A3 — medium (succ e funz)

Note Title 9/7/2017

Scambret

→ Successioni → ripasso → nuovo → "analisi"

→ Funzionali → ripasso → tecniche

Successioni

- Ripasso

$$a_{n+2} = \lambda a_{n+1} + \mu a_n \quad n \geq 0 \quad \boxed{\begin{matrix} a_0 \\ a_1 \end{matrix}} \qquad (1)$$

OMOG

$$x^2 = \lambda x + \mu \longrightarrow R_1 \text{ e } R_2$$

Perché ?

$$a_n = R_1^n \cdot c_1 + R_2^n \cdot c_2 \qquad (2)$$

$\overline{a_n}$, a_n^*

$\boxed{c_1 \overline{a_n} + c_2 a_n^*}$ risolve (1)

$$\left.\begin{matrix} \overline{a_n} + a_n^* \Rightarrow (1) \\ c\overline{a_n} \to (1) \end{matrix}\right\}$$

$$a_n = R^n$$

$$R^2 = \lambda R + \mu$$

- Novità

$$a_{n+2} = \lambda a_{n+1} + \mu a_n + f(n) \qquad (3)$$

NON OMOG

$\overline{a_n}$ e a_n^* risolvono (3)

$\Rightarrow \overline{a_n} - a_n^*$ risolve l'equazione omogenea (1)

b_n

$$\overline{a_{n+2}} - a_{n+2}^* = \lambda(\overline{a_{n+1}} - a_{n+1}^*) + \mu(\overline{a_n} - a_n^*)$$

$\bar{a}_n$ sol. nella (3) e a_n^* nella (3) $\Rightarrow \bar{a}_n - a_n^*$ (1)

$$\bar{a}_n = (\bar{a}_n - a_n^*) + a_n^*$$

$$\Uparrow$$

$$\left\{ \begin{matrix} \text{Soluzione} \\ \text{generale} \\ \text{della} \\ \text{non omogenea} \end{matrix} \right\} = \left\{ \begin{matrix} \text{Soluzione} \\ \text{generale} \\ \text{della} \\ \text{omogenea} \end{matrix} \right\} + \left\{ \begin{matrix} \text{Soluzione} \\ \text{particolare} \\ \text{della non} \\ \text{omogenea} \end{matrix} \right\}$$

- <u>Esempi</u>

$$a_{n+1} = ca_n + d \quad \Big\} \quad a_0 = \alpha$$

1) Induzione

$$a_0 = \alpha$$
$$a_1 = \alpha c + d$$
$$a_2 = \alpha c^2 + cd + d$$
$$a_3 = \alpha c^3 + c^2 d + cd + d$$

$$\Rightarrow a_n = \alpha c^n + d(c^{n-1} + \dots + 1)$$

$$a_n = \alpha c^n + d\,\frac{c^n - 1}{c-1} \quad (\text{se } c \neq 1)$$

$$a_n = \alpha + dn \quad (\text{se } c = 1)$$

2) Shift $\quad b_n = a_n - \underline{l} \qquad b_{n+1} = cb_n$

$$\begin{cases} a_{n+1} - l = c(a_n - l) \\ a_{n+1} = ca_n + d \end{cases}$$

$$d = l - cl \qquad c \neq 1$$

$$l = \frac{d}{1-c}$$

3) Usare i potenti mezzi

$$a_{n+1} = c a_n + \underline{d}$$

$\underline{a_n}$ che risolve la omogenea generale

a_{n*} che risolve la non omogenea (basta una particolare)

