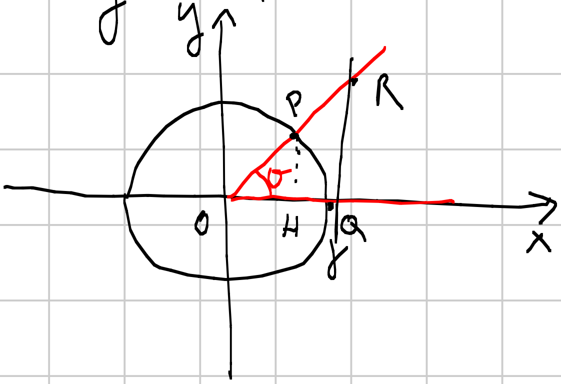
G1 - basic - Senior '15 - Gioacchino

Titolo nota 22/08/2015

Consideriamo la circonferenza goniometrica: (γ)



Sia θ un angolo.

1) Misuriamo, a partire dal semiasse positivo delle x in senso antiorario, un angolo θ (es. $\theta = 30°$) e traccio una semiretta

Sia P l'int. di questa con la circ. gon.

$PH \overset{def}{=} \sin\theta$

$OH \overset{def}{=} \cos\theta$ $\leadsto$ $P(\cos\theta, \sin\theta)$

C.1 Pitagora su $P\hat{O}H$: $PH^2 + OH^2 = 1 \rightarrow \boxed{\sin^2\theta + \cos^2\theta = 1}$

$\boxed{Tg}$ Traccio da Q la $\perp$ all'asse x. Interseca la semiretta in R.

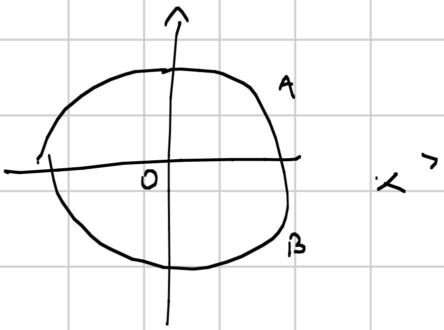
$QR \overset{def}{=} \tan\theta$

C.2 $P\hat{O}H \sim R\hat{O}Q \rightarrow \dfrac{PH}{OH} = \dfrac{QR}{OQ} \rightarrow \boxed{\dfrac{\sin\theta}{\cos\theta} = \tan\theta}$

Inciso $\pi \leadsto 180°$

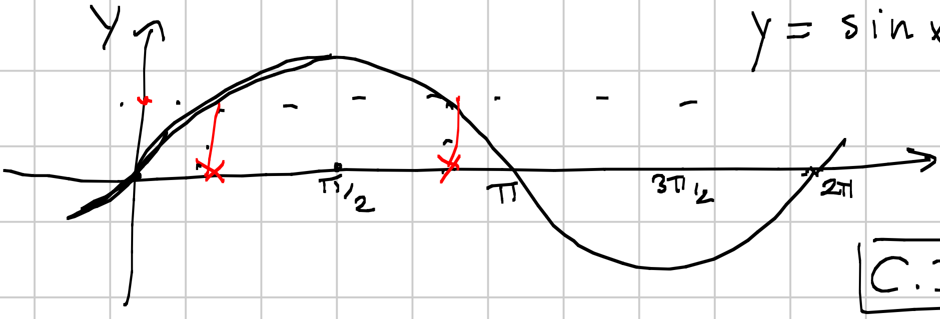
$\pi : 180° =$ ang. in rad : ang. in gradi

Per chi volesse ...



Angoli	sin	cos	tan
0	0	1	0
$\frac{\pi}{6}$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3} = \sqrt{3}/3$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	n.d.

$y = \sin x$

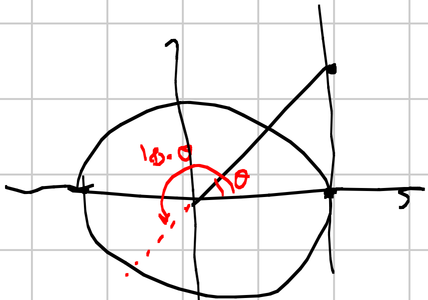


$2\pi : 360°$

$\boxed{C.3}$ Il seno ha periodicità 2π

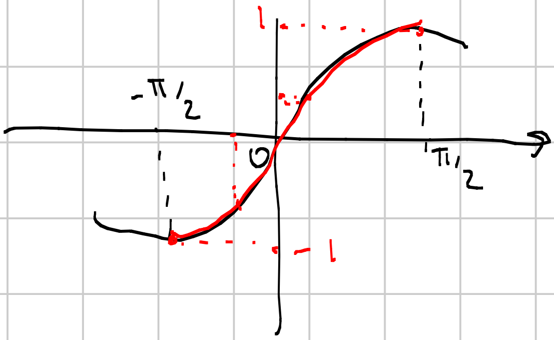
$\sin(x+2\pi) = \sin x \quad \forall x \in \mathbb{R}$

e analogamente vale per il coseno.



