ALGEBRA 2 - BASIC

Titolo nota 05/09/2018

DISUGUAGLIANZE

- $a^2+b^2+c^2 \geq ab+bc+ca \qquad a,b,c \in \mathbb{R}$
- $\dfrac{a}{b+c}+\dfrac{b}{a+c}+\dfrac{c}{a+b} \geq \dfrac{3}{2}$ [Nesbitt] $a,b,c \in \mathbb{R}^+$
- $xz+yz \leq C(x^2+y^2+z^2)$

  ↑ trovare la migliore costante

NOTAZIONE

in 3 var, a, b, c $\qquad \sum_{cyc} f(a,b,c) = f(a,b,c)+f(b,c,a)+f(c,a,b)$

$$\sum_{cyc} a = a+b+c \qquad \sum_{cyc} ab = ab+bc+ca$$

in n variabili $\qquad \sum_{cyc} f(a_1,a_2,\dots a_n) = f(a_1,\dots)+f(a_2,\dots)+\dots+f(a_n,\dots)$

$$\sum_{sym} f(a,b,c) = f(a,b,c)+f(a,c,b)+f(b,c,a)+f(b,a,c)+f(c,a,b)+f(c,b,a)$$

$$\sum_{sym} a = a+a+b+b+c+c = 2(a+b+c)$$

$$\sum_{sym} ab = 2\sum_{cyc} ab$$

$$\sum_{sym} a^2 b = a^2 b + a^2 c + b^2 a + b^2 c + c^2 a + c^2 b$$

$$\sum_{cyc} a^2 b = a^2 b + b^2 c + c^2 a$$

$$[2,1,0] = \sum_{sym} a^2 b \qquad [i,j,k] = \sum_{sym} a^i b^j c^k$$

SOS

$$x^2 \geq 0$$

$$x = a - b \Rightarrow (a-b)^2 \geq 0 \quad a^2 + b^2 \geq 2ab$$

$$\sum_{cyc} (a-b)^2 \geq 0$$

$$\parallel$$

$$\sum_{cyc} a^2 + b^2 - 2ab = 2\sum_{cyc} a^2 - 2\sum_{cyc} ab$$

$$\sum_{cyc} a^2 \geq \sum_{cyc} ab$$

RIARRANGIAMENTO

con le cifre $5, 2, 3, 7$ – numero più grande? 7532

– piccolo? 2357

$a_1 \geq \dots \geq a_n$

$b_1 \geq \dots \geq b_n$

$$\sum_{i=1}^{n} a_i b_i \geq \sum a_i b_{\sigma(i)} \geq \sum a_i b_{n-i+1}$$

dim.

FATTO: $x \geq a,\ y \geq b \quad (x-a)(y-b) \geq 0$

$$xy + ab \geq xb + ya$$

$(1) = \sum a_i b_{\sigma(i)} \qquad \sigma \neq id \qquad i < j$ t.c. $\sigma(i) > \sigma(j)$

$$\underbrace{a_i b_{\sigma(i)} + a_j b_{\sigma(j)}}$$

$a_i \geq a_j$

$b_{\sigma(j)} \geq b_{\sigma(i)}$

$$a_i b_{\sigma(j)} + a_j b_{\sigma(i)} \geq$$

FORMALMENTE: induzione sulla "lunghezza" di σ

quella inversa si fa uguale, oppure

prendiamo $b_i' = -b_i$

CHEBYCHEFF

$a_1 \geq \dots \geq a_n$

$b_1 \geq \dots \geq b_n$

$$\frac{\sum a_i b_i}{n} \geq \left(\frac{\sum a_i}{n}\right)\left(\frac{\sum b_i}{n}\right)$$

dim. $\left(\sum a_i\right)\left(\sum b_i\right) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i b_{i+j}$

