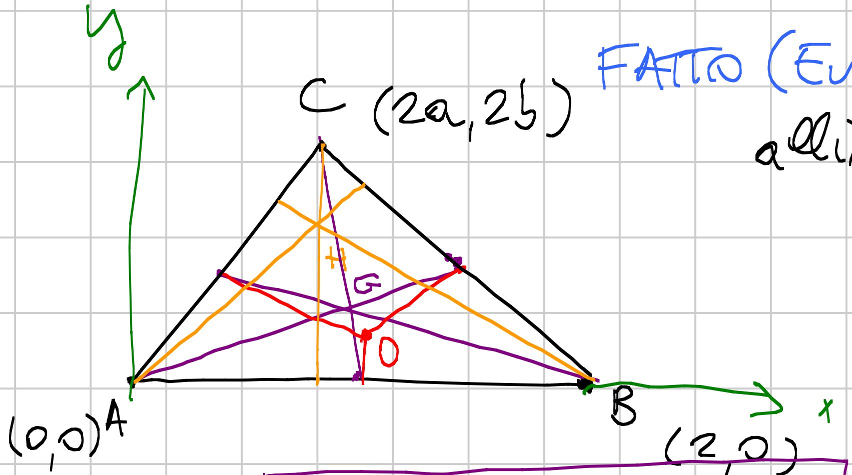
Geometria 2

φ



FATTO (Eulero) H, G, O allineati.

$$\boxed{G \equiv \left(\frac{2}{3}(a+1),\ \frac{2}{3}b\right)}$$

H sta su h_C $\qquad x_H = x_C = 2a$

retta AC $\qquad y = \frac{b}{a}x$

H sta su h_B $\qquad y_H = -\frac{a}{b}x_H + \frac{2a}{b}$

$$\boxed{H \equiv \left(2a,\ -\frac{2a^2}{b} + \frac{2a}{b}\right)}$$

O sta sull'asse di AB $\to$ $x_O = 1$

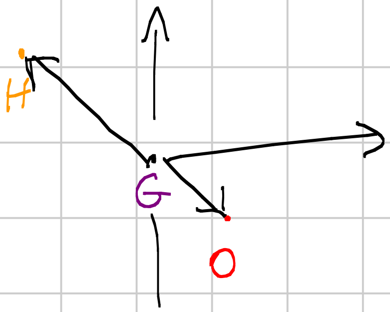
O sta sull'asse di AC $\to$ $y_O = -\frac{a}{b}x_O + \frac{a^2}{b} + b$

$$\boxed{O \equiv \left(1,\ -\frac{a}{b} + \frac{a^2}{b} + b\right) \equiv \left(1,\ \frac{a(a-1)}{b} + b\right)}$$

concludere: allineamento!

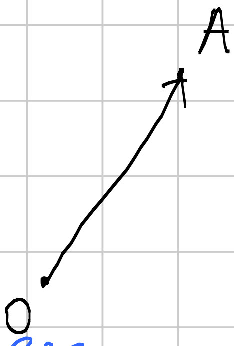
$$\frac{x_O - x_G}{y_O - y_G} = \frac{x_H - x_G}{y_H - y_G}$$

$$\frac{-\frac{2}{3}a + \frac{1}{3}}{\frac{a(a-1)}{b} + \frac{1}{3}b} \boxed{\overset{?}{=}} \frac{\frac{4}{3}a - \frac{2}{3}}{-\frac{2a(a-1)}{b} - \frac{2}{3}b}$$



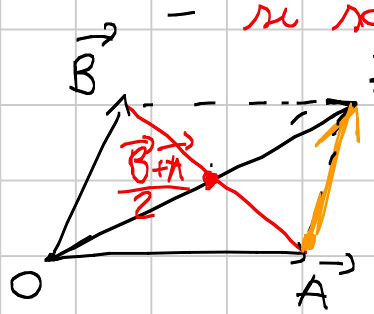
$x_H = -2x_O$
$y_H = -2y_O$

VETTORI



$\vec{A}$

- si sommano

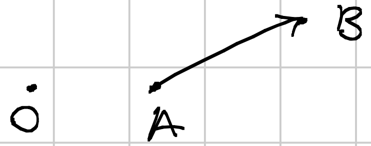


$\vec{A}+\vec{B}$ (traslazione)

