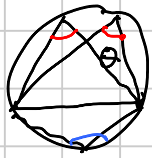
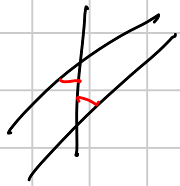
Geometria 3 – Basic

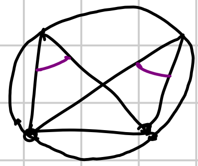
Kfp

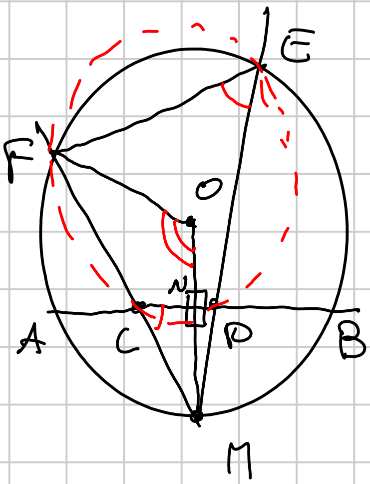
Note Title 9/7/2017

– Angoli
- somma angoli in $\triangle$ è π
- 

  $\widehat{BCD} = \pi - \theta$
- 

Problema di riscaldamento

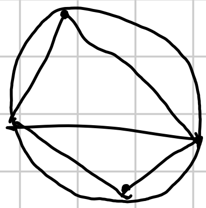




TS: CDEF

OM $\perp$ AB

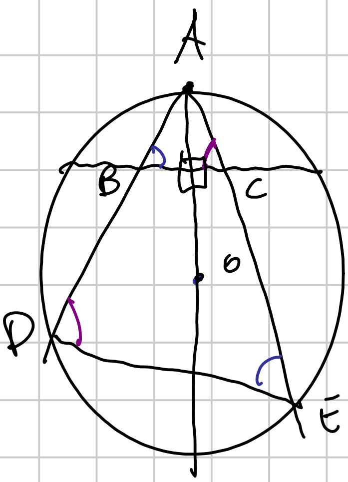
$\widehat{MOF} = 2\widehat{MEF}$



$\triangle MOF$ è isoscele

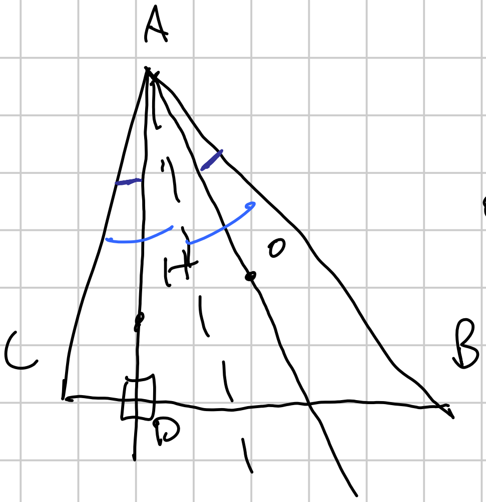
$\widehat{OMF} = \widehat{OFM}$ $\quad \widehat{OMF} = \dfrac{\pi - \widehat{MOF}}{2} = \dfrac{\pi}{2} - \widehat{MEF}$

$\boxed{\widehat{NCM}} = \dfrac{\pi}{2} - \widehat{OMF} = \dfrac{\pi}{2} - \dfrac{\pi}{2} + \widehat{MEF} = \boxed{\widehat{MEF}}$



AO è altezza di $\triangle ABC$

e passa per il circocentro di $\triangle ADE$



$$\widehat{AOB} = 2\,\widehat{ACB} = 2\gamma$$

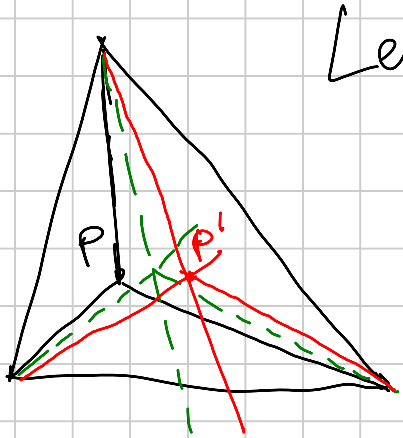
$$\widehat{OAB} = \frac{\pi}{2} - \gamma$$

$$\widehat{CAD} = \frac{\pi}{2} - \gamma$$

O e H sono coniugati isogonali

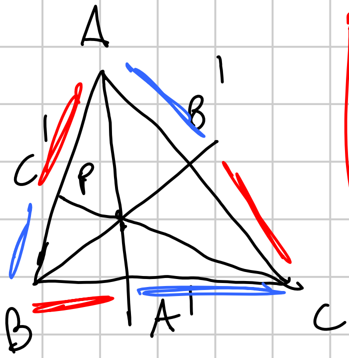
$I \longleftrightarrow I$

$G \longleftrightarrow$ Lemoine



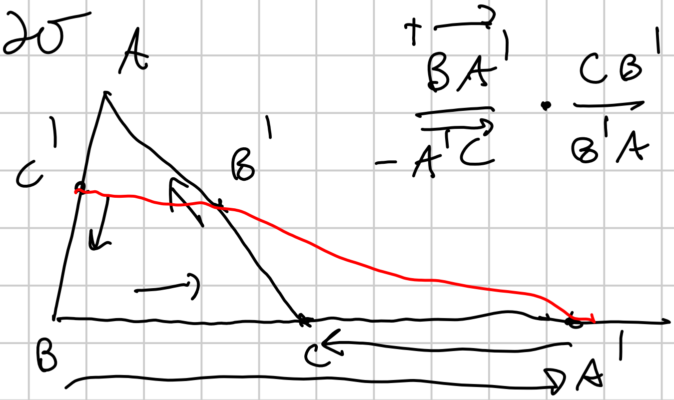
Lemma: $\forall$ P nel piano, $\exists$ P' coniugato isogonale.

