G1 basic (solo esercizi) Pol senior 17

Note Title 9/4/2017

Es. 3 p.3

$$\sin\left(2\vartheta+\frac{\pi}{4}\right)=\underline{\sin 2\vartheta}\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\underline{\cos 2\vartheta}=$$

$$=\underline{2\sin\vartheta\cos\vartheta}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(\underline{\cos^2\vartheta-\sin^2\vartheta}\right)$$

$$\sin 3\vartheta=\sin(\underline{2\vartheta+\vartheta})=\sin 2\vartheta\cos\vartheta+\sin\vartheta\cos 2\vartheta=$$

$$=2\sin\vartheta\cos\vartheta\cos\vartheta+\sin\vartheta\left(\cos^2\vartheta-\sin^2\vartheta\right)=$$

$$=3\sin\vartheta\cos^2\vartheta-\sin^3\vartheta$$

$$\cos 3\vartheta=\cos^3\vartheta-3\sin^2\vartheta\cos\vartheta$$

$$e^{i\cdot 3\vartheta}=\left(e^{i\vartheta}\right)^3$$

$$\underline{\cos 3\vartheta}+\underline{i}\,\underline{\sin 3\vartheta}=\left(\cos\vartheta+i\sin\vartheta\right)^3=\dots$$

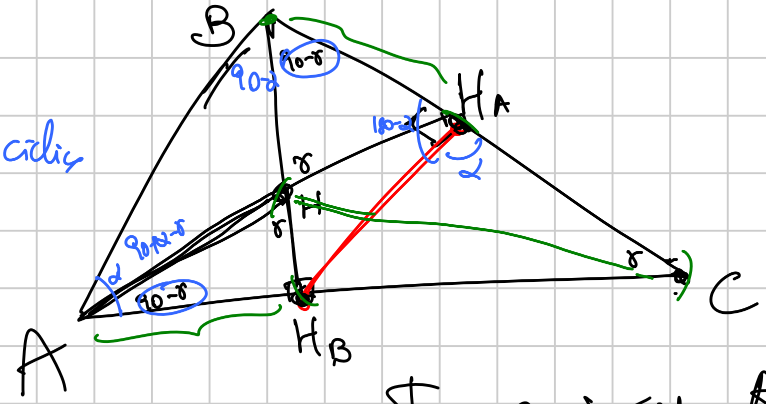
$$\underline{\cos^3\vartheta}+\underline{3i\cos^2\vartheta\sin\vartheta}+\underline{3i^2\cos\vartheta\sin^2\vartheta}$$

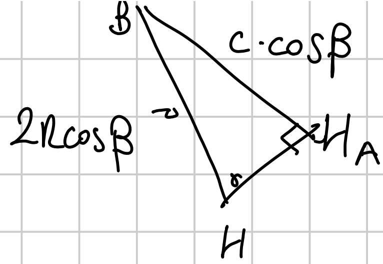
$$+\underline{i^3\sin^3\vartheta}$$

$$\tan 4\vartheta=\frac{\sin(2\vartheta+2\vartheta)}{\cos(2\vartheta+2\vartheta)}=\dots$$

Es. 7 $\boxed{AH}$

$ABH_AH_B$ ciclic




Teo. seni su $AHH_B$:

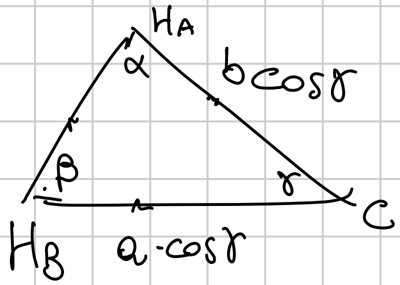
$AH_B = c\cos\alpha$

$$\frac{c\cos\alpha}{\sin\gamma} = \frac{AH}{1}$$

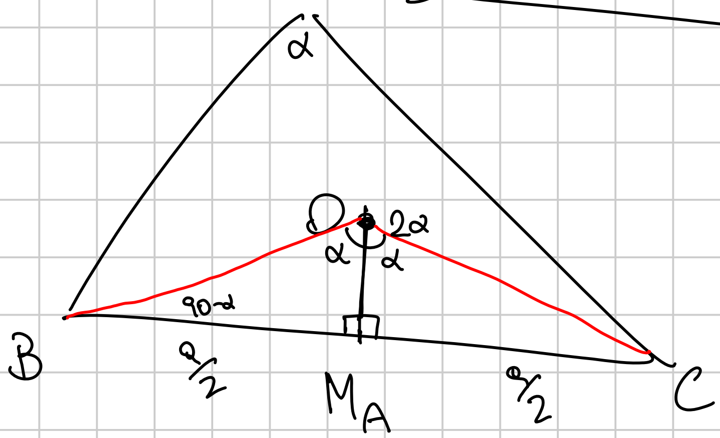
$$AH = \frac{c\cos\alpha}{\sin\gamma} = \frac{b\cos\alpha}{\sin\beta} = \boxed{2R\cos\alpha}$$

$$HH_a = HB\cdot\cos\gamma = 2R\cos\beta\cos\gamma$$ (simmetrico in $\beta,\gamma$)