Remind: f iniettiva $\iff$ $f(x)=f(y) \Rightarrow x=y$

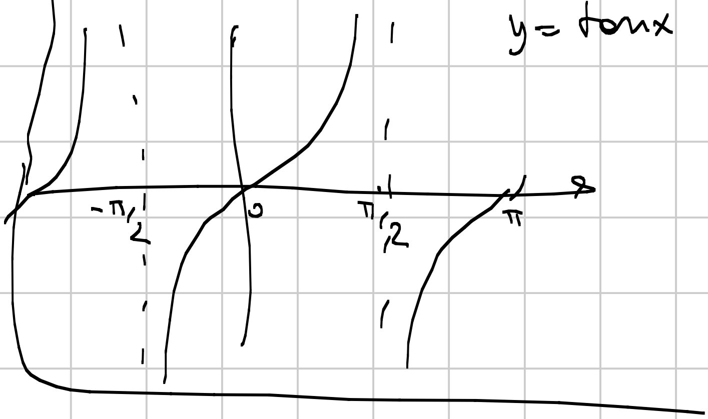
Oss: sin, cos, tan non sono iniettive! (su $\mathbb{R}$...)



$\boxed{C.4}$ La tan ha periodicità π.

$\tan(x+\pi) = \tan x \quad \forall x \in \mathbb{R}$

$y = \tan x$

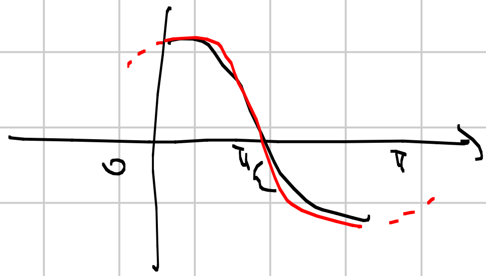


$f(x) = \sin x$ fra $-\pi/2$ e $\pi/2$ è iniettiva, quindi invertibile.

La sua inversa si chiama funzione arcsin.

$f(x) = \cos x$ fra 0 e π è inv.

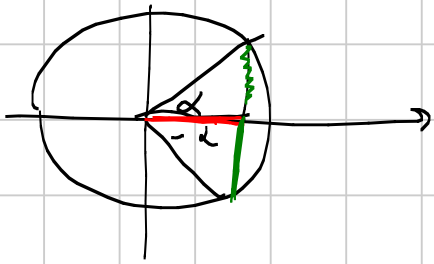
La sua inversa si chiama funzione arccos.



$f(x) = \tan x$ fra $-\pi/2$ e $\pi/2$ ...

$\boxed{\text{ESERCIZIO}}$ Disegnare i grafici delle funz. inverse.

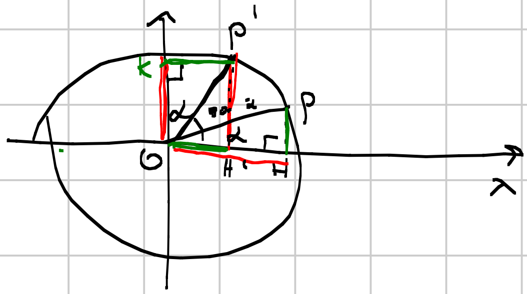
Oss. su archi associati



$\cos(-\alpha) = \cos\alpha = \cos(2\pi - \alpha)$

$\sin(-\alpha) = -\sin\alpha = \sin(2\pi - \alpha)$

$\tan(-\alpha) = -\tan\alpha$



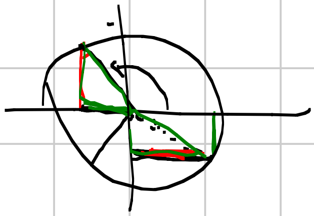
$\cos(90-\alpha) = OH' = P'K \underset{}{\overset{\text{cong.}}{\cong}} PH = \sin\alpha$

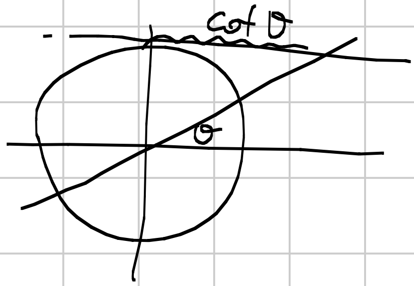
$\sin(90-\alpha) = P'H' = OK = OH = \cos\alpha$

$\hat{OKP'} \ ? \ \hat{OHP}$

$K\hat{O}P = \alpha$

$\tan(90-\alpha) = \dfrac{\cos\alpha}{\sin\alpha} = \dfrac{1}{\tan\alpha} = \cot\alpha$





Dimostra

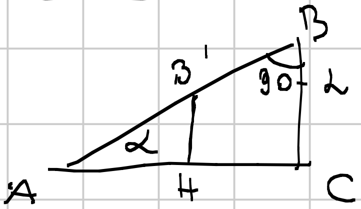
Esercizio

Le funzioni goniometriche di $90+\alpha$, $180-\alpha$.

Rifare la dim. per $90-\alpha$ con $\alpha > 90$.