Ceva

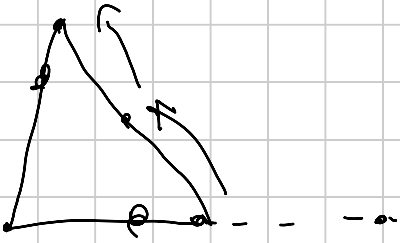


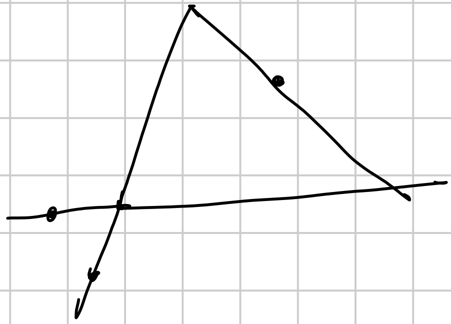
$$\boxed{\frac{BA'}{A'C} \cdot \frac{CB'}{B'A} \cdot \frac{AC'}{C'B} = 1}$$

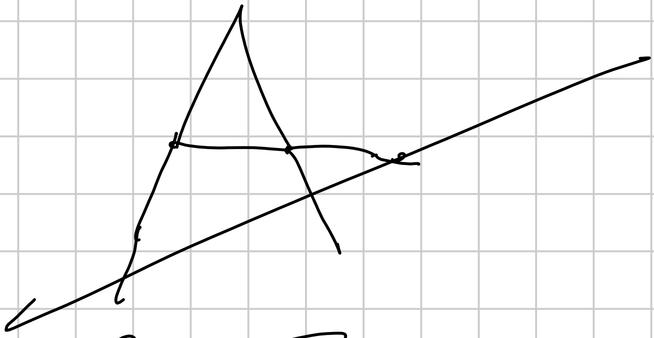
Menelao



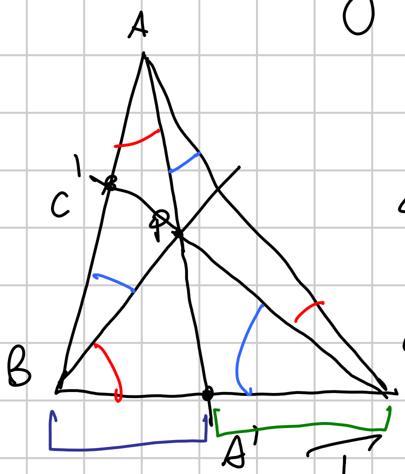
$$\frac{+\overrightarrow{BA'}}{-\overrightarrow{A'C}} \cdot \frac{\overrightarrow{CB'}}{\overrightarrow{B'A}} \cdot \frac{\overrightarrow{AC'}}{\overrightarrow{C'B}} = -1$$







Ceva Trigonometrico



AP, BP, CP concorrono

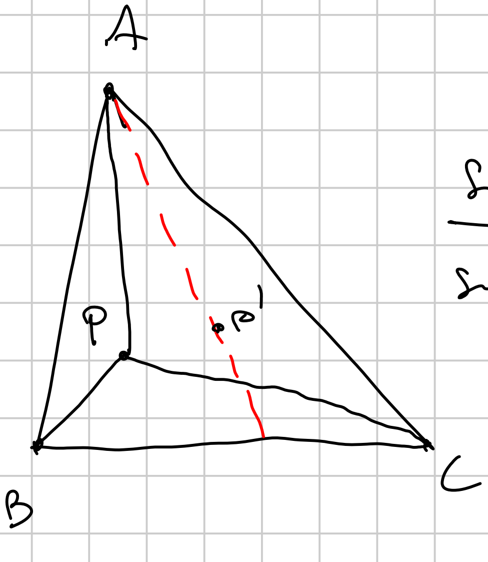
$\Longleftrightarrow$

$$\boxed{\frac{\sin \widehat{BAP}}{\sin \widehat{PAC}} \cdot \frac{\sin \widehat{ACP}}{\sin \widehat{PCB}} \cdot \frac{\sin \widehat{CBP}}{\sin \widehat{PBA}} = 1}$$

$\triangle ABA'$

Traccia di dim: $\dfrac{BA'}{A'A} = \dfrac{\sin \widehat{BAA'}}{\sin \beta}$

$$BA' = \boxed{AA'} \, \frac{\sin \widehat{BAA'}}{\sin \beta}$$



$$\frac{\sin \widehat{BAP}}{\sin \widehat{PAC}} \cdot \frac{\sin \widehat{ACP}}{\sin \widehat{PCB}} \cdot \frac{\sin \widehat{CBP}}{\sin \widehat{PBA}} = 1$$