- si moltiplicano per scalari

→ PTO MEDIO di A e B $\quad \dfrac{\vec{A}+\vec{B}}{2}$

→ retta per A e B



$(\vec{B}-\vec{A})\lambda + \vec{A} \qquad \lambda\vec{B} + (1-\lambda)\vec{A}$

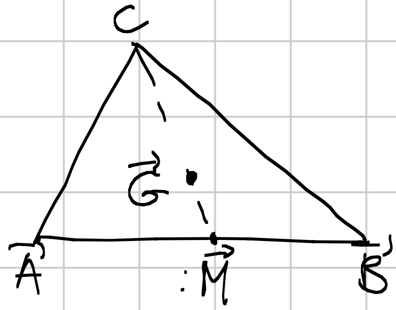
→ segmento per A, B $\quad \lambda \in [0,1]$

$\overline{PA}/\overline{AB} = \lambda$

$(1-\lambda)\overline{AB}$

$\vec{P} = \lambda\vec{B} + (1-\lambda)\vec{A}$

Baricentro di ABC



$$\vec{M} = \frac{\vec{A}+\vec{B}}{2}$$

$$\vec{G} = \frac{1}{3}\vec{C} + \frac{2}{3}\vec{M} = \frac{\vec{A}+\vec{B}+\vec{C}}{3}$$

Ortocentro di ABC

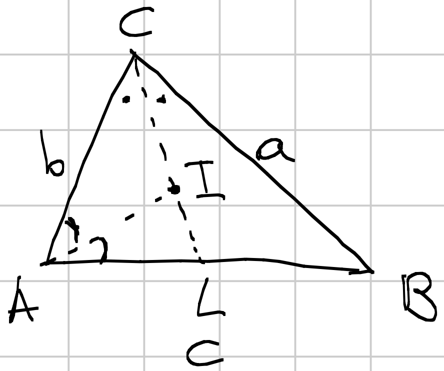
$$\vec H - \vec G = -2(\vec O - \vec G)$$

$$\vec H = 3\vec G - 2\vec O = \vec A + \vec B + \vec C - 2\vec O$$

se metto l'origine in O

$$\vec H = \vec A + \vec B + \vec C$$

Incentro di ABC



$$\vec L = \lambda \vec A + (1-\lambda)\vec B$$

$$\lambda = \frac{BL}{AB} = \frac{BC}{BC+AC} = \frac{a}{a+b}$$

$$\vec L = \frac{a}{a+b}\vec A + \frac{b}{a+b}\vec B$$

Teorema della bisettrice su ALC

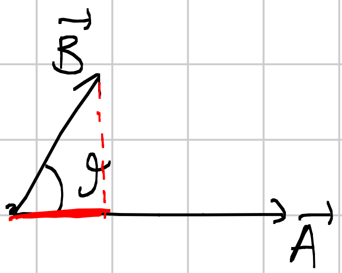
$$\frac{IL}{CL} = \frac{AL}{AL+AC} = \frac{c\cdot\frac{b}{a+b}}{\frac{cb}{a+b}+b} = \frac{c\cancel{b}}{c\cancel{b}+a\cancel{b}+b^{\cancel{2}}} =$$

$$= \frac{c}{a+b+c}$$

$$\vec I = \vec C \cdot \frac{c}{a+b+c} + \vec L\,\frac{b+c}{a+b+c} = \frac{a\vec A + b\vec B + c\vec C}{a+b+c}$$

i vettori...

hanno il PRODOTTO SCALARE



$$\langle \vec A, \vec B\rangle = |A||B|\cos\vartheta$$

se A, B allineati

$$\langle \vec A, \vec B\rangle = |A||B| \quad (\langle \vec A, \vec A\rangle = |A|^2)$$

$$\langle \lambda\vec{A}, \vec{B}\rangle = \lambda\langle \vec{A}, \vec{B}\rangle \qquad \text{se } \vec{A} \perp \vec{B} \quad \langle \vec{A}, \vec{B}\rangle = 0$$

$$\langle (x_A, y_A), (x_B, y_B)\rangle = x_A x_B + y_A y_B$$

$$|\langle \vec{A}, \vec{B}\rangle| \le |\vec{A}|\,|\vec{B}| \qquad CS$$

esercizio

$\vec{H} = \vec{A} + \vec{B} + \vec{C}$ (origine è in O)

