$\frac{65\pi}{8}$	$\frac{79\pi}{12}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{65\pi}{8}$
	$\frac{11\pi}{8}$	$\frac{5\pi}{8}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{75\pi}{12}$	$\frac{67\pi}{8}$	$\frac{81\pi}{12}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{67\pi}{8}$
	$\frac{13\pi}{8}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{77\pi}{12}$	$\frac{69\pi}{8}$	$\frac{83\pi}{12}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{69\pi}{8}$
	$\frac{17\pi}{12}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{79\pi}{12}$	$\frac{71\pi}{8}$	$\frac{85\pi}{12}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{71\pi}{8}$
	$\frac{13\pi}{6}$	$\frac{13\pi}{4}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{81\pi}{12}$	$\frac{73\pi}{8}$	$\frac{87\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{73\pi}{8}$
	$\frac{13\pi}{4}$	$\frac{13\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{83\pi}{12}$	$\frac{75\pi}{8}$	$\frac{89\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{75\pi}{8}$
	$\frac{13\pi}{2}$	$\pi$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\pi, \frac{1}{2}$	$\frac{85\pi}{12}$	$\frac{77\pi}{8}$	$\frac{91\pi}{12}$	$\pi, \frac{1}{2}$	$\frac{77\pi}{8}$
	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{87\pi}{12}$	$\frac{79\pi}{8}$	$\frac{93\pi}{12}$	$\frac{7\pi}{3}, \frac{1}{2}$	$\frac{79\pi}{8}$
	$\frac{11\pi}{8}$	$\frac{5\pi}{8}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{89\pi}{12}$	$\frac{81\pi}{8}$	$\frac{95\pi}{12}$	$\frac{5\pi}{3}, \frac{1}{2}$	$\frac{81\pi}{8}$
	$\frac{13\pi}{8}$	$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}, \frac{1}{2}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{91\pi}{12}$	$\frac{83\pi}{8}$	$\frac{97\pi}{12}$	$\frac{4\pi}{3}, \frac{1}{2}$	$\frac{83\pi}{8}$
	$\frac{17\pi}{12}$	$\frac{\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{93\pi}{12}$	$\frac{85\pi}{8}$	$\frac{99\pi}{12}$	$\frac{3\pi}{2}, \frac{1}{2}$	$\frac{85\pi}{8}$
	$\frac{13\pi}{6}$	$\frac{13\pi}{4}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{95\pi}{12}$	$\frac{87\pi}{8}$	$\frac{101\pi}{12}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{87\pi}{8}$
	$\frac{13\pi}{4}$	$\frac{13\pi}{2}$	$\frac{1}{\sqrt{3}}, \frac{1}{2}$	$\frac{13\pi}{2}, \frac{1}{2}$	$\frac{97\pi}{12}$	$\frac{89\$			



$$y = \sin x$$

$$2\pi = 360^\circ$$

C.3 Il seno ha periodicità  $2\pi$

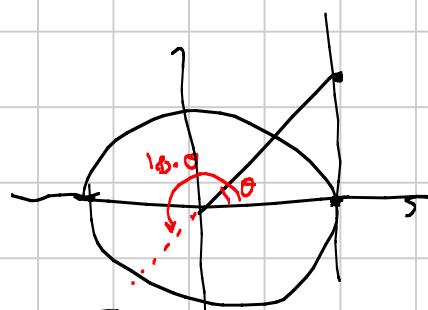
$\sin(x + 2\pi) = \sin x \quad \forall x \in \mathbb{R}$   
e analogamente vale per le coseno.



Ricordando  $f$  iniettiva

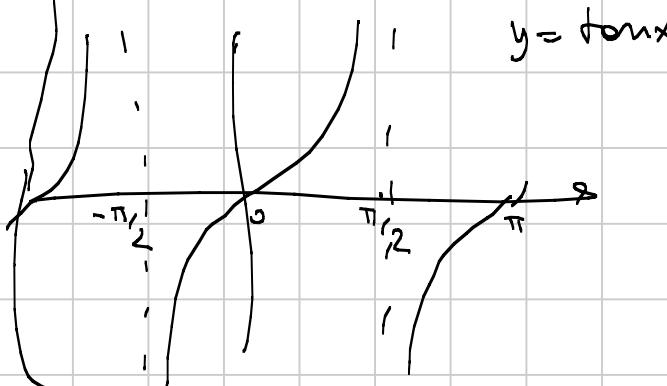
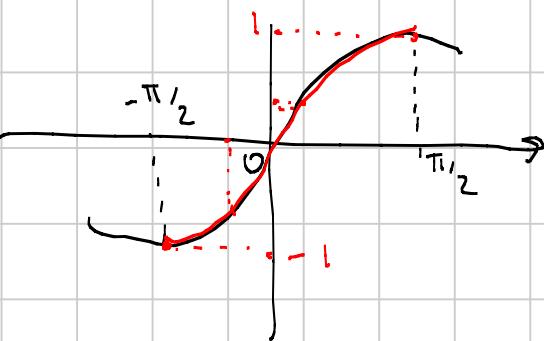
$$f(x) = f(y) \Rightarrow x = y$$

Ora sin, cos, tan non sono iniettive! ( $\mathbb{R} \setminus \dots$ )



C.4 La tan ha periodicità  $\pi$ .

