

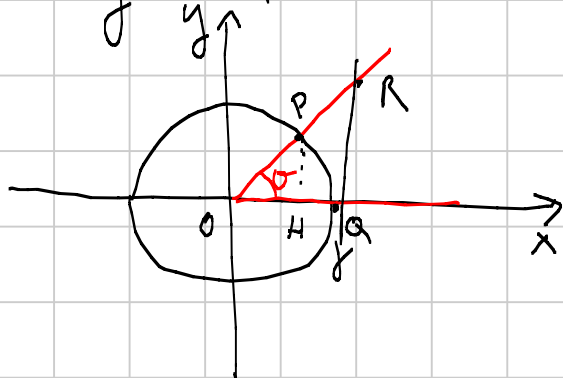
G1 - basic - Senior '15 - Gioacchino

Titolo nota

22/08/2015

Consideriamo la circonferenza

geometrica: (f)



Sia θ un angolo.

1) Misuriamo, a partire
 del semiasse positivo delle
 x in senso antiorario,
 un angolo θ (es. $\theta = 30^\circ$)
 e tracciamo una semiretta

Sia P l'int. di questa con la cir. geom.

$$\begin{aligned} PH &\stackrel{\text{def}}{=} \sin \theta \\ OH &\stackrel{\text{def}}{=} \cos \theta \end{aligned} \quad \sim P(\cos \theta, \sin \theta)$$

(C.1) Pitagora su $\triangle POH$: $PH^2 + OH^2 = 1 \rightarrow \boxed{\sin^2 \theta + \cos^2 \theta = 1}$

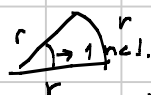
(Tg) Traccio da Q la \perp all'asse x . Interseca
 la semiretta in R .

$$QR \stackrel{\text{def}}{=} \tan \theta$$

(C.2) $\triangle POH \sim \triangle ROQ \rightarrow \frac{PH}{OH} = \frac{QR}{OQ} \rightarrow \boxed{\frac{\sin \theta}{\cos \theta} = \tan \theta}$

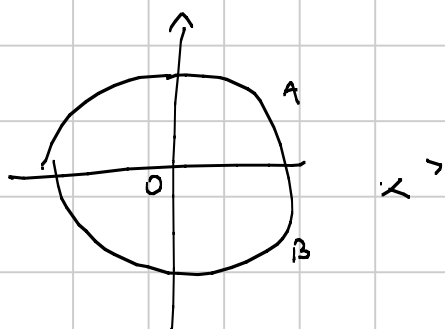
Inciso $\pi \rightsquigarrow 180^\circ$
 $\pi : 180^\circ = \text{ang. in rad} : \text{ang. in gradi}$

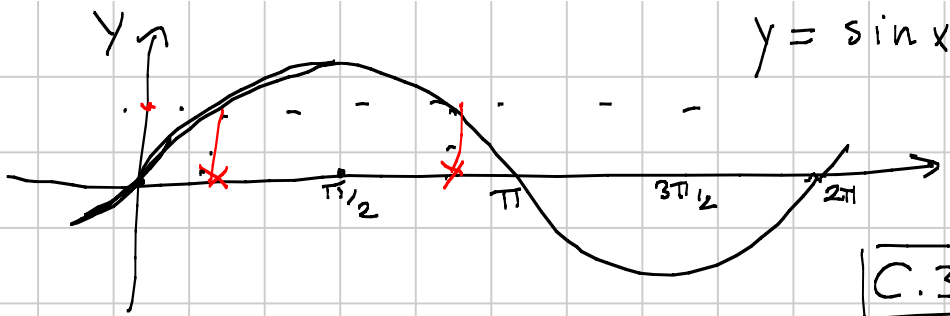
Per chi volene ...



Angoli

Angoli	\sin	\cos	\tan
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	n.d.

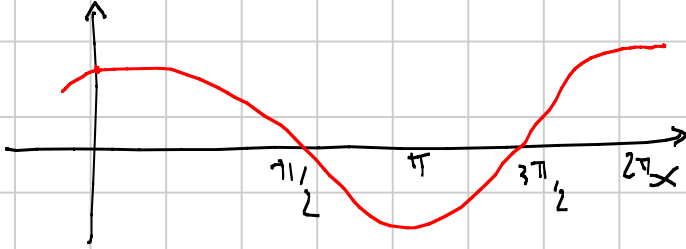




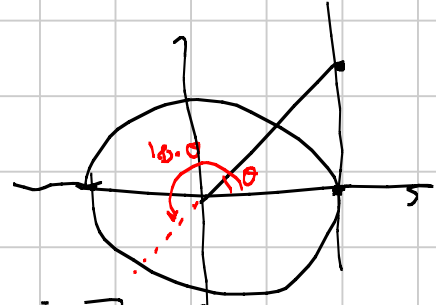
$$y = \sin x$$

$$2\pi = 360^\circ$$

C.3 Il seno ha
periodicità 2π
 $\sin(x + 2\pi) = \sin x \quad \forall x \in \mathbb{R}$
e monotonamente
per il coseno.

