G1 medium - Contazzi

Note Title 9/3/2017

TI 2017: 10) $z_1 = 18+83i \qquad z_2 = 18+39i$

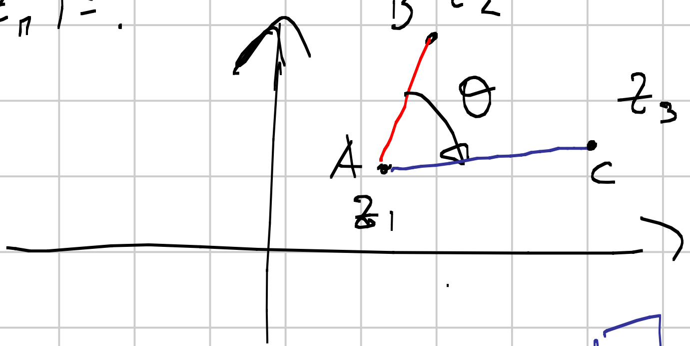
$$z_3 = 78+99i$$

$$S = \left\{ z \in \mathbb{C} : \frac{z_3 - z_1}{z_2 - z_1} \cdot \frac{z - z_2}{z - z_3} \in \mathbb{R} \right\}.$$

$z_\Pi \in S$ è "quello con la parte immaginaria maggiore".

$Re(z_\Pi) = ?$

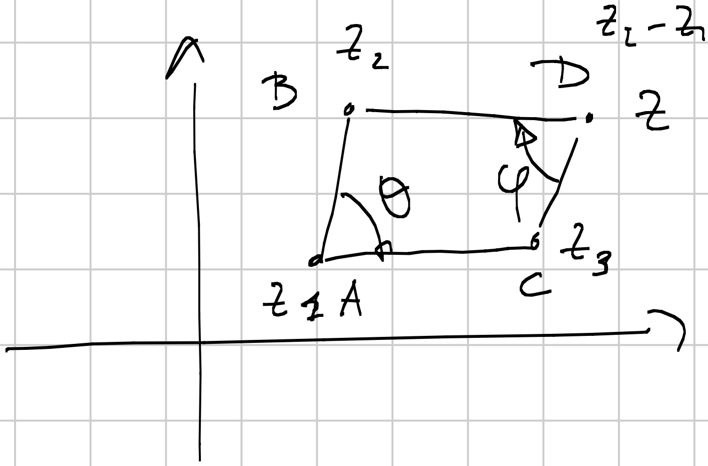
Sol:



$$x \longrightarrow \frac{x - z_1}{z_2 - z_1}$$

$$\frac{z_3 - z_1}{z_2 - z_1} = \boxed{\frac{AC}{AB}} e^{i\theta}$$

$\in \mathbb{R}$



$\mathbb{R}$

$$\frac{z - z_2}{z - z_3} = \boxed{\frac{DB}{DC}} e^{i\varphi}$$

$$z \in S$$
$$\Updownarrow$$
$$e^{i\theta} \cdot e^{i\varphi} \in \mathbb{R}$$
$$\|$$
$$\theta + \varphi = k\pi_{\ k \in \mathbb{Z}} \iff e^{i(\theta+\varphi)} \in \mathbb{R}$$

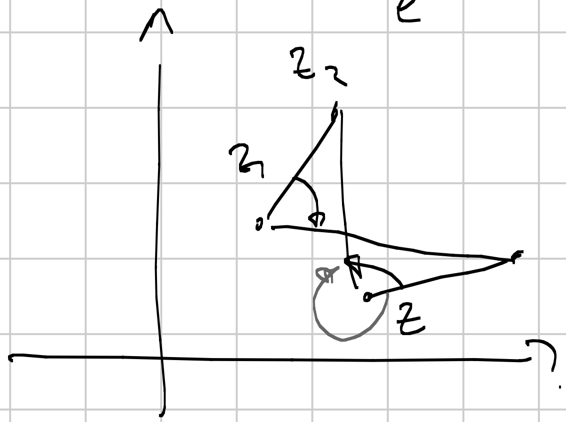
$\theta + \varphi = \pi \iff z_1, z$ stanno da parti opp. di BC.

e ABCD ciclico

$\theta + \varphi = \underset{0}{2\pi} \iff z_1, z$ stanno dalla stessa parte di BC

e $\widehat{BAC} = \widehat{BDC}$

$\iff$ ABCD ciclico.



$\Rightarrow \quad S = \text{cfr per } z_1, z_2, z_3$

<u>Oss</u>: Se z_1, z_2, z_3 sono allineati, S diventa.... una retta!

$$\Downarrow$$

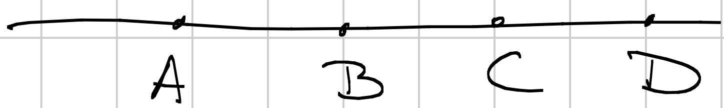
$$\frac{z_3 - z_1}{z_2 - z_1} \in \mathbb{R}$$