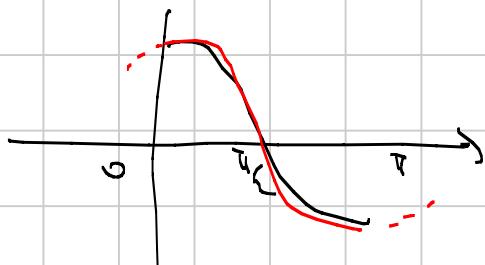
$$\tan(x + \pi) = \tan x \quad \forall x \in \mathbb{R}$$



$f(x) = \sin x$  fra  $-\pi/2$  e  $\pi/2$  è iniettivo, quindi invertibile.  
La sua inversa si chiama funzione arc sin.

$f(x) = \cos x$  fra  $0$  e  $\pi$  è iniettivo.  
La sua inversa si chiama funzione arc cos.

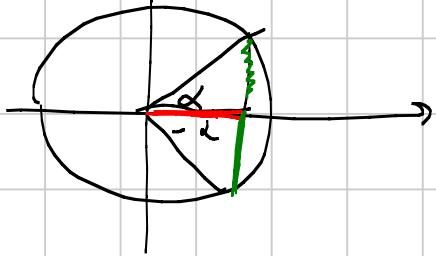
$f(x) = \tan x$  fra  $-\pi/2$  e  $\pi/2$  ...



ESEMPIO

Daremo i grafici delle funzioni inverse.

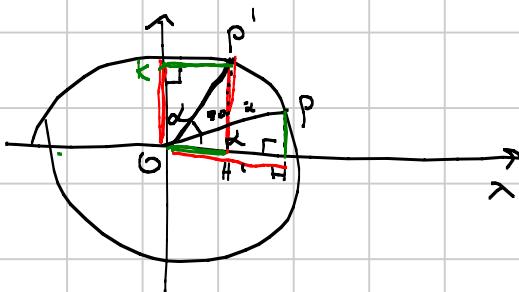
## Oss. sui archi associati



$$\cos(-\alpha) = \cos \alpha = \cos(2\pi - \alpha)$$

$$\sin(-\alpha) = -\sin \alpha = \sin(2\pi - \alpha)$$

$$\tan(-\alpha) = -\tan \alpha$$

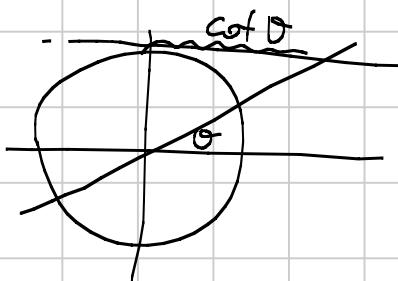
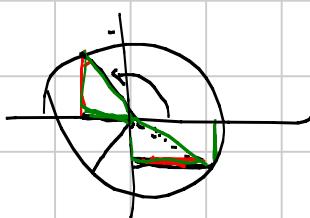


$$\cos(90 - \alpha) = OH' = P'K \approx PH = \sin \alpha$$

$$\sin(90 - \alpha) = P'H' = OK = OH = \cos \alpha$$

$$OKP' ? \stackrel{?}{=} OH'$$

$$\tan(90 - \alpha) = \frac{\cos \alpha}{\sin \alpha} = \frac{OK}{OH} = \cot \alpha$$



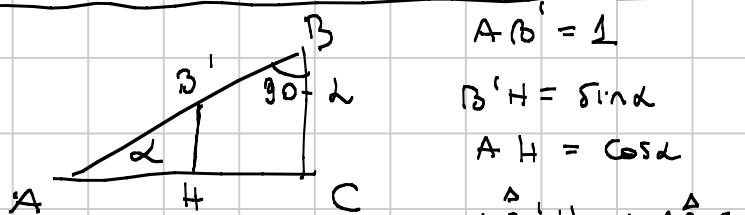
Dimostra

Esercizio ~~Le funzioni goniometriche~~  
di  $90 + \alpha$ ,  $180 - \alpha$ .

Rifare la dim. per  $90 - \alpha$  con  $\alpha > 90$ .

Q. Come si fa il resto di una formula.

INCIOSO



$$AB' = 1$$

$$B'H = \sin \alpha$$

$$AH = \cos \alpha$$

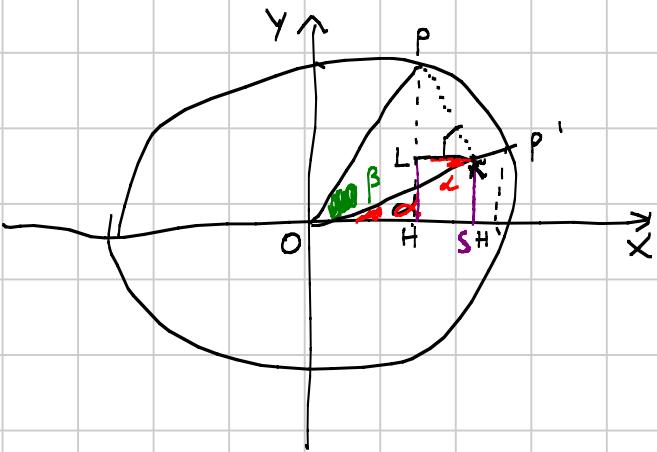
$$\stackrel{\Delta}{AB'}H \sim \stackrel{\Delta}{ABC}$$

$$\frac{AB}{AB'}H = \frac{BC}{B'H}$$

$$AB = \frac{BC}{\sin \alpha}$$

$$BC = AB \sin \alpha$$

Analogamente  $AC = AB \cos \alpha$



$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\begin{cases} Pk = \sin \beta \\ Ok = \cos \beta \\ \hat{\angle kO} = \alpha \\ \hat{\angle kL} = 90^\circ - \alpha \end{cases}$$

$$PL = Pk \sin(90^\circ - \alpha) = \sin \beta \cos \alpha$$

↓  
metri.  
PLk

$\sin(\alpha + \beta)$

!!

