G1 Basic

Danilo

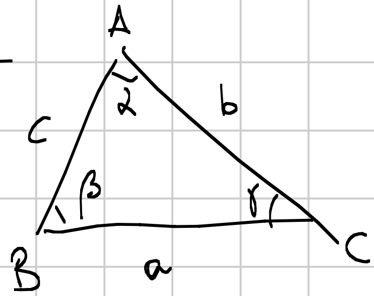
Note Title 9/2/2016

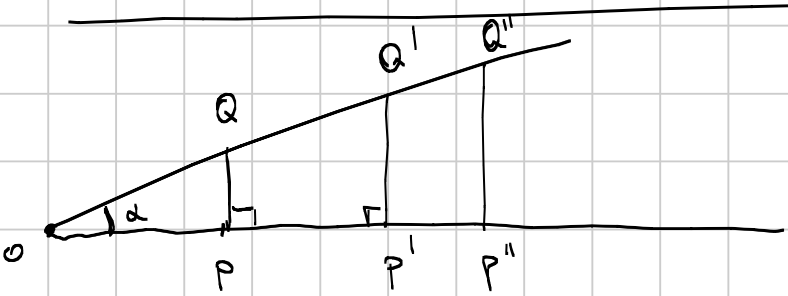
G1 - Trigonometria

G2 - Metodi analitici

G3 - Sintetica

Notazione

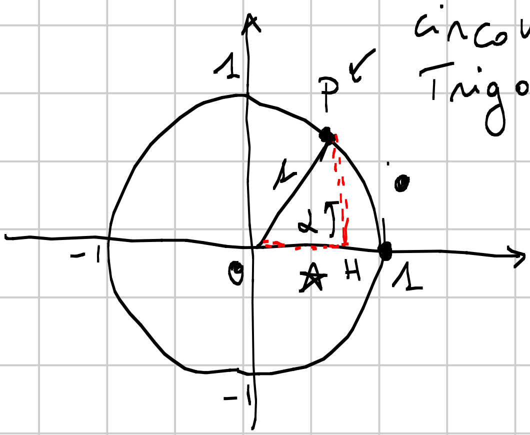




$$\frac{OP}{OQ} = \frac{OP'}{OQ'} = \bigstar$$

$$\frac{QP}{OQ} = \frac{Q'P'}{OQ'} = \bullet$$

circonferenza Trigonometrica



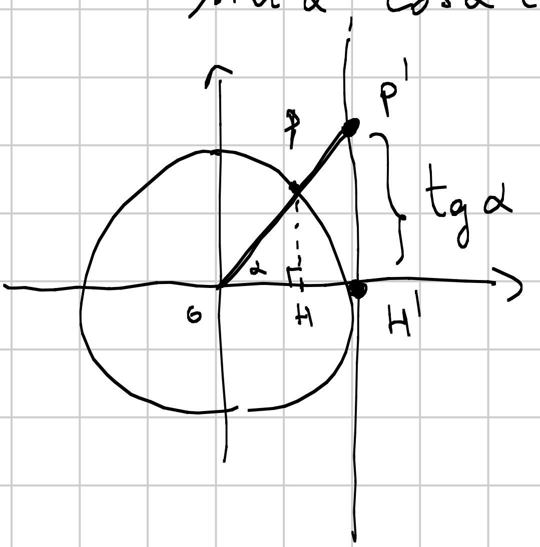
$\bigstar = \cos\alpha$

$\bullet = \operatorname{sen}\alpha$

$$\overline{OP}^2 = 1^2 = \overline{OH}^2 + \overline{PH}^2 = \cos^2\alpha + \sin^2\alpha$$

$$1 = \cos^2\alpha + \sin^2\alpha$$

$\sin\alpha, \cos\alpha \in [-1, 1]$



$$\operatorname{tg}\alpha = \frac{\overline{P'H'}}{\overline{OH'}} = \frac{\overline{PH}}{\overline{OH}} = \frac{\sin\alpha}{\cos\alpha}$$