MOD n

usiamo riarr. $\sum_{k=1}^{n} a_k b_k \geq \sum_{j=1}^{n} a_i b_{i+j}$

$$n \sum a_i b_i \geq \left(\sum a_i\right)\left(\sum b_i\right)$$

$$a^2+b^2+c^2 \geq ab+bc+ca$$

WLOG $a \geq b \geq c$

$$\begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix} \geq \begin{pmatrix} a & b & c \\ b & c & a \end{pmatrix}$$

$$a^2+b^2+c^2 \geq ab+bc+ca$$

NESBITT $\dfrac{a}{b+c}+\dfrac{b}{c+a}+\dfrac{c}{a+b} \geq \dfrac{3}{2}$

WLOG $[a \geq b \geq c]$ $a+b \geq a+c \geq b+c$

$$\left[\frac{1}{a+b} \leq \frac{1}{a+c} \leq \frac{1}{b+c}\right]$$

$$\sum_{cyc} \frac{a}{b+c} \geq \sum_{cyc} \frac{b}{b+c}$$

$$\sum_{cyc} \frac{a}{b+c} \geq \sum_{cyc} \frac{c}{b+c}$$

$$+ \implies 2\sum_{cyc} \frac{a}{b+c} \geq \sum \frac{b+c}{b+c} = 3$$

CAUCHY-SCHWARZ (CS PER GLI AMICI)

$x_1, \dots, x_n$
$y_1, \dots, y_n$

$$\left(\sum x_i y_i\right)^2 \le \left(\sum x_i^2\right)\left(\sum y_i^2\right)$$

dim_1

$$f(t) = \sum (x_i - t y_i)^2 = t^2\left(\sum y_i^2\right) - 2t\left(\sum x_i y_i\right) + \sum x_i^2$$

$f(t) \ge 0 \quad \forall t \in \mathbb{R}$

$\Updownarrow$

$\Delta \le 0$

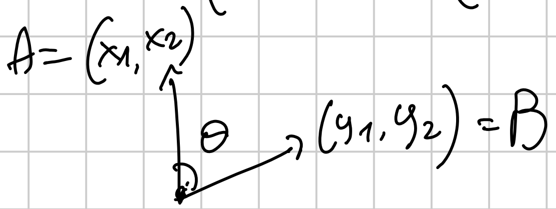
ma $\Delta = 4\left(\sum x_i y_i\right)^2 - 4\left(\sum y_i^2\right)\left(\sum x_i^2\right)$

caso di uguaglianza?

$\Delta = 0$, ovvero $f(t)$ ha una radice λ

$f(\lambda) = 0 \iff \boxed{x_i - \lambda y_i = 0} \quad \forall i = 1, \dots, n$

$\frac{x_i}{y_i} = \lambda$



$x_1^2 + x_2^2 = \|A\|^2$ $\qquad$ $x_1 y_1 + x_2 y_2 = \langle A, B \rangle$

$y_1^2 + y_2^2 = \|B\|^2$

$$|\langle A, B\rangle| = \|A\| \cdot \|B\| \cdot \cos\theta$$

e' = in CS si ha per $\cos\theta = 1$ ovvero $\theta = 0$

TITU

$a_1, \dots, a_n \in \mathbb{R}$

$x_1, \dots, x_n \in \mathbb{R}^+$

$$\sum \frac{a_i^2}{x_i} \geq \frac{\left(\sum a_i\right)^2}{\sum x_i}$$

dim. $\left(\sum a_i\right)^2 \leq \left(\sum \frac{a_i^2}{x_i}\right)\left(\sum x_i\right)$

che è CS su $\left(\frac{a_1}{\sqrt{x_1}}, \dots \frac{a_n}{\sqrt{x_n}}\right)$ e $\left(\sqrt{x_1}, \dots, \sqrt{x_n}\right)$

rifacciamo Nesbitt

$$\sum \frac{a}{b+c} \overset{\text{TITU}}{\geq} \frac{\left(\sum \sqrt{a}\right)^2}{\sum b+c} = \frac{\sum a + 2\sum \sqrt{ab}}{\cancel{2}\sum a} \overset{?}{\geq} \frac{3}{\cancel{2}}$$

$$\sum a + 2\sum \sqrt{ab} \overset{?}{\geq} 3\sum a \qquad \sum \sqrt{ab} \overset{?}{\geq} \sum a$$