$$z \in S \iff \frac{z - z_2}{z - z_3} \in \mathbb{R}$$

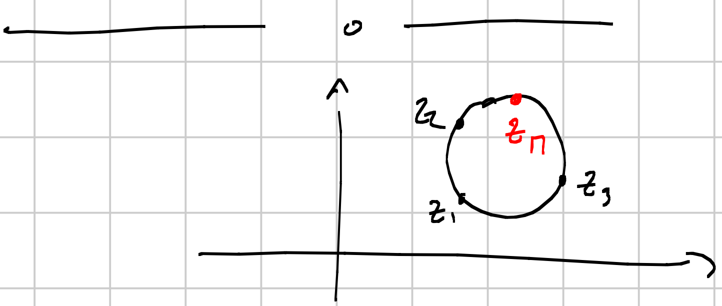
$$\Updownarrow$$

$$\varphi = 0, \pi$$

$$\frac{z_3 - z_1}{z_2 - z_1} \cdot \frac{z - z_2}{z - z_3}$$

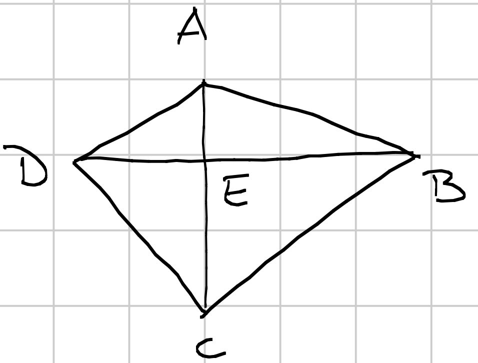


$$\frac{AC}{AB} \cdot \frac{BD}{CD}$$ birapporto $(A, D; B C)$



$\mathrm{Re}(\text{centro}) = 56$

TI '17 - 11)



i simmetrici di E risp. ai lati sono conciclici.

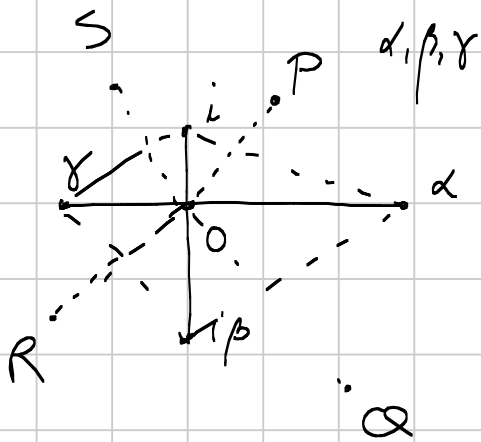
$\boxed{V/F?}$

rimetto tutto a posto.

$$x \longrightarrow x-a \longrightarrow \frac{x-a}{b-a} \longrightarrow \frac{\bar{x}-\bar{a}}{\bar{b}-\bar{a}} \longrightarrow \frac{\bar{x}-\bar{a}}{\bar{b}-\bar{a}}(b-a)$$

traslo A in 0 — porto B in 1 — coniugo

$$\frac{\bar{x}-\bar{a}}{\bar{b}-\bar{a}}(b-a)+a$$



$\alpha, \beta, \gamma \in \mathbb{R}$

$$P = \frac{i}{\alpha+i}(\alpha-i)+i$$

$$Q = \frac{i\beta}{\alpha+i\beta}(\alpha-i\beta)+i\beta$$

$$R = \frac{i\beta}{\gamma+i\beta}(\gamma-i\beta)+i\beta$$

$$S = \frac{i}{\gamma+i}(\gamma-i)+i$$

$$P = \frac{i}{\alpha+i}(\alpha-i)+i = i\left(\frac{\alpha-i+\alpha+i}{\alpha+i}\right) = \frac{2\alpha i}{\alpha+i}$$

$$Q = \frac{2\alpha\beta i}{\alpha+i\beta} \qquad R = \frac{2\gamma\beta i}{\gamma+i\beta} \qquad S = \frac{2\gamma i}{\gamma+i}$$

$$\frac{p-q}{p-r} \cdot \frac{s-r}{s-q} \in \mathbb{R} \quad \leftarrow \text{per casa}$$

<u>Oss</u>: se $a, b \in$ cf. unitaria

$$\frac{\bar{x}-\bar{a}}{\bar{b}-\bar{a}}(b-a)+a = \frac{\bar{x}-\frac{1}{a}}{\frac{1}{b}-\frac{1}{a}}(b-a)+a =$$

$$= \frac{\frac{\bar{x}a-1}{\cancel{a}}}{\frac{\cancel{a-b}}{\cancel{a}b}}\underbrace{\cancel{(b-a)}}_{}^{-1}+a =$$

$$= b - \bar{x}ab + a = a + b - ab\bar{x}$$

—— • ——

<u>Vettori</u>:

1) Combinazioni convesse
2) Prodotto scalare
3) Prodotto vettore

1)

$$A \quad P \qquad\qquad B$$

$$\boxed{\frac{AP}{PB}} = \lambda \qquad \vec{P} = ?$$

$$\vec{P} = \frac{\lambda\vec{B}+\vec{A}}{\lambda+1}$$

$$\vec{P}-\vec{A} = \lambda(\vec{B}-\vec{P})$$

retta AB

$$\boxed{\vec{P} = \frac{\alpha\vec{A}+\beta\vec{B}}{\alpha+\beta} = h\vec{A}+k\vec{B} \quad \boxed{h+k=1}}$$

$$\boxed{\vec{P} = \alpha\vec{A}+\beta\vec{B}, \quad \alpha,\beta\in\mathbb{R}}$$ tutti i punti del piano

$$\vec{P}-\vec{A} = \frac{\alpha A+\beta B-\alpha A-\beta A}{\alpha+\beta} = \frac{\beta}{\alpha+\beta}(B-A)$$

$$B-P = \frac{B\alpha+B\beta-\alpha A-\beta B}{\alpha+\beta} = \frac{\alpha}{\alpha+\beta}(B-A)$$

$$\boxed{\frac{AP}{PB} = \frac{\beta}{\alpha}}$$

$$\vec{P} = \frac{\alpha\vec{A} + \beta\vec{B} + \gamma\vec{C}}{\alpha+\beta+\gamma}$$