$\underline{a_n} = R^n$ $\qquad R^{n+1} = c R^n$

$$\boxed{c = R}$$

$$\underline{a_n} = \lambda \cdot c^n$$

$a_{n*} = \cancel{k}$

$$k = ck + d$$

$$\Rightarrow k = \frac{d}{1-c} = a_{n*}$$

$$a_n = \lambda \cdot c^n + \frac{d}{1-c}$$

$$a_0 = \alpha$$

<u>Es. 2</u> $\qquad a_{n+2} = 3a_{n+1} - 2a_n + n$

$$\underline{a_n} = \lambda_1 \cdot 1^n + \lambda_2 \cdot 2^n$$

$$a_{n*} = bn + c$$

$$b(n+2)+c = 3b(n+1)+3c-2bn-2c+n$$

$$\lambda_1 + \lambda_2 \cdot 2^n + bn + c$$

<u>Es. 3</u>

$$a_{n+2} = 3a_{n+1} - 2a_n + 3^n$$

$$x^2 = 3x - 2$$

$$a_{n_*} = \mu \cdot 3^n$$

$$\mu \cdot 3^{n+2} = 3\mu \cdot 3^{n+1} - 2\mu \cdot 3^n + 3^n$$

$$\cancel{9\mu} = \cancel{9\mu} - 2\mu + 1 \Rightarrow \mu = \frac{1}{2}$$

$$a_n = \lambda_1 + \lambda_2 \cdot 2^n + \frac{1}{2} \cdot 3^n$$

<u>Es. 4</u>

$$a_{n+2} = 3a_{n+1} - 2a_n + 2^n$$

$$\underline{a_n} = \lambda_1 + \lambda \cdot 2^n$$

$$a_{n_*} = \mu \cdot 2^n$$

$$4\mu = 6\mu - 2\mu + 1 \Rightarrow 0 = 1$$

$$2^n \cdot p(n)$$ dove $\deg(p(n)) =$ molteplicità di 2 nell'eq. omogenea (m)

$$(An+B) \cdot 2^n$$

$$\underline{An \cdot 2^n} \qquad \mu\, 2^n \cdot n^m$$

$$a_{n+2} = 4a_{n+1} - 4a_n + 2^n$$

$$An^2 2^n \rightsquigarrow$$

Se k risolve l'eq. omogenea con molteplicità m, poniamo $a_{n*} = \underline{c \cdot n^m \cdot k^n}$

Esiste sempre un c? PROVATELO...

$$a_{n+w} = \dots\dots + k^n$$

$x^w = \dots$ ha m volte la radice k

$$\boxed{w \geq m}$$

Esercizi

BMO 04/1

$$\begin{cases} a_{2004} = ? \\ a_{m+n} + a_{m-n} - m + n - 1 = \frac{1}{2}(a_{2m} + a_{2n}) \end{cases}$$

$m \geq n \geq 0$

$a_1 = 3$

$\vdots$

$$a_{m+2} - 2a_{m+1} + a_m = \underline{2 \cdot 1^m}$$

$a_0 = 1$

$a_1 = 3$

$\underline{a_n} = \lambda_1 \cdot 1^n + \lambda_2 \cdot n \cdot 1^n$

$a_{n_0} = c$

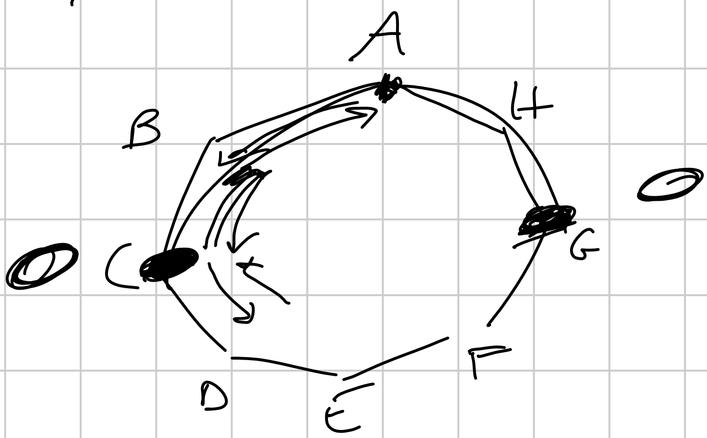
$cn + d$

$a_{n_*} = 1^n \cdot n^2 \cdot c$

$$a_n = cn^2 + \lambda_2 n + \lambda_1$$

$$\boxed{a_n = n^2 + n + 1}$$

IMO 79/6



P_n

$P_{2n} \equiv 0$

A_n

O_n

$A_{2n+1} \equiv 0$

$O_{2n+1} \equiv 0$

$$\begin{cases} A_{2n+2} = 2A_{2n} + O_{2n} \\ O_{2n+2} = 2O_{2n} + 2A_{2n} \end{cases}$$

A_{2n} $\quad O_{2n}$ $\longrightarrow$ npoodn...

$$\boxed{x_{n+1} = \sqrt{5x_n - 6}}$$

$x_0 = 2017$

$x_{n+1} = 2x_n^2 - 1$

$\ell = 3$

$f(x) = \sqrt{5x-6}$

$$\begin{cases} i) & 3 \le x_n \le 2017 \\ ii) & x_{n+1} \le x_n \\ iii) & x_n \to \ell \\ iv) & \ell = 3 \end{cases}$$