Imkos ...

↑

$$\boxed{\text{Similare} \quad LH = \sin \alpha \cos \beta}$$

$$PH = PL + LH = \sin \beta \cos \alpha + \sin \alpha \cos \beta$$

■

$$\begin{aligned} \sin(\alpha - \beta) &= \sin(\alpha + (-\beta)) = \sin \alpha \cos(-\beta) + \sin(-\beta) \cos \alpha = \\ &= \sin \alpha \cos \beta - \sin \beta \cos \alpha \end{aligned}$$

$$\begin{aligned} \cos(\alpha + \beta) &\stackrel{?}{=} \sin(90^\circ - (\alpha + \beta)) = \sin(90^\circ - \alpha - \beta) = \\ &= \sin(90^\circ - \alpha) \cos \beta - \sin \beta \cos(90^\circ - \alpha) \\ &= \cos \alpha \cos \beta - \sin \beta \sin \alpha \end{aligned}$$

$$\cos(\alpha - \beta) = \dots$$

$$\stackrel{?}{=} \cos \alpha \cos \beta + \sin \beta \sin \alpha$$

Esercizio:  $\tan(\alpha + \beta) = ?$

DUPPLICAZIONE

$$\sin 2\alpha = \sin(\alpha + \alpha) = \sin \alpha \cos \alpha + \sin \alpha \cos \alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos(\alpha + \alpha) = \cos \alpha \cos \alpha - \sin \alpha \sin \alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos^2 \alpha + \sin^2 \alpha - 2 \sin^2 \alpha$$

$$\frac{1 - 2 \sin^2 \alpha}{1 + 2 \sin^2 \alpha}$$

$$2 \cos^2 \alpha -$$

$$-(\cos^2 \alpha + \sin^2 \alpha)$$

$$\frac{2 \cos^2 \alpha - 1}{2 \cos^2 \alpha + 1}$$

In conclusione

$$\begin{aligned} \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1 & \tan 2\alpha = \frac{2 \tan \alpha}{\tan^2 \alpha + 1} \\ \text{BISSEGNALE} \\ \sin \frac{\alpha}{2}, \cos \frac{\alpha}{2} & \end{aligned}$$

Le ricaviamo da III

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\alpha \rightarrow \alpha/2$$

$$\cos 2(\alpha/2) = 1 - 2 \sin^2 \alpha/2$$

$$\sin^2 \alpha/2 = \frac{1 - \cos \alpha}{2}$$

$$\sin \alpha/2 = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

In un triangolo  $\oplus$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\alpha \rightarrow \alpha/2$$

$$\cos^2 \alpha/2 = \frac{1 + \cos \alpha}{2}$$

$$\tan \frac{\alpha}{2} = \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \frac{2 \sin \alpha/2 \cdot \sin \alpha/2}{2 \sin \alpha/2 \cdot \cos \alpha/2} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$\frac{2 \cos \alpha/2 \cdot \sin \alpha/2}{2 \cos \alpha/2 \cdot \cos \alpha/2} = \frac{\sin \alpha}{1 + \cos \alpha}$$

Parametriche

Idea: Esprimere seno e coseno  
in funzione di  $\tan \alpha/2$ .

$$t = \tan \frac{\alpha}{2}$$

$$t = \frac{1 - c}{s} \quad (1)$$

$\alpha \neq 0, 180^\circ$

$$s = \sin \alpha$$

$$t = \frac{s}{1 + c} \quad (2)$$

$$c = \cos \alpha$$

$$(1) \cdot s$$

$$st = 1 - c$$
$$c = 1 - st \quad (3)$$

$$(2) \cdot (1 + c) \quad t + tc = s$$

Usa la (3)

$$t + t(1 - st) = s$$

$$2t - st^2 = s$$

$$2t = st^2 + s = s(t^2 + 1)$$

$$s = \frac{2t}{t^2 + 1}$$

Polla 3  $c = 1 - \frac{2t}{t^2 + 1} = \frac{t^2 + 1 - 2t^2}{t^2 + 1} = \boxed{\frac{1 - t^2}{1 + t^2}}$

Randi

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{\tan^2 \frac{\alpha}{2} + 1}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

Esercizio: Controlla le c.d.e. per ogni formula...