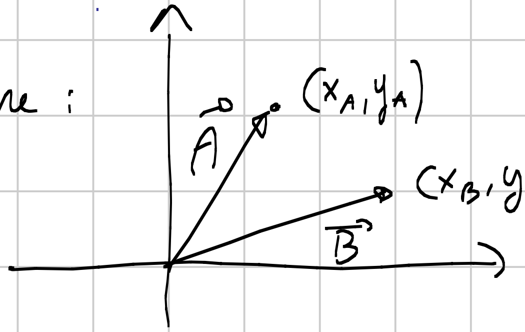
$$\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$$

$$\vec{I}_A = \frac{-a\vec{A} + b\vec{B} + c\vec{C}}{-a+b+c}$$

$\vdots$

$$\alpha = \beta = \gamma \longrightarrow \vec{G} = \frac{\vec{A}+\vec{B}+\vec{C}}{3}$$

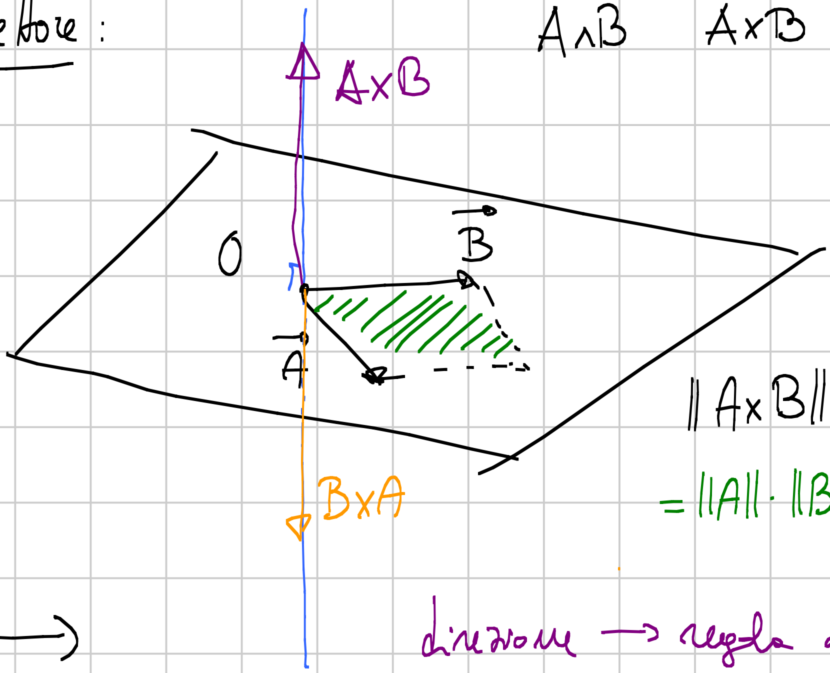
2) Prodotto scalare:



$$\vec{A}\cdot\vec{B} = x_A x_B + y_A y_B \qquad |AB|^2 = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B}) =$$
$$= \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} - 2\vec{A}\cdot\vec{B}$$

3) Prodotto vettore:

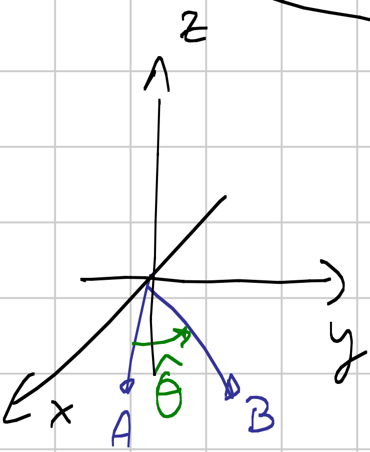
$A \wedge B \qquad A \times B$



$\|A\times B\|$ = area verde =

$= \|A\|\cdot\|B\|\cdot|\sin \widehat{AB}|$

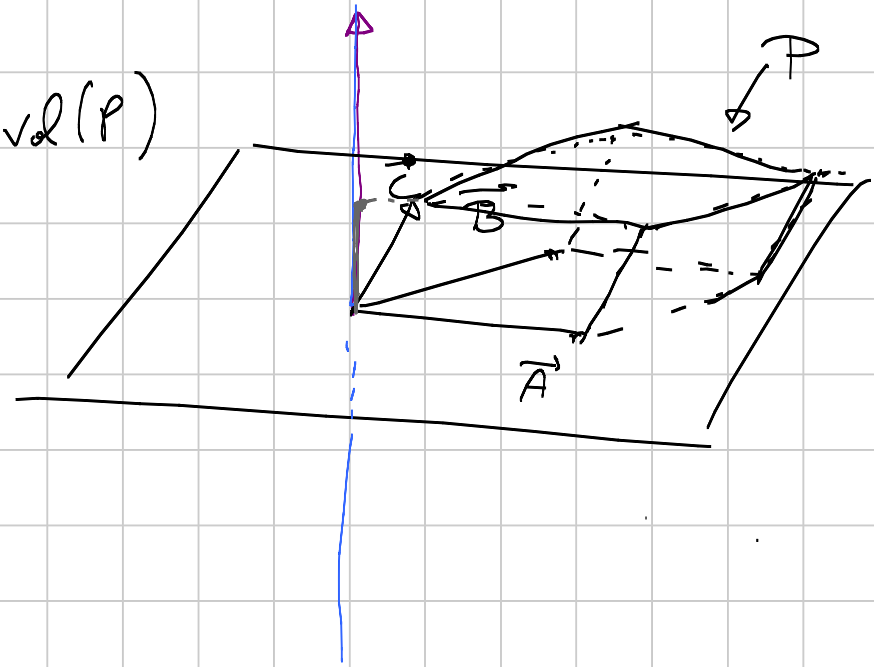
direzione → regola della mano dx



θ = angolo orientato da $\vec{A}$ verso $\vec{B}$

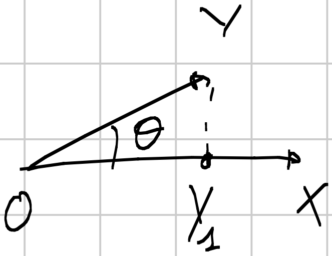
$$A\times B = (0,\ 0,\ \|A\|\cdot\|B\|\cdot\sin\theta)$$