(era vero??)

$$\langle \vec{B}-\vec{A}, \vec{H}-\vec{C}\rangle = \langle \vec{B}-\vec{A}, \vec{A}+\vec{B}\rangle =$$
$$= \cancel{\langle \vec{B}, \vec{A}\rangle} + \langle \vec{B}, \vec{B}\rangle - \langle \vec{A}, \vec{A}\rangle - \cancel{\langle \vec{A}, \vec{B}\rangle}$$
$$= R^2 - R^2 = 0$$

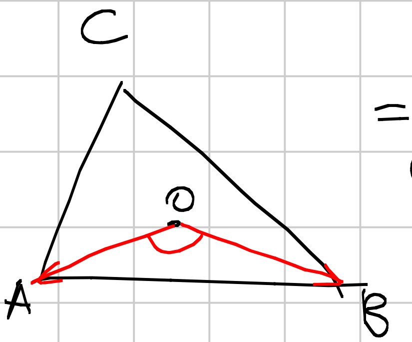
$\overline{OI}^2$ in funzione di r, R — metto l'origine in O

$$\left\langle \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}, \text{''} \right\rangle =$$

$$= \frac{1}{(2p)^2}\langle a\vec{A} + b\vec{B} + c\vec{C}, a\vec{A} + b\vec{B} + c\vec{C}\rangle$$

$$= \frac{1}{(2p)^2}\Big[\sum_{cyc} a^2 \underbrace{\langle \vec{A}, \vec{A}\rangle}_{R^2} + \sum_{cyc} 2ab\,\langle \vec{A}, \vec{B}\rangle\Big] =$$

$$\sum_{cyc} 2ab R^2 \cos 2\gamma =$$



$$= \frac{R^2}{(2p)^2}\Big[\sum_{cyc} a^2 + \sum_{cyc} 2ab - \sum_{cyc} 4ab\,\text{sen}^2\gamma\Big]$$

$\cos^2\gamma - \text{sen}^2\gamma =$
$= 1 - 2\,\text{sen}^2\gamma$

$$= \frac{R^2}{\cancel{(2p)^2}}\cancel{(2p)^2} - \frac{4R^2}{(2p)^2}\, 2S \sum_{cyc} \text{sen}A =$$

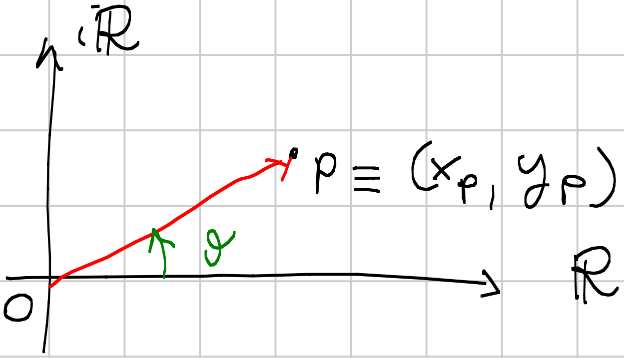
$$= R^2 - \frac{4R^{\cancel{2}}\cancel{2}S}{(2p)^2}(a+b+c) =$$