Prostafenesi, Werner (Es. 8 pagina 3)

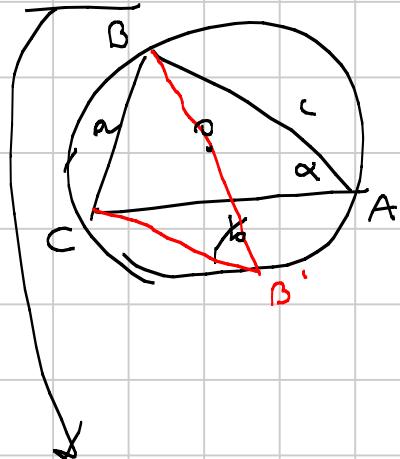
$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\rightarrow \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$


---

## Trigonometria

Th. 1



Triangolo  $A, B, C$  ✓

$a, b, c$  i lati opposti ad  $A, B, C$   
 $R$  il raggio della circonference  
 $r$  il " " " " " inscritta  
 $\alpha, \beta, \gamma$  gli angoli in  $A, B, C$

Teorema della corda : In un triangolo  
 (semi)

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

P.m.  $BB' = 2R$   
 $\widehat{B'CB} = 90^\circ$

$$BC = BB' \cdot \sin \widehat{B'CB} =$$

$$= BB' \cdot \sin \alpha$$

↓

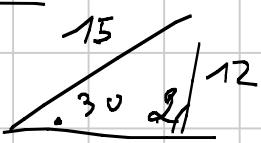
$$a = 2R \cdot \sin \alpha$$

$$\frac{a}{\sin \alpha} = 2R$$

Cor. Semi : In un triangolo il rapporto fra un lato

e il seno dell'angolo opposto è costante  
 $(= 2R)$

E.s.



$$\frac{15}{\sin \alpha} = \frac{12}{\sin 30} = 24$$

$$\sin \alpha = \frac{15}{24} = \frac{5}{8}$$

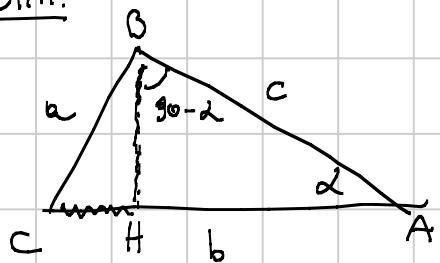
Th. di Carnot (coseni)

$$\rightarrow \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos \gamma = \frac{b^2 + a^2 - c^2}{2ab}$$

Dim.



$$AH = AB \cos \alpha = c \cos \alpha$$

$$BH = c \sin \alpha$$

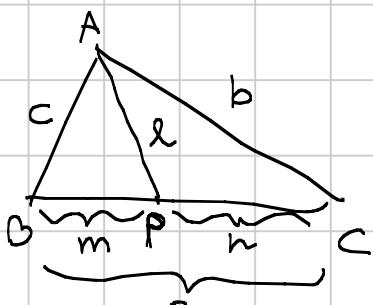
$$CH = AC - AH = b - c \cos \alpha$$

Saranno più facili im  $\hat{BCH}$

$$\begin{aligned} BC^2 &= CH^2 + BH^2 = (b - c \cos \alpha)^2 + (c \sin \alpha)^2 = \\ Q^2 &= " = b^2 - 2bc \cos \alpha + c^2 \cos^2 \alpha + c^2 \sin^2 \alpha = \\ &= b^2 - 2bc \cos \alpha + c^2 \end{aligned}$$

$$2bc \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

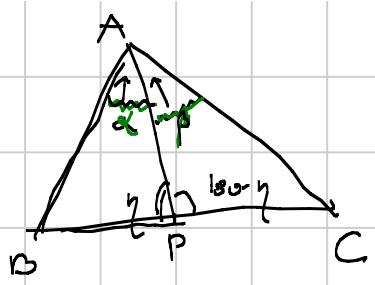
Th. Stewart (E.s. 9 pagina 3)



$$\frac{b^2m + c^2n}{l} = a(l^2 + mn)$$

E.s. Dim. Stewart

e ricava la lung. della mediana ( $= \sqrt{\frac{2(b^2 + c^2) - a^2}{4}}$ ) e bisettrice



Oss.

- Th. dei seni su  $\triangle ABP$

$$\frac{AB}{\sin \gamma} = \frac{BP}{\sin \alpha} \quad (1)$$

- Th. dei seni su  $\triangle APC$

$$\frac{AC}{\sin(180-\gamma)} = \frac{AC}{\sin \beta} = \frac{PC}{\sin \alpha} \quad (2)$$

Divido (1) / (2)

$$\frac{AB/\sin \gamma}{AC/\sin \beta} = \frac{BP/\sin \alpha}{PC/\sin \alpha}$$

$$\frac{AB}{AC} = \frac{BP}{PC} \frac{\sin \beta}{\sin \alpha}$$

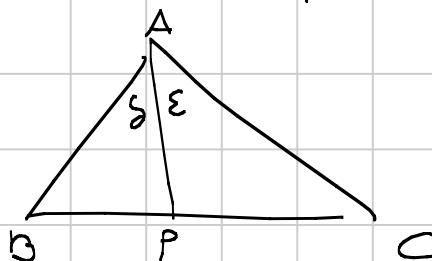
$$\frac{BP}{PC} = \frac{c}{b} \frac{\sin \alpha}{\sin \beta}$$

Attenzione!

$\alpha$  e  $\beta$   
non im  
mat.  
standard!