Oss: $(A\times B)\cdot C = \pm \text{vol}(P)$



$X\cdot Y = \|X\|\cdot\|Y\|\cdot\cos\widehat{XY} =$

$= \|X\|\cdot\|\text{proiez. di } Y \text{ in } X\|$



$$X\cdot Y = OY_1\cdot OX$$

$$(A\times B)\cdot C = \det(A,B,C)$$

1) dimostro che $(A_1+A_2)\times B =$

$$= A_1\times B + A_2\times B$$

1b) dimostro che $(kA)\times B = k(A\times B) \quad k\in\mathbb{R}$

2) $\hat{\imath} = (1,0,0) \qquad \hat{\jmath} = (0,1,0) \qquad \hat{k} = (0,0,1)$

$$\hat{\imath}\times\hat{\jmath} = \hat{k} \qquad \hat{\jmath}\times\hat{k} = \hat{\imath} \qquad \hat{k}\times\hat{\imath} = \hat{\jmath}$$

3) $(x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k})\times(x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k})$

$$\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{pmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{pmatrix} \end{array} \rightsquigarrow (y_1z_2 - y_2z_1)\hat{\imath} - (x_1z_2 - x_2z_1)\hat{\jmath} +$$

$$+ (x_1y_2 - x_2y_1)\hat{k}$$

$$\begin{array}{ccc} + & - & + \\ \end{array}$$
$$\begin{pmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{pmatrix} \qquad \begin{pmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{pmatrix}$$

Es $(1,2,3) \times (1,2,1) = (-4,2,0)$

$$\begin{pmatrix} i & j & k \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix} = i(-4) - j(-2) + k(0) =$$
$$= -4i + 2j + 0k$$

$$\Rightarrow \det(A, B, C) = (A \times B) \cdot C =$$
$$= x_C(y_A z_B - y_B z_A) - y_C(x_A z_B - x_B z_A) + z_C(x_A y_B - x_B y_A)$$

$$\det \begin{pmatrix} x_A & x_B & x_C \\ y_A & y_B & y_C \\ z_A & z_B & z_C \end{pmatrix} = x_A y_B z_C + x_B y_C z_A + y_A z_B x_C -$$
$$- x_C y_B z_A - y_C z_B x_A - x_B y_A z_C$$

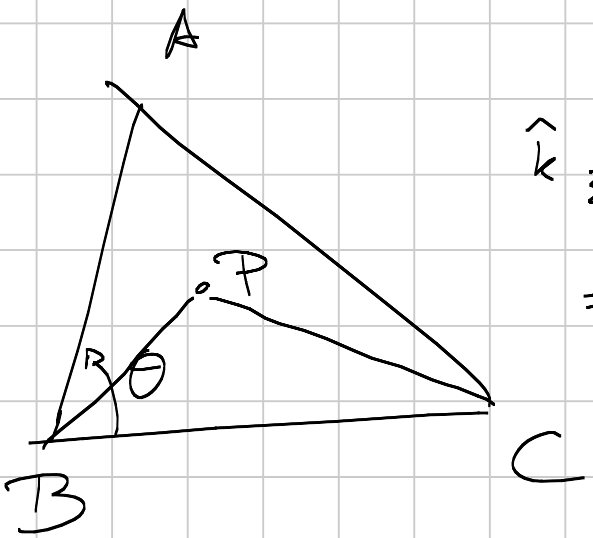
$$\boxed{A = (a_{ij})_{\substack{i=1\dots n \\ j=1\dots n}} \qquad \det A = \sum_{\sigma \in S_n} (-1)^{|\sigma|} \prod_{i=1}^{n} a_{i\sigma(i)}}$$

Es: $\det \begin{pmatrix} a & -a & a \\ -b & b & b \\ c & c & -c \end{pmatrix} = -4abc$

$\det \begin{pmatrix} a & -b & c \\ a & b & -c \\ 1 & 0 & 0 \end{pmatrix} = 0$

Ponte: $\overline{P} = \dfrac{\alpha A + \beta B + \gamma C}{\alpha + \beta + \gamma}$

$\frac{1}{2}\|A \times B\| = \text{Area}(A\overset{\Delta}{O}B)$
compatibile con aree orientate



$\hat{k}\ \frac{1}{2} BC \cdot BA \sin\theta =$

$= \frac{1}{2}(C-B)\times(A-B)$

$$[PBC] = \frac{1}{2}(C-B)\times(P-B) =$$

$$= \frac{1}{2}(C-B)\times\left(\frac{\alpha A + \beta B + \gamma C}{\alpha+\beta+\gamma} - B\right) =$$

$$= \frac{1}{2}(C-B)\times\left(\frac{\alpha(A-B)+\gamma(C-B)}{\alpha+\beta+\gamma}\right) =$$

$$= [ABC]\cdot\frac{\alpha}{\alpha+\beta+\gamma}$$

$$[PAB] = \frac{\gamma}{\alpha+\beta+\gamma}[ABC] \qquad [APC] = \frac{\beta}{\alpha+\beta+\gamma}[ABC]$$