IN REALTÀ È FALSA

è $\sum a^2 \geq \sum ab$ con le radici, al contrario

$$\sum \frac{a}{b+c} = \sum \frac{a^2}{ab+ac} \overset{\text{TITU}}{\geq} \frac{\left(\sum a\right)^2}{\sum ab+ac} = \frac{\sum a^2 + 2\sum ab}{\cancel{2}\sum ab} \overset{?}{\geq} \frac{3}{\cancel{2}}$$

$\sum a^2 \geq \sum ab$ ✓ la Nesbitt è vera dim. 2 CS

x_i, y_i $\qquad \sum_{i,j} (x_i y_j - x_j y_i)^2 \geq 0$

$$\sum_{i,j} x_i^2 y_j^2 + x_j^2 y_i^2 - 2 x_i x_j y_i y_j \geq 0$$

$$\cancel{2} \sum_{i,j} x_i^2 y_j^2 - \cancel{2} \sum_{i,j} \underbrace{x_i x_j y_i y_j}_{(x_i y_i)(x_j y_j)} \geq 0$$

$$\overset{\shortparallel}{\left(\sum x_i^2\right)\left(\sum y_j^2\right)} - \left(\sum x_i y_i\right)^2 \geq 0$$

$$\sum_{i,j} x_i^2 y_j^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i^2 y_j^2 = \sum_{i=1}^{n} x_i^2 \cdot \left(\sum y_j^2\right) = \left(\sum y_j^2\right)\left(\sum x_i^2\right)$$

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max $\quad x + 2y + 3z \quad$ t.c. $\quad x^2 + y^2 + z^2 = 1$

$\uparrow$

$(x, y, z) \cdot (1, 2, 3)$

$$\text{CS}: \quad (x + 2y + 3z)^2 \leq \underbrace{(x^2 + y^2 + z^2)}_{1} \cdot (1^2 + 2^2 + 3^2)$$

$$x + 2y + 3z \leq \sqrt{14}$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \Rightarrow \quad y = 2x \quad z = 3x \quad x^2 + 2^2 x^2 + 3^2 x^2 = 1$$

$$x = \frac{1}{\sqrt{14}}$$

$$x + 2y + 3z = x + 2^2 x + 3^2 x = \frac{14}{\sqrt{14}} = \sqrt{14}$$

$$\sum_{cyc} \sqrt{x(3x+y)} \leq 2(x+y+z)$$

$$\sum_{cyc} \sqrt{x} \cdot \sqrt{3x+y} \leq \sqrt{\sum_{cyc} (\sqrt{x})^2} \cdot \sqrt{\sum_{cyc} (\sqrt{3x+y})^2}$$

$$= \sqrt{\sum_{cyc} x} \cdot \sqrt{\sum_{cyc} 4 \cdot x} = 2\sqrt{\left(\sum_{cyc} x\right)^2} = 2\sum_{cyc} x$$

$$\sum_{cyc} 3x+y = (3x+y)+(3y+z)+(3z+x) = 4(x+y+z)$$

$$\left(\sum x_i y_i\right)^2 \leq \left(\sum x_i^2\right)\left(\sum y_i^2\right)$$

$$\sum x_i y_i \leq \sqrt{\sum x_i^2} \cdot \sqrt{\sum y_i^2}$$ è sempre CS, scritta con le radici.