$x \to \sqrt{5x-6}$

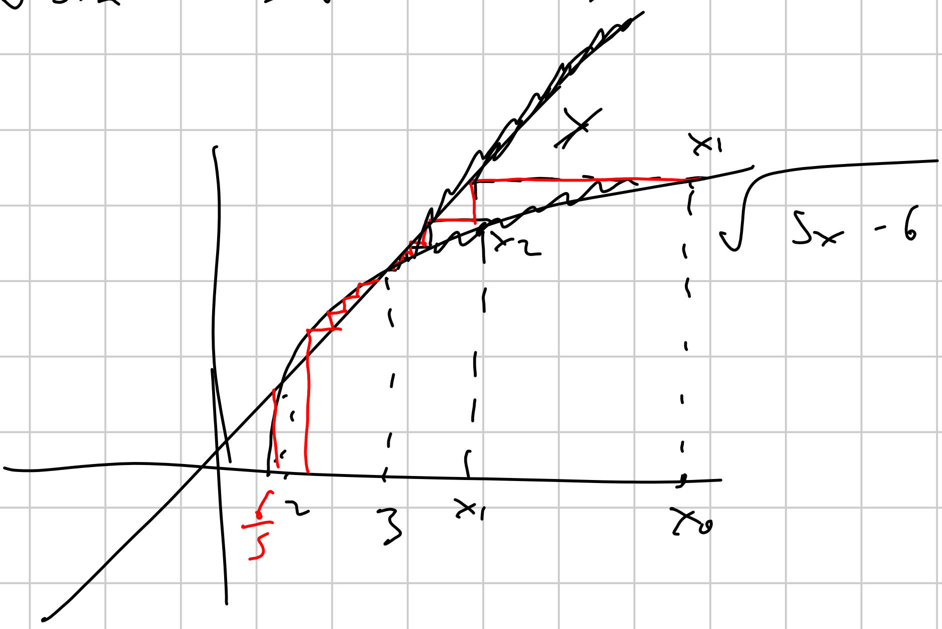
$x_0 = 2017$

ver

$x_{n+1} = \sqrt{5x_n - 6} \le \sqrt{5 \cdot 2017 - 6} \le 2017$

$x_0 = 2017 \ge 3$

$x_{n+1} = \sqrt{5x_n - 6} \ge \sqrt{5 \cdot 3 - 6} \ge 3$



$x_n \to \ell$

$\ell = \sqrt{5\ell - 6}$

$\ell = 2$

$\ell = 3$

$x_{n+1} = x_n^3$

a) $x_0 = \boxed{2}$

i) $x_n \geq 2$

ii) $x_{n+1} \geq x_n$

iii) l esiste ✓

iv) $l \to +\infty$ ✓

b) $x_0 = \frac{1}{2}$

i) $0 \leq x_n \leq \frac{1}{2}$

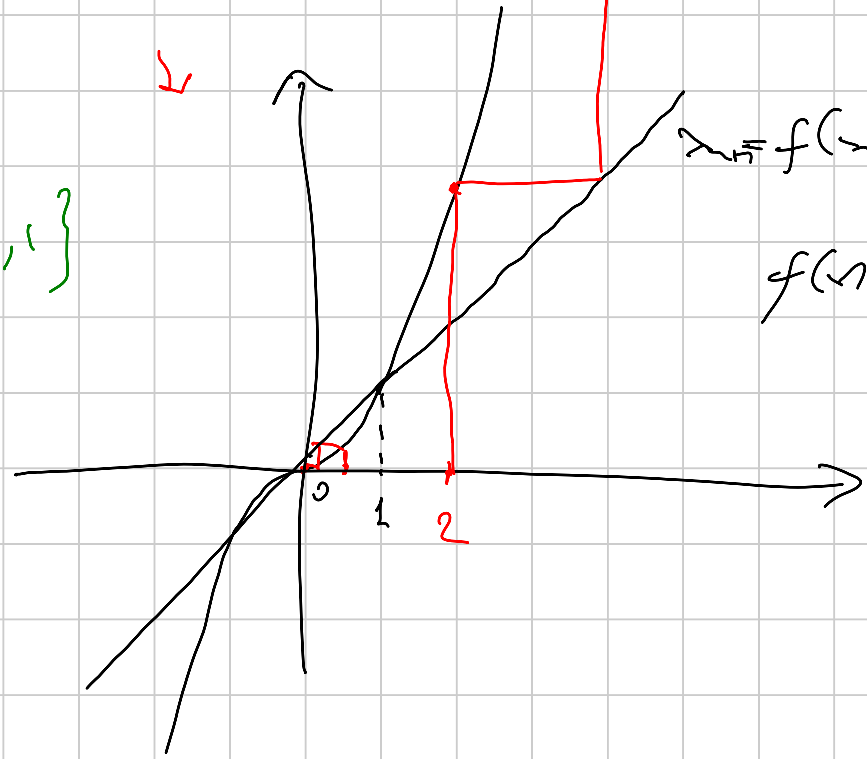
ii) $x_{n+1} \leq x_n$

iii) l ✓

iv) $l \to 0$

$l = l^3$

$l = \{-1, 0, 1\}$



$x_{n+1} = f(x_n)$

$f(x) = x^3$

T1 2017/2

$x = \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}}}$