Q. Come si fa il seno di una somma.

Inciso



$AB' = 1$

$B'H = \sin\alpha$

$AH = \cos\alpha$

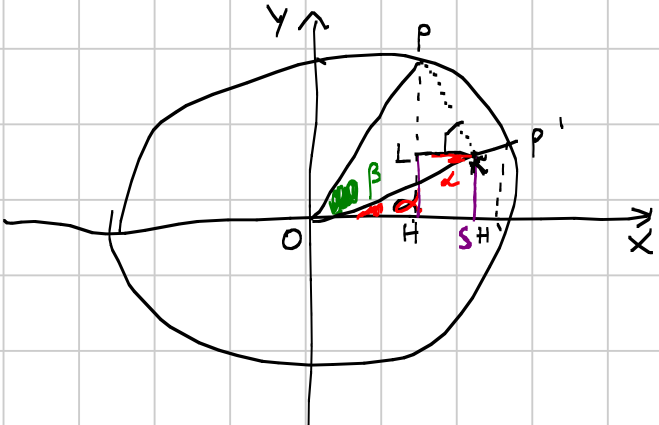
$\hat{AB'H} \sim \hat{ABC}$

$\dfrac{AB}{AB'} = \dfrac{BC}{B'H}$

$AB = \dfrac{BC}{\sin\alpha}$

$\boxed{BC = AB\sin\alpha}$

Analogamente $\boxed{AC = AB\cos\alpha}$



$$\boxed{\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha}$$

$$\begin{cases} PK = \sin\beta \\ OK = \cos\beta \end{cases}$$

$L\hat{K}O = \alpha$

$P\hat{K}L = 90 - \alpha$

$PL = PK\sin(90-\alpha) = \sin\beta\cos\alpha$

nel tr. $P\hat{L}K$

Imkôs ...

$$\boxed{\text{Si mostra} \quad LH = \sin\alpha\cos\beta}$$

$$\underset{\displaystyle PH}{\overset{\sin(\alpha+\beta)}{\parallel}} = PL + LH = \sin\beta\cos\alpha + \sin\alpha\cos\beta$$

$$\sin(\alpha-\beta) = \sin(\alpha+(-\beta)) = \sin\alpha\cos(-\beta) + \sin(-\beta)\cos\alpha =$$
$$= \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

$$\cos(\alpha+\beta) \overset{?}{=} \sin(90-(\alpha+\beta)) = \sin((90-\alpha)-\beta) =$$
$$= \sin(90-\alpha)\cos\beta - \sin\beta\cos(90-\alpha)$$
$$= \cos\alpha\cos\beta - \sin\beta\sin\alpha$$

$$\cos(\alpha-\beta) = \ldots \quad \overset{\circ}{=} \quad \cos\alpha\cos\beta + \sin\beta\sin\alpha$$

$$\boxed{\text{Esercizio: } \tan(\alpha+\beta) = ?}$$

DUPLICAZIONE

$$\sin 2\alpha = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos^2\alpha + \sin^2\alpha - 2\sin^2\alpha = \boxed{1 - 2\sin^2\alpha}$$

$$2\cos^2\alpha - (\cos^2\alpha + \sin^2\alpha) = \boxed{2\cos^2\alpha - 1}$$

In conclusione

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{\tan^2\alpha + 1}$$

BISEZIONE

$$\sin\frac{\alpha}{2}, \quad \cos\frac{\alpha}{2}$$

Le ricaviamo da

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\alpha \to \alpha/2$$

$$\cos 2(\alpha/2) = 1 - 2\sin^2 \alpha/2$$

$$\sin^2 \alpha/2 = \frac{1-\cos\alpha}{2}$$

$$\sin \alpha/2 = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

In un triangolo $\oplus$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\alpha \to \alpha/2$$

$$\vdots$$

$$\cos^2 \alpha/2 = \frac{1+\cos\alpha}{2}$$

$$\tan\frac{\alpha}{2} \stackrel{\text{def}}{=} \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{2\sin\alpha/2 \cdot \sin\alpha/2}{2\sin\alpha/2 \cdot \cos\alpha/2} = \frac{1-\cos\alpha}{\sin\alpha}$$

$$\| $$

$$\frac{2\cos\alpha/2 \cdot \sin\alpha/2}{2\cos\alpha/2 \cdot \cos\alpha/2} = \frac{\sin\alpha}{1+\cos\alpha}$$

Parametriche

Idea: Esprimere seno e coseno in funzione di $\tan \alpha/2$.

$t = \tan\frac{\alpha}{2}$

$s = \sin\alpha$

$c = \cos\alpha$

$$t = \frac{1-c}{s} \quad (1)$$

$$t = \frac{s}{1+c} \quad (2)$$

$\alpha \neq 0, 180°$

$(1)\cdot s$ $\quad st = 1 - c$

$$\boxed{c = 1 - st} \quad (3)$$

$(2)\cdot(1+c)$ $\quad t + tc = s$

Uso la (3) $\quad t + t(1-st) = s$

$$2t - st^2 = s$$

$$2t = st^2 + s = s(t^2+1)$$

$$\boxed{s = \frac{2t}{t^2+1}}$$

Dalla 3 $\quad c = 1 - \frac{2t}{t^2+1}\cdot t = \frac{t^2+1-2t^2}{t^2+1} = \boxed{\frac{1-t^2}{1+t^2}}$

Quindi $\sin\alpha = \dfrac{2\tan\frac{\alpha}{2}}{\tan^2\frac{\alpha}{2}+1}$

$\cos\alpha = \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

Esercizio: controlla il c.d.e. per ogni formula...