Coordinate baricentriche di P rispetto ad $A\overset{\Delta}{B}C$

$([PBC], [APC], [ABP])$ o un suo multiplo

notazione alternativa: $[\alpha : \beta : \gamma] \leftarrow$ terna omogenea

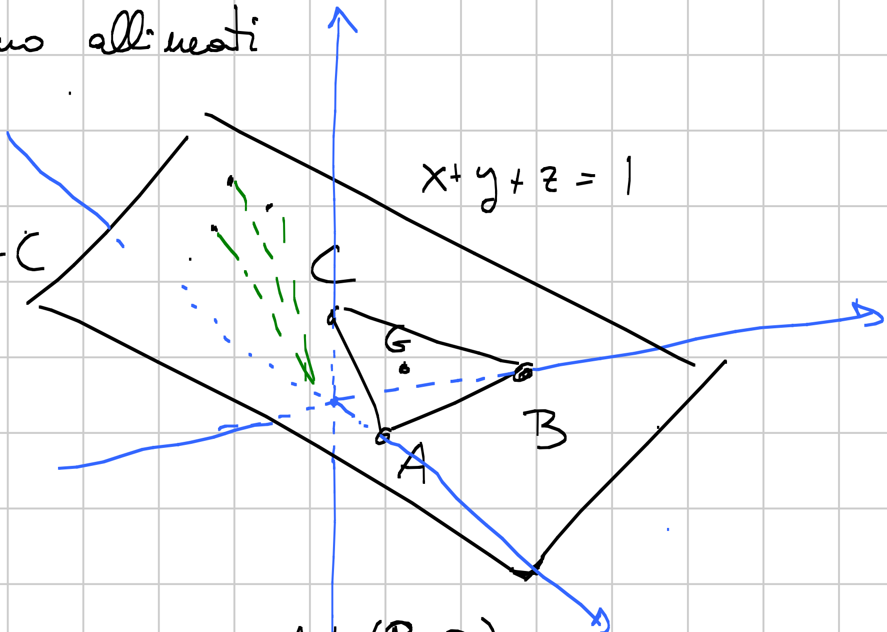
Coord. esatte: $\left(\dfrac{[PBC]}{[ABC]}, \dfrac{[APC]}{[ABC]}, \dfrac{[ABP]}{[ABC]}\right)$

Es: G baricentro di $ABC \Rightarrow G = (1,1,1)$

Es: I incentro di $ABC \Rightarrow I = (a,b,c)$

I_A A-excentro $\Rightarrow I_A = (-a,b,c)$

Es: I_C, A, I_B sono allineati



$$P = \frac{\alpha}{\alpha+\beta+\gamma} A + \frac{\beta}{\alpha+\beta+\gamma} B + \frac{\gamma}{\alpha+\beta+\gamma} C$$

P, Q, R allineati

$\Downarrow$

$\mathrm{vol}(OPQR) = 0 \iff \det(PQR) = 0$

$$\Rightarrow I_C, A, I_B \text{ allineati} \iff \det\begin{pmatrix} a & b & -c \\ a & -b & c \\ 1 & 0 & 0 \end{pmatrix} = 0$$

che è vero!

Eq. di una retta

Es: mediana $A = (1,0,0)$, $G = (1,1,1)$

$$\left\{ (x,y,z) \text{ omogenee t.c. } \det\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ x & y & z \end{pmatrix} = 0 \right\}$$

$z - y = 0 \iff z = y$

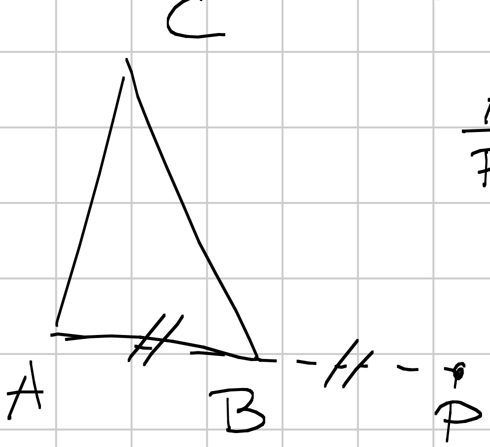
$$\Rightarrow \text{pt. medio di } BC \quad \begin{cases} z = y \\ x = 0 \end{cases} \longrightarrow (0, \lambda, \lambda) \quad \lambda \in \mathbb{R}$$

$\to (0,1,1)$

$$\frac{AP}{PB} = \frac{\lambda}{\mu} \Rightarrow$$

$$\Rightarrow P = \frac{\mu A + \lambda B}{\lambda + \mu}$$

Es: Simmetrico di A rispetto a B



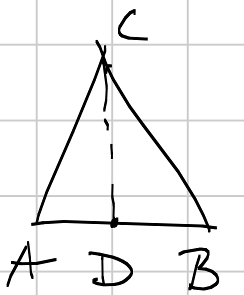
$$\frac{AP}{PB} = -2$$

$$\vec{P} = \frac{-2\vec{B} + \vec{A}}{-1} = 2\vec{B} - \vec{A}$$

$A = (1,0,0)$
$B = (0,1,0)$ somma $= 1$ $\Rightarrow P = 2(0,1,0) - (1,0,0) =$

$= (-1,2,0)$

Es: Piede della bisettrice



$$\frac{AD}{DB} = \frac{b}{a} \qquad \vec{D} = \frac{a\vec{A} + b\vec{B}}{a+b}$$

$$\vec{D} = \left(\frac{a}{a+b}, \frac{b}{a+b}, 0\right)$$

$= (a, b, 0) \leftarrow$ perché posso molt. per $(a+b)$ per omogeneità.