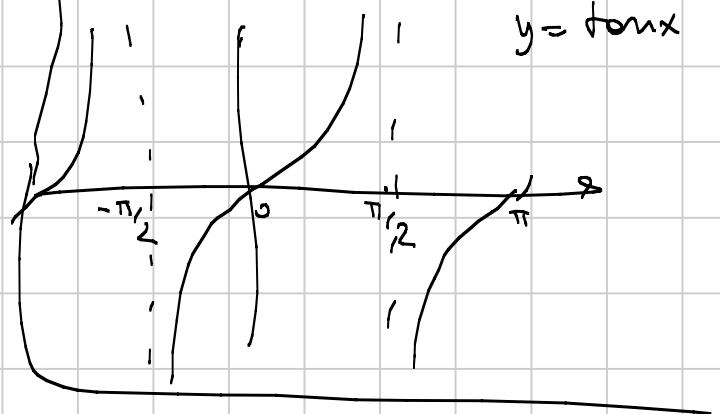
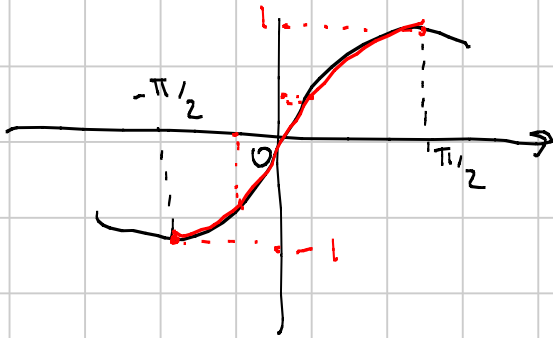


Remind f iniettiva
 \Leftrightarrow
 $f(x) = f(y) \Rightarrow x = y$



C.4 La tangente ha
periodicità π .
 $\tan(x + \pi) = \tan x \quad \forall x \in \mathbb{R}$

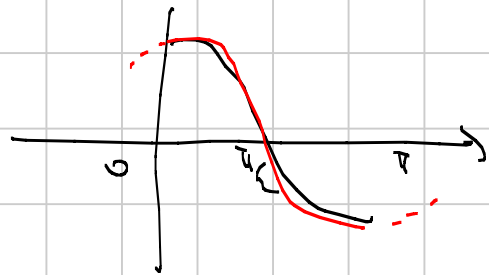
On \sin, \cos, \tan non
sono iniettive! ($\mathbb{R} \rightarrow \mathbb{R} \dots$)



$f(x) = \sin x$ fca $-\pi/2$ e $\pi/2$ e
iniettiva, quindi invertibile

La sua inversa si chiama funzione
arcsin.

$f(x) = \cos x$ fca 0 e π e inv.
La sua inversa si chiama
funzione arccos.

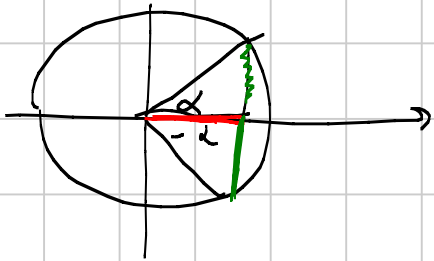


$f(x) = \tan x$ fca $-\pi/2$ e $\pi/2$...

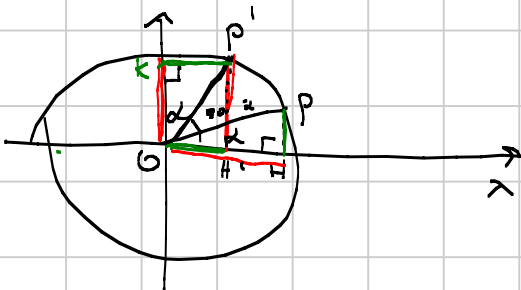
ESERCIZIO

Disegnare i grafici delle f inv.
inverte.

Obs. su archi associati



$$\begin{aligned} \cos(-\alpha) &= \cos \alpha = \cos(2\pi - \alpha) \\ \sin(-\alpha) &= -\sin \alpha = \sin(2\pi - \alpha) \\ \tan(-\alpha) &= -\tan \alpha \end{aligned}$$

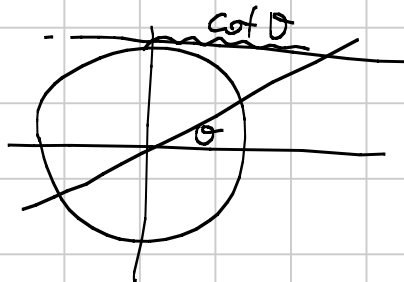
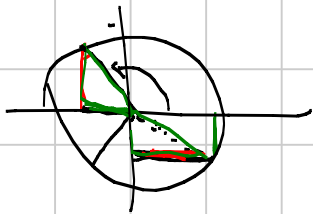


$$\begin{aligned} \cos(90-\alpha) &= OH' = P'K \stackrel{\text{cong.}}{=} PH = \sin \alpha \\ \sin(90-\alpha) &= P'H' = OK = OH = \cos \alpha \end{aligned}$$

$$\hat{OKP'} = \hat{OHP}$$

$$\hat{KOP} = \alpha$$

$$\tan(90-\alpha) = \frac{\cos \alpha}{\sin \alpha} = \cot \alpha$$



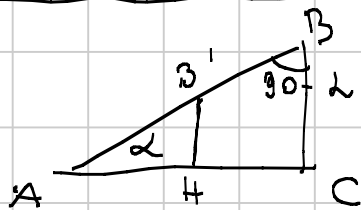
Dimostrare

Esercizio Le funzioni goniometriche di $90+\alpha$, $180-\alpha$.