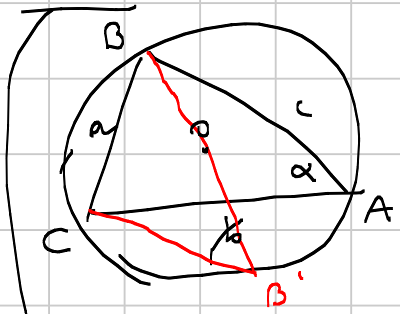
Prostaferesi, Werner (Es. 8 pagina 3)

$$\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\longrightarrow \sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$

Trigonometria

Th. 1



Triangolo A, B, C

a, b, c i lati opposti ad A, B, C

R il raggio della cfr. circoscritta

r il " " " inscritta

α, β, γ gli angoli in A, B, C

Teorema della corda (seni): In un triangolo $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} = 2R$

D.m. $BB' = 2R$ $\quad BC = BB' \cdot \sin B\hat{B}'C =$

$B\hat{C}B' = 90°$ $\quad = BB' \cdot \sin\alpha$

$\downarrow$

$a = 2R \cdot \sin\alpha$

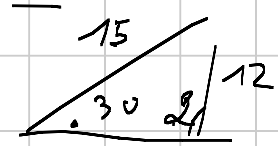
$\dfrac{a}{\sin\alpha} = 2R$ $\blacksquare$

Cor.

Seni: In un triangolo il rapporto fra un lato

e il seno dell'angolo opposto è costante
$(= 2R)$

Es.



$$\frac{15}{\sin\alpha} = \frac{12}{\sin 30} = 24$$

$$\sin\alpha = \frac{15}{24} = \frac{5}{8}$$

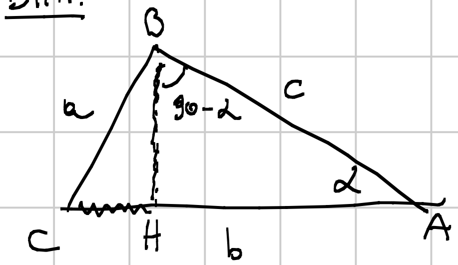
Th. di Carnot (coseni)

$$\to \cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\beta = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos\gamma = \frac{b^2+a^2-c^2}{2ab}$$

Dim.



$AH = AB\cos\alpha = c\cos\alpha$

$BH = c\sin\alpha$

$CH = AC - AH = b - c\cos\alpha$

Scrivo Pitagora in $B\hat{C}H$

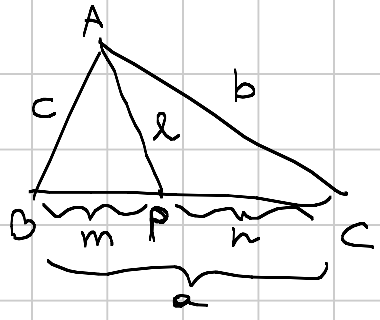
$$BC^2 = CH^2 + BH^2 = (b - c\cos\alpha)^2 + (c\sin\alpha)^2 =$$

$$a^2 = \quad '' \quad = b^2 - 2bc\cos\alpha + \underbrace{c^2\cos^2\alpha + c^2\sin^2\alpha} =$$

$$= b^2 - 2bc\cos\alpha + c^2$$

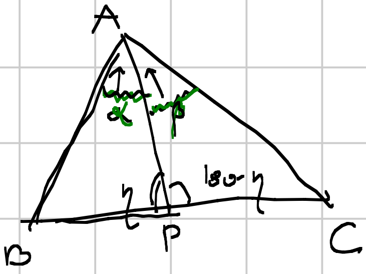
$$\cancel{2b}c\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$$

Th. Stewart (ES. 3 pagina 3)



$$b^2 m + c^2 n = a(l^2 + mn)$$

Es. Dim. Stewart.
e ricava la lungh. della mediana $\left(= \sqrt{\frac{2(b^2+c^2)-a^2}{4}}\right)$
e bisettrice



Oss.

Th. dei seni su $A\hat{B}P$

$$\frac{AB}{\sin\eta} = \frac{BP}{\sin\alpha} \qquad (1)$$

$\sin(180-\eta) = \sin\eta$

$\cos(180-\eta) = -\cos\eta$

Th. dei seni su $A\hat{P}C$

$$\frac{AC}{\sin(180-\eta)} = \frac{AC}{\sin\eta} = \frac{PC}{\sin\beta} \qquad (2)$$

Divido (1) / (2)

