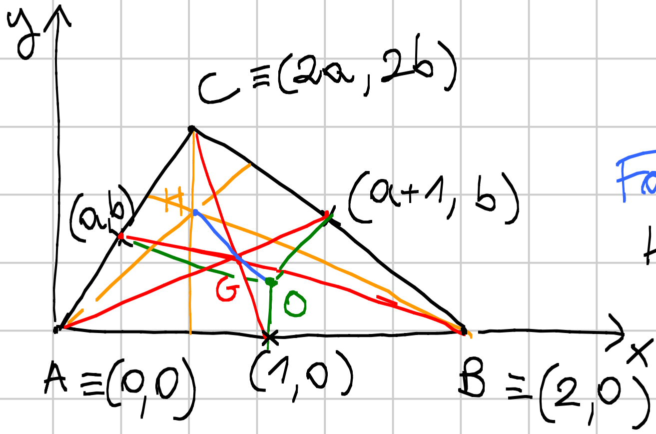
Geometria 2

4



Fatto (Eulero)
H, G, O allineati

calcolo $G \equiv \left(\frac{2}{3}(1+a), \frac{2}{3}b\right)$

calcolo H

sta su h_C $\quad x = 2a$

sta su h_A $\quad y = \frac{1-a}{b}x$

$\left(2a, \frac{2a-2a^2}{b}\right)$

coeff. ang. di CB $\frac{2b}{2a-2} = \frac{b}{a-1}$

calcolo O

sta sull'asse di AB : $x = 1$

sta sull'asse di AC : $y = -\frac{a}{b}x + \frac{a^2}{b} + b$

$\left(1, -\frac{a}{b} + \frac{a^2}{b} + b\right)$

coeff. ang. di HG

$$\frac{\frac{1}{b}\left(2a - 2a^2 - \frac{2}{3}b^2\right)}{2a - \frac{2}{3} - \frac{2}{3}a} = \frac{\frac{2}{b}\left(a - a^2 - \frac{1}{3}b^2\right)}{\frac{2}{3}(2a-1)}$$

$\overset{?}{=}$

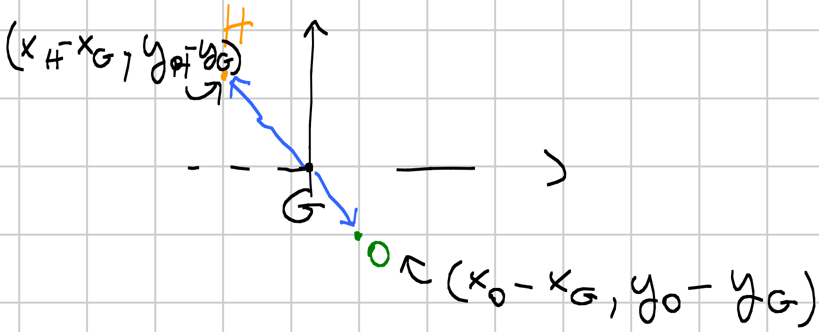
coeff. ang. di OG

$$\frac{\frac{1}{b}\left(-a + a^2 + b^2 - \frac{2}{3}b^2\right)}{1 - \frac{2}{3} - \frac{2}{3}a} = \frac{-\frac{1}{b}\left(a - a^2 - \frac{1}{3}b^2\right)}{-\frac{1}{3}(2a-1)}$$

= YES!

BONUS :

$$y_H - y_G = -2(y_O - y_G)$$
$$x_H - x_G = -2(x_O - x_G)$$

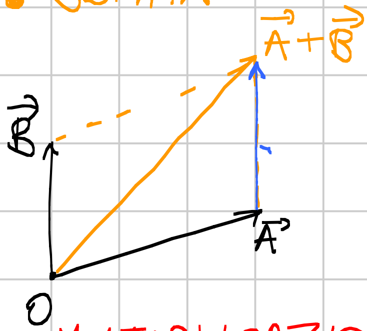


$\rightarrow$ $\boxed{\overline{HG} = 2\overline{GO} \, !}$

VETTORI

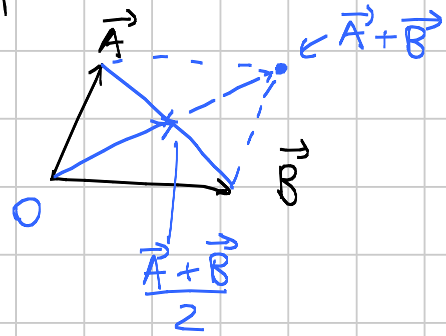
cosa posso fare con vettori?