Sono gli  
angoli staccati  
delle cerchi

Th.



$$\boxed{\frac{BP}{PC} = \left(\frac{c}{b}\right) \frac{\sin \beta}{\sin \gamma}}$$

Ese 5 pag. 3

$$1 - \cot 23^\circ = ? \quad \frac{2}{1 - \cot 22^\circ}$$

$$\frac{\sin 23^\circ - \cos 23^\circ}{\sin 23^\circ} = ? \quad \frac{2 \sin 22^\circ}{\sin 22^\circ - \cos 22^\circ}$$

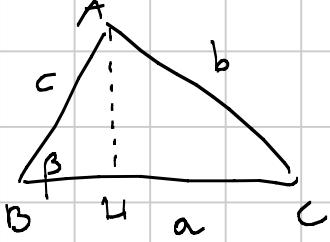
$$\sin 23^\circ \sin 22^\circ - \sin 23^\circ \cos 22^\circ - \cos 23^\circ \sin 22^\circ + \cos 22^\circ \cos 23^\circ = ? \quad \sin 77^\circ \cos 23^\circ$$

$$\cos 22^\circ \cos 23^\circ - \sin 77^\circ \sin 23^\circ = ? \quad \sin 23^\circ \sin 22^\circ + \sin 22^\circ \cos 23^\circ$$

$$\cos 45^\circ$$

$$= \frac{\downarrow}{\sin 45^\circ}$$

## Area di un triangolo



$S, A$

$$S = \frac{BC \cdot AH}{2} = \frac{ac \sin \beta}{2}$$

$$S = \frac{1}{2} ac \sin \beta$$

$$\frac{1}{2} ab \sin \gamma$$

$$\frac{1}{2} bc \sin \alpha$$

A parole: l'area è il  $\frac{1}{2}$  prodotto fra le due e il seno dell'angolo compreso fra loro.

$$S = \frac{1}{2} ac \sin \beta$$

$$\frac{1}{2} ac \frac{b}{2R} = \frac{abc}{4R}$$

$$\frac{b}{\sin \beta} = 2R$$

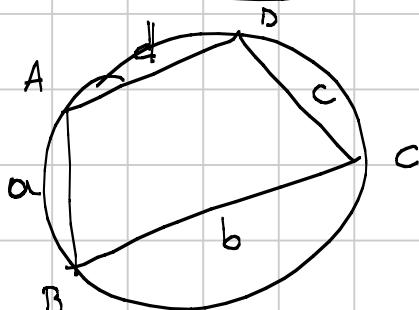
$$\sin \beta = \frac{b}{2R}$$

$$S = \frac{abc}{4R}$$

$$R = \frac{abc}{4S}$$

[E.D.]

## Gener. Erone



Brahmagupta

$$s = \frac{a+b+c+d}{2}$$

$$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

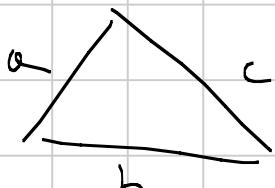
$$d \rightarrow 0$$

$$s = \frac{a+b+c}{2}$$

$$p = \frac{a+b+c}{2}$$

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

Erone



# NUMERI COMPLESSI

$$z = a + bi$$

↓      ↓  
 ∈ ℝ      ∈ ℝ  
 parte reale      parte immaginaria

$$i^2 = -1$$

SOMMA

$$a + bi + c + di =$$

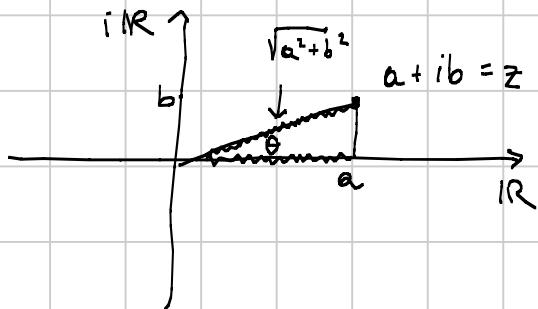
$$= (a+c) + (b+d)i$$

PRODOTTO

$$(a+bi)(c+di) = ac + adi + bci - bd =$$

$$= (ac - bd) + (ad + bc)i$$

Rapp. sul piano di Gauss.



RAPP. POLARE

$$a + bi = \sqrt{a^2 + b^2} \left( \frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} i \right)$$

$$\downarrow$$

$$(\cos \theta + \sin \theta i)$$

NORMA DI Z

$$|z| = \rho = \sqrt{a^2 + b^2}$$

MODULO del n. complesso

θ = argomento

$$\theta = 2,70 \dots$$

$$z = \rho (\cos \theta + i \sin \theta) = \rho e^{i \theta}$$

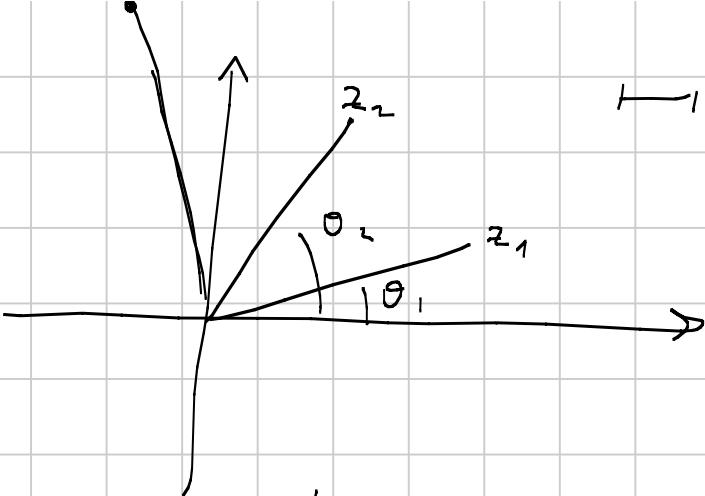
$e^{i\theta}$   
 $\cos \theta$   
 $\sin \theta$