$H_BH_C$: su $H_AH_BC$:



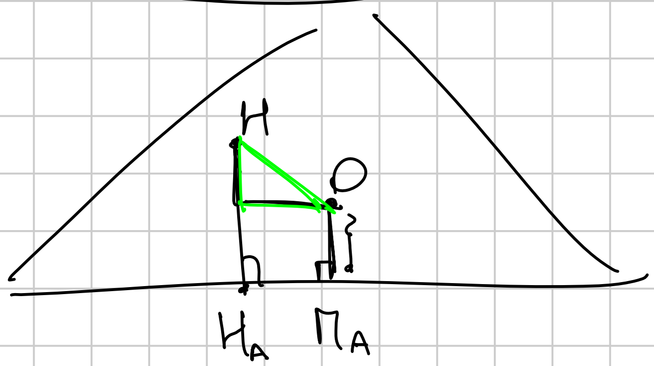

$$\frac{H_AH_B}{\sin\gamma} = \frac{b\cos\gamma}{\sin\beta} = \frac{b}{\sin\beta}\cos\gamma\sin\gamma = 2R\cos\gamma\sin\gamma$$
$$= R\sin 2\gamma$$



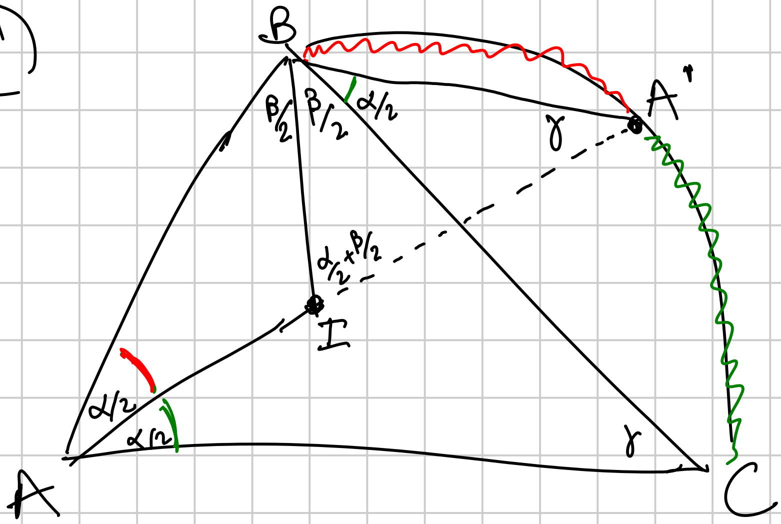

$OB = R$

$$BM_A = \boxed{\frac{a}{2} = R\cdot\sin\alpha}$$

$$OM_A = R\cdot\cos\alpha$$




$\boxed{AI}$ $\boxed{IA'}$




$IA' = BA'$

$$\frac{AI}{\sin\beta/2} = \frac{c}{\sin\left(\pi-\frac{\alpha}{2}-\frac{\beta}{2}\right)} = \frac{c}{\sin\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)} =$$

$$= \frac{c}{\sin\left(90-\frac{\gamma}{2}\right)} = \frac{c}{\cos\gamma/2}$$

$$AI = \frac{c}{\cos\gamma/2}\cdot\sin\beta/2 = \frac{2R\cdot\boxed{\sin\gamma}}{\cos\gamma/2}\cdot\sin\beta/2 =$$

$$= \frac{2R\cdot 2\sin\gamma/2\,\cancel{\cos\gamma/2}}{\cancel{\cos\gamma/2}}\cdot\sin\beta/2 = 4R\sin\gamma/2\,\sin\beta/2$$

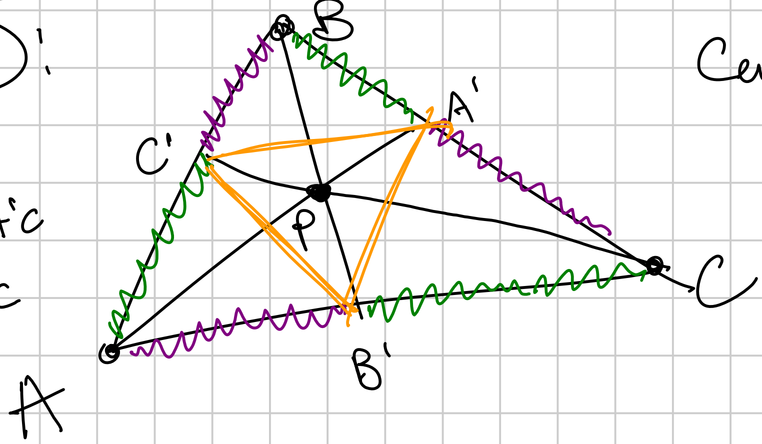
seni su $BIA'$: $\dfrac{BI}{\sin\gamma} = \dfrac{IA'}{\sin\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)} = \dfrac{IA'}{\sin\left(90-\frac{\gamma}{2}\right)}$