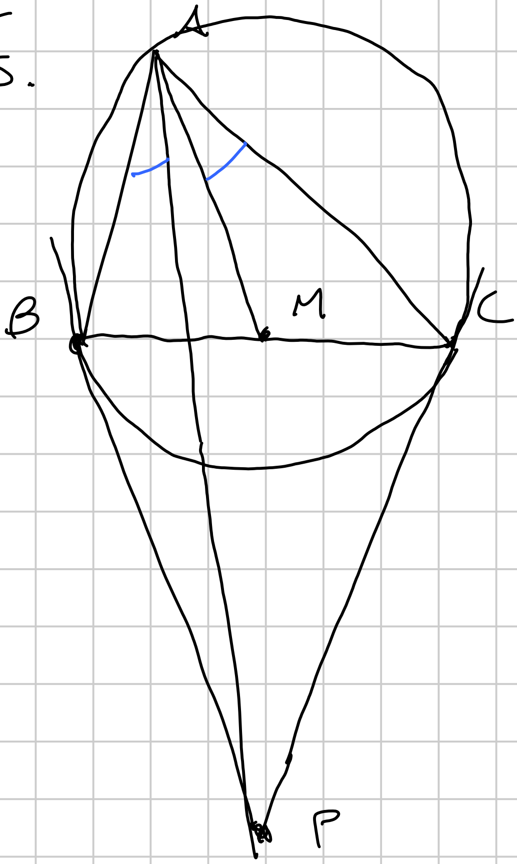
$\triangle$

$\widehat{BAP} = \widehat{P'AC}$

$\widehat{PAC} = \widehat{BAP'}$

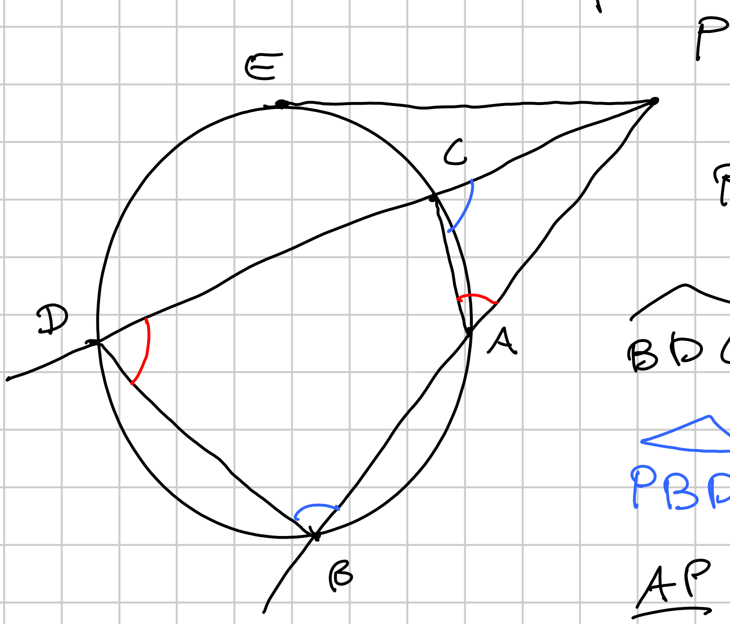
$$\frac{\sin \widehat{BAP'}}{\sin \widehat{P'AC}} = \left(\frac{\sin \widehat{BAP}}{\sin \widehat{PAC}} \right)^{-1}$$

Es.



$\widehat{BAP} = \widehat{MAC}$

Potenza di un punto



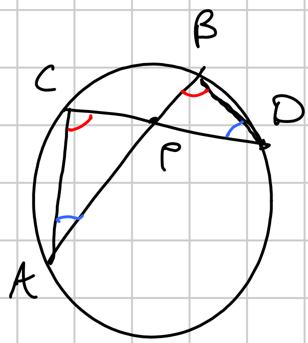
$PA \cdot PB = PC \cdot PD$

$\widehat{BDC} = \pi - \widehat{CAB} = \widehat{CAP}$

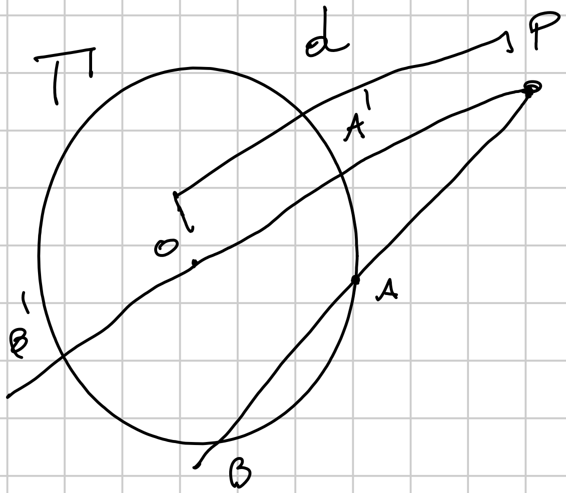
$\overset{\triangle}{PBD} \sim \overset{\triangle}{PCA}$

$\frac{AP}{PC} = \frac{PD}{PB}$

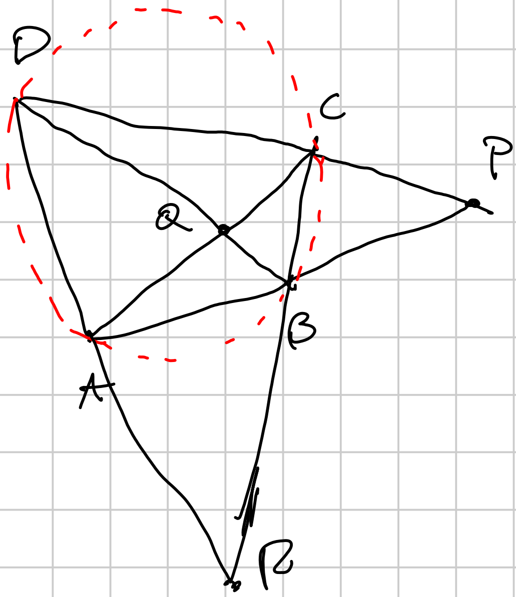
$PA \cdot PB = PE^2$



$PC \cdot PD = PB \cdot PA$



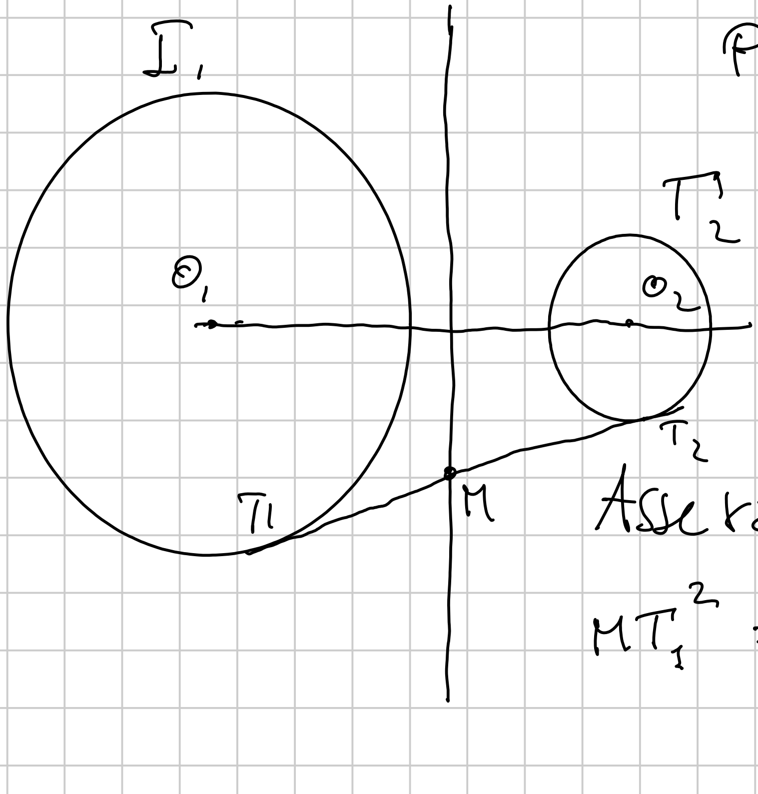
$$pow_{\Pi}(P) = PA' \cdot PB' =$$
$$= (d-R)(d+R) = d^2 - R^2$$