$$z_1 = \rho_1 e^{i \theta_1}$$

$$z_2 = \rho_2 e^{i \theta_2}$$

$$z_1 z_2 = \rho_1 \rho_2 e^{i (\theta_1 + \theta_2)}$$

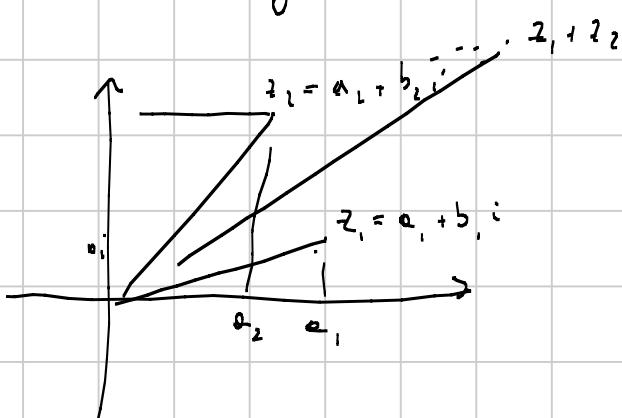
$$z_1 z_2$$



Cosa vuol dire, sul piano di Gauss moltiplicare per un numero complesso  $z$  di lunghezza  $|z| (\rho)$

= ROTAZIONE

dell'origine di  $z$



SOMMA = Regole del parallelogramma

DE-MOLIRE

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \quad \sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 3\alpha = ?$$

$$\sin 3\alpha = ?$$

$$\cos n\alpha = ?$$

$$\sin n\alpha = ?$$

$$z = e^{i\theta} = \cos \theta + i \sin \theta$$

$$z^n = e^{in\theta} = \cos(n\theta) + i \sin(n\theta) \quad \downarrow$$

$$z^n = (\cos \theta + i \sin \theta)^n = \sum_{k=0}^n \binom{n}{k} \cos^k \theta (i \sin \theta)^{n-k}$$

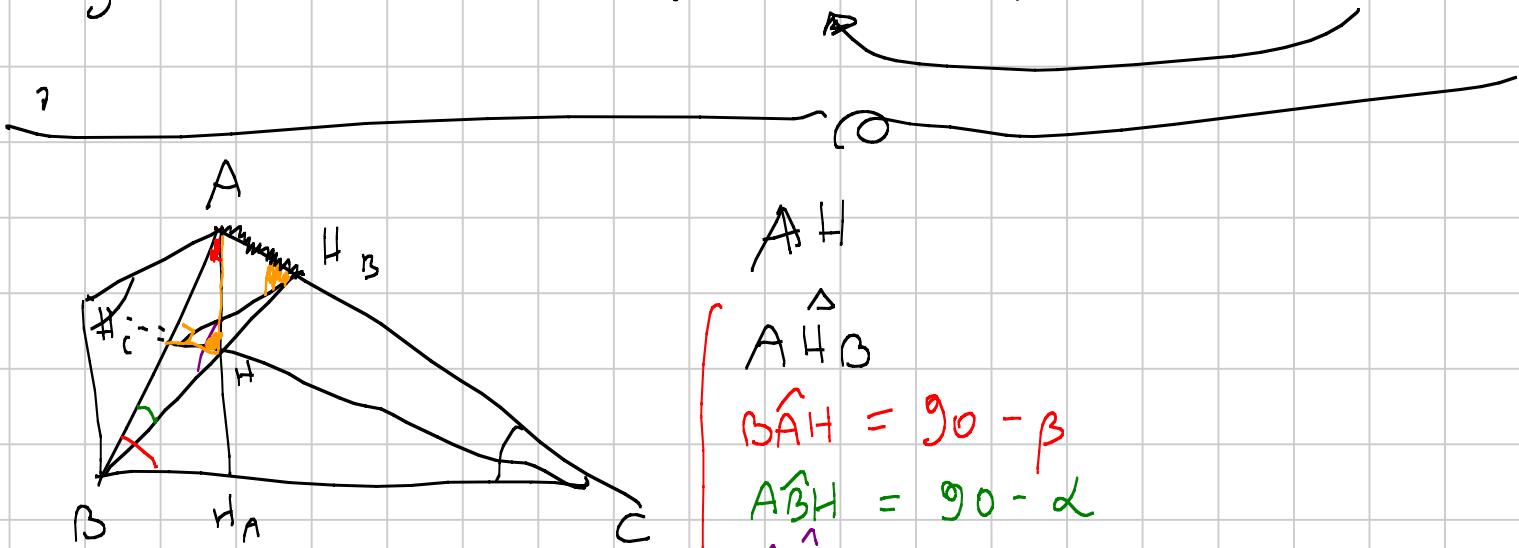
$$= \underline{\underline{A}} + i \underline{\underline{B}} \quad \downarrow$$

$$\cos(n\theta) = \binom{n}{0} \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \dots$$

$$\sin(n\theta) = \binom{n}{1} \cos^{n-1} \theta \sin \theta - \binom{n}{3} \cos^{n-3} \theta \sin^3 \theta - \dots$$

Pog. 3 7 - 8 - 9 - 10 (B rotondeggiare)