Chiamiamo: "α che?"

$$\frac{\alpha}{360^\circ} = \frac{\alpha_{rad}}{2\pi}$$

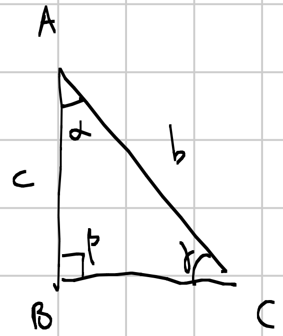
$2\pi \longleftrightarrow 360$

$\pi \longleftrightarrow 180$

$\frac{\pi}{2} \longleftrightarrow 90$

$\pi/3 \longleftrightarrow 60$

...



$\beta = 90^\circ = \pi/2$

$c = b \cos\alpha$

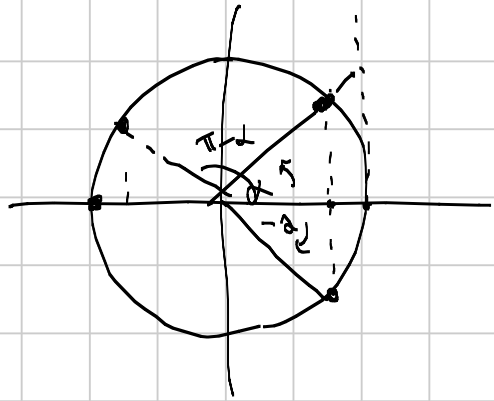
$a = b \sin\alpha = b \cos\gamma$

ma $\beta = 90^\circ \Rightarrow \gamma = 90 - \alpha$

$b \sin\alpha = b\cos\gamma = b \cos(90-\alpha)$

$\Longrightarrow \sin\alpha = \cos(90-\alpha)$

"A volte ritornano"



$\sin(2\pi+\alpha) = \sin(\alpha)$

$\cos(2\pi+\alpha) = \cos(\alpha)$

$\sin(\alpha) = \cos(90-\alpha)$

$\beta = \alpha + 90 \longleftrightarrow \beta - 90 = \alpha$

$\sin(\beta-90) = \cos(\beta)$

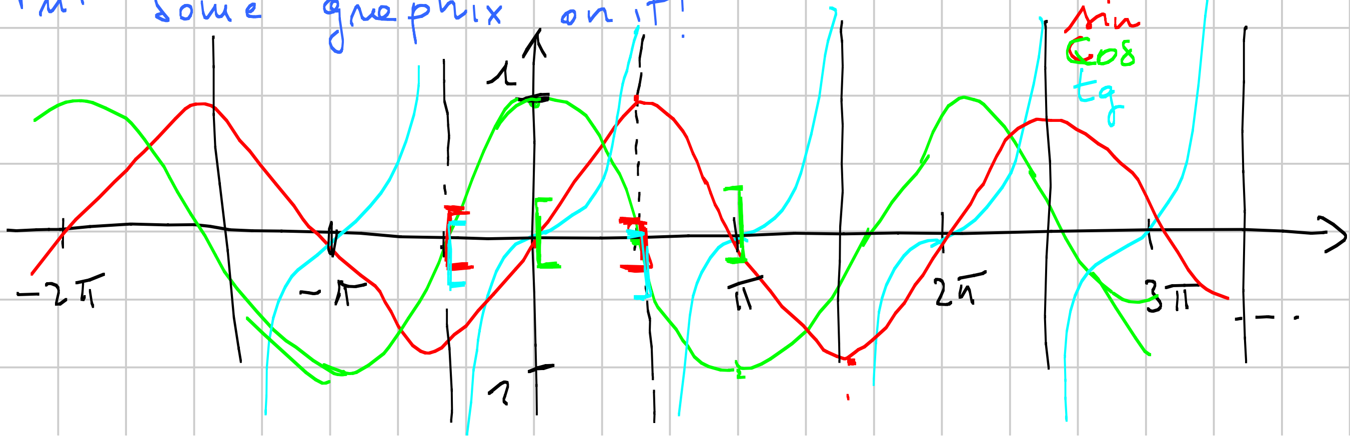
$\sin(-\alpha) = -\sin\alpha$ ← funzioni dispari

$\cos(-\alpha) = \cos\alpha$ ← funzioni pari

$\cos(\pi-\alpha) = -\cos\alpha$

$\sin(\pi-\alpha) = \sin\alpha$

"Put some graphix on it!"