Ripete la dim. per $90-\alpha$ con $\alpha > 90$.

Q. Come si fa il seno di una penna.

INCLUSO



$$AB' = 1$$

$$B'H = \sin \alpha$$

$$AH = \cos \alpha$$

$$\triangle AB'H \sim \triangle ABC$$

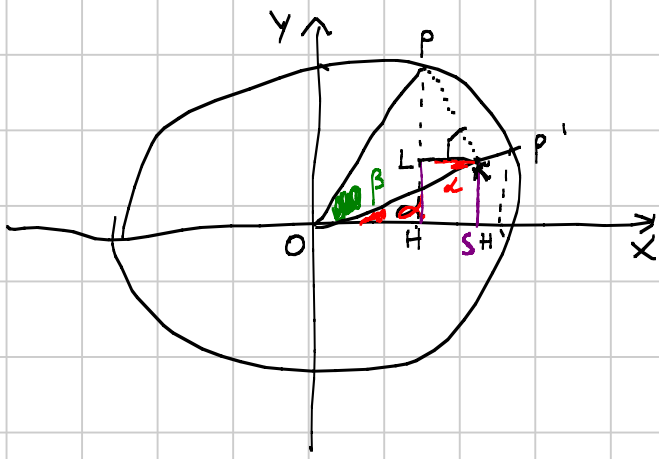
$$\frac{AB}{AB'} = \frac{BC}{B'H}$$

$$AB = \frac{BC}{\sin \alpha}$$

$$BC = AB \sin \alpha$$

Analogamente

$$AC = AB \cos \alpha$$



$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \sin\beta \cos\alpha$$

$$\begin{cases} PK = \sin\beta \\ OK = \cos\beta \end{cases}$$

$$\begin{aligned} \angle KOP &= \alpha \\ \angle PKL &= 90 - \alpha \end{aligned}$$

$$PL = PK \sin(90 - \alpha) = \sin\beta \cos\alpha$$

Imkos...

$$\text{Similia} \quad LH = \sin\alpha \cos\beta$$

$$\sin(\alpha + \beta)$$

$$PH = PL + LH = \sin\beta \cos\alpha + \sin\alpha \cos\beta$$

$$\sin(\alpha - \beta) = \sin(\alpha + (-\beta)) = \sin\alpha \cos(-\beta) + \sin(-\beta) \cos\alpha = \sin\alpha \cos\beta - \sin\beta \cos\alpha$$

$$\begin{aligned} \cos(\alpha + \beta) &\stackrel{?}{=} \sin(90 - (\alpha + \beta)) = \sin(\underbrace{90 - \alpha}_{\beta} - \beta) = \\ &= \sin(90 - \alpha) \cos\beta - \sin\beta \cos(90 - \alpha) \\ &= \cos\alpha \cos\beta - \sin\beta \sin\alpha \end{aligned}$$

$$\cos(\alpha - \beta) = \dots = \cos\alpha \cos\beta + \sin\beta \sin\alpha$$

DUPLICAZIONE

$$\sin 2\alpha = \sin(\alpha + \alpha) = \sin\alpha \cos\alpha + \sin\alpha \cos\alpha = 2 \sin\alpha \cos\alpha$$

$$\cos 2\alpha = \cos(\alpha + \alpha) = \cos\alpha \cos\alpha - \sin\alpha \sin\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\begin{aligned} \cos^2\alpha + \sin^2\alpha - 2\sin^2\alpha &= 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - (\cos^2\alpha + \sin^2\alpha) &= 2\cos^2\alpha - 1 \end{aligned}$$

Im conclusione

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{\tan^2\alpha + 1}$$

BISEZIONE

$$\sin \frac{\alpha}{2}, \cos \frac{\alpha}{2}$$

Le ricaviamo da \square

$$\cos 2\alpha = 1 - 2\sin^2 \alpha$$

$$\alpha \rightarrow \alpha/2$$

$$\cos 2(\alpha/2) = 1 - 2\sin^2 \alpha/2$$

$$\sin^2 \alpha/2 = \frac{1 - \cos \alpha}{2}$$

$$\sin \alpha/2 = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

Im un triangolo \oplus

$$\cos 2\alpha = 2\cos^2 \alpha - 1$$

$$\alpha \rightarrow \alpha/2$$

$$\cos^2 \alpha/2 = \frac{1 + \cos \alpha}{2}$$

$$\tan \frac{\alpha}{2} = \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \frac{2 \sin \alpha/2 \cdot \sin \alpha/2}{2 \sin \alpha/2 \cdot \cos \alpha/2} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$\frac{2 \cos \alpha/2 \cdot \sin \alpha/2}{2 \cos \alpha/2 \cdot \cos \alpha/2} = \frac{\sin \alpha}{1 + \cos \alpha}$$