$$PB \cdot PA = PC \cdot PD$$
$$QB \cdot QD = QA \cdot QC$$

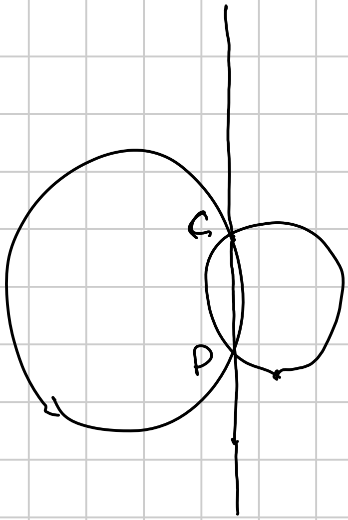
Asse radicale

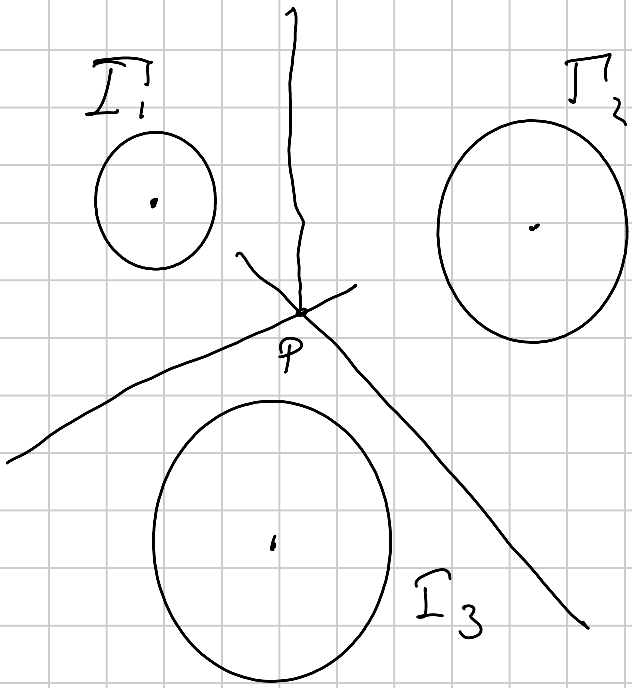
P: $pow_{\Gamma_1}(P) = pow_{\Gamma_2}(P)$



Asse radicale $\perp O_1O_2$

$$MT_1^2 = MT_2^2 \Rightarrow MT_1 = MT_2$$





r_1 As. rad. Γ_2 , Γ_3

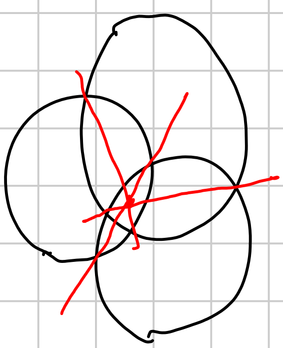
r_2

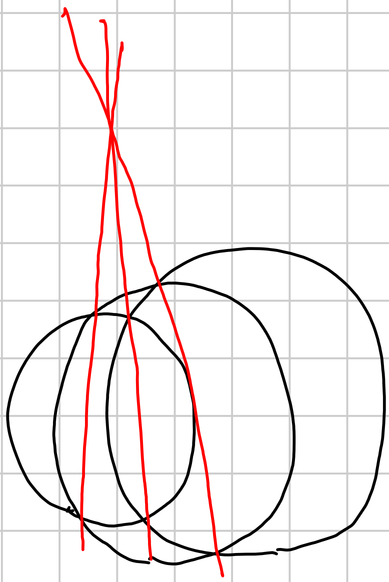
r_3

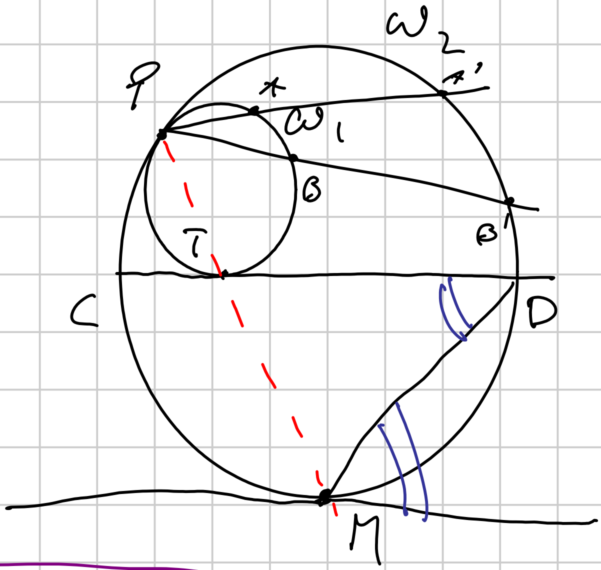
r_1, r_2, r_3 concorrono

$$\mathrm{pow}_{\Gamma_1}(P) = \mathrm{pow}_{\Gamma_3}(P) \qquad \mathrm{pow}_{\Gamma_3}(P) = \mathrm{pow}_{\Gamma_2}(P)$$

$$\boxed{\mathrm{pow}_{\Gamma_1}(P) = \mathrm{pow}_{\Gamma_2}(P)}$$







P, T, M allineati

$$\frac{PA}{PA'} = \frac{PB}{PB'}$$

P, A, A'

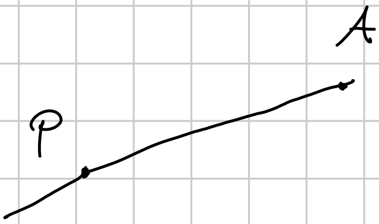
CD tange $\omega_1 \implies C'D'$ tange ω_2