$$= R^2 - \frac{4RS}{2p} = R^2 - 2Rr$$

$$= R(R-2r)$$

BONUS: $R \geq 2r$

NUMERI COMPLESSI



$$p = x_p + i y_p$$
$$= |p|\, e^{i\vartheta}$$

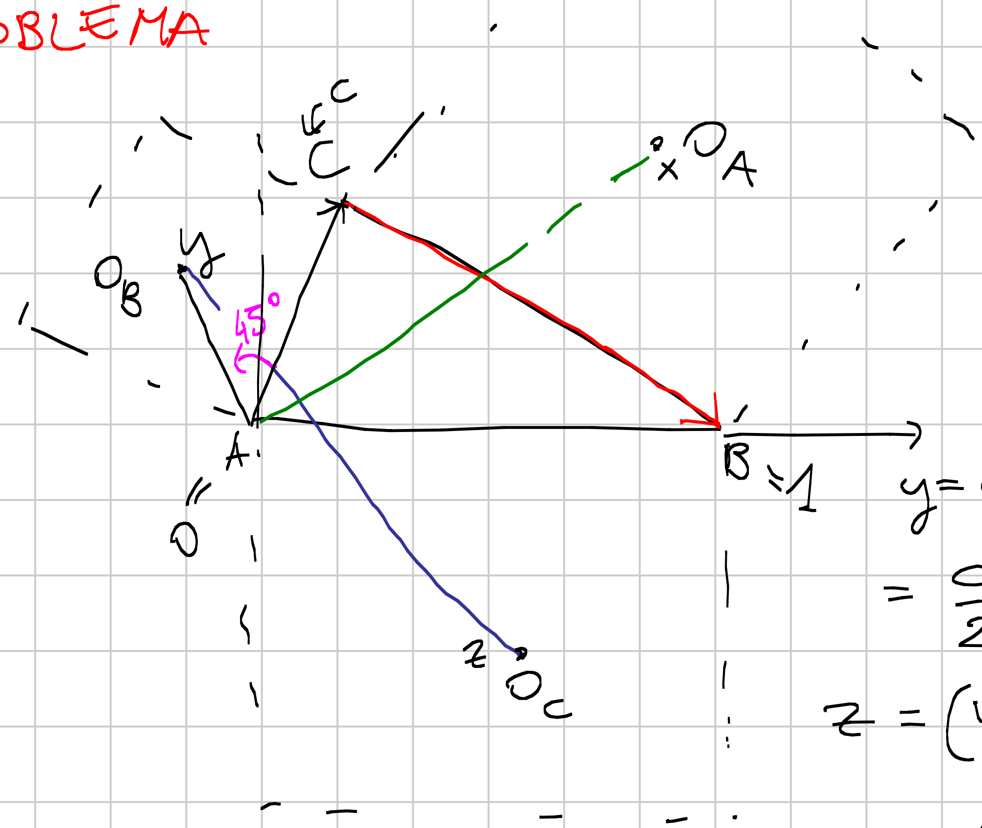
- si sommano (esattamente come vettori/pti in coordinate)
- si moltiplicano "per scalare" (per numero reale)
- si moltiplicano

$$x = |x|\, e^{i\vartheta} \qquad y = |y|\, e^{i\varphi}$$

$$xy = |x||y|\, e^{i(\vartheta+\varphi)}$$

$$e^{i\varphi} = \cos\varphi + i\,\mathrm{sen}\,\varphi$$

PROBLEMA



$$y = c\cdot\left(\frac{\sqrt2}{2}+i\frac{\sqrt2}{2}\right)\frac{\sqrt2}{2}$$
$$= \frac{c}{2}(1+i)$$

$$z = \left(\frac{\sqrt2}{2}-i\frac{\sqrt2}{2}\right)\frac{\sqrt2}{2} =$$
$$= \frac{1}{2}(1-i)$$

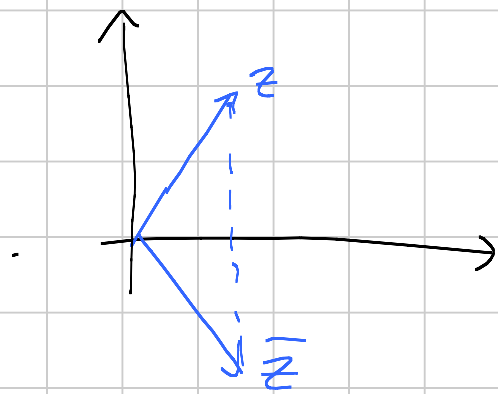
$$x = (1-c)\left(\frac{\sqrt2}{2}+i\frac{\sqrt2}{2}\right)\frac{\sqrt2}{2}+c =$$
$$= \frac{(1-c)(1+i)+2c}{2} = \frac{(1+c)}{2}+\frac{i}{2}(1-c)$$

$$z-y = \frac{(1-c)}{2}+\frac{1}{2}(-1-c)i$$

$$i(z-y) = \frac{1}{2}(1+c)+i\left(\frac{1-c}{2}\right) = x$$

$AO_A \perp O_BO_C \to$ altezze $\to$ concorrono ...