Parametriche

Idea: Esprimere seno e coseno in funzione di $\tan \alpha/2$.

$$t = \tan \frac{\alpha}{2}$$

$$s = \sin \alpha$$

$$c = \cos \alpha$$

$$t = \frac{1-c}{s} \quad (1)$$

$$t = \frac{s}{1+c} \quad (2)$$

$$\alpha \neq 0, 180^\circ$$

$$(1) \cdot s$$

$$st = 1 - c$$

$$c = 1 - st \quad (3)$$

$$(2) \cdot (1+c)$$

$$t + tc = s$$

Uso la (3)

$$t + t(1-st) = s$$

$$2t - st^2 = s$$

$$2t = st^2 + s = s(t^2 + 1)$$

$$s = \frac{2t}{t^2 + 1}$$

$$\text{Pella 3 } c = 1 - \frac{2t}{t^2 + 1} \cdot t = \frac{t^2 + 1 - 2t^2}{t^2 + 1} = \frac{1 - t^2}{1 + t^2}$$

Quindi

$$\sin \alpha = \frac{2 \tan \frac{\alpha}{2}}{\tan^2 \frac{\alpha}{2} + 1}$$

$$\cos \alpha = \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

Esercizio: controlla le c.d.e. per ogni formula...

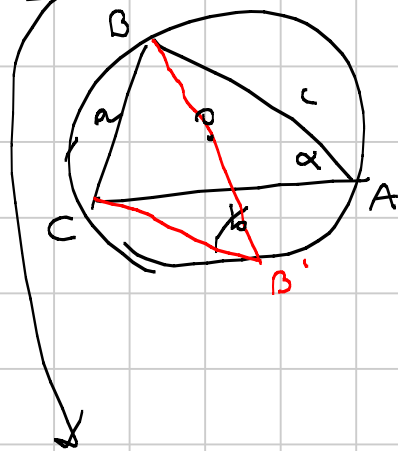
Prostaferesi, Werner (Es. 8 pagina 3)

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\rightarrow \sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

Trigonometria

Th. 1



Triangolo A, B, C \checkmark

a, b, c i lati opposti ad A, B, C

R il raggio della cfr. circoscritta

r il " " " " inscritta

α, β, γ gli angoli in A, B, C

Teorema della corda (seni): In un triangolo

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

D.m.

$$BB' = 2R$$

$$\widehat{BCB'} = 90^\circ$$

$$BC = BB' \cdot \sin \widehat{BB'C} =$$

$$= BB' \cdot \sin \alpha$$

↓

$$a = 2R \cdot \sin \alpha$$

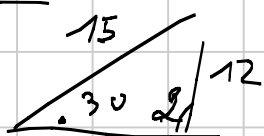
$$\frac{a}{\sin \alpha} = 2R \quad \square$$

Cor.

Seni: In un triangolo il rapporto fra un lato

è il seno dell'angolo opposto è costante
(= 2R)

Es.



$$\frac{15}{\sin \alpha} = \frac{12}{\sin 30} = 24$$

$$\sin \alpha = \frac{15}{24} = \frac{5}{8}$$

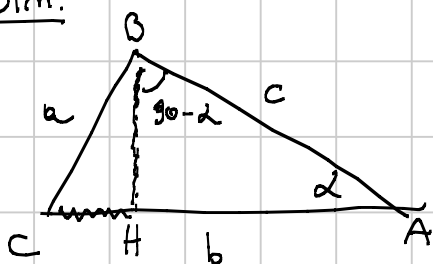
Th. di Carnot (coseni)

$$\rightarrow \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \beta = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos \gamma = \frac{b^2 + a^2 - c^2}{2ab}$$

Dim.



$$AH = AB \cos \alpha = c \cos \alpha$$

$$BH = c \sin \alpha$$

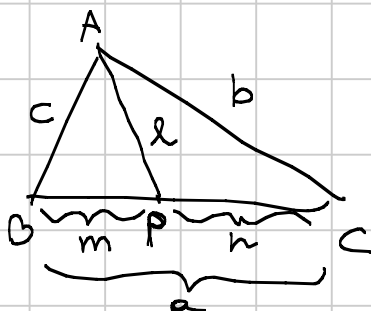
$$CH = AC - AH = b - c \cos \alpha$$

Scrivo Pitagora in $\triangle BCH$

$$\begin{aligned} BC^2 &= CH^2 + BH^2 = (b - c \cos \alpha)^2 + (c \sin \alpha)^2 = \\ a^2 &= \quad \quad \quad = b^2 - 2bc \cos \alpha + c^2 \cos^2 \alpha + c^2 \sin^2 \alpha = \\ &= b^2 - 2bc \cos \alpha + c^2 \end{aligned}$$

$$2bc \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} \quad \square$$

Th. Stewart (Es. 9 pagina 3)

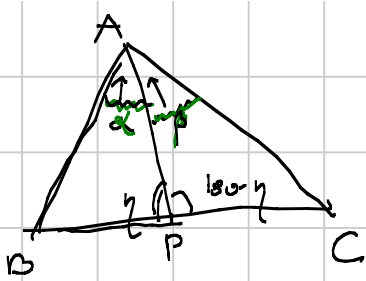


$$\underline{b^2 m} + \underline{c^2 n} = a(\underline{d^2} + mn)$$

Es.