$= \dfrac{IA'}{\cos\gamma/2}$ $\qquad IA' = \dfrac{BI}{\sin\gamma}\cdot\cos\gamma/2 =$

$$= \frac{BI}{2\sin\gamma/2\,\cancel{\cos\gamma/2}}\,\cancel{\cos\gamma/2} = \frac{BI}{2\sin\gamma/2} = \frac{4R\sin\alpha/2\,\sin\gamma/2}{2\sin\gamma/2}$$

$$= 2R\sin\frac{\alpha}{2}$$

(12):




$a = BA' + A'C$

$b = AB' + B'C$

Ceva: le tre ceviane si incontrano $\Leftrightarrow$ prodotto verdi = prodotto viola

$$2R\cdot\boxed{S'} = \underbrace{AB'\cdot BC'\cdot CA'}_{\text{prod. viola}} =$$

$$2R\cdot S' = \left(S_{ABC} - S_{AB'C'} - S_{BA'C'} - S_{CA'B'}\right)\cdot 2R =$$

$$= 2R\cdot\left(S_{ABC} - \frac{AC'\cdot AB'\cdot\sin\alpha}{2} - \frac{BA'\cdot BC'\cdot\sin\beta}{2} - \frac{CA'\cdot CB'\cdot\sin\gamma}{2}\right)$$

$$= 2R\cdot S - AC'\cdot AB'\cdot R\sin\alpha - (\text{cicliche}) =$$ $\qquad 2R = \dfrac{a}{\sin\alpha}$

$= \underline{2 \cdot R \cdot S} - \underline{AC'} \cdot \underline{AB'} \cdot \frac{a}{2}$ – cicliche

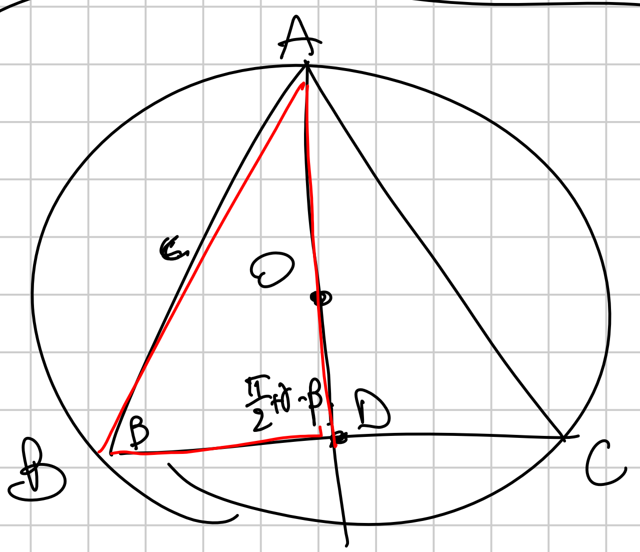
$= \frac{abc}{2} - AC' \cdot AB' \cdot \frac{a}{2}$ – cicliche $\qquad R = \frac{abc}{4S}$

$= \frac{1}{2}\left[ (AB' + B'C) \cdot (BA' + A'C) \cdot (AC' + C'B) - AC' \cdot AB' \cdot (BA' + A'C) - \text{cicliche} \right]$

6 di questi 8 prodotti si semplificano

$= \frac{1}{2}\left[ AB' \cdot BC' \cdot CA' + AC' \cdot BA' \cdot CB' \right] = AB' \cdot BC' \cdot CA'$

Ceva




$$\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{2}{R}$$

Step 1: mi trovo formule per AD

$AD = ..$ (teo. seni su ABD)

$$\frac{AD}{\sin\gamma} = \frac{b}{\sin\left(\frac{\pi}{2} + \beta - \gamma\right)} = \frac{b}{\cos(\beta - \gamma)} = \frac{b}{\cos\beta\cos\gamma + \sin\beta\sin\gamma}$$

$BE =$ (ciclico)

$CF =$ ciclica

... sommate ...

Dovrebbe venire una formula equivalente a

$$\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$$

che è sempre vera se $\alpha, \beta, \gamma$ sono i lati di un triangolo (es. 11 pag. 3)

(dim: usa $\gamma = \pi - \alpha - \beta$ + formule somma tangente)