$x_{n+1} = \sqrt{1+x_n}$

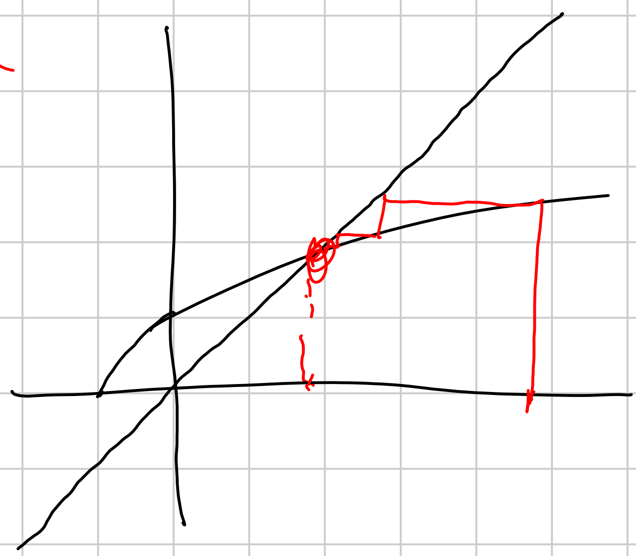
$x_0 = \alpha$

$x_{n+5} = x_n$

$\sqrt{1+x} < x$

$\sqrt{1+\sqrt{1+x}} > x$

$x = \sqrt{1+x}$



Funzionali

$f: S \to S \qquad f(x+y) = f(x) + f(y)$

$S \subseteq \mathbb{R}$

- $S = \mathbb{Q}$ Risposta: $f(x) = f(1) \cdot x$

$P(x,y): \quad f(x+y) = f(x) + f(y)$

$P(0,0): \quad f(0) = 0$

$P(-x,x): \quad f$ è dispari

$P(n,1): \quad f(n) = nf(1) \qquad f(\sum x) = \sum f(x)$

$P(n,x): \quad f(nx) = nf(x)$

$P(n, \frac{m}{n}): \quad mf(1) = nf(\frac{m}{n})$

Se $S = \mathbb{R}$

- f continua $\quad \boxed{\checkmark \mathbb{Q}}$
- f monotona
- f limitata
- $\exists$ un rettangolo in $\mathbb{R}^2$ t.c. non ci sono punti $(x, f(x))$