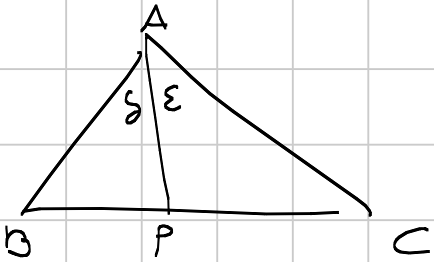
$$\frac{AB/\cancel{\sin\eta}}{AC/\cancel{\sin\eta}} = \frac{BP/\sin\alpha}{PC/\sin\beta}$$

$$\frac{AB}{AC} = \frac{BP}{PC}\,\frac{\sin\beta}{\sin\alpha}$$

$$\frac{BP}{PC} = \frac{c}{b}\,\frac{\sin\alpha}{\sin\beta}$$

Attenzione! α e β non in not. standard! Sono gli angoli staccati dalla ceviana

Th.



$$\boxed{\frac{BP}{PC} = \left(\frac{c}{b}\right)\frac{\sin\delta}{\sin\varepsilon}}$$

Es 3 pag. 3

$$1-\cot 23 \overset{?}{=} \frac{2}{1-\cot 22}$$

$$\frac{\sin 23-\cos 23}{\sin 23} \overset{?}{=} \frac{2\sin 22}{\sin 22-\cos 22}$$

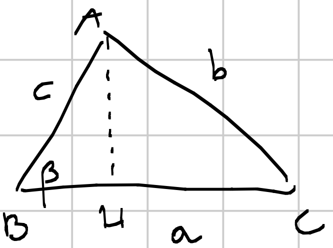
$$\cancel{\sin 23\sin 22} - \sin 23\cos 22 - \cos 23\sin 22 + \cos 22\cos 23 \overset{?}{=} \cancel{2}\sin 22\sin 23$$

$$\underbrace{\cos 22\cos 23 - \sin 22\sin 23}_{\cos 45} \overset{?}{=} \underbrace{\sin 23\cos 22 + \sin 22\cos 23}_{\sin 45}$$

$$\cos 45 \overset{!}{=} \sin 45$$

■

Area di un triangolo



S , A

$$S = \frac{BC \cdot AH}{2} = \frac{ac \sin\beta}{2}$$

$$S = \frac{1}{2} ac \sin\beta$$

$$\frac{1}{2} ab \sin\gamma$$

$$\frac{1}{2} bc \sin\alpha$$

A parole: l'area è il ½ prodotto fra due lati e il seno dell'angolo compreso fra loro.

$$S = \frac{1}{2} ac \sin\beta$$

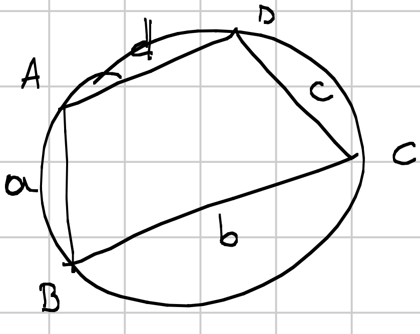
$$\frac{b}{\sin\beta} = 2R \qquad \sin\beta = \frac{b}{2R}$$

$$\frac{1}{2} ac \frac{b}{2R} = \frac{abc}{4R}$$

$$S = \frac{abc}{4R} \longrightarrow R = \frac{abc}{4S}$$

ES.

Gener. Erone



Brahamogupta

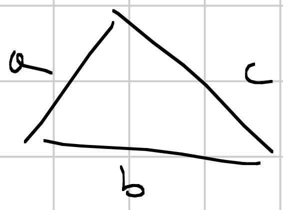
$$s = \frac{a+b+c+d}{2}$$

$$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$d \to 0$$

$$s = \frac{a+b+c}{2}$$

Erone



$$p = \frac{a+b+c}{2}$$

$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

NUMERI COMPLESSI

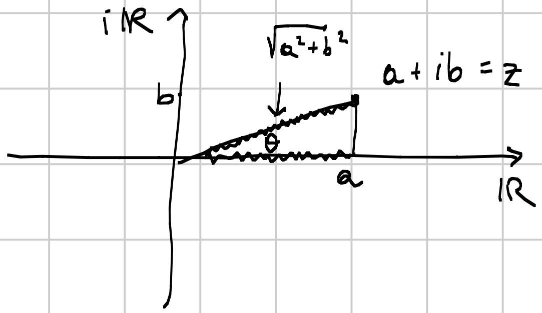
$\boxed{z = a + bi}$ $\qquad i^2 = -1$

$a \in \mathbb{R}$ parte reale, $b \in \mathbb{R}$ parte immaginaria

SOMMA $\quad a+bi+c+di =$
$= (a+c) + (b+d)i$

PRODOTTO $\quad (a+bi)(c+di) = ac + adi + bci - bd =$
$= (ac-bd) + (ad+bc)i$

Rapp. sul piano di Gauss.