Dim. Stewart.

e ricava la lunghezza della mediana ($= \sqrt{\frac{2(b^2 + c^2) - a^2}{4}}$)
e birettuce



On.

Th. dei seni su $\triangle ABP$

$$\frac{AB}{\sin \gamma} = \frac{BP}{\sin \alpha} \quad (1)$$

Th. dei seni su $\triangle APC$

$$\frac{AC}{\sin(180-\gamma)} = \frac{PC}{\sin \alpha} = \frac{PC}{\sin \beta} \quad (2)$$

Divido (1)/(2)

$$\frac{AB/\sin \gamma}{AC/\sin \gamma} = \frac{BP/\sin \alpha}{PC/\sin \beta}$$

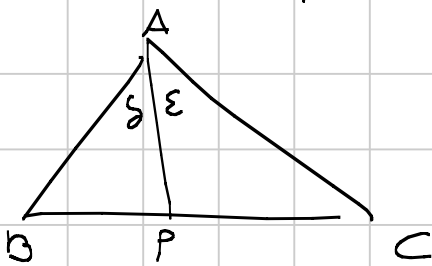
$$\frac{AB}{AC} = \frac{BP}{PC} \frac{\sin \beta}{\sin \alpha}$$

$$\frac{BP}{PC} = \frac{c}{b} \frac{\sin \alpha}{\sin \beta}$$

Attenzione!
 α e β
 non in
 not.
 standard!

Sono gli
 angoli staccati
 dalle cenero

Th.



$$\boxed{\frac{BP}{PC} = \left(\frac{c}{b}\right) \frac{\sin \beta}{\sin \alpha}}$$

Es 3 pag. 3

$$1 - \cot 23 = \frac{2}{1 - \cot 22}$$

$$\frac{\sin 23 - \cot 23}{\sin 23} = \frac{2 \sin 22}{\sin 22 \cdot \cos 22}$$

$$\cancel{\sin 23 \sin 22} - \sin 23 \cos 22 - \cos 22 \sin 22 + \cos 22 \cos 23 = \frac{2}{\sin 22 \cos 22}$$

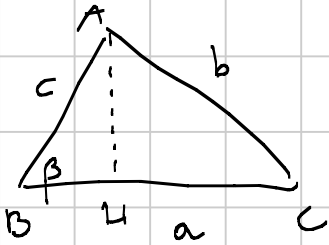
$$\cos 22 \cos 23 - \sin 22 \sin 23 = \frac{2}{\sin 22 \cos 22}$$

$$\cos 45$$

$$= \frac{1}{\sin 45}$$



Area di un triangolo



S, A

$$S = \frac{BC \cdot AH}{2} = \frac{ac \sin \beta}{2}$$

$$S = \frac{1}{2} ac \sin \beta$$

$$\frac{1}{2} ab \sin \gamma$$

$$\frac{1}{2} bc \sin \alpha$$

A parole: L'area è il $\frac{1}{2}$ prodotto fra due lati e il seno dell'angolo compreso fra loro.

$$S = \frac{1}{2} ac \sin \beta$$

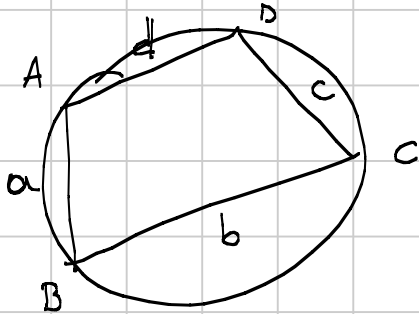
$$\frac{1}{2} ac \stackrel{||}{=} \frac{b}{2R} = \frac{abc}{4R}$$

$$\left| \begin{aligned} \frac{b}{\sin \beta} &= 2R \\ \sin \beta &= \frac{b}{2R} \end{aligned} \right.$$

$$S = \frac{abc}{4R} \rightarrow R = \frac{abc}{4S}$$

ED.

Gener. Erone



Brachmagupta

$$s = \frac{a+b+c+d}{2}$$

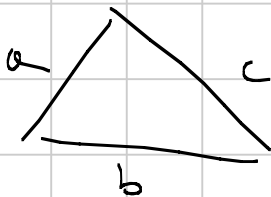
$$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$d \rightarrow 0$$

$$s = \frac{a+b+c}{2}$$

$$p = \frac{a+b+c}{2}$$

Erone



$$S = \sqrt{p(p-a)(p-b)(p-c)}$$

NUMERI COMPLESSI

$$z = a + bi$$

$$i^2 = -1$$

\downarrow
 $\in \mathbb{R}$
 parte reale

\downarrow
 $\in \mathbb{R}$
 parte immaginaria

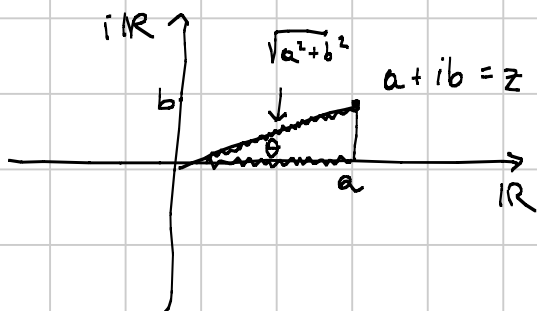
SOMMA

$$a + bi + c + di = (a+c) + (b+d)i$$

PRODOTTO

$$(a+bi)(c+di) = ac + adi + bci - bd = (ac - bd) + (ad + bc)i$$

Rapp. sul piano di Gauss.