Se f è monotona $\Rightarrow f(x) = kx$

wlog crescente

$f(q) = kq \qquad q \in \mathbb{Q}$

$x: \quad f(x) = bx \qquad b \neq k$

$b < k$

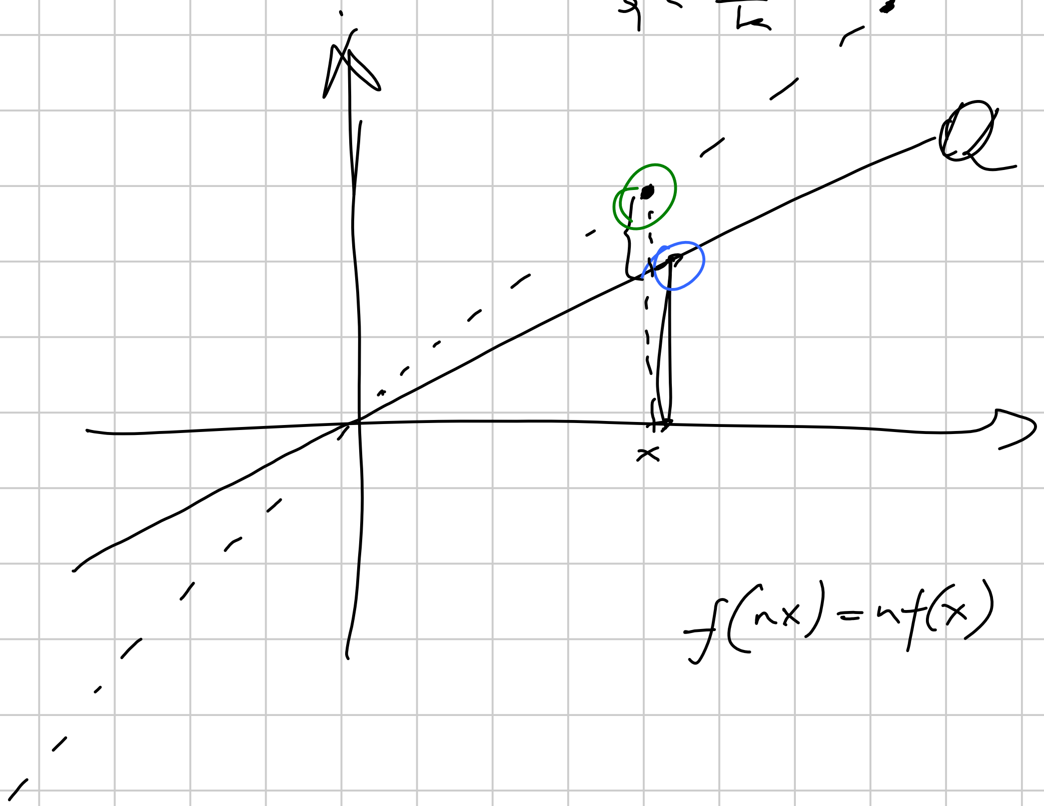
$\boxed{\frac{bx}{k}} < q < \boxed{x}$ ——

$f(q) < f(x)$

$\begin{array}{ccc} f(q) & < & f(x) \\ \parallel & & \wedge \\ qk & & bx \end{array}$

$qk < bx$

$q < \frac{bx}{k}$



$f(nx) = nf(x)$

$P(a,b)$: $f(a+b^2) = f(a) + f(b^2)$ $\quad a,b \in \mathbb{Q}$

$P(0,0)$: $f(0) = 0$

$P(-b^2,b)$: f è dispari su $[\mathbb{Q}]^2$ $\quad f(x^2) = -f(-x^2)$ $\quad x \in \mathbb{Q}$

$P(nb^2,b)$: $f(nb^2) = n f(b^2)$ $\quad Q(n,b)$

$f(n) = n f(1) = nk \Leftarrow Q(n,1)$

$\underline{Q\left(q^2, \frac{p}{q}\right)}$ $\quad f(p^2) = q^2 f\left(\left(\frac{p}{q}\right)^2\right) \Rightarrow f(x^2) = kx^2$

$$f(nb^2) = n f(b^2) = k \cdot nb^2$$

$(n = pq,\ b = \frac{1}{q})$

$Q(pq, \frac{1}{q})$:

$$f\left(pq \cdot \frac{1}{q^2}\right) = k \cdot pq \cdot \frac{1}{q^2}$$

$$f\left(\frac{p}{q}\right) = k \cdot \frac{p}{q}$$

Iniettività e surgettività

IMO SL 02 / 1

$f : \mathbb{R} \to \mathbb{R}$

$$f(f(x)+y) = 2x + f(f(y)-x).$$

$$f(\ldots) = \mathbb{R}$$

z

$\Big[\ f(f(n)) = n+2 \qquad f: \mathbb{N}_{\geq 0} \to \mathbb{N}_{\geq 0}\ \Big]$

$\boxed{y = -f(x)}$

$z \quad z, -f(z)$

$$f(0) - 2x = f(f(-f(x)) - x)$$

f surgettiva

$$\exists x_0 : f(x_0) = 0$$

$$f(f(y)) = 2f(y)$$

$$f(z) = 2z$$

$x = x_0$

$$f(y) = 2x_0 + f(f(y) - x_0)$$

$$f(y) = z$$

TT 2017.3 (BST 2012 / 4)

$f: \mathbb{Q} \to \mathbb{Q}$ $\quad f(x+f(y+f(z))) = \underline{y+f(x+z)}$

$x=z=0$ $\quad f(f(y+f(0))) = y+f(0)$

$f(f(y)) = y$

$\cancel{f}(x+f(y+f(z))) = y+f(x+z) = y+f(z+x)$

$= \cancel{f}(z+f(y+f(x)))$

$f(y_1)=f(y_2) \Rightarrow y_1 = y_2$

$f(f(x)) = x$

$x+f(y+f(z)) = z+f(y+f(x))$

$x \to f(x)$

$f(x)+f(y+z) = f(z)+f(y+x)$

$z=0$ $\quad f(x)+f(y) = f(0)+f(x+y)$

$-f(0) \quad -f(0) \qquad -2f(0)$

$g(x) = f(x)-f(0)$

$f(x) = Ax+B$

$f(x)=x \quad \text{o} \quad f(x) = -x+c$

Sostituzioni (furbe?)