$\widehat{CDM}$ sta su $\widehat{CM}$

l'altro sta su $\widehat{DM}$

Omotetie

P centro $\lambda \neq 0$, $\lambda \in \mathbb{R}$



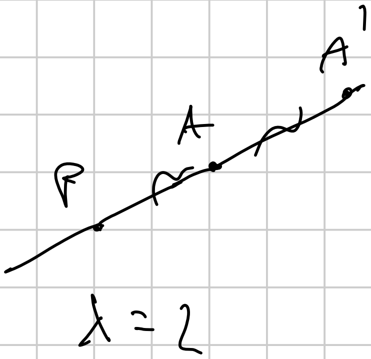
$A \to A'$

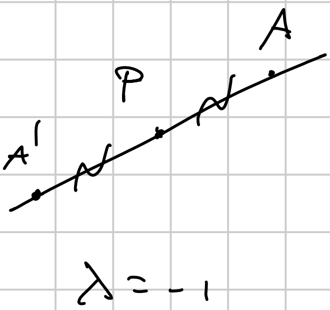
$A' \in AP$

$\lambda > 0$ semiretta PA

$\lambda < 0$ altra semiretta

$|A'P| = |\lambda| \cdot |AP|$





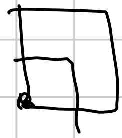
rette $\to$ rette parallele

Conserva angoli

Conserva un po' tutto

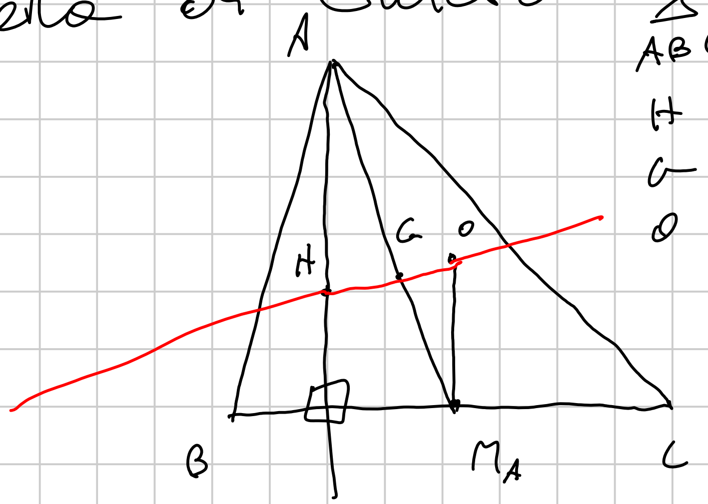
A PARTE lunghezze e aree

$\overline{AB}$ $\quad \overline{A'B'} = |\lambda| \overline{AB}$



Aree $\longrightarrow$ fattore λ^2

Retta di Eulero



$\overset{\triangle}{ABC}$ triangolo

H ortocentro

G baricentro

O circocentro

$\overline{AG} = 2\,\overline{GM_A}$

Omotetia centro G $\quad \lambda = -\frac{1}{2}$

$A \longrightarrow M_A$

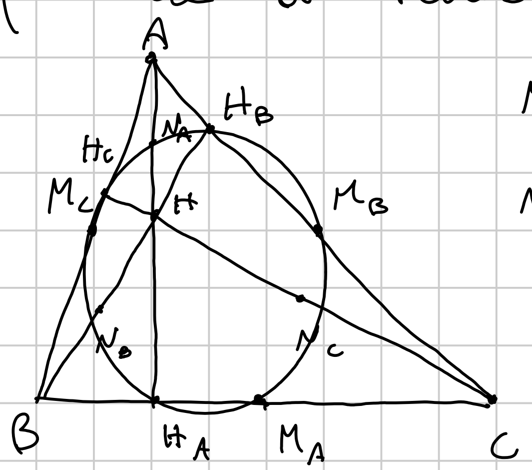
$AH \longrightarrow M_A O$ $\quad H \longrightarrow O$

$BH \longrightarrow M_B O$ $\quad H, G, O$ allineati

$CH \longrightarrow M_C O$

$\overline{GH} = 2\,\overline{OG}$

Circonferenza di Feuerbach

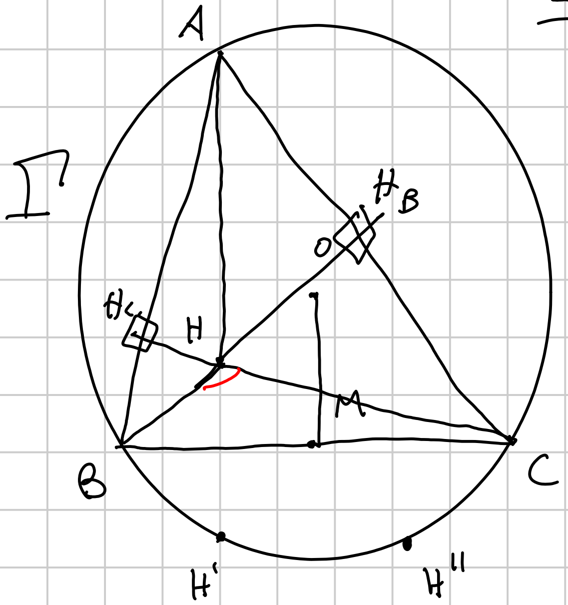


$M_A, M_B, M_C, H_A, H_B, H_C$

N_A, N_B, N_C

N_A pto medio di AH

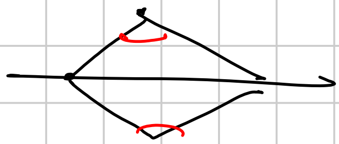
Lemmi



$\underline{1}$ • H' sym di H wrt BC

• H'' sym di H wrt M

$H', H'' \in \Gamma$



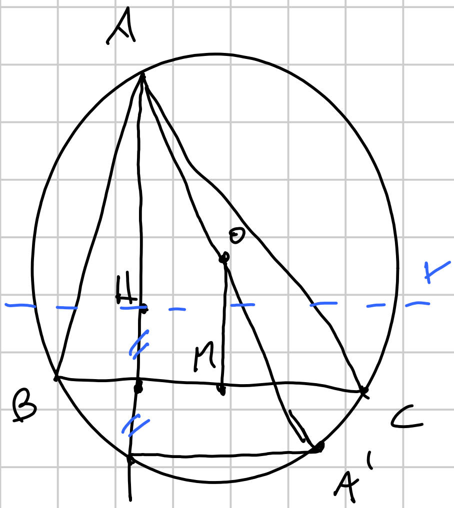
$$H' \in \Gamma \iff \widehat{BH'C} + \widehat{BAC} = \pi$$