RAPP. POLARE $\quad a+bi = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}} + \frac{b}{\sqrt{a^2+b^2}}i\right)$

$(\cos\theta + \sin\theta\, i)$

NORMA DI Z

$|z| = \rho = \sqrt{a^2+b^2}$ MODULO del N. COMPLESSO

θ = argomento

$e = 2{,}78\ldots$

$z = \rho(\cos\theta + i\sin\theta) = \rho e^{i\theta}$

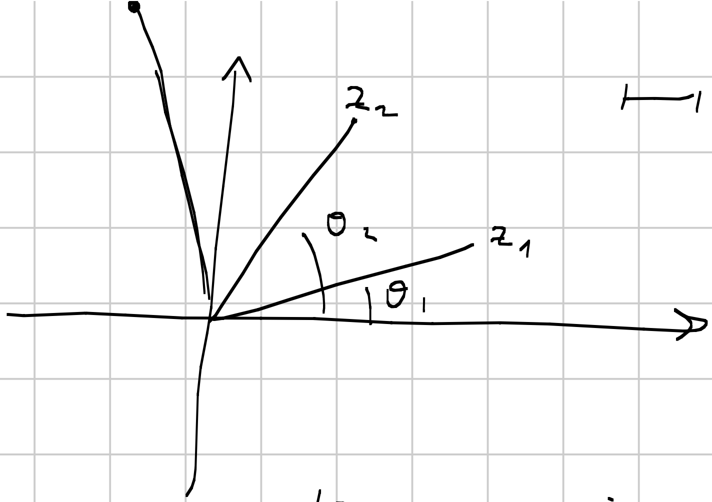
e^x
cos
sin

$z_1 = \rho_1 e^{i\theta_1}$

$z_2 = \rho_2 e^{i\theta_2}$

$z_1 z_2 = \rho_1\rho_2\, e^{i(\theta_1+\theta_2)}$

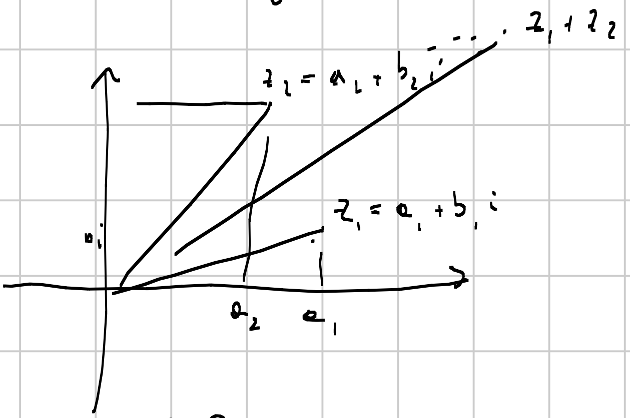
$z_1 z_2$



Cosa vuol dire, sul piano di Gauß moltiplicare per un numero complesso z

= ROTOMOTETIA → di ragione $|z|$ (ρ)

dell'argomento di z



Somma = Regola del parallelogramma

DE-MOIVRE

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\qquad$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 3\alpha = ?$ $\qquad$ $\sin 3\alpha = ?$

$\cos n\alpha = ?$ $\qquad$ $\sin n\alpha = ?$

$$z = e^{i\theta} = \cos\theta + i\sin\theta$$

$$z^n = e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$$

$$z^n = (\cos\theta + i\sin\theta)^n = \sum_{k=0}^{n} \binom{n}{k} \cos^{n-k}\theta\,(i\sin\theta)^k$$

$$= \underline{\underline{A}} + iB$$

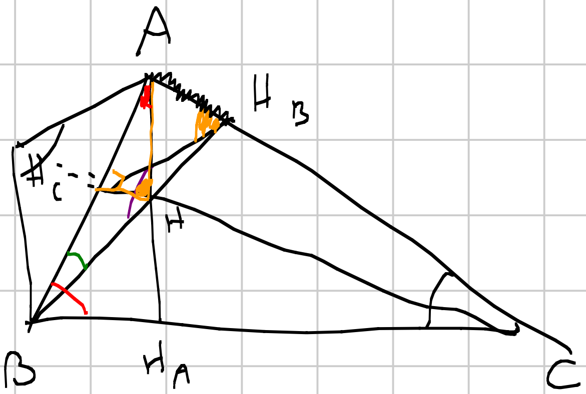
$$\cos(n\theta) = \binom{n}{0}\cos^n\theta - \binom{n}{2}\cos^{n-2}\theta\sin^2\theta + \dots$$

$$\sin(n\theta) = \binom{n}{1}\cos^{n-1}\theta\sin\theta - \binom{n}{3}\cos^{n-3}\theta\sin^3\theta - \dots$$

Pag. 3 7 - 8 - 9 - 10 (B rohomogyla)

Pag. 32 10 - 11 - 13 (1 - 5) - 14 (1 - 3 - 6 - 8)

7



AH

$A\hat{H}B$

$B\hat{A}H = 90 - \beta$

$A\hat{B}H = 90 - \alpha$

$A\hat{H}B = \alpha + \beta = 180 - \gamma$

Seni: $\dfrac{AH}{\sin(90-\alpha)} = \dfrac{AB}{\sin(180-\gamma)}$

$$AH = \frac{c\cos\alpha}{\sin\gamma} = 2R\cos\alpha$$

H_BH_C

$A\hat{H}_BH_C$

$A\hat{H}_BH_C = A\hat{H}H_C = \beta$ perché AH_CHH_B è un quad. ciclico

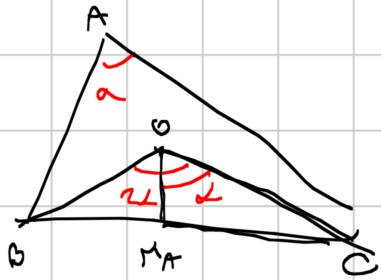
$A\hat{H}_BH_C = \beta$

$A\hat{H}_CH_B = \gamma$

Seni: $\dfrac{AH_B}{\sin A\hat{H}_CH_B} = \dfrac{H_BH_C}{\sin\alpha} \to H_BH_C = \dfrac{AH_B\sin\alpha}{\sin\gamma}$