- SOMMA

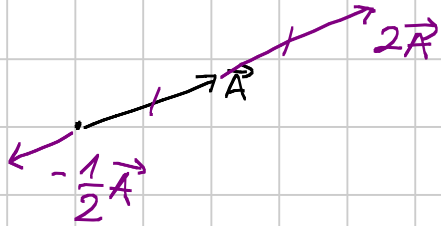


Si scrivono bene:

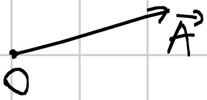
- pto medio di AB



- MOLTIPLICAZIONE PER "SCALARI"

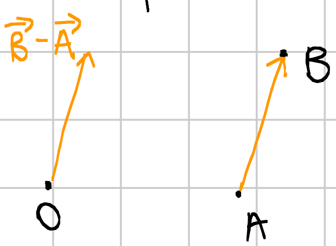


- retta per O e A



$\lambda \vec{A}$ ← numero reale

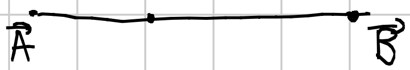
- retta per A e B



$$\lambda(\vec{B}-\vec{A}) + \vec{A} = \lambda\vec{B} + (1-\lambda)\vec{A}$$

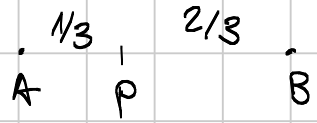
- segmento AB $\quad \lambda \in [0,1]$

$$\vec{p} = \lambda \vec{B} + (1-\lambda)\vec{A}$$



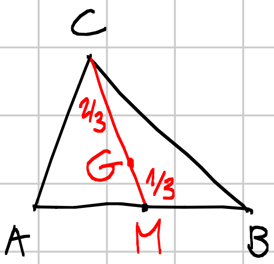
$$\frac{\overline{AP}}{\overline{AB}} = \lambda$$

esempio



$$\vec{p} = \frac{1}{3}\vec{B} + \frac{2}{3}\vec{A}$$

Baricentro di ABC



$$\vec{M} = \frac{\vec{A}+\vec{B}}{2} \qquad \vec{G} = \frac{2}{3}\vec{M} + \frac{1}{3}\vec{C} =$$

$$= \frac{1}{3}\vec{A} + \frac{1}{3}\vec{B} + \frac{1}{3}\vec{C}$$

Ortocentro di ABC

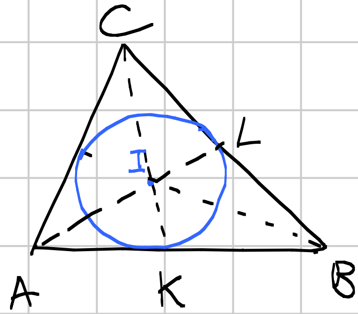
$$\vec{H} - \vec{G} = -2(\vec{O} - \vec{G})$$

$$\vec{H} = 3\vec{G} - 2\vec{O} = \vec{A} + \vec{B} + \vec{C} - 2\vec{O}$$

se metto l'origine in $\vec{O}$

$$\rightsquigarrow \vec{H} = \vec{A} + \vec{B} + \vec{C}$$

Incentro di ABC



$$\frac{\overline{AK}}{\overline{KB}} = \frac{\overline{AC}}{\overline{CB}} = \frac{b}{a} \qquad \frac{\overline{AK}}{\overline{AB}} = \frac{b}{a+b}$$