MEDIE

$a_1, \dots, a_n > 0$

$$AM(a_1, \dots a_n) = \frac{a_1 + \dots + a_n}{n} = \frac{\sum a_i}{n}$$

$$GM(a_1, \dots, a_n) = \sqrt[n]{a_1 \cdots a_n}$$

$$HM(a_1, \dots a_n) = \frac{n}{\frac{1}{a_1} + \dots + \frac{1}{a_n}}$$

$$QM(a_1, \dots, a_n) = \sqrt{\frac{a_1^2 + \dots + a_n^2}{n}}$$

$\forall a_1 \dots a_n > 0$

$$HM \leq GM \leq AM \leq QM$$

l'= vale solo per $a_1 = a_2 = \dots = a_n$

dim $HM \leq AM$

$$\frac{a_1 + \dots + a_n}{n} \geq \frac{n}{\frac{1}{a_1} + \dots + \frac{1}{a_n}} \qquad \left(\sum a_i\right) \cdot \left(\sum \frac{1}{a_i}\right) \geq n^2$$

CS su $(\sqrt{a_1} \dots \sqrt{a_n})$ e $\left(\frac{1}{\sqrt{a_1}}, \dots, \frac{1}{\sqrt{a_n}}\right)$

dim $AM \leq QM$

$$\frac{a_1 + \dots + a_n}{n} \leq \sqrt{\frac{a_1^2 + \dots + a_n^2}{n}}$$

$$(a_1 + \dots + a_n)^2 \leq n \cdot (a_1^2 + \dots + a_n^2)$$

CS su $(1, 1, \dots 1)$ e $(a_1, \dots a_n)$

dim $AM \geq GM$

lemma: se $x_1, \dots, x_n > 0$ $\sum \frac{x_{i+1}}{x_i} \geq n$

dim: riarr.

supp. $x_1 \geq \dots \geq x_n$

$\frac{1}{x_1} \leq \dots \leq \frac{1}{x_n}$

$\frac{x_2}{x_1} + \frac{x_3}{x_2} + \dots \frac{x_n}{x_{n-1}} + \frac{x_1}{x_n}$

allora $\sum \frac{x_{i+1}}{x_i}$ è una permutazione

$$m = \sum x_i \cdot \frac{1}{x_i}$$ è la perm. minima

$$g = GM(a_1 \dots a_n) = \sqrt[n]{a_1 \cdots a_n}$$

$$x_i = \frac{a_1 \cdots a_i}{g^i} \qquad x_1 = \frac{a_1}{g} \quad x_2 = \frac{a_1 \cdot a_2}{g^2}, \; \dots \; x_n = \frac{a_1 \cdots}{g^n} = 1$$

Lemma: $\sum \frac{x_{i+1}}{x_i} \geq n$

vale $\frac{x_{i+1}}{x_i} = \frac{a_{i+1}}{g}$ $\qquad \frac{a_1}{g} + \frac{a_2}{g} + \dots + \frac{a_n}{g} \geq n$

$$\frac{a_1 + \dots + a_n}{n} \geq g$$

es:

min $a^4 + b^2 + c$ t.c. $abc = 1$ $\quad a, b, c > 0$

$$\frac{a^4 + b^2 + c}{3} \geq \sqrt[3]{a^4 b^2 c} = \sqrt[3]{a^3 b} = ?$$

$$\frac{a^4 + \frac{b^2}{2} + \frac{b^2}{2} + \frac{c}{4} + \frac{c}{4} + \frac{c}{4} + \frac{c}{4}}{7} \geq \sqrt[7]{a^4 \left(\frac{b^2}{2}\right)^2 \left(\frac{c}{4}\right)^4}$$

$$= \sqrt[7]{\frac{a^4 b^4 c^4}{2^2 4^4}} = \sqrt[7]{\frac{1}{2^{10}}}$$

$$a^4 + b^2 + c \geq 7 \cdot \sqrt[7]{\frac{1}{2^{10}}}$$

quando l'=? per $a^4 = \frac{b^2}{2} = \frac{c}{4}$

trova C t.c. $xz+yz \le C(x^2+y^2+z^2)$ $\quad x,y,z \in \mathbb{R}^+$

$\forall C' < C \ \exists x_0, y_0, z_0$ t.c. $(\ ) \ge C'(\ )$

$xz \le \frac{x^2+z^2}{2} \quad yz \le \frac{y^2+z^2}{2} \qquad xz+yz \le \frac{1}{2}(x^2+y^2+2z^2)$