$$\widehat{BH'C} = \widehat{BHC}$$

$$\widehat{CBH_B} = \frac{\pi}{2} - \gamma \qquad \widehat{BCH_C} = \frac{\pi}{2} - \beta$$

$$\widehat{BHC} = \pi - \widehat{CBH_B} - \widehat{BCH_C} = \beta + \gamma$$

$$\widehat{BHC} + \widehat{BAC} = \beta + \gamma + \alpha = \pi$$



Omotetia di centro A' e fattore 2

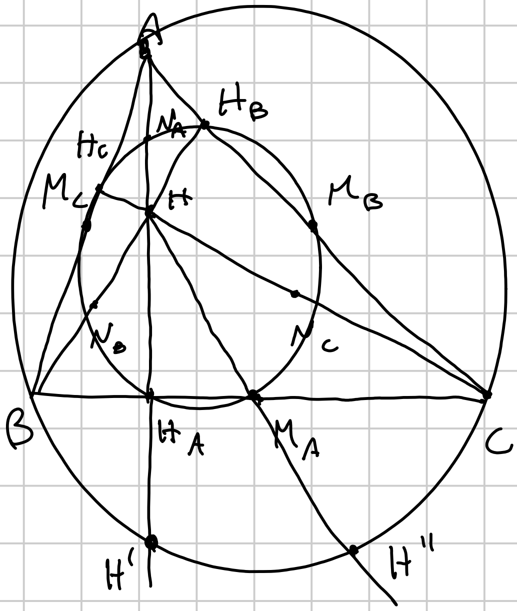
$O \longrightarrow A$

$OM \longrightarrow AH$

$(\cap M) \longrightarrow H$

$BC \longrightarrow r$

$$\boxed{\begin{array}{c} r \xrightarrow{f} r' \\ s \xrightarrow{f} s' \\ r \cap s = P \\ r' \cap s' = f(P) \end{array}}$$



$HH_A = H_A H'$

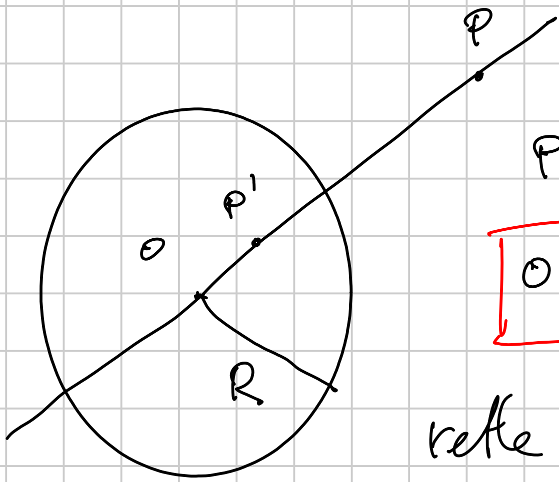
$HM_A = M_A H''$

Omotetia di centro H e fattore $\frac{1}{2}$

A, B, C, H', H'' ciclici pre-omotetia.

$N_A, N_B, N_C, \begin{matrix} H_A \\ H_B \\ H_C \end{matrix}, \begin{matrix} M_A \\ M_B \\ M_C \end{matrix}$ ciclici

Inversione (♡)



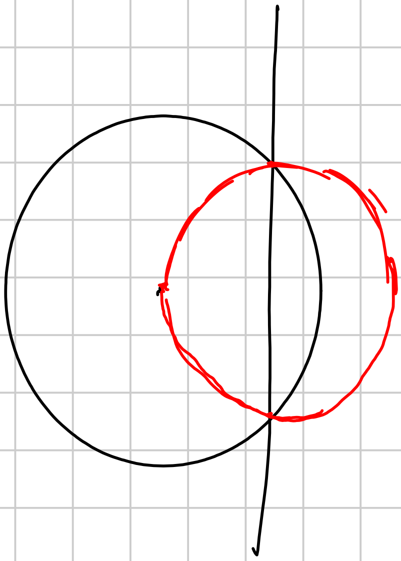
$P \to$ semiretta OP

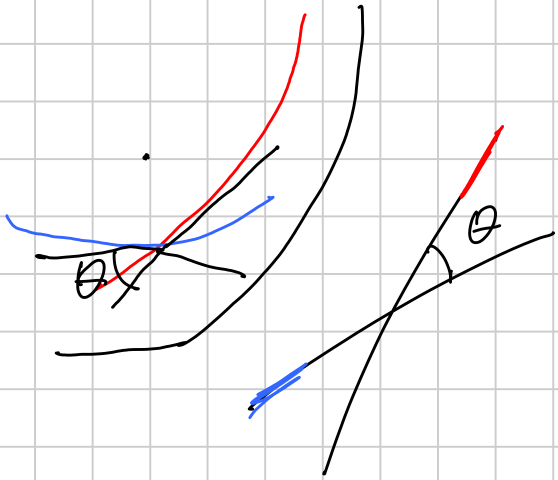
$\boxed{OP \cdot OP' = R^2}$ (involuzione)

rette passanti per $O \to$ rette passanti per O

Circonferenze non passanti per O in circ. non passanti per O.

rette non passanti per $O \longleftrightarrow$ Circonferenze per O

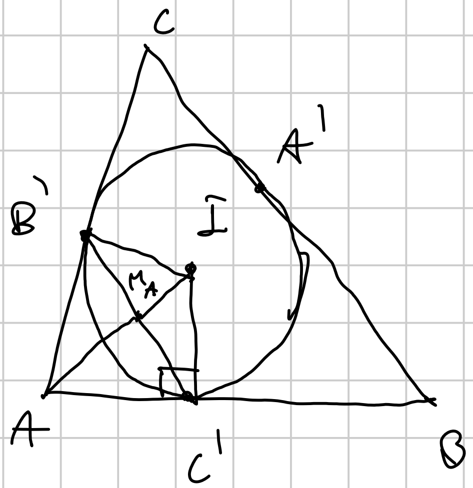




Euristica: se tanti cerchi passano per P, inverti in P.