$$\vec{K} = \frac{b}{a+b}\vec{B} + \frac{a}{a+b}\vec{A}$$

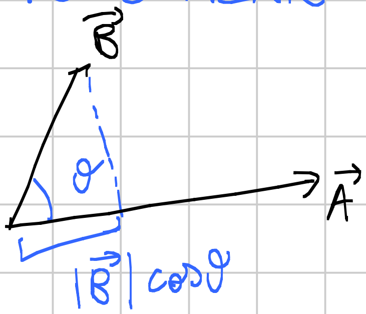
$$\frac{\overline{CI}}{\overline{CK}} = \frac{\overline{AC}}{\overline{AC}+\overline{AK}} = \frac{b}{b+\overline{AK}} =$$

$$= \frac{b}{b + \frac{cb}{a+b}} =$$

$$= \frac{(a+b)\cancel{b}}{c\cancel{b} + a\cancel{b} + b^{\cancel{2}}}$$

$$\vec{I} = \vec{K} \cdot \frac{a+b}{a+b+c} + \vec{C} \cdot \frac{c}{a+b+c} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$$

• PRODOTTO SCALARE



$$\langle \vec{A}, \vec{B} \rangle = |\vec{A}||\vec{B}| \cdot \cos\vartheta$$

$$\langle \vec{A}+\vec{B}, \vec{C} \rangle = \langle \vec{A}, \vec{C} \rangle + \langle \vec{B}, \vec{C} \rangle$$

$$\langle \lambda\vec{A}, \vec{B} \rangle = \lambda \langle \vec{A}, \vec{B} \rangle$$

$$\vec{A} \parallel \vec{B} \quad \langle \vec{A}, \vec{B} \rangle = |\vec{A}||\vec{B}|$$

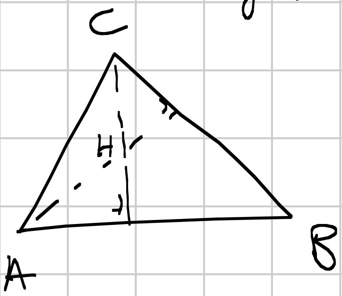
$$\vec{A} \perp \vec{B} \quad \langle \vec{A}, \vec{B} \rangle = 0$$

(nota: $\langle \vec{A}, \vec{A} \rangle = |\vec{A}|^2$)

in coordinate: $\langle \vec{A}, \vec{B} \rangle = x_A x_B + y_A y_B$

esempio: $\vec{H} = \vec{A} + \vec{B} + \vec{C}$ ... è vero?

(origine in $\vec{O}$)



$$\vec{H}-\vec{C} \overset{?}{\perp} \vec{B}-\vec{A}$$

$$\langle \vec{H}-\vec{C}, \vec{B}-\vec{A} \rangle =$$

$$= \langle \vec{A}+\vec{B}, \vec{B}-\vec{A} \rangle =$$

$$= \cancel{\langle \vec{A}, \vec{B} \rangle} + \langle \vec{B}, \vec{B} \rangle - \langle \vec{A}, \vec{A} \rangle - \cancel{\langle \vec{A}, \vec{B} \rangle}$$

$$= R^2 - R^2 = 0$$

altro esempio: $\overline{OI}$ in funzione di R e r

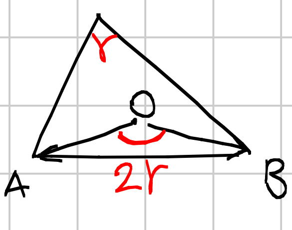
mettiamo origine in O

$$\overline{OI}^2 = \langle \vec{I}, \vec{I} \rangle = \left(\frac{1}{a+b+c}\right)^2 \langle a\vec{A}+b\vec{B}+c\vec{C}, \quad " \quad \rangle$$

$$= \left(\frac{1}{2p}\right)^2 \left(\sum_{cyc} a^2 \langle \vec{A}, \vec{A} \rangle + \sum_{cyc} 2ab \langle \vec{A}, \vec{B} \rangle \right) =$$

$$= \left(\frac{1}{2p}\right)^2 \left[R^2(a^2+b^2+c^2) + \sum_{cyc} 2ab R^2 \underbrace{\cos 2\gamma} \right] =$$

$\cos^2\gamma - \operatorname{sen}^2\gamma =$
$= 1 - 2\operatorname{sen}^2\gamma$



$$= \left(\frac{1}{2p}\right)^2 \left[R^2(a^2+b^2+c^2) + \sum_{cyc} 2abR^2 - 4\sum_{cyc} ab R^2 \operatorname{sen}^2\gamma \right]$$

$$= \frac{1}{(2p)^2}\left[R^2(a+b+c)^2 - 4\sum_{cyc} R\frac{c}{\cancel{2}}\cancel{2}S\right] =$$