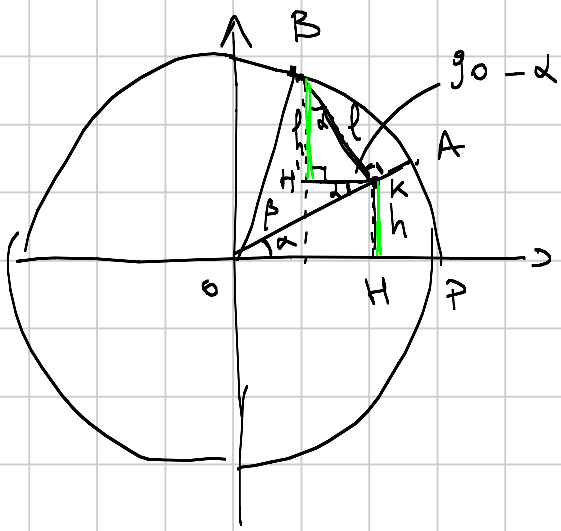
Cerchiamo quando sono iniettive, cioè dove vale " $f(x) = f(y) \iff x = y$ ".

$\sin$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightsquigarrow \arcsin : [-1,1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos$ in $[0, \pi] \rightsquigarrow \arccos : [-1,1] \to [0,\pi]$

tg in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightsquigarrow \mathrm{arctg} : \mathbb{R} \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

"Abracadabra"



$\sin(\alpha+\beta) = \cancel{\sin\alpha + \sin\beta}$

$\sin(\alpha+\beta) = h + h'$

$\ell = \underbrace{\overline{OB}}_{1} \sin\beta$

$h' = \ell \cos\alpha = \sin\beta\cos\alpha$

$h = \sin\alpha\cos\beta$

$\Rightarrow \quad \sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$

$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

se $\alpha=\beta$

f di duplicaz.

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos^2\alpha + (\sin^2\alpha - \sin^2\alpha) - \sin^2\alpha =$

$= 1 - 2\sin^2\alpha =$

$= 2\cos^2\alpha - 1$

f di bisez.

$\sin\alpha = \pm\sqrt{\frac{1-\cos 2\alpha}{2}} \rightsquigarrow \sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos\alpha}{2}}$

$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

$$tg(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} =$$

$$= \frac{\underbrace{\frac{\sin\alpha\,\cancel{\cos\beta}}{\cos\alpha\,\cancel{\cos\beta}}}_{tg\alpha} + \underbrace{\frac{\sin\beta\,\cancel{\cos\alpha}}{\cancel{\cos\alpha}\,\cos\beta}}_{tg\beta}}{1 - \underbrace{\frac{\sin\alpha}{\cos\alpha}}_{tg\alpha}\underbrace{\frac{\sin\beta}{\cos\beta}}_{tg\beta}} = \frac{tg(\alpha)+tg(\beta)}{1 - tg\alpha\, tg\beta}$$

$$\boxed{tg(2\alpha) = \frac{2tg(\alpha)}{1-tg^2(\alpha)}}$$

Prostaferesi, Werner, parametriche

pr $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}$

w $\sin\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

par

$$t = tg\frac{\alpha}{2}$$

$$tg\,\alpha = \frac{2t}{1-t^2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1}{\frac{1}{\cos^2\alpha/2}} = \frac{1}{\frac{\sin^2\alpha/2 + \cos^2\alpha/2}{\cos^2\alpha/2}} = \frac{1}{t^2+1}$$