Notazione di Conway

$$S = 2[ABC], \qquad S_\theta = S \cdot \cot\theta \qquad S_{\theta\varphi} = S_\theta \cdot S_\varphi$$

$$S_A = S \cdot \cot A = 2[ABC]\frac{\cos A}{\sin A} = bc \cdot \cos A =$$

$$= \cancel{bc}\,\frac{b^2+c^2-a^2}{2\cancel{bc}} = \frac{b^2+c^2-a^2}{2}$$

$$S_B = \frac{a^2+c^2-b^2}{2}$$

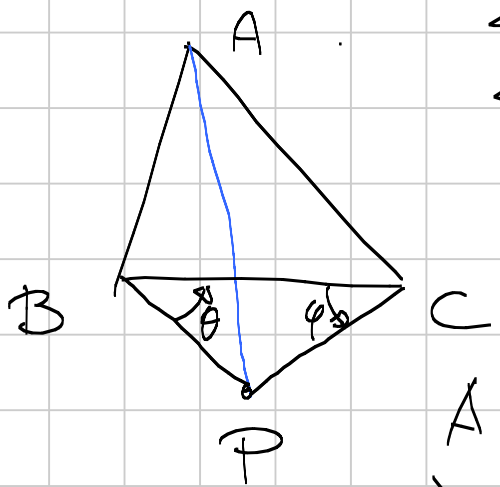
$$S_C = \frac{a^2+b^2-c^2}{2}$$

Oss: $S_B + S_C = a^2$

$$A+B+C=\pi$$

$$S_{AB}+S_{BC}+S_{CA} = S^2 \qquad (\cot A\cot B + \cot B \cot C + \cot C \cot A = 1)$$

Formula di Conway

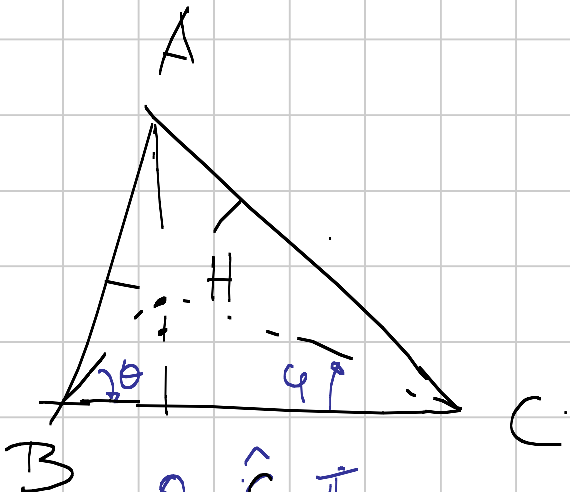


$\sphericalangle PBC = \theta$

$\sphericalangle BCP = \varphi$

$$\Rightarrow P: (-a^2, S_C + S_\varphi, S_B + S_\theta)$$

Es: Ortocentro



$\theta = \hat{C} - \frac{\pi}{2}$ $\qquad \cot\theta = -\operatorname{tg}\hat{C}$

$\varphi = \hat{B} - \frac{\pi}{2}$ $\qquad \cot\varphi = -\operatorname{tg}\hat{B}$

$$H: (-a^2; S_C + S_\varphi; S_B + S_\theta) =$$

$$S_\theta = S\cdot\cot\theta = -S\operatorname{tg}\hat{C} = -\frac{S^2}{S_C}$$

$$= \left(-a^2; S_C - \frac{S^2}{S_B}; S_B - \frac{S^2}{S_C}\right) =$$

$$=\left(-a^2 S_B S_C : S_C(S_C S_B - S^2) : S_B(S_B S_C - S^2)\right)=$$

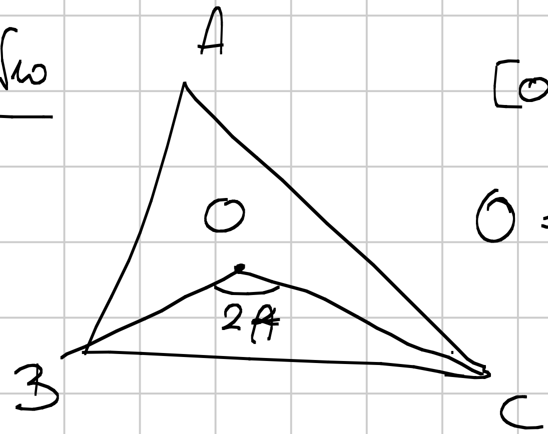
$$=\left(-(S_B+S_C)S_B S_C : S_C(-S_{AC}-S_{AB}) : S_B(-S_{AC}-S_{AB})\right)=$$

$$=\left((S_B+S_C)S_{BC} : S_{AC}(S_C+S_B) : S_{AB}(S_C+S_B)\right)=$$

$$=\left(S_{BC} : S_{AC} : S_{AB}\right)=\left(\cot B\cot C : \cot A\cot C : \cot A\cot B\right)=$$

$$=\left(\frac{1}{\cot A} : \frac{1}{\cot B} : \frac{1}{\cot C}\right)=\left(\mathrm{tg}\,A : \mathrm{tg}\,B : \mathrm{tg}\,C\right)$$

Es: O circocentro



$$[OBC]=\frac{1}{2}R^2\sin 2A$$

$$O=(\sin 2A : \sin 2B : \sin 2C)$$

Es: Centro della circonf. di Feuerbach?