$x^2+y^2+z^2 = \left(x^2+\frac{z^2}{2}\right)+\left(y^2+\frac{z^2}{2}\right) \ge \frac{2xz}{\sqrt{2}} + \frac{2yz}{\sqrt{2}}$

$\frac{\sqrt{2}}{\sqrt{2}}(xz+yz) \le \frac{1}{\sqrt{2}}x^2+y^2+z^2$

con $C = \frac{1}{\sqrt{2}}$, la disuguaglianza vale

$a^2+b^2 \ge 2ab$

$x^2 \quad \left(\frac{z}{\sqrt{2}}\right)^2$

$=$ c'è per $x = \frac{z}{\sqrt{2}} = y$ (es: $z=\sqrt{2}$, $x=1$, $y=1$)

$xz+yz = 2\sqrt{2} \qquad \frac{1}{\sqrt{2}}\cdot(1+1+2) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$\sum_{sym} a^3 \ge \sum_{sym} a^2 b$

$2(a^3+b^3+c^3) \qquad \underbrace{a^2b}+\underbrace{a^2c}+\dots$

$\frac{a^3+a^3+b^3}{3} \ge \sqrt[3]{a^6b^3} = a^2b$

$\frac{a^3+a^3+c^3}{3} \ge a^2c$

$$\frac{a^3}{\;} \geq c^2 a$$

$$\frac{a^3}{\;} \geq b^2 a$$

a sinistra abbiamo usato $\frac{6 \cdot a^3}{3} = 2a^3$

si fa anche per murr. WLOG $a \geq b \geq c$

$(a^2, a^2, b^2, b^2, c^2, c^2)$ e $(a, a, b, b, c, c) \leftarrow$ LHS

$(b, c, a, c, a, b) \leftarrow$ RHS

$$\sum_{cyc} a^3 \geq \sum_{cyc} ab$$ $\leftarrow$ NON È OMOGENEA

$f(a,b,c)$ si dice omogenea (di grado d)

se $f(\lambda a, \lambda b, \lambda c) = \lambda^d f(a,b,c)$

$a^2+b^2+c^2$ è omo di grado 2

$\square M(\lambda a_1 \dots \lambda a_n) = \lambda \cdot M(a_1 \dots a_n)$

varrà per $a=b=c$? $3a^3 \geq 3a^2$. per $a \geq 0$

$a \geq 1$

per $a \leq 1$? $3a^3 \leq 3a^2$

Back to medie:

dato $p \in \mathbb{R}$ $\quad a_1 \dots a_n \geq 0$

$$M_p(a_1 \dots a_n) = \left(\frac{\sum a_i^p}{n} \right)^{1/p} \qquad (\text{per } p \neq 0)$$

$M_1 = AM \quad M_2 = QM \quad M_{-1} = HM$

$GM = M_0$

$M_2 \geq M_1 \geq M_0 \geq M_{-1}$

$p \geq q \Rightarrow M_p \geq M_q \qquad M_3 \geq M_{-5}$

$M_{\frac{1}{2}} \geq M_{-\frac{1}{2}}$

SCHUR

$$x^3 + y^3 + z^3 + 3xyz \geq \sum_{sym} x^2 y$$

$$\Updownarrow$$

$$\sum_{cyc} x(x-y)(x-z) \geq 0$$

WLOG $x \geq y \geq z$

$$x(\underbrace{x-y}_{\geq 0})(x-z) + y(\underbrace{y-z}_{\geq 0})(\underbrace{y-x}_{\leq 0}) + z(\underbrace{z-x}_{\leq 0})(\underbrace{z-y}_{\leq 0}) \overset{?}{\geq} 0$$

$$-y(y-z)(x-y) \qquad \underbrace{z(x-z)(y-z)}_{\geq 0}$$

$$= \underset{\geq 0}{(x-y)}\Big[\underbrace{x(x-z)-y(y-z)}_{\geq 0}\Big] + z(\ )(\ ) \geq 0 \quad \checkmark$$