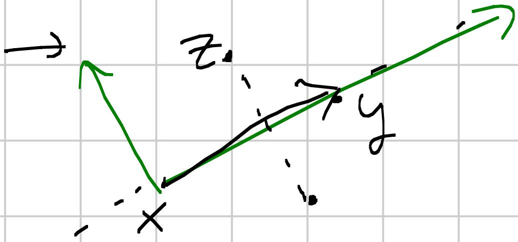
numero complesso z



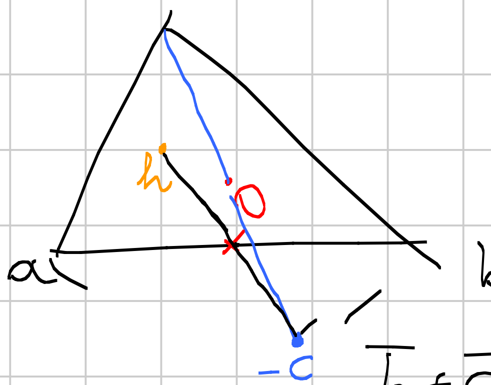
$$z = a+ib$$
$$\bar z = a-ib$$

$$\operatorname{Re} z = \frac{z+\bar z}{2} \qquad |z|^2 = z\bar z$$

$$\operatorname{Im} z = \frac{z-\bar z}{2}$$



$$\frac{1}{2}z + \frac{1}{2}\left[\overline{\left(\frac{z-x}{y-x}\right)}(y-x)+x\right]$$



$h = a+b+c$

$a\bar a = b\bar b = c\bar c = 1$

$$\overline{\left(\frac{\cancel{a}+b+c-\cancel{a}}{b-a}\right)}\cdot(b-a)+a$$

$$= \frac{\bar b+\bar c}{\bar b-\bar a}(b-a)+a =$$

$$= \frac{\cancel{b\bar b}+\bar c b-\cancel{a\bar b}-a\bar c+\cancel{a\bar b}-\cancel{a\bar a}}{\bar b-\bar a}$$

$$= \bar c\,\frac{b-a}{\overline{b-a}}$$ ← ha modulo 1!

pto medio $\frac{a+b}{2}$ $\quad (-1)\left(a+b+c-\frac{a+b}{2}\right)+\frac{a+b}{2} =$

$= -c$

- Similitudini

se nel primo verso

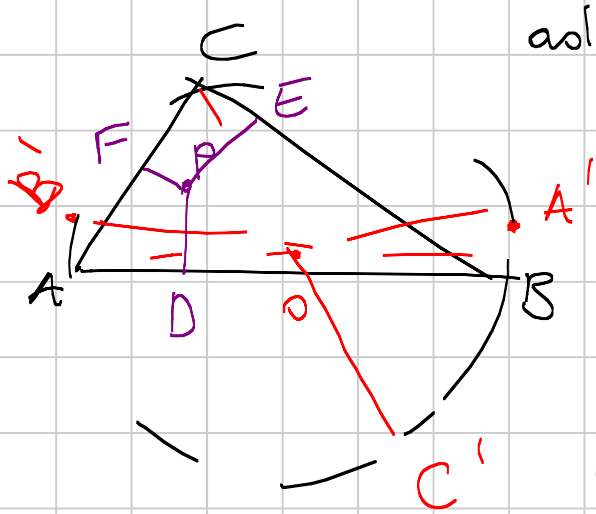
$$\frac{c-a}{b-a} = \frac{z-x}{y-x}$$

$$= \frac{\bar z-\bar x}{\bar y-\bar x}$$

← se riflesso

triangolo ABC

A', B', C' diam. opposti ad A, B, C, Γ



D, E, F proiezioni di un P sui lati.