Ma $AH_B = c\cos\alpha \to H_BH_C = \dfrac{c\cos\alpha\sin\alpha}{\sin\gamma} = 2R\cos\alpha\sin\alpha = R\sin 2\alpha$

OM_A



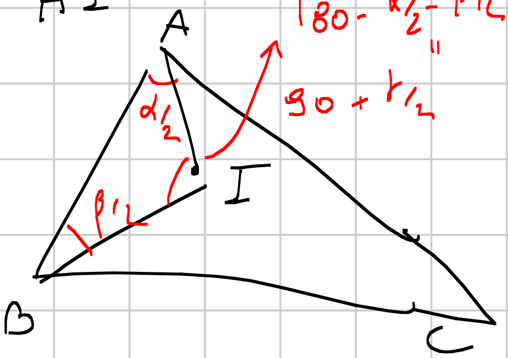
$O\hat{M}_A C$

Dicono $\sqrt{R^2-\frac{a^2}{4}}$

$OM_A = R\cos\alpha$ (la metà di AH..)

$OM_A = \frac{AH}{2}$

AI



$\alpha/2+\beta/2+\gamma/2 = 90$

$\sin(90+\alpha) = \cos\alpha$

$A\hat{B}I$

$$\frac{AI}{\sin\beta/2} = \frac{AB}{\sin(90+\gamma/2)} = \frac{AB}{\cos\gamma/2}$$

$$AI = \frac{c\sin\beta/2}{\cos\gamma/2} = \frac{2c\sin\beta/2\sin\gamma/2}{\sin\gamma}$$

$$= 4R\sin\beta/2\sin\gamma/2$$

G3 + Pow $\longrightarrow$ OI

$OI^2 = R^2 - 2Rr.$

Es. 8

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

Verifica truth

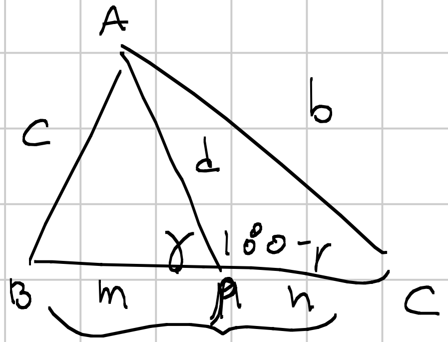
$$\sin\alpha+\sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$\alpha+\beta = p$ $\quad$ $\alpha = \frac{p+q}{2}$

$\alpha-\beta = q$ $\quad$ $\beta = \frac{p-q}{2}$

$$2\sin\frac{p+q}{2}\cos\frac{p-q}{2} = \frac{1}{\cancel{2}}(\sin p+\sin q)$$

Es. 9 Stewart



$$man + dad = bmb + cnc$$

$$a(mn+d^2) = b^2 m + c^2 n$$

Idea: $c^2 = d^2 + m^2 - 2md\cos\gamma$

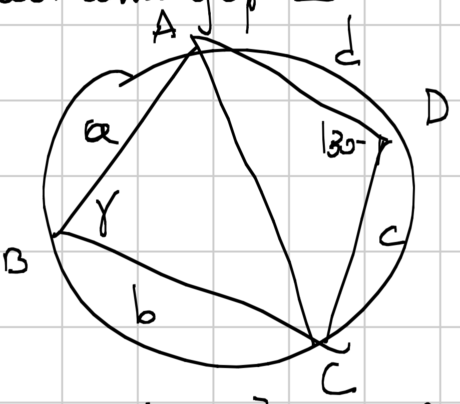
$b^2 = d^2 + n^2 + 2nd\cos\gamma$

Sostituisco e viene.

$$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Es 5.10 gen.

Brahamagupta



$$AC^2 = a^2+b^2-2ab\cos\gamma$$

$$\parallel$$

$$d^2+c^2+2dc\cos\gamma$$

$$a^2+b^2-2ab\cos\gamma = d^2+c^2+2dc\cos\gamma$$

$$\cos\gamma = \frac{a^2+b^2-c^2-d^2}{2(ab+cd)}$$

$$S = \underbrace{\frac{1}{2}ab\sin\gamma}_{[ABC]} + \underbrace{\frac{1}{2}cd\,\sin(180-\gamma)}_{[ADC]} =$$

$$\sin(180-\gamma) = \sin\gamma$$

$$= \frac{1}{2}(ab+cd)\sin\gamma$$

$$\sin^2\gamma = 1-\cos^2\gamma = 1 - \frac{(a^2+b^2-c^2-d^2)^2}{4(ab+cd)^2} =$$

$$= \frac{[2(ab+cd)]^2 - (a^2+b^2-c^2-d^2)^2}{4(ab+cd)^2} =$$

$$= \frac{(\underbrace{a^2+2ab+b^2}-\underbrace{c^2+2cd-d^2})(c^2+d^2+2cd-a^2-b^2+2ab)}{4(ab+cd)^2} =$$