Argentina TST 10/13 (modificato)

$f: \mathbb{R}^- \to \mathbb{R}$

$$f(x+xy+f(y)) = \left(f(x)+\tfrac{1}{2}\right)\left(f(y)+\tfrac{1}{2}\right)$$

$y=-1$

$$\underbrace{f(f(-1))}_{0} = \left(f(x)+\tfrac{1}{2}\right)\underbrace{\left(f(-1)+\tfrac{1}{2}\right)}_{0}$$

$$f(x)=c$$

$$f(-1) = -\tfrac{1}{2} \Rightarrow \boxed{f\left(-\tfrac{1}{2}\right)=0}$$

$\boxed{OK}$ magari non si può davvero fare!!!

$f: \mathbb{R} \to \mathbb{R}$

$$\boxed{f(\alpha) = -\tfrac{1}{2} \iff \alpha = -1}$$

$y=\alpha$

$$f\Big(\underbrace{(\alpha+1)x - \tfrac{1}{2}}_{\in \mathbb{R}}\Big) = 0 \qquad \alpha+1=0$$

$$x+xy+f(y) = -\tfrac{1}{2} \qquad \boxed{x = \frac{-\frac{1}{2}-f(y)}{y+1}}, \; y \neq -1$$

$$0 = f\left(-\tfrac{1}{2}\right) = \left(f(x)+\tfrac{1}{2}\right)\underline{\left(f(y)+\tfrac{1}{2}\right)}$$

$x = -1$

$\frac{1}{2} + f(y) = y + 1$ ✓

IMO SL 16/4 $\qquad f: \mathbb{R}^+ \to \mathbb{R}^+$

$$x f(x^2) f(f(y)) + f(y f(x)) = f(xy)\left(f(f(x^2)) + f(f(y^2)) \right)$$

$$x f(x^2) f(f(y)) + f(y f(x)) = y f(y^2) f(f(x)) + f(x f(y))$$

$x = y = 1 \qquad \boxed{f(1) = 1}$

$y = 1 \qquad x f(x^2) = f(x)$

$x = 1 \qquad f(f(y)) = f(y) f(f(y^2))$

$f(f(y)) = f(y) f(f(y)^2)$ $\qquad x = f(y)$

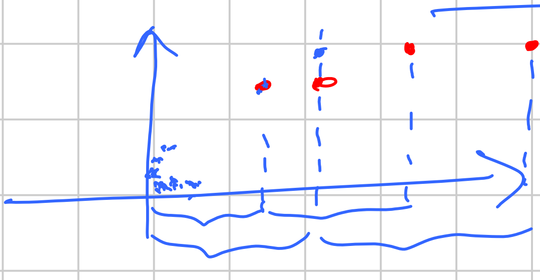
$$\cancel{f}(f(y^2)) = \cancel{f}(f(y)^2)$$

$$\frac{f(x)}{x} = f(x)^2 \quad ✓$$

Se $f(x_1) = f(x_2)$ e vogliamo $x_1 = x_2$

$\Rightarrow f(x_1^2) = f(x_2^2)$

$f(x_1) = f(x_2) \Rightarrow f(x_1 y) = f(x_2 y)$



$$y = \frac{x_2}{x_1} \qquad f(x_1) = f(x_2) = f\left(\frac{x_2^2}{x_1}\right)$$
$$= f\left(\frac{x_2^3}{x_1^2}\right)$$

$$y = x_1 \qquad f(x_1^2) = f(x_1 x_2) = f(x_2^2)$$

Lavorare coi positivi

MO S2 07/4

$$f: \mathbb{R}^+ \to \mathbb{R}^+$$

$$f(x + f(y)) = f(xy) + f(y)$$

$$f(\ldots) \neq 0 \qquad f(X) = f(Y) + f(Z)$$
$$X = Y$$

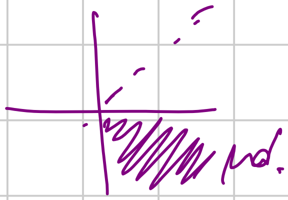
$x + f(y) = x + y \Rightarrow f(y) = 0$ no! $f(y) \neq y$

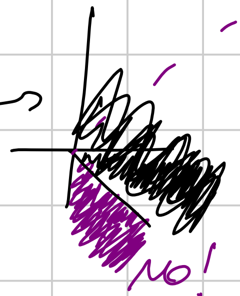
$x + f(y) = y \Rightarrow f(xy) = 0 \Rightarrow x = y - f(y) > 0$ no!