$$\cos\alpha = 2\cos^2\alpha/2 - 1 = \frac{2}{t^2+1} - 1 = \frac{1-t^2}{1+t^2}$$

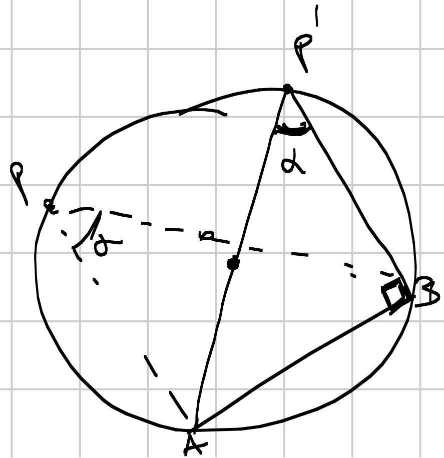
$(1+t^2) > 0$ sempre 4 evah

$$\sin\alpha = \cos\alpha\, tg\,\alpha = \frac{2t}{t^2+1}$$

Pasticci sui triangoli

Teorema (dei seni)

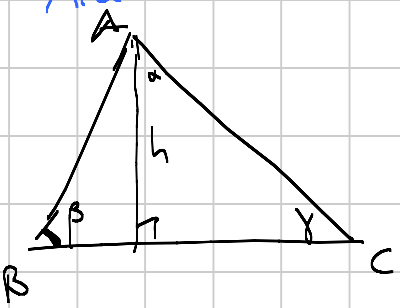
$$\overline{AB} = 2R\sin\alpha$$



$\exists !$ $$\boxed{2R = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

□

Area



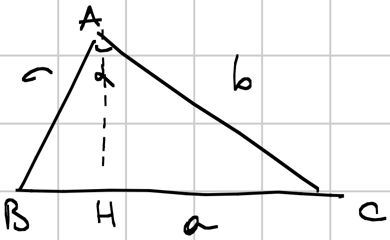
$$A_{ABC} = \frac{1}{2} b h = \frac{1}{2} ac \sin\beta =$$

$$\sin\beta = \frac{b}{2R}$$

$$\Rightarrow A_{ABC} = \frac{1}{2} ac\sin\beta = \frac{abc}{4R}$$

□

Teorema (di Carnot o dei coseni)



$$\boxed{a^2 = b^2 + c^2 - 2bc\cos\alpha}$$

Dim $b^2+c^2-a^2 = (AH^2 + CH^2) + (BH^2 + AH^2) - (BH+CH)^2 =$

$$= 2AH^2 + \cancel{CH^2} + \cancel{BH^2} - \cancel{CH^2} - \cancel{BH^2} - 2BH\,CH =$$

$$= 2\,AH\,AH - 2\,BH\,CH =$$

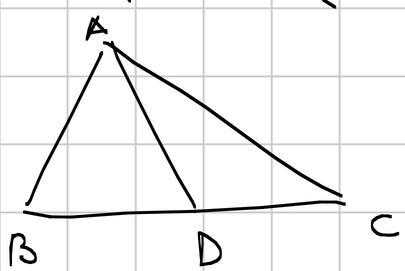
$$= 2\,c\sin\beta\, b\sin\gamma - 2\,c\cos\beta\, b\cos\gamma = -2bc(\ldots ? \ldots)$$

$$= -2bc\cos(\beta+\gamma) \underset{\hat{\Delta}}{=} -2bc\cos(\pi-\alpha) = 2bc\cos\alpha$$