RAPP. POLARE

$$a + bi = \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} + \frac{b}{\sqrt{a^2 + b^2}} i \right)$$

\downarrow
 $(\cos \theta + i \sin \theta)$

NORMA DI z

$$|z| = \rho = \sqrt{a^2 + b^2} \quad \text{MODULO del N. COMPLESSO}$$

$\theta =$ argomento

$$e = 2,718 \dots$$

$$z = \rho (\cos \theta + i \sin \theta) = \rho e^{i\theta}$$

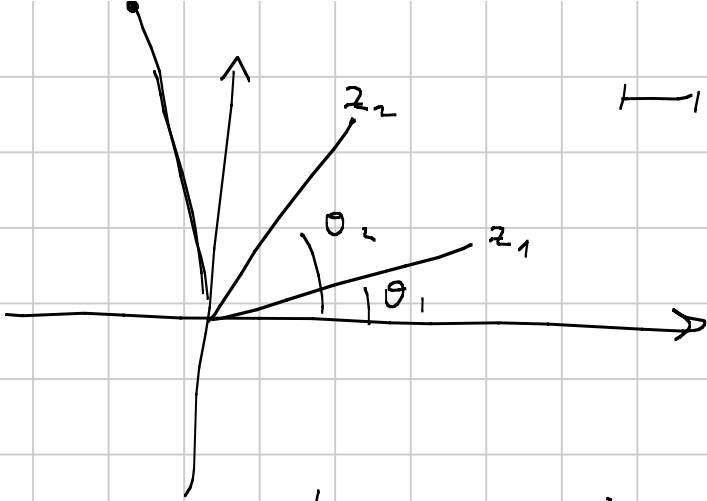
e^x
 $e^{i\theta}$
 $e^{-i\theta}$

$$z_1 = \rho_1 e^{i\theta_1}$$

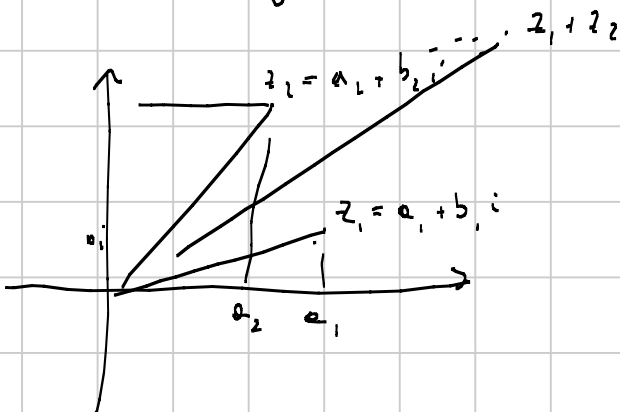
$$z_2 = \rho_2 e^{i\theta_2}$$

$$z_1 z_2 = \rho_1 \rho_2 e^{i(\theta_1 + \theta_2)}$$

$z_1 z_2$



Cosa vuol dire, sul piano di Gauss moltiplicare per un numero complesso z di modulo $|z|$ (ρ)
 = ROTOTETIA
 dell'argomento di z



Somma = Regola del parallelogramma

DE-MOIVRE

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 3\alpha = ?$$

$$\cos n\alpha = ?$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\sin 3\alpha = ?$$

$$\sin n\alpha = ?$$

$$z = e^{i\theta} = \cos \theta + i \sin \theta$$

$$z^n = e^{in\theta} = \cos(n\theta) + i \sin(n\theta)$$

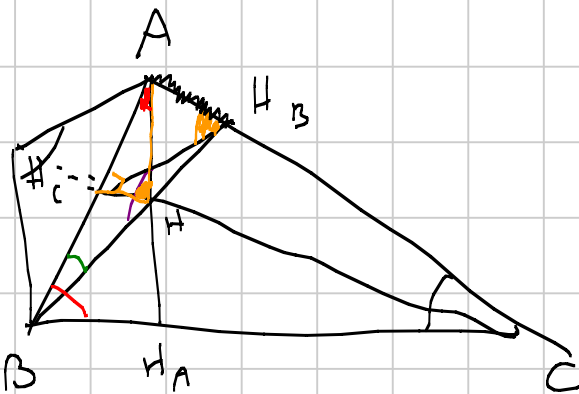
$$z^n = (\cos \theta + i \sin \theta)^n = \sum_{k=0}^n \binom{n}{k} \cos^k \theta (i \sin \theta)^{n-k} = A + i B$$

$$\cos(n\theta) = \binom{n}{0} \cos^n \theta - \binom{n}{2} \cos^{n-2} \theta \sin^2 \theta + \dots$$

$$\sin(n\theta) = \binom{n}{1} \cos^{n-1} \theta \sin \theta - \binom{n}{3} \cos^{n-3} \theta \sin^3 \theta + \dots$$