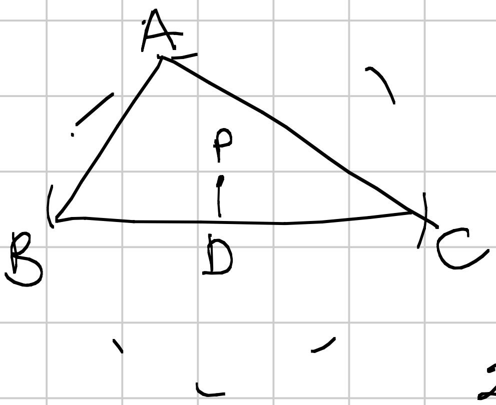
X, Y, Z simmetrici di C', A', B' risp a D, E, F

TESI: $\widehat{XYZ} \sim \widehat{ABC}$

Problemi: 1, 2, 3 "analitica"

$\boxed{6,16}$ ← + difficili e "importanti": fateli prima o poi!!

$\boxed{10,8,9}$ ← + facili, di riscaldamento



$$\frac{1}{2}\left(\overline{\frac{p-b}{c-b}}\right)(c-b)+\frac{b}{2}+\frac{p}{2} = d$$

$$2d = \frac{\bar p c - \bar b c - \bar p b + \cancel{\bar b b} + b\bar c - \cancel{b\bar b}}{\bar c - \bar b}$$

$\bar c = 1/c$

$\bar b = 1/b$

$$= \frac{bc\left(\bar p c - c/b - \bar p b + b/c + p/c - p/b\right)}{b-c} =$$

$+p\bar c - p\bar b$

$$= \frac{bc\left(\bar p(c-b) + p\,\frac{b-c}{bc} + \frac{b^2-c^2}{bc}\right)}{b-c} =$$

$$= bc(-\bar{p}) + p + b + c = p + b + c - bc\bar{p}$$

$$2d = p + b + c - bc\bar{p}$$

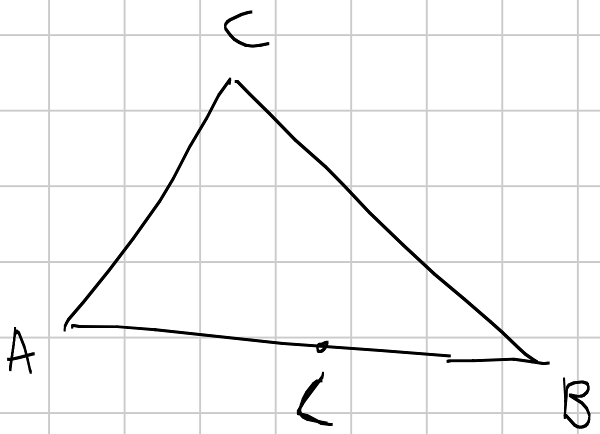
$$d = \frac{x-a}{2} \iff 2d = x - a$$

$$x = 2d + a =$$

$$= a + b + c - bc\bar{p} + p$$

$$\frac{y-x}{z-x} = \frac{-ab\bar{p} + bc\bar{p}}{-ac\bar{p} + bc\bar{p}} = \frac{b(c-a)}{c(b-a)} =$$

$$= \frac{c}{b}\,\frac{\frac{a-c}{ac}}{\frac{a-b}{ab}} = \frac{a-c}{a-b}$$



$$\vec{L} = \frac{\vec{A} + \vec{B}}{2}$$

$$\langle \vec{L} - \vec{C}, \vec{L} - \vec{C} \rangle =$$

$$= \left\langle \frac{\vec{A}}{2} + \frac{\vec{B}}{2} - \vec{C}, \frac{\vec{A}}{2} + \frac{\vec{B}}{2} - \vec{C} \right\rangle$$

$$= \frac{R^2}{4} + \frac{R^2}{4} + R^2 + \frac{|A||B|}{2}\cos 2\gamma - |A||C|\cos 2\beta - |B||C|\cos 2\alpha$$

$$= \cancel{\frac{R^2}{2}} + \cancel{R^2} + \cancel{\frac{R^2}{2}} - \cancel{2R^2} + \cancel{2}\frac{R^2}{\cancel{2}}\,\mathrm{sen}^2\gamma - 2R^2\,\mathrm{sen}^2\beta - 2R^2\,\mathrm{sen}^2\alpha$$

$$= \frac{c^2}{4} - \frac{b^2}{2} - \frac{a^2}{2}$$