□

Esercizi

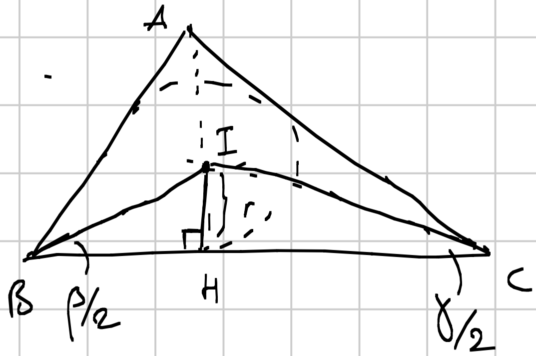
- Teorema (di Stewart)



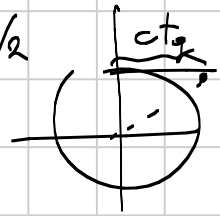
$$a(BD\,CD + AD^2) = b^2 BD + c^2 CD$$

Hint: Carnot su $\widehat{ABD}$ e $\widehat{ADC}$

- Raggio C. inscritta



$$\frac{IH}{BH} = tg\,\beta/2 \qquad \frac{IH}{CH} = tg\,\gamma/2$$



$$r = IH$$

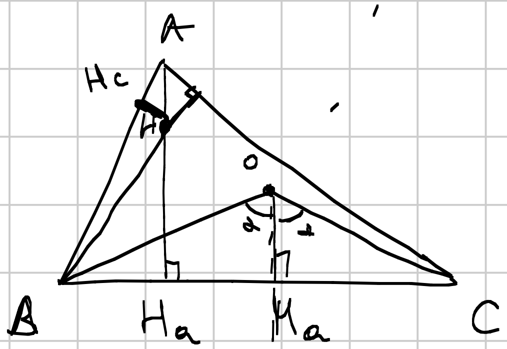
$$\frac{BH}{IH} = \frac{1}{tg\,\beta/2} = \frac{\cos\beta/2}{\sin\beta/2} = ctg\,\beta/2$$

$$\frac{a}{r} = \frac{BH + CH}{r} = ctg\,\beta/2 + ctg\,\frac{\gamma}{2}$$

$$\Rightarrow \quad r = \frac{a}{ctg\,\beta/2 + ctg\,\frac{\gamma}{2}}$$

esiste?

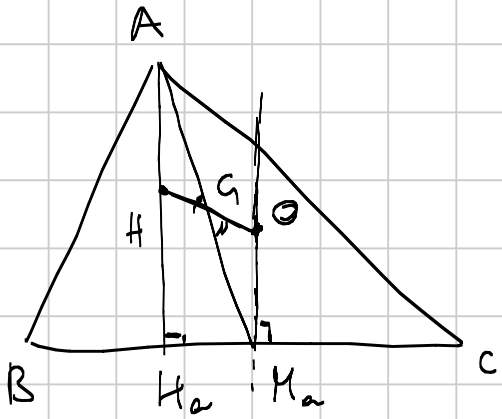
- Ortocentro, Baricentro e circocentro sono allineati



$$\overline{OM_a} = OB \cos\alpha = R\cos\alpha$$

$$\overline{AH} = \frac{AH_c}{\sin\beta} = \frac{b\cos\alpha}{\sin\beta} = 2R\cos\alpha$$

$$\Rightarrow 2OM_a = AH$$



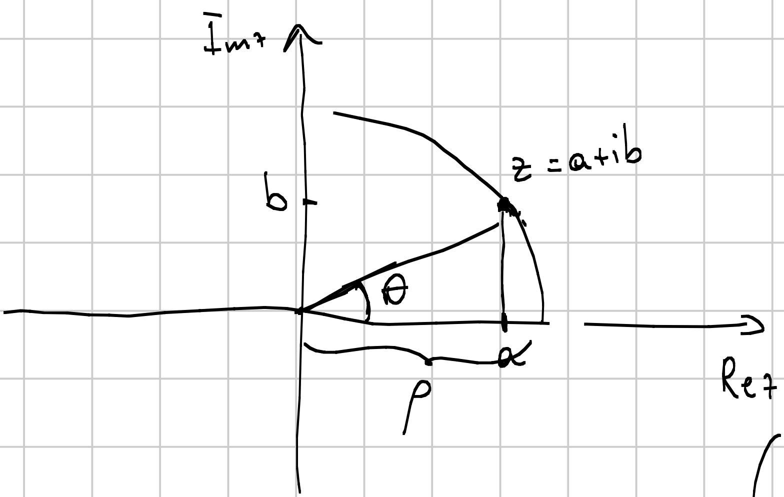
$$\overline{AG} = 2\,\overline{GM_a}$$ è proprio il baricentro!