$$f(y) \geq y$$

$$f(y) > y > 0$$
$$f(y) > y$$

$$g(y) = \boxed{f(y) - y} \qquad g: \mathbb{R}^+ \to \mathbb{R}^+ \qquad \mathbb{R}^+ \to$$





$$f(x+y+f(y)) = f(x+y)+y \qquad x,y\in\mathbb{R}^+ \quad f:\mathbb{R}^+\to\mathbb{R}^+$$

$$f(a+f(b)) = f(a)+b \qquad a>b>0$$

$f(b_1)=f(b_2) \Rightarrow b_1=b_2$ mmmm... $\quad a=\max\{b_1,b_2\}+2017$

$\boxed{f \text{ iniettiva}}$

$$f(a+f(b)) = \underline{f(a)+b}$$

$$f(a+f(b+c)) = \underbrace{f(a)+b}+c$$
$$= f(a+f(b))+c$$
$$= f(a+f(b)+f(c))$$

$f(a+f(b)+f(c))$

Immagini

MO SL 05/2 $\qquad f:\mathbb{R}^+\to\mathbb{R}^+$

$$f(x)f(y) = 2f(x+yf(x))$$

$$x+yf(x) = y+xf(y) \qquad f(x)=Ax+1$$

$x=x+yf(x)$ no

$y=x+yf(x) \qquad y=\dfrac{x}{1-f(x)}>0 \Rightarrow f(x)=2$

$1-f(x)>0$

$f(x)<1$

$$\boxed{f(x) \geq 1}$$

$$f(x)f(y) = 2f(\ldots)$$

$$a, b \in \mathrm{Im} f \Rightarrow \frac{ab}{2} \in \mathrm{Im} f$$

$\times$ $(1, +\infty]$

Brutal mode

$$m < 2$$

$$\frac{m^2}{2} < m \Rightarrow m \geq 2$$

$$\frac{f(x)}{2} \cdot \frac{f(y)}{2} = f\,\frac{(x+yf(x))}{2}$$

$$\frac{f(x)}{2} \geq \frac{1}{2}$$

$$a, b \in \mathrm{Im}\,\frac{(f)}{2} \Rightarrow ab \in \frac{\mathrm{Im}}{2}(f)$$

$$a^n \in \mathrm{Im}\,\frac{(f)}{2}$$

$$a < 1 \quad e \quad a \in \mathrm{Im}\,\frac{(f)}{2} \Rightarrow a^n \in \mathrm{Im}\,\frac{(f)}{2}$$

$$f(x) \geq 2$$

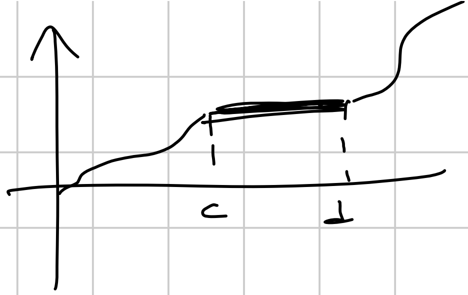
$$f(x)f(y) \geq 2f(x)$$

$$\shortparallel$$

$$2f(x+yf(x))$$

$f(x+\ldots) \geq f(x) \Rightarrow$ f crescente

$$x \in (c, d): \; f(c) = f(x) = f(d)$$



$x = c$

$$\cancel{f(c)}\,f(y) = 2\,\cancel{f(\underbrace{c+yf(c)})}$$

$c \le c + yf(c) \le d$

$y \le \frac{d-c}{f(c)}$

$f(y) = 2$ $\qquad 0 < y < \underbrace{\frac{d-c}{f(c)}}_{\alpha}$

$x = y: f(x) = 2 \quad \Rightarrow \quad f(3x) = 2$

$0 < x < 3^k\alpha \to f(x) = 2$

TST 06/3 $\qquad f: \mathbb{Z} \to \mathbb{Z}$

$$f(m-n+f(n)) = f(m) + f(n)$$

$$m - n + f(n) = n - m + f(m)$$

$$\underbrace{f(n) - 2n}_{c} = \underbrace{f(m) - 2m}_{c}$$

$f(n) = 2n + c$

$a, b \in \mathrm{Im}\, f \Rightarrow a + b \in \mathrm{Im}\, f$

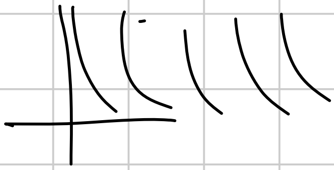
$a \in \mathrm{Im}\, f \Rightarrow na \in \mathrm{Im}\, f$