$$= \frac{[(a+b)^2-(c-d)^2][(c+d)^2-(a-b)^2]}{4(ab+cd)^2} =$$

$$= \frac{(a+b+c-d)(a+b-c+d)(c+d+a-b)(c+d+b-a)}{4(ab+cd)^2} =$$

$$s-d = \frac{a+b+c+d}{2} - d = \frac{a+b+c-d}{2}$$

$$= \frac{\cancel{4}(s-d)(s-c)(s-b)(s-a)}{(ab+cd)^2}$$

$$\sin\gamma = \frac{2\sqrt{(s-d)(s-c)(s-b)(s-a)}}{(ab+cd)}$$

Quindi $S = \frac{1}{\cancel{2}}(\cancel{ab+cd})\frac{\cancel{2}\sqrt{(s-a)(s-b)(s-c)(s-d)}}{(\cancel{ab+cd})}$

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Erone $S = \sqrt{p(p-a)(p-b)(p-c)}$

Triangolo $16S^2 = -a^4-b^4-c^4+2a^2b^2+2b^2c^2+2c^2a^2$

Es. 13

$r = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} = \frac{1-\frac{b^2+c^2-a^2}{2bc}}{2} = \frac{2bc+a^2-b^2-c^2}{4bc} =$$

$$= \frac{a^2-(b-c)^2}{4bc} =$$

$$= \frac{(a+b-c)(a+c-b)}{4bc} =$$

$$= \frac{\cancel{2}(p-c)\cancel{2}(p-b)}{\cancel{4}bc} =$$

$$= \frac{(p-c)(p-b)}{bc}$$

$p = \frac{a+b+c}{2}$

$p-c = \frac{a+b+c}{2} - c =$

$= \frac{a+b-c}{2}$

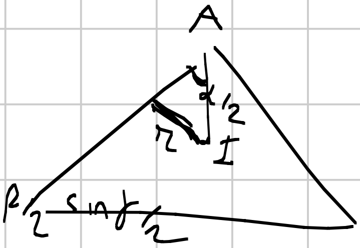
$\sin\frac{\alpha}{2} = \sqrt{\frac{(p-c)(p-b)}{bc}}$ $\quad$ $\sin\frac{\beta}{2} = \sqrt{\frac{(p-a)(p-c)}{ac}}$

$$4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} = 4R\sqrt{\frac{(p-c)(p-b)}{bc}}\sqrt{\frac{(p-a)(p-c)}{ac}}\sqrt{\frac{(p-b)(p-a)}{ab}}$$

$$= 4R\frac{(p-a)(p-b)(p-c)}{abc} =$$

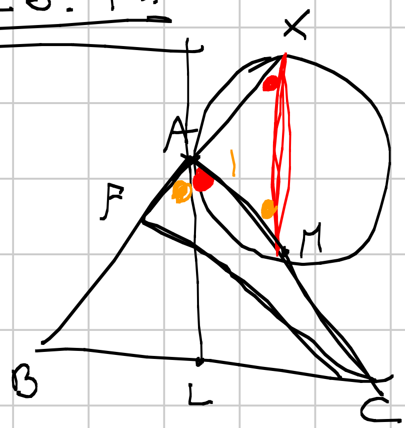
$$= \cancel{4R} \frac{S^2}{\cancel{p}abc} = \frac{S}{p} = r$$

$$AI = 4R \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$$



$$r = AI \cdot \sin\frac{\alpha}{2} = 4R \sin\alpha/_2 \sin\beta/_2 \sin\gamma/_2$$

ES. 11.



Minimizzare $\frac{BX}{CF}$

$L\hat{A}M = A\hat{X}M$

perché insistono su $\overset{\frown}{AM}$

$X\hat{M}A = B\hat{A}L$

in $A\hat{X}M$

$$\frac{AX}{\sin \bullet} = \frac{AM}{\sin \bullet}$$

$$AX = \frac{b}{2} \frac{\sin \bullet}{\sin \bullet} = \frac{b^2}{2c}$$

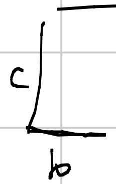
$\frac{\sin \bullet}{\sin \bullet} = \frac{b}{c}$ Usando la formula che vi ho dato.

$$BX = AB + AX = c + \frac{b^2}{2c} = \frac{2c^2 + b^2}{2c}$$

$$CF = b\sin\alpha$$

Infine $\frac{BX}{CF} = \frac{b^2 + 2c^2}{2bc \sin\alpha}$

$\alpha = \pi/2$ (90) perché in questo modo $\frac{1}{\sin\alpha}$ è minimo.



$$\boxed{\frac{b^2 + 2c^2}{2bc} \geq \sqrt{2}}$$

$$b^2 - 2\sqrt{2}bc + 2c^2 \geq 0$$

$$(b - \sqrt{2}c)^2 \geq 0$$

se $b - \sqrt{2}c = 0$

L'uguaglianza si ha se $\alpha = 90°$, $b/c = \sqrt{2}$.