□

NUMERI COMPLESSI

$$\mathbb{C} = \{ a + ib \mid a, b \in \mathbb{R},\ i^2 = -1 \}$$

$$z = \underset{\substack{\uparrow \\ \mathrm{Re}\,z}}{a} + i\underset{\substack{\uparrow \\ \mathrm{Im}\,z}}{b}$$



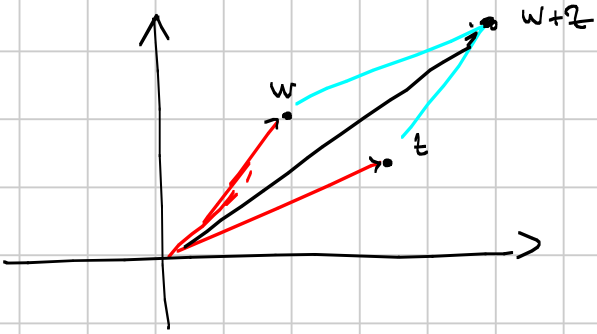
$$z = \rho\cos\theta + i\rho\sin\theta =$$
$$= \underline{\rho(\cos\theta + i\sin\theta)}$$

forma polare

$$(\rho e^{i\theta}) \leftarrow$$

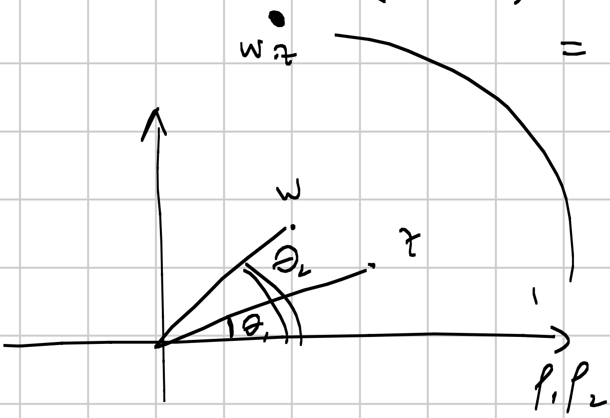
Operazioni

SOMMA : $w + z = (a + ib) + (c + id) = (a+c) + i(b+d)$



PRODOTTO

$$w \cdot z = (a+ib)(c+id) = ac + aid + ibc + (-1)bd =$$
$$= (ac - bd) + i(ad + bc)$$



$$w \cdot z = \rho_1(\cos\theta_1 + i\sin\theta_1)\rho_2(\cos\theta_2 + i\sin\theta_2) =$$
$$= \rho_1\rho_2\big(\underline{\cos\theta_1\cos\theta_2} + i\,\underline{\sin\theta_2\cos\theta_1} +$$
$$+ i\,\underline{\sin\theta_1\cos\theta_2} + \underline{(-1)\sin\theta_1\sin\theta_2}\big)$$

$$= \rho_1\rho_2\big(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\big)$$

(θ si chiama argomento
ρ si chiama modulo)

$$zw = \rho_1 \rho_2 \left(\cos \quad - \quad \right) \overset{?}{=} \rho_1 \rho_2 \, e^{i(\theta_1+\theta_2)}$$