$x \geq y$

$x-z \geq y-z$

BUNCHING

$[2,1,0] = \sum_{sym} a^2 b$. $\qquad [i,j,k] = \sum_{sym} a^i b^j c^k$

$[a_1, \dots, a_n] \geq [b_1, \dots b_n]$

se valgono

$a_1 \geq b_1$

$a_1 + a_2 \geq b_1 + b_2$

$a_1 + a_2 + a_3 \geq b_1 + b_2 + b_3$

$\vdots$

$a_1 + \dots + a_n = b_1 + \dots + b_n$

es: $[3,0,0] \geq [2,1,0] \geq [1,1,1]$

$[3,0,0] + [1,1,1] \geq 2[2,1,0]$

ESERCIZI

ES. 86, 87, 88, 89 PAG. 16

ES. 1, 4, 5, 9, 10 PAG. 217

- $a^3b^6 + b^3c^6 + c^3a^6 + 3a^3b^3c^3 \geq abc(a^3b^3 + b^3c^3 + c^3a^3) + a^2b^2c^2(a^3 + b^3 + c^3)$
- $\sum_{cyc} \frac{a+b}{c} \geq 6$ con $a, b, c > 0$
- trovare la più piccola costante K t.c. $(4x + 5y + 3z)^2 \leq K(3x^2 + 4y^2 + 5z^2)$

CORREZIONE

ES. 88 $\quad \frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 3 \quad a, b, c > 0$

$$\frac{a^2c + b^2a + c^2b}{3} \geq \frac{\cancel{3}abc}{\cancel{3}}$$

AM-GM : $\sqrt[3]{a^2cb^2ac^2b} = \sqrt[3]{a^3b^3c^3}$

ES. 89

min $\frac{xy}{z} + \frac{yz}{x} + \frac{xz}{y}$ con $x + y + z = 17$

$$\frac{1}{2}\left(\frac{xy}{z} + \frac{xz}{y}\right) \geq \sqrt{\frac{xy}{z} \cdot \frac{xz}{y}} = x$$

$$\sum \frac{xy}{z} \geq x + y + z = 17$$

con $x = y = z = \frac{17}{3}$

c'è l'$=$

$\min \quad x+8y+4z \qquad$ con $\quad 4\frac{1}{x} + \frac{2}{y} + z = 3$

$$4\frac{1}{x} + \frac{2}{y} = 3 - z$$

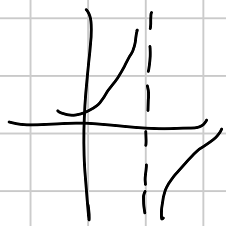
$$x + 8y = \underbrace{\frac{\frac{x}{2} + \frac{x}{2} + 2y + 2y + 2y + 2y}{6}} \geq \frac{6}{2\cdot\frac{2}{x} + 4\cdot\frac{1}{2y}}$$

$$x + 8y \geq \frac{6^2}{\frac{4}{x}+\frac{2}{y}} = \frac{6^2}{3-z}$$

$$x+8y+4z \geq \frac{6^2}{3-z} + 4z$$

il min per z si ha in $z=0$

$$= 6\cdot 2$$



$$\frac{x}{2} = 2y$$

Es. 1)