- Inscritta

- Centro in un vertice (A) e raggio $\sqrt{AB \cdot AC}$ (+ simmetria wrt bisettrice in A)

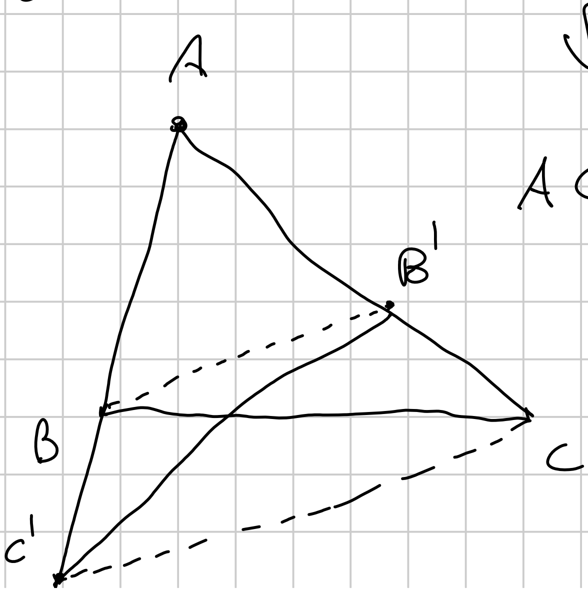


M pto medio B'C'

$A \leftrightarrow M$

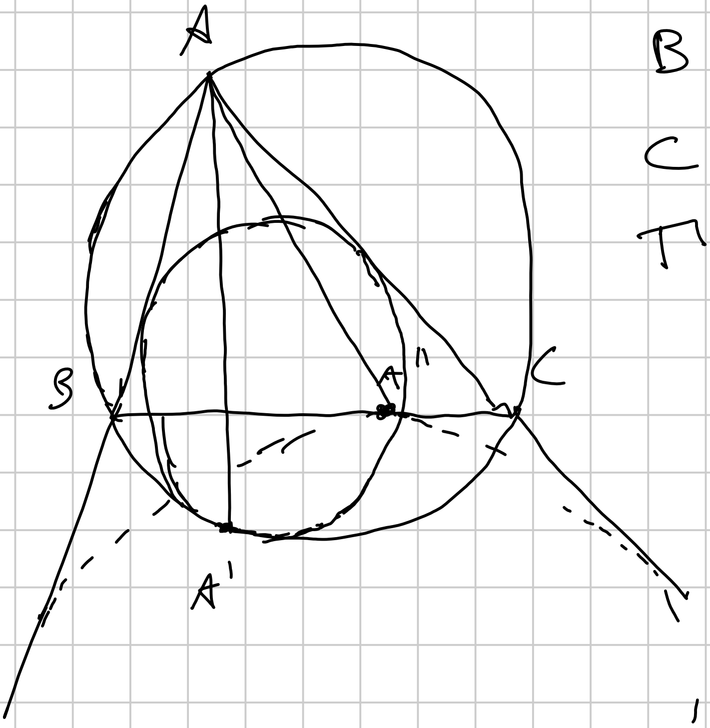
$IM \cdot IA = r^2$

11



$\sqrt{AB \cdot AC}$

$AC' \cdot \cancel{AC} = AB \cdot \cancel{AC}$



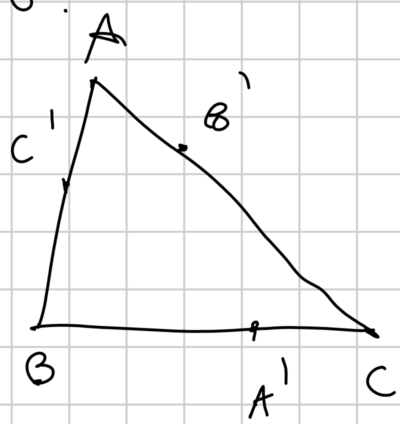
$B \longrightarrow C$

$C \longrightarrow B$

$T \longrightarrow BC$

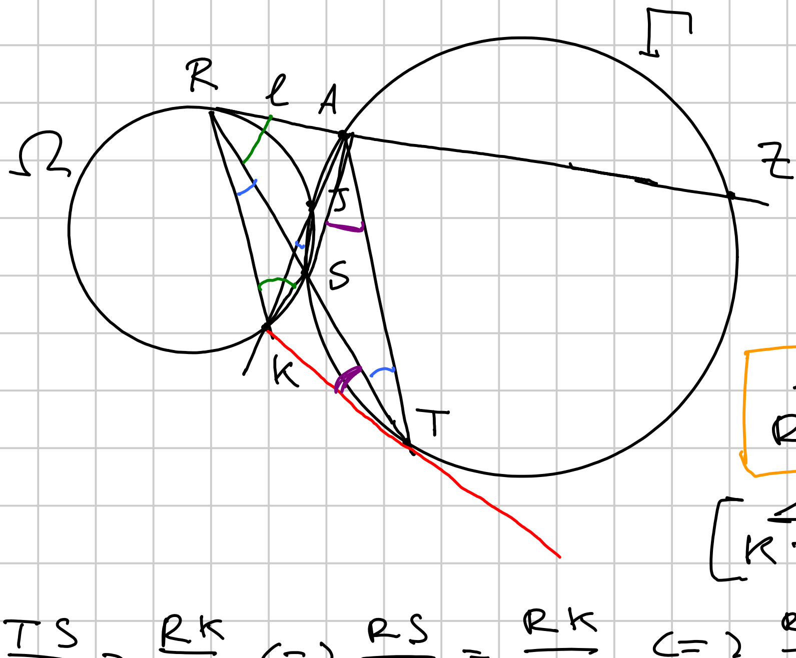
$\widehat{BAA'} = \widehat{A''AC}$

- Esercizio 36.
- (Miquel)