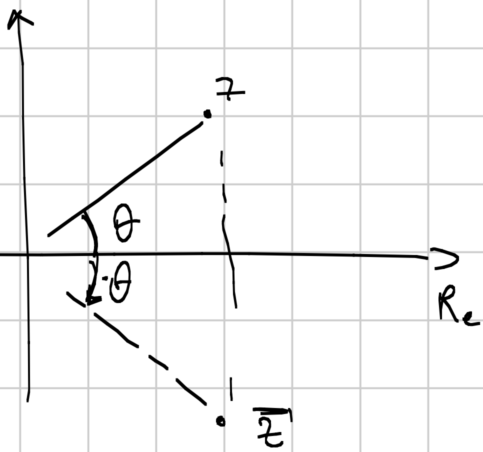
sì

e ci piace

- coniugio

$z = a + ib$

$\bar{z} = a - ib$



$$\bar{z} = \rho\left(\cos\theta + i\sin(-\theta)\right) = \rho\left(\cos\theta - i\sin\theta\right)$$

Come calcoliamo il modulo?

$$z\bar{z} = (a+ib)(a-ib) = a^2+b^2 = \rho^2\cos^2\theta + \rho^2\sin^2\theta = \rho^2\left(\sin^2\theta + \cos^2\theta\right) = \rho^2 = |z|^2$$

Pag 3 n° 1, 2, 4, 7, 11

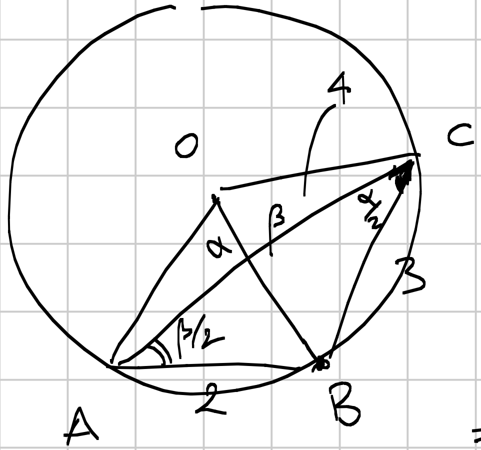
Pag 32 n° 1, 4, 8, 9

CORREZIONE

1 $$\sum_{n=0}^{90} \sin^2(n) = \sum_{n=0}^{44}\Big[\underbrace{\sin^2(90-n)}_{\cos^2 n} + \sin^2(n)\Big] + \sin^2(45)$$

$$= 45 + \frac{1}{2}$$

④



Teorema di Carnot su $\widehat{ABC}$

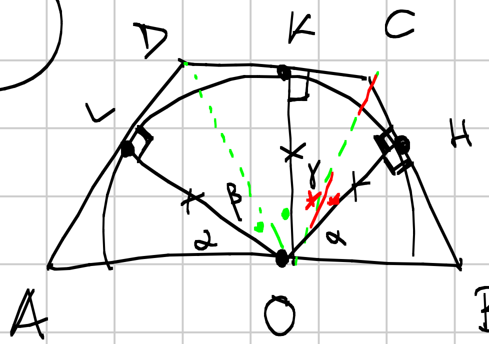
$$2^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cos \alpha/2$$

$$\Rightarrow \cos \frac{\alpha}{2} = \frac{7}{8}$$

$$\Rightarrow \cos \alpha = 2 \cos^2 \frac{\alpha}{2} - 1 = 2 \cdot \frac{7^2}{8^2} - 1$$

$$= \frac{49 - 32}{32} = \frac{17}{32}$$

⑨



$$AB^2 = 4\, BC \cdot AD$$

$$AB = AO + OB$$

$$BC = CH + HB$$

$$AD = AL + LD$$

$$\left.\begin{aligned} CH &= \frac{r \sin\gamma}{\cos\gamma} = r \tan\gamma \\ HB &= AL = r \tan\alpha \\ LD &= r \tan\beta \\ AO &= OB = \frac{r}{\cos\alpha} \end{aligned}\right\} \Rightarrow \begin{aligned} AB &= \frac{2r}{\cos\alpha} \\ BC &= r(\tan\alpha + \tan\gamma) \\ AD &= r(\tan\alpha + \tan\beta) \end{aligned}$$

$$\frac{\cancel{4}\cancel{r^2}}{\cos^2\alpha} = \cancel{4}\cancel{r^2}(\tan\alpha + \tan\beta)(\tan\alpha + \tan\gamma)$$