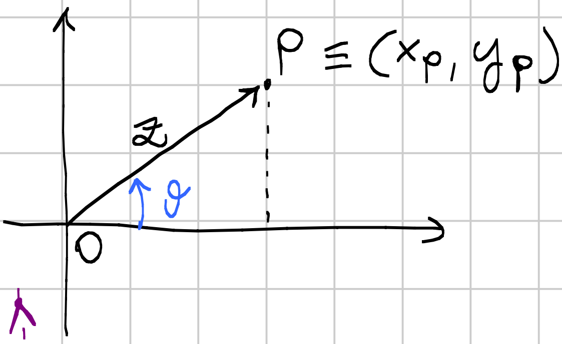
$$= \left(\frac{1}{2p}\right)^2\left[R^2(a+b+c)^2 - 4RS(a+b+c)\right]$$

$$= R^2 - \frac{\overset{2}{\cancel{4}}RS}{\cancel{2}p} = R^2 - 2Rr$$

BONUS: $R^2 - 2Rr \geq 0 \to R(R-2r) \geq 0$

$R \geq 2r$

Complessi

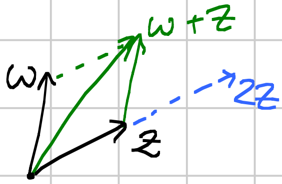


$z = x_p + iy_p =$

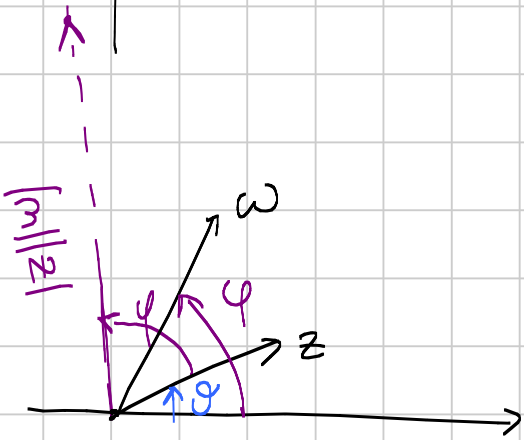
$= |z|(\cos\vartheta + i\,\text{sen}\,\vartheta)$

$= |z|e^{i\vartheta}$

- si sommano

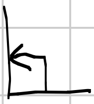


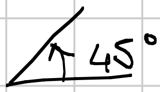
- si moltiplicano
(per numeri reali
E per altri complessi)



$$zw = |z||w|\,e^{i\vartheta}e^{i\varphi} = |z||w|\,e^{i(\vartheta+\varphi)}$$

Rotazioni di angoli particolari si scrivono estremamente bene:

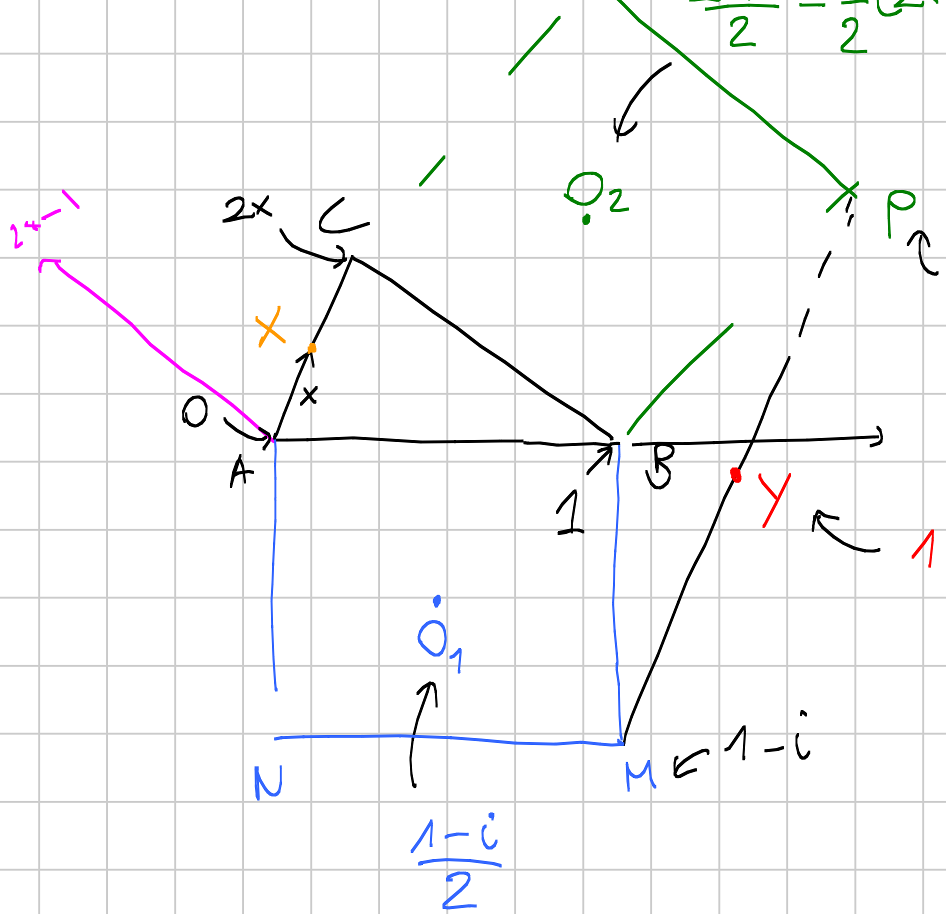
 → moltiplico per i



→ per $\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

...

Problema 10



$Q = \frac{2x+1}{2} - \frac{i}{2}(2x-1)$

TESI: $O_1 Y O_2 X$ è un quadrato

P: $\frac{2x-1}{i} + 1 = 1 - i(2x-1)$

Y: $1 - i\frac{(2x-1+1)}{2} = 1 - ix$

$O_1 = \frac{1-i}{2}$

$$\vec{XO_2} = \vec{XO_1} \text{ ruotato di } 90°$$

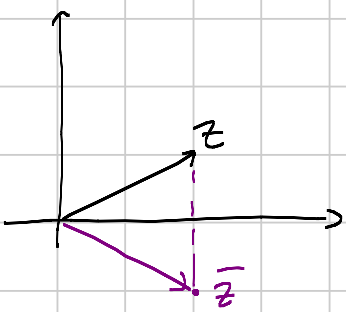
$$\left(\frac{1-i}{2} - x\right) i \overset{?}{=} \frac{2x+1}{2} - \frac{i}{2}(2x-1) - x$$

$$i\frac{1}{2} + \frac{1}{2} - ix = \frac{1}{2} - ix + \frac{i}{2}$$

Ok!

faccio lo stesso con altri due lati del quadrato ...

— ... si "coniugano"