$$O=(a^2 S_A : cyc : cyc)$$

$$H=(S_B S_C : S_A S_C : S_B S_A)$$

$$O=(S_A S_B + S_A S_C : cyc : cyc)$$

$$H \to \Sigma = S^2$$

$$O \to \Sigma = 2S^2$$

$$\cot A=\frac{\cos A}{\sin A}$$

$$\cot A \cdot \sin^2 A=\frac{1}{2}\sin 2A$$

$$S_A \cdot \sin^2 A=\frac{S}{2}\sin 2A$$

$$S_A \frac{a^2}{4R^2}=\frac{S}{2}\sin 2A$$

$$S_A a^2=\boxed{2R^2 S}\sin 2A$$

$$\frac{1}{2}\left(\left(S_{BC}:S_{AC}:S_{AB}\right)+\left(\frac{S_{AB}+S_{AC}}{2}:\frac{S_{BA}+S_{BC}}{2}:\frac{S_{CA}+S_{CB}}{2}\right)\right)=$$

$$=\left(\frac{S_{AB}+S_{AC}+2S_{BC}}{4}:\ cyc\ :\ cyc\right)=$$

$$=\left(S^2+S_{BC}:\ cyc\ :\ cyc\right)$$

$$S^2(1+\cot B\cot C)=$$

$$=S^2\frac{(\sin B\sin C+\cos B\cos C)}{\sin B\sin C}=$$

$$=\frac{S^2}{\sin B\sin C}\left(\cos(B-C)\right)$$

$$=ac\,ab\left(\cos(B-C)\right)=$$

$$=\underset{\text{simm}}{\boxed{abc}}\cdot a\cos(B-C)$$

(basta cyc)

$$=\left(a\cos(B-C):\ cyc\ :\ cyc\right)$$

Es: Punto medio di AH.

$H=(S_{BC}:S_{AC}:S_{AB})$

$\Sigma=S^2$

$A:(S^2:0:0)$

pt. medio di $AH=$

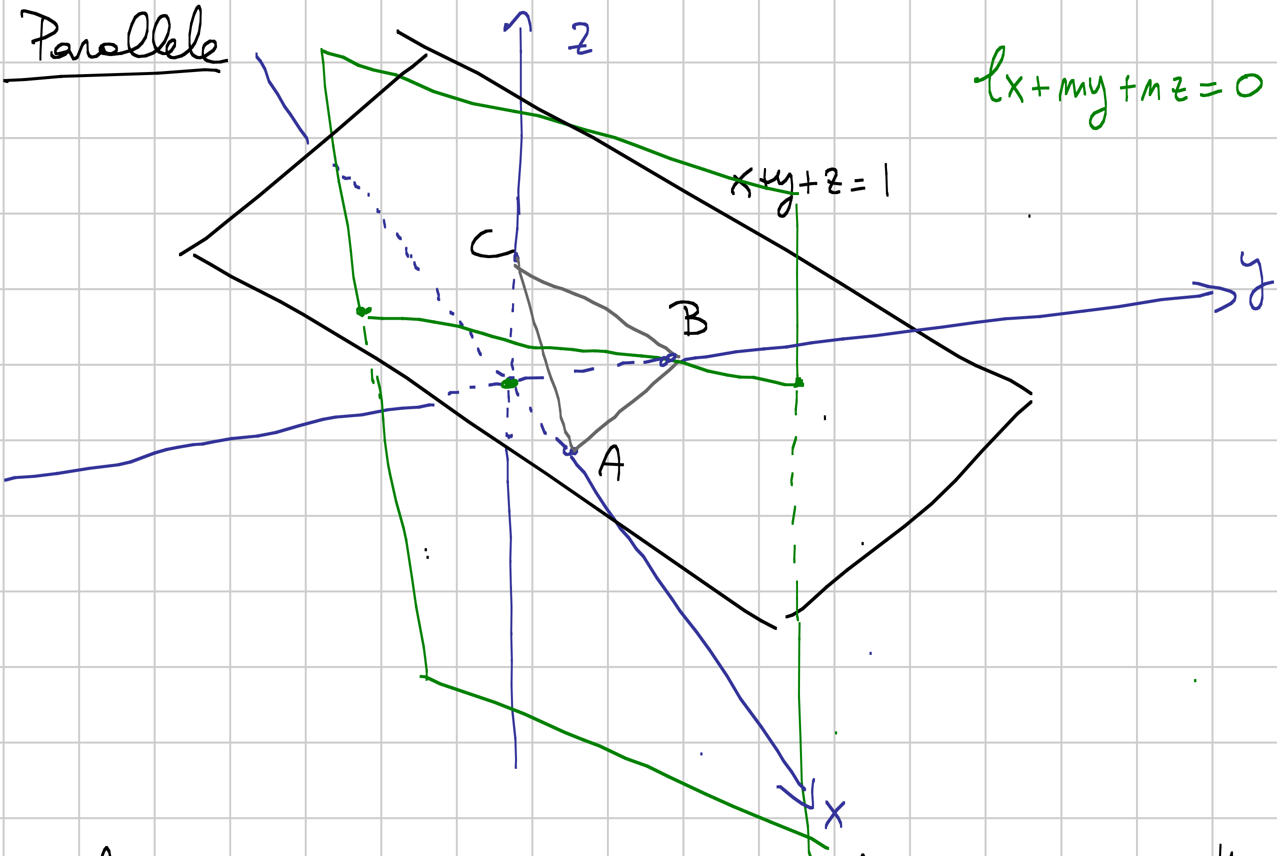
$$=\cancel{\tfrac{1}{2}}\left(S^2+S_{BC}:S_{AC}:S_{AB}\right)=$$

$$=\left(\cancel{abc}\,a\cos(B-C):\underset{\cancel{abc}\cdot b}{\boxed{\frac{S^2}{\sin A\sin C}}}\cos A\cos C:\underset{\cancel{abc}\cdot c}{\boxed{\frac{S^2}{\sin A\sin B}}}\cos A\cos B\right)=$$

$$= \left(a\cos(B-C) : b\cos A\cos C : c\cos A\cos B \right)$$

Rette parallele e perpendicolari

1) Parallele



Oss: il piano $x+y+z=0$ non corrisponde a nessuna retta nel nuovo piano.