Pog. 32 10 - 11 - 13 (1 - 5) - 14 (1 - 3 - 6 - 8)



$$\begin{aligned} & A \hat{H} \\ & \Delta \\ & A \hat{H} B \\ & \widehat{B A H} = 90 - \beta \\ & \widehat{A B H} = 90 - \gamma \\ & \widehat{A H B} = \alpha + \beta = 180 - \gamma \end{aligned}$$

$$\text{Semi: } \frac{AH}{\sin(90 - \alpha)} = \frac{AB}{\sin(180 - \gamma)}$$

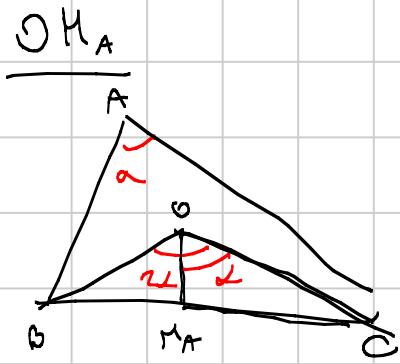
$$AH = \frac{C \cos \alpha}{\sin \gamma} = 2R \cos \alpha$$

$$\left\{ \begin{array}{l} H_B \hat{H}_C \\ A \hat{H}_B \hat{H}_C \\ A \hat{H}_B \hat{H}_C = \beta \\ A \hat{H}_C \hat{H}_B = \gamma \end{array} \right.$$

$\widehat{A H_B H_C} = \beta$        $A \hat{H}_B \hat{H}_C < A \hat{H}_C \hat{H}_B$       perché  $A \hat{H}_C \hat{H}_B$  è un quad.  
 ciclico

$$\text{Lemi: } \frac{AH_B}{\sin AH_C H_B} = \frac{H_B H_C}{\sin \alpha} \rightarrow H_B H_C = \frac{AH_B \sin \alpha}{\sin \gamma}$$

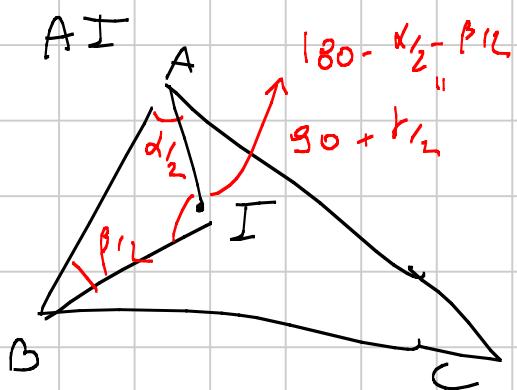
$$\text{Ma } AH_B = C \cos \alpha \rightarrow H_B H_C = \frac{C \cos \alpha \sin \alpha}{\sin \gamma} = \frac{2R \cos \alpha \sin \alpha}{\sin \gamma} = R \sin 2\alpha$$



$$OM_A = \sqrt{R^2 - \frac{\alpha^2}{4}}$$

$OM_A = R \cos \alpha$  (la metà di  $AH\dots$ )

$$\underline{OM_A = \frac{AH}{2}}$$



$$\alpha_2 + \beta_2 + \gamma_2 = 90^\circ$$

$$\sin(90^\circ - \alpha) = \cos \alpha$$

$$\underline{\frac{AI}{\sin \beta_2} = \frac{AB}{\sin(90^\circ + \gamma_2)} = \frac{AB}{\cos \gamma_2}}$$

$$\begin{aligned} AI &= \frac{c \sin \beta_2}{\cos \gamma_2} = \frac{2c \sin \beta_2 \sin \gamma_2}{\sin \gamma} \\ &= 4R \sin \frac{\beta}{2} \sin \frac{\gamma}{2} \end{aligned}$$

$$\underline{G3} + Pow \rightarrow GI$$

$$GI^2 = R^2 - 2Rr.$$

$$\underline{\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]}$$

Verifica sulle

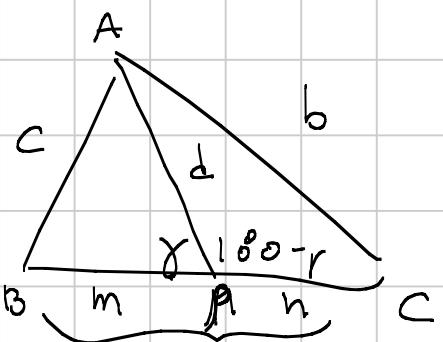
$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\begin{aligned} \alpha + \beta &= p \\ \alpha - \beta &= q \end{aligned}$$

$$\begin{aligned} \alpha &= \frac{p+q}{2} \\ \beta &= \frac{p-q}{2} \end{aligned}$$

$$2 \sin \frac{p+q}{2} \cos \frac{p-q}{2} = \frac{1}{2} (\sin p + \sin q)$$

Es. 9 Stewart



$$m^2n + d^2d = b^2m^2 + c^2n^2$$

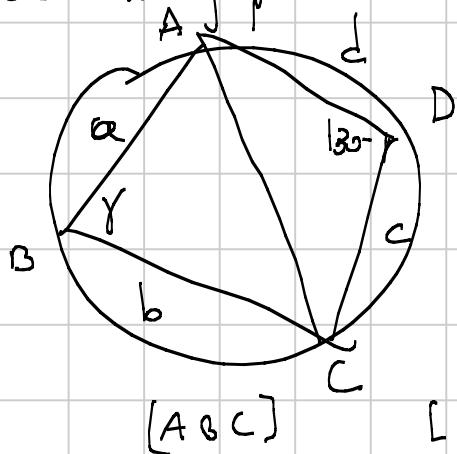
$$2(mn + d^2) = b^2m^2 + c^2n^2$$

Idea:  $c^2 = d^2 + m^2 - 2md \cos \gamma$   
 $b^2 = d^2 + n^2 + 2nd \cos \gamma$

Sostituiendo e viene.