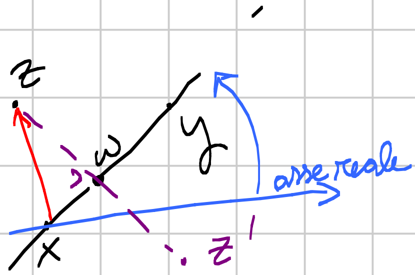


$$z = a + ib$$
$$\bar{z} = a - ib$$

$$\operatorname{Re} z = \frac{z + \bar{z}}{2}$$

$$\operatorname{Im} z = \frac{z - \bar{z}}{2}$$

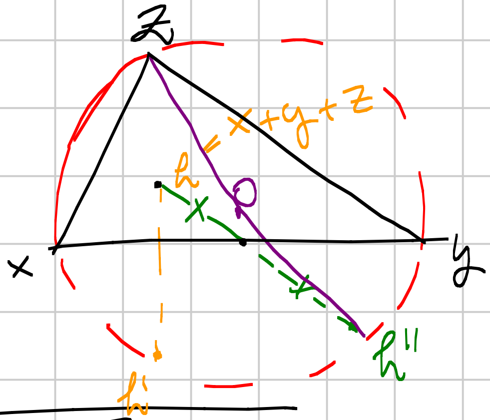
$$z\bar{z} = |z|^2$$



$$\overline{\left[\frac{(z-x)}{(y-x)}\right]}(y-x)+x = z'$$

$$w = \frac{z+z'}{2}$$

Esempio



$$x\bar{x} = y\bar{y} = z\bar{z} = 1$$

$$\overline{\left[\frac{h-x}{y-x}\right]}(y-x)+x =$$

$$= \overline{\left[\frac{x+y+z-x}{y-x}\right]}(y-x)+x =$$

$$= \frac{\bar{y}+\bar{z}}{\bar{y}-\bar{x}}(y-x)+x = \frac{(\bar{y}+\bar{z})(y-x)+x(\bar{y}-\bar{x})}{\bar{y}-\bar{x}}$$

$$= \frac{\cancel{\bar{y}y} + \bar{z}y - \cancel{\bar{y}x} - \bar{z}x + \cancel{x\bar{y}} - \cancel{x\bar{x}}}{\bar{y}-\bar{x}} =$$

$$= \frac{\bar{z}(y-x)}{\overline{(y-x)}}$$ ha modulo 1!

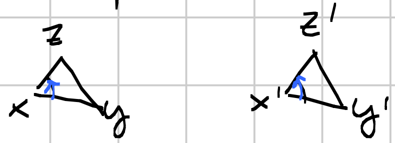
Calcolo h''

$$-\left(h-\frac{x+y}{2}\right)+\frac{x+y}{2} = -x-y-z+x+y =$$
$$= -z$$

importanti!

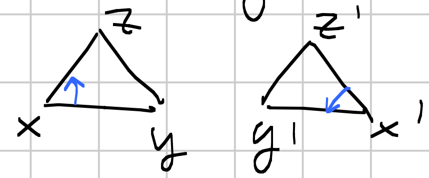
8, 6, 16, 20, 9

Parentesi bonus: come si può scrivere la condizione di similitudine di due triangoli in complessi?



6. $\vec{M} = \dfrac{\vec{A}+\vec{B}}{2}$

$$\frac{z-x}{y-x} = \frac{z'-x'}{y'-x'}$$



$\vec{CM} \qquad \vec{M}-\vec{C} = \dfrac{\vec{A}+\vec{B}-2\vec{C}}{2}$

$$|\vec{CM}|^2 = \left\langle \frac{\vec{A}+\vec{B}-2\vec{C}}{2}, \frac{\vec{A}+\vec{B}-2\vec{C}}{2} \right\rangle$$

origine nel circocentro

$$\rightarrow \frac{1}{4}\left(\langle\vec{A},\vec{A}\rangle + \langle\vec{B},\vec{B}\rangle + 4\langle\vec{C},\vec{C}\rangle + 2\langle\vec{A},\vec{B}\rangle - 4\langle\vec{A},\vec{C}\rangle - 4\langle\vec{B},\vec{C}\rangle\right.$$

$$= \frac{1}{4}\left(6R^2 + 2R^2\cos 2\gamma - 4R^2\cos 2\beta - 4R^2\cos 2\alpha\right) =$$

$$= \frac{1}{4}\left(-2R^2 \operatorname{sen}^2\gamma + 4R^2\operatorname{sen}^2\beta + 4R^2\operatorname{sen}^2\alpha\right) =$$

$$= \frac{1}{4}\left(-\cancel{2}\frac{c^2}{\cancel{2}} + 4\frac{b^2}{2} + 4\frac{a^2}{2}\right) =$$

$$= \frac{a^2}{2} + \frac{b^2}{2} - \frac{c^2}{4}$$

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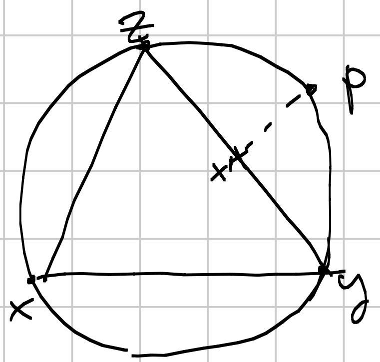

$$x\bar{x} = y\bar{y} = z\bar{z} = 1 = p\bar{p}$$