$$\emptyset$$

$$(A\cap B)\cap(C\cap B) = (A\cap C)\cap B$$

retta nel nuovo piano

$\emptyset$

retta nel nuovo piano

retta per l'origine

nuovo piano.

$$= \emptyset \iff A\cap C \subseteq x+y+z=0$$

Due rette sono parallele $\iff$ la soluzione del sistema rispetta $\boxed{x+y+z=0}$

$$\begin{cases} lx + my + nz = 0 \\ l'x + m'y + n'z = 0 \end{cases}$$

Es: AH $\qquad A: (1:0:0) \qquad H = (\operatorname{tg} A : \operatorname{tg} B : \operatorname{tg} C)$

$$0 = \det\begin{pmatrix} 1 & 0 & 0 \\ \operatorname{tg} A & \operatorname{tg} B & \operatorname{tg} C \\ x & y & z \end{pmatrix} \iff \operatorname{tg} B \cdot z = \operatorname{tg} C \cdot y$$

$O\Pi_A \qquad \Pi_A: (0:1:1) \qquad O: (\sin 2A : \sin 2B : \sin 2C)$

$$0 = \det\begin{pmatrix} 0 & 1 & 1 \\ \sin 2A & \sin 2B & \sin 2C \\ x & y & z \end{pmatrix} = \sin 2C \cdot x + \sin 2A \cdot y - \sin 2B \cdot x - \sin 2A \cdot z =$$

$$= x(\sin 2C - \sin 2B) + y \sin 2A - z \sin 2A$$

$$\begin{cases} \operatorname{tg} B \cdot z = \operatorname{tg} C \cdot y \\ x(\sin 2C - \sin 2B) = \sin 2A (z - y) \end{cases}$$

$$z = \frac{\operatorname{tg} C}{\operatorname{tg} B} y$$

$$x = \frac{\sin 2A}{\sin 2C - \sin 2B}\left(\frac{\operatorname{tg} C - \operatorname{tg} B}{\operatorname{tg} B}\right) y$$

$$\left(\frac{\sin 2A}{\sin 2C - \sin 2B} \cdot (\operatorname{tg} C - \operatorname{tg} B) ; \operatorname{tg} B : \operatorname{tg} C\right)$$

$$\boxed{\text{Due rette sono parallele} \iff \text{intersezione} \in x + y + z = 0}$$

Digressione: come si intersecano due piani in $\mathbb{R}^3$ per l'origine.

$\alpha x + \beta y + \gamma z = 0$ $\qquad$ $lx + my + nz = 0$

$\Downarrow$ $\qquad$ $\Downarrow$ piano $\perp$ (l, m, n)

$(x,y,z) \cdot (\alpha,\beta,\gamma) = 0$

$\Downarrow$

piano $\perp$ a (α, β, γ)

$\Rightarrow$ la loro intersezione è la retta generata da $(\alpha,\beta,\gamma) \times (l, m, n)$

$\Rightarrow$ il punto di intersez. delle rette corrisp. in coord. baricentriche è $(\alpha,\beta,\gamma) \times (l, m, n)$

voglio sapere se questo punto appartiene alla retta $px + qy + rz = 0$

$\Downarrow$

vett t.c.

$(p,q,r) \cdot (x,y,z) = 0$

$\Rightarrow$ voglio sapere se $((\alpha,\beta,\gamma) \times (l,m,n)) \cdot (p,q,r) = 0$

$\Updownarrow$

$$\det \begin{pmatrix} \alpha & l & p \\ \beta & m & q \\ \gamma & n & r \end{pmatrix} = 0 \iff \begin{cases} \alpha x + \beta y + \gamma z = 0 \\ lx + my + nz = 0 \\ px + qy + rz = 0 \end{cases} \text{ concorrono}$$

Corollario: due rette sono $\parallel$

$\Updownarrow$

$$\det \begin{pmatrix} l & l' & 1 \\ m & m' & 1 \\ n & n' & 1 \end{pmatrix} = 0$$

Rette per la parallela

Es: parallela a BC per I

1) scrivo BC $\quad x=0$
2) trovo il "punto all'infinito" $\begin{cases} x=0 \\ x+y+z=0 \end{cases} \to (0:1:-1) = P_\infty$
3) Trovo la retta per I e P_∞

$$0=\det\begin{pmatrix} a & b & c \\ 0 & 1 & -1 \\ x & y & z \end{pmatrix} = az - bx - cx + ay =$$

$$= -(b+c)x + a(y+z)$$

$$x(b+c) = a(y+z)$$

2) Perpendicolari: 2a) casi speciali

perpendicolari ai lati
$\|$
parallele alle altezze

2b) formula generale [lasciatela perdere]

Extra: Circonferenze | Coniugati isogonali

$\updownarrow$

distanze e angoli

$$(x:y:z) \to \left(\frac{a^2}{x} : \frac{b^2}{y} : \frac{c^2}{z}\right)$$