E s. 10 segu.

Brahmagupta



$$AC^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$d^2 + c^2 + 2dc \cos \gamma$$

$$a^2 + b^2 - 2ab \cos \gamma = d^2 + c^2 + 2dc \cos \gamma$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}$$

$$\begin{aligned} S &= \frac{1}{2} ab \sin \gamma + \frac{1}{2} cd \sin((180 - \gamma)) = \\ &= \frac{1}{2} (ab + cd) \sin \gamma \end{aligned}$$

$$\begin{aligned} \sin^2 \gamma &= 1 - \cos^2 \gamma = 1 - \frac{(a^2 + b^2 - c^2 - d^2)^2}{4(ab + cd)^2} = \\ &= \frac{[2(ab + cd)]^2 - (a^2 + b^2 - c^2 - d^2)^2}{4(ab + cd)^2} = \\ &= \frac{(a^2 + 2ab + b^2 - c^2 + 2cd - d^2) / (c^2 + d^2 + 2cd - a^2 - b^2 + 2ab)}{4(ab + cd)^2} = \\ &= \frac{[(a+b)^2 - (c-d)^2][(c+d)^2 - (a-b)^2]}{4(ab + cd)^2} = \\ &= \frac{(a+b+c-d)(a+b-c+d)(c+d+a-b)(c+d+b-a)}{4(ab + cd)^2} = \end{aligned}$$

$$r - d = \left( \frac{a+b+c+d}{2} - d \right) = \frac{a+b+c-d}{2}$$

$$= \frac{2(s-d)(s-c)(s-b)(s-a)}{(a+b+c+d)^2}$$

$$\sin f = 2 \frac{\sqrt{(s-d)(s-c)(s-b)(s-a)}}{(a+b+c+d)}$$

$$\text{Quindi } S = \frac{1}{2} \cancel{(a+b+c+d)} \frac{\cancel{2\sqrt{(s-a)(s-b)(s-c)(s-d)}}}{\cancel{(a+b+c+d)}} \\ = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Esempio  $\downarrow$   $S = \sqrt{p(p-a)(p-b)(p-c)}$

Tramppo  $16S^2 = -a^4 - b^4 - c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2$

E s. 13

$$\{ r = L R \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} = \frac{1 - \frac{b^2 + c^2 - a^2}{2bc}}{2} = \frac{2bc + a^2 - b^2 - c^2}{4bc} =$$

$$= \frac{a^2 - (b-c)^2}{4bc} =$$

$$= \frac{(a+b-c)(a+c-b)}{4bc} =$$

$$= \frac{2(p-c)p(b)}{4bc} =$$

$$= \frac{(p-c)(p-b)}{bc}$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{(p-c)(p-b)}{bc}}$$

$$\sin \frac{\beta}{2} = \sqrt{\frac{(p-a)(p-c)}{bc}}$$

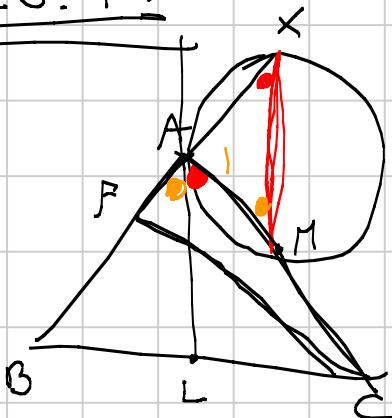
$$LR \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} = LR \frac{\sqrt{\frac{(p-c)(p-b)}{bc}} \sqrt{\frac{(p-a)(p-c)}{bc}} \sqrt{\frac{(p-b)(p-a)}{ab}}}{abc} \\ = LR \frac{\cancel{(p-a)(p-b)(p-c)}}{abc} =$$

$$= 4R \frac{s^2}{P_{abc}} = \frac{S}{P} = r$$

$$AI = 4R \sin \frac{\alpha}{2} \sin \frac{\beta}{2}$$

$$r = AI \cdot \sin \frac{\gamma}{2} = R \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$$

E.S. 11.



minimizza  $\frac{BX}{CF}$

$$\hat{L}AM = \hat{A}X\hat{M}$$

perché insieme su  $\hat{A}M$

$$\hat{XMA} = \hat{BAL}$$

in  $A\hat{X}\hat{M}$

$$\frac{AX}{\sin \bullet} = \frac{AM}{\sin \circ}$$

$$AX = \frac{b}{2} \frac{\sin \bullet}{\sin \circ} = \frac{b^2}{2c}$$

$$\left| \frac{\sin \bullet}{\sin \circ} = \frac{b}{c} \right. \quad \text{Usando la formula del seno}$$

$$BX = AB + AX = b + \frac{b^2}{2c} = \frac{2c^2 + b^2}{2c}$$

$$CF = b \sin \alpha$$

$$\text{Inoltre } \frac{BX}{CF} = \frac{b^2 + 2c^2}{2bc \sin \alpha}$$

$\alpha = \pi/2 (90^\circ)$   
perché più piccolo  
modo  $\frac{1}{\sin \alpha}$  è min.

$$\frac{b^2 + 2c^2}{2bc} \geq \sqrt{2}$$

$$b^2 - 2\sqrt{2}bc + 2c^2 \geq 0$$

$$(b - \sqrt{2}c)^2 \geq 0$$

L'ugualanza si ha se  $\alpha = 90^\circ$ ,  $b/c = \sqrt{2}$ . ■