Pag. 3 7-8-9-10 (B zohomegyula)

Pag. 32 10-11-13 (1-5) - 14 (1-3-6-8)



AH

$\hat{A}H_b$

$$\hat{B}A H = 90 - \beta$$

$$\hat{A}B H = 90 - \alpha$$

$$\hat{A}H_b = \alpha + \beta = 180 - \gamma$$

Semi: $\frac{AH}{\sin(90 - \alpha)} = \frac{AB}{\sin(180 - \gamma)}$

$$AH = \frac{c \cos \alpha}{\sin \gamma} = 2R \cos \alpha$$

$$\left\{ \begin{array}{l} H_b H_c \\ \hat{A}H_b H_c \\ \hat{A}H_b H_c = \beta \\ \hat{A}H_c H_b = \gamma \end{array} \right.$$

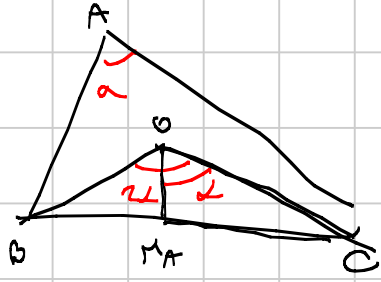
$$\hat{A}H_b H_c = \hat{A}H_c H_b = \beta$$

perché $A H_c H_b$ è un quad. ciclico

Semi: $\frac{AH_b}{\sin \hat{A}H_c H_b} = \frac{H_b H_c}{\sin \alpha} \rightarrow H_b H_c = \frac{AH_b \sin \alpha}{\sin \gamma}$

Ma $AH_b = c \cos \alpha \rightarrow H_b H_c = \frac{c \cos \alpha \sin \alpha}{\sin \gamma} = \frac{2R \cos \alpha \sin \alpha}{\sin \gamma} = R \sin 2\alpha$

OH_A



OH_A
 Dico $\sqrt{R^2 - \frac{a^2}{4}}$

$OH_A = R \cos \alpha$ (la metà di AH...)

$OH_A = \frac{AH}{2}$

$\alpha_2 + \beta_2 + \frac{\gamma}{2} = 90$

$\sin(90 + \alpha) = \cos \alpha$

$\frac{AI}{\sin \beta_2} = \frac{AB}{\sin(90 + \frac{\gamma}{2})} = \frac{AB}{\cos \frac{\gamma}{2}}$

$AI = \frac{c \sin \beta_2}{\cos \frac{\gamma}{2}} = \frac{2c \sin \beta_2 \sin \frac{\gamma}{2}}{\sin \gamma}$

$= 4R \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$

G_3 + Pow \rightarrow OI

$OI^2 = R^2 - 2Rr$

Es. 8

$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

Verifica brutta

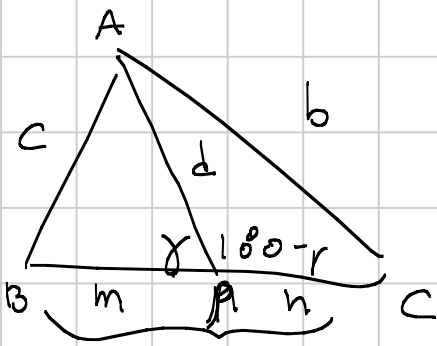
$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$

$\alpha + \beta = p$
 $\alpha - \beta = q$

$\alpha = \frac{p+q}{2}$
 $\beta = \frac{p-q}{2}$

$2 \sin \frac{p+q}{2} \cos \frac{p-q}{2} = \frac{1}{2} (\sin p + \sin q)$

Es. 9 Stewart



$$m^2 + d^2 = b^2 + c^2$$

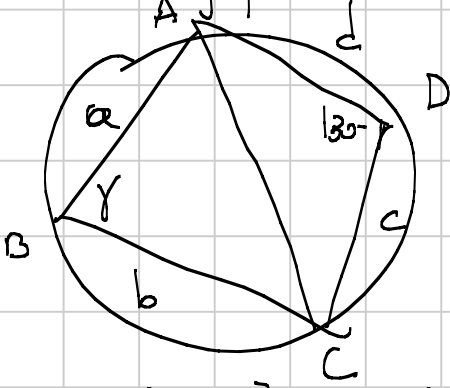
$$2(mn + d^2) = b^2 + c^2$$

Idea: $c^2 = d^2 + m^2 - 2md \cos \gamma$
 $b^2 = d^2 + n^2 + 2nd \cos \gamma$

Sostituisco e viene.

E s. 10 Egeu.