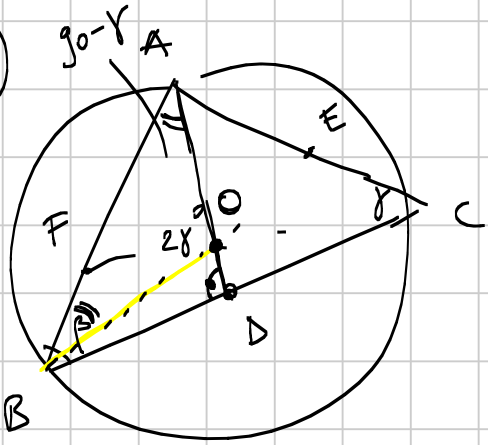
$$\frac{1}{\cos^2\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\cos^2\alpha} = \tan^2\alpha + 1$$

$$\left[\cancel{tg^2}\alpha+1 = \cancel{tg^2\alpha} + tg\,\alpha\, tg\,\beta + tg\,\alpha\, tg\,\gamma + tg\,\beta\, tg\,\gamma\right]$$

$$\alpha+\beta+\gamma = 90^\circ \qquad [\text{es } 13 \text{ p.} 33\, !]$$

$\square$

(8)



$$\frac{1}{AD}+\frac{1}{BE}+\frac{1}{CF}=\frac{2}{AO}$$

$)\!)$ vale $\dfrac{180-2\gamma}{2} = 90-\gamma$

$\bullet$ vale $180-(90-\gamma)-\beta = 90-\beta+\gamma$

Uso teorema dei seni

$$\frac{AD}{\sin\beta} = \frac{AB}{\sin(\bullet)} = \frac{AB}{\sin(90-\beta+\gamma)} = \frac{AB}{\cos(\beta-\gamma)} = \frac{2R\sin\gamma}{\cos(\beta-\gamma)}$$

$$AD = \frac{2R\sin\gamma\sin\beta}{\cos(\beta-\gamma)}$$

gli altri due si trovano allo stesso modo

$$\frac{\cos(\beta-\gamma)}{2\cancel{R}\sin\beta\sin\gamma}+\frac{\cos(\alpha-\beta)}{2\cancel{R}\sin\alpha\sin\beta}+\frac{\cos(\gamma-\alpha)}{2\cancel{R}\sin\alpha\sin\gamma}=\frac{2}{\cancel{R}}$$

$$\frac{\sin\alpha\cos(\beta-\gamma)+\sin\beta\cos(\alpha-\gamma)+\sin\gamma\cos(\beta-\alpha)}{\sin\alpha\sin\beta\sin\gamma}=4$$

$$\frac{\sin\alpha\cos\beta\cos\gamma+\sin\alpha\sin\beta\sin\gamma+\underline{\qquad}}{\cos\alpha\cos\beta\cos\gamma}=\frac{4\sin\alpha\sin\beta\sin\gamma}{\cos\alpha\cos\beta\cos\gamma}$$

$$\frac{\sin\alpha}{\cos\alpha} + \cancel{\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma} + \operatorname{tg}\beta + \cancel{\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma} +$$

$$+ \operatorname{tg}\gamma + \cancel{\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma} = \cancel{4}\,1\; \operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma$$

$$\operatorname{tg}\alpha\operatorname{tg}\beta\operatorname{tg}\gamma = \operatorname{tg}\alpha + \operatorname{tg}\beta + \operatorname{tg}\gamma$$

$\alpha+\beta+\gamma = 180^\circ$ [Es 11. p3 !]

$\square$

BUON PRANZO!