$a \neq 0 \Rightarrow |\mathrm{Im}\, f| = +\infty$

$a = 0 \Rightarrow f \equiv 0$

$f : \mathbb{Z} \to \mathbb{Z}$ e f periodica $\Rightarrow$ f limitata

$$f(x) = \frac{1}{x} \qquad [0, 1)$$



$$f(m - n + f(n)) = f(m) + f(n)$$

$a < b \qquad f(a) = f(b)$

$n = a \qquad f(m - a + f(a)) = f(m) + f(a)$

$$= f(m) + f(b)$$

$$= f(m - b + f(b))$$

$$m = m + a - f(a)$$

$$f(m) = f(m + a - \cancel{f(a)} - b + \cancel{f(b)})$$

$\Rightarrow f$ limitata $\Rightarrow \quad f(x) = 0$

BMO 07/2 $\qquad f: \mathbb{R} \to \mathbb{R}$

$$f(f(x)+y) = \underline{f(f(x)-y)} + \underline{4f(x)y}$$

$y = f(x) \qquad f(2f(x)) = [2f(x)]^2 + f(0)$

$$f(z) = z^2 + f(0)$$

NO!!!!!! per $z \in \mathbb{R}$

SI :) per $z \in 2\,\mathrm{Im} f$

$$f(x) - y = 2f(w)$$

$$y = f(x) - 2f(w)$$

$$f(2(f(x)-f(w))) = [2(f(x)-f(w))]^2 + f(0)$$

$$\boxed{f(z) = z^2 + f(0)}$$

SI :) $z \in 2\mathrm{Im} f - 2\mathrm{Im} f$

Se $f(x) = 0$ $\qquad$ ok $\leftarrow$

$\exists x_0 : f(x_0) \neq 0 \qquad \leftarrow$

$$f(\quad) - f(\quad) = 4f(x)y$$

$\mathrm{Im} f - \mathrm{Im} f = \mathbb{R}$ $\qquad \underbrace{f(x_0)}_{= \mathbb{R}}$

IMO 09/5

$f: \mathbb{N}^+ \to \mathbb{N}^+$

a $f(b), f(b+f(a)-1)$

$a=1$ $1, f(b), f(b+f(1)-1)$

$$f(b)=f(b+f(1)-1)$$

$\boxed{f(1)=1}$

$$2+a=f(f(a))$$
$$a=f(f(a))$$

NO! per ragioni di iniettività e surgettività

$f(a), f(b), f(a+b-1)$ ←

$a, b, f(f(a)+f(b)-1)$ ←

$$f(a)+f(b) \geq f(a+b-1)+1$$

$$f(a)+f(2) \geq f(a+1)+1$$

$$a, b, f(f(a)+f(b)-1)$$

$a=b=2$ $f(2f(2)-1) \leq 4$

$f(f(2f(2)-1))=f(k)$ $k \leq 3$

$$2f(2)-1=f(k)$$

$$f(3) = 2f(2) - 1$$

$a=3,\ b=2$ $\qquad f(3f(2)-2) = k \qquad k \leq 4$

$$3f(2) - 2 = f(k)$$

$$3f(2)-2 = 3f(2)-1 \quad \Rightarrow ?\ f(2)=1$$

$$f(4) = 3f(2) - 2$$

$$f(n) = (n-1)f(2) - (n-2) \qquad n \geq 2$$

$a=n,\ b=2$

$$f(n) + f(2) - 1 = f(k) \qquad k \leq n+1$$

$$nf(2) - (n-1) = f(k)$$

Se $k \neq n+1$, $k \leq n$

$$nf(2) - (n-1) = (k-1)f(2) - (k-2) \qquad \text{No!}$$

$$\underline{n}(f(2)-1) = (k-1)(f(2)-1) - f(2) + 1$$

Cioè...

$$kf(2) - 1) - 1$$

$$f(2) \geq 2$$

$$f(n) = (n-1)f(2) - (n-2)$$

$$= n(f(2)-1) + \underbrace{2 - f(2)}_{c} = n$$

$f(2) = h \geq 3$ $\qquad f(n) = \underline{n(h-1) + 2 - h}$

$An + B$

$A \neq 1$ per assurdo.

$A = 2$

$2n + B$ $\quad$ $2n + 1 + B$

$2n + 2 + B$

$f(2) = 2$