Brahmagupta



$$S = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$AC^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$d^2 + c^2 + 2dc \cos \gamma$$

$$a^2 + b^2 - 2ab \cos \gamma = d^2 + c^2 + 2dc \cos \gamma$$

$$\cos \gamma = \frac{a^2 + b^2 - c^2 - d^2}{2(ab + cd)}$$

$$S = \frac{1}{2} ab \sin \gamma + \frac{1}{2} cd \sin(180 - \gamma) = \frac{1}{2} (ab + cd) \sin \gamma$$

$$\sin^2 \gamma = 1 - \cos^2 \gamma = 1 - \frac{(a^2 + b^2 - c^2 - d^2)^2}{4(ab + cd)^2} =$$

$$= \frac{[2(ab + cd)]^2 - (a^2 + b^2 - c^2 - d^2)^2}{4(ab + cd)^2} =$$

$$= \frac{(a^2 + 2ab + b^2 - c^2 + 2cd - d^2)(c^2 + d^2 + 2cd - a^2 - b^2)}{4(ab + cd)^2} =$$

$$= \frac{[(a+b)^2 - (c-d)^2][(c+d)^2 - (a-b)^2]}{4(ab + cd)^2} =$$

$$= \frac{(a+b+c-d)(a+b-c+d)(c+d+a-b)(c+d+b-a)}{4(ab + cd)^2}$$

$$s - d = \frac{a+b+c+d}{2} - d = \frac{a+b+c-d}{2}$$

$$= \frac{2(s-d)(s-c)(s-b)(s-a)}{(ab+cd)^2}$$

$$\sin f = \frac{2 \sqrt{(s-d)(s-c)(s-b)(s-a)}}{(ab+cd)}$$

$$\text{Quindi } S = \frac{1}{2} (ab+cd) \frac{\sqrt{(s-a)(s-b)(s-c)(s-d)}}{(ab+cd)}$$

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Elonne \swarrow $S = \sqrt{p(p-a)(p-b)(p-c)}$

Thorp $16S^2 = -a^4 - b^4 - c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2$

Es. 13

$$\{r = 4R \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} = \frac{1 - \frac{b^2 + c^2 - a^2}{2bc}}{2} = \frac{2bc + a^2 - b^2 - c^2}{4bc} =$$

$$= \frac{a^2 - (b-c)^2}{4bc} =$$

$$p = \frac{a+b+c}{2}$$

$$p-c = \frac{a+b+c}{2} - c =$$

$$= \frac{a+b-c}{2}$$

$$= \frac{(a+b-c)(a+c-b)}{4bc} =$$

$$= \frac{2(p-c) \cdot 2(p-b)}{4bc} =$$

$$= \frac{(p-c)(p-b)}{bc}$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{(p-c)(p-b)}{bc}}$$

$$\sin \frac{\beta}{2} = \sqrt{\frac{(p-a)(p-c)}{ac}}$$

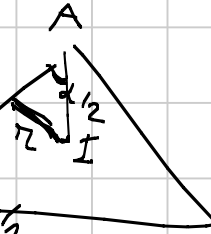
$$4R \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2} = 4R \sqrt{\frac{(p-c)(p-b)}{bc}} \sqrt{\frac{(p-a)(p-c)}{ac}} \sqrt{\frac{(p-b)(p-a)}{ab}}$$

$$= 4R \frac{(p-a)(p-b)(p-c)}{abc} =$$

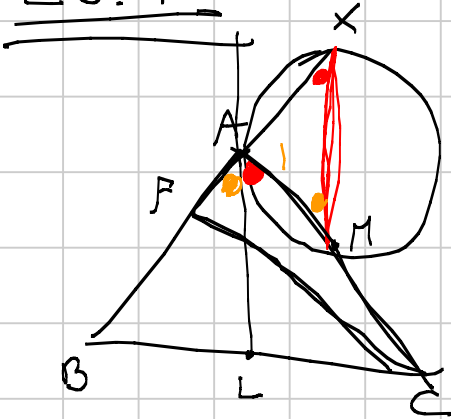
$$= 4R \frac{S^2}{abc} = \frac{S}{p} = r$$

$$AI = 4R \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$$

$$r = AI \cdot \sin \frac{\alpha}{2} = 4R \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \sin \frac{\gamma}{2}$$



ES. 11.



Minimizza $\frac{BX}{CF}$

$$\widehat{LAM} = \widehat{AXM}$$

perché inscrivono su \widehat{AM}

$$\widehat{XMA} = \widehat{BAL}$$

in \widehat{AXM}

$$\frac{AX}{\sin \alpha} = \frac{AM}{\sin \alpha}$$

$$AX = \frac{b}{2} \frac{\sin \alpha}{\sin \alpha} = \frac{b^2}{2c}$$

$\left| \frac{\sin \alpha}{\sin \alpha} = \frac{b}{c} \right.$ Usando la formula che viene data.

$$BX = AB + AX = b + \frac{b^2}{2c} = \frac{2c^2 + b^2}{2c}$$

$$CF = b \sin \alpha$$

Infine $\frac{BX}{CF} = \frac{b^2 + 2c^2}{2bc \sin \alpha}$

$\alpha = \frac{\pi}{2}$ (90°)
perché $\frac{1}{\sin \alpha}$ è max.

$$\frac{b^2 + 2c^2}{2bc} \geq \sqrt{2}$$

$$b^2 - 2\sqrt{2}bc + 2c^2 \geq 0$$

$$(b - \sqrt{2}c)^2 \geq 0$$

L'uguaglianza si ha se $\alpha = 90^\circ$, $\frac{b}{c} = \sqrt{2}$.