$\odot(AB'C')$
$\odot(BA'C')$ concorrono
$\odot(CA'B')$

IMO 4 - 2017



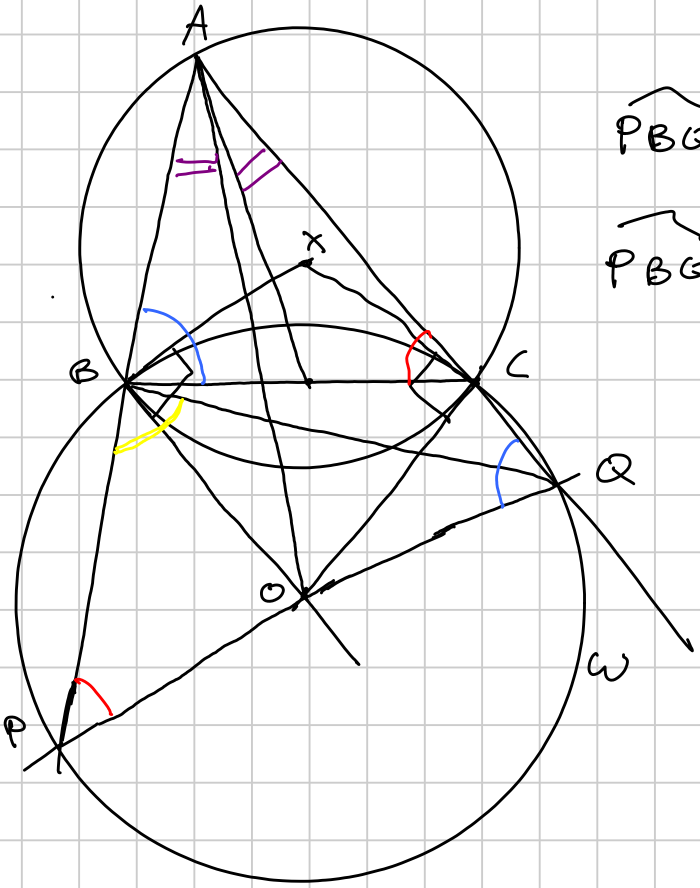
$\widehat{KRS} = \widehat{KJS} = \widehat{ATS}$

$\Rightarrow RK \parallel AT$

$$\boxed{\triangle RKS \sim \triangle TRA}$$

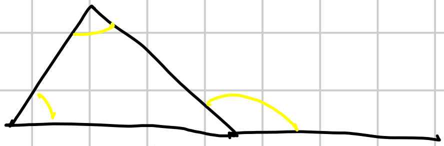
$[\triangle KTR \sim \triangle SAT]$ Equiv. alle tesi

$$\Leftrightarrow \frac{TS}{TA} = \frac{RK}{RT} \Leftrightarrow \frac{RS}{TA} = \frac{RK}{RT} \Leftrightarrow \frac{RS}{RK} = \frac{TA}{RT}$$



$\widehat{PBQ} = \frac{\pi}{2}$

$\widehat{PBQ} = \widehat{BQC} + \widehat{BAC}$



$\widehat{BQC} = \frac{\widehat{BOC}}{2}$

$\widehat{BXC} = 2\alpha$

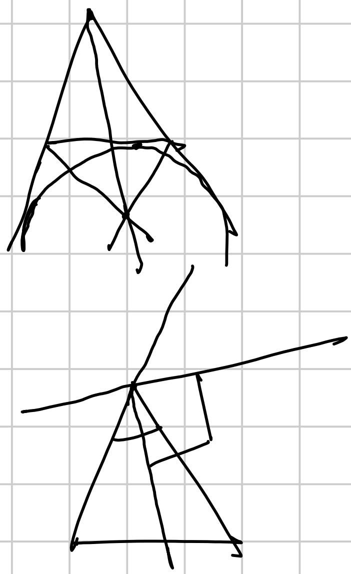
$XBOC \Rightarrow \widehat{BOC} = \pi - 2\alpha$

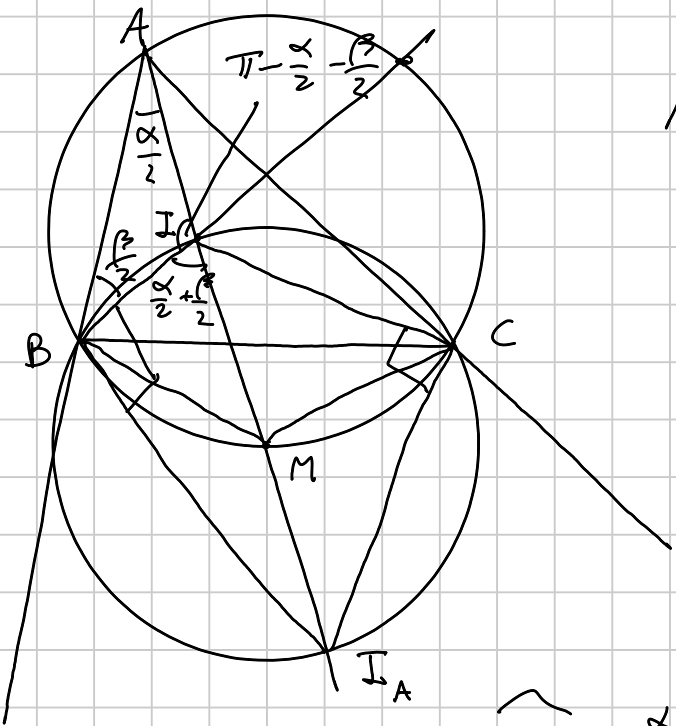
$\widehat{BQC} = \frac{\pi}{2} - \alpha$

$\widehat{PBQ} = \frac{\pi}{2} - \alpha + \underbrace{\widehat{BAC}}_{\alpha} = \frac{\pi}{2}$

$\Rightarrow AO$ mediana di $\overset{\triangle}{APQ}$

$\overset{\triangle}{APQ} \sim \overset{\triangle}{ACB}$





$M \in$ asse BI

(CI)

$\overset{\triangle}{BIM}$ isoscele di vertice M

$\widehat{IBM} = \frac{\alpha}{2} + \frac{\beta}{2}$