$x_1, \dots x_n \qquad AM = 8, \; GM = 7, \; HM = 6$

$$y_i = \prod_{j\neq i} x_j = \frac{\prod x_j}{x_i} = \frac{7^n}{x_1} \qquad y_i' = \frac{y_i}{7^n}$$

$$M_1\left(\frac{1}{x_i}\right) = \frac{\frac{1}{x_1} + \dots + \frac{1}{x_n}}{n} = HM^{-1}$$

$$M_{-1}\left(\frac{1}{x_i}\right) = \frac{n}{x_1 + \dots + x_n} = AM^{-1}$$

$$M_0\left(\frac{1}{x_i}\right) = \sqrt[n]{\frac{1}{x_1} \dots \frac{1}{x_n}} = GM^{-1}$$

ES. 4

$\max\ x^5yz$ con $x+y+z=1$

$$\frac{5\cdot\frac{x}{5}+y+z}{7} \geq \sqrt[7]{\frac{x^5}{5^5}yz} \qquad x^5yz \leq \frac{5^5}{7^7}$$

ES. 5

$$\frac{\sum x_i\sqrt{y_i}}{n} = 9 \qquad \frac{\sum y_i}{n} = 8$$

$\uparrow$

SEMBRA IL PROD. DI CS

$$\frac{\left(\sum x_i\sqrt{y_i}\right)^2}{n^2} \leq \frac{\left(\sum x_i^2\right)}{n}\cdot\frac{\left(\sum y_i\right)}{n}$$

$$9^2 \leq \frac{\sum x_i^2}{n}\cdot 8 \qquad \sqrt{\frac{\sum x_i^2}{n}} \geq \frac{9}{\sqrt{8}}$$

ES. 9

$$\sum \frac{x_i}{\sqrt{1-x_i}} \geq \sqrt{\frac{n}{n-1}} \qquad \text{con } x_1+\dots+x_n=1$$

Chebychev su $(x_1\dots x_n),\left(\frac{1}{\sqrt{1-x_1}}\dots\right)$

$$n\sum\frac{x_i}{\sqrt{1-x_i}} \geq \underbrace{\left(\sum x_i\right)}_{=1}\cdot\left(\sum\frac{1}{\sqrt{1-x_i}}\right)$$

$$\text{LHS} \geq AM\left(\frac{1}{\sqrt{1-x_i}}\right) \geq M_{-2}\left(\frac{1}{\sqrt{1-x_i}}\right)$$

$$M_{-2}(\ ) = \left(\frac{\left(\frac{1}{\sqrt{1-x_1}}\right)^{-2} + \dots}{n}\right)^{-\frac{1}{2}} = \left(\frac{\sum 1-x_1}{n}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{n-1}{n}\right)^{-\frac{1}{2}} = \sqrt{\frac{n}{n-1}} = RHS$$

Es. 10

$a+b=z \quad b+c=x \quad c+a=y$

$(a+b)+(c+a)-(b+c)=2a \Rightarrow a = \dfrac{y+z-x}{2}$

NESBITT:
$$\sum \frac{y+z-x}{\cancel{2}x} \geq \frac{3}{\cancel{2}}$$

$$\Vert$$

$$\sum \frac{y}{x} + \frac{z}{x} - 1 \geq 3 \qquad \sum_{sym} \frac{x}{y} \geq 6$$

$$\frac{x}{y} + \frac{x}{z} + \frac{y}{x} + \frac{y}{z} + \frac{z}{x} + \frac{z}{y} \geq 6$$

AMGM $\quad \dfrac{x}{y} + \dfrac{y}{x} \geq 2 \quad \left(\text{in gen } t + \frac{1}{t} \geq 2\right)$

trovare le migliori c_1, c_2 t.c.

$$c_1 \leq \frac{a}{a+b} + \frac{b}{b+c} + \frac{c}{c+a} \leq c_2$$

prove: $-\ a=b=c \rightarrow \frac{3}{2}$

$-\ (1, M, M^2) \rightarrow \underset{\substack{\text{si} \\ 0}}{\dfrac{1}{1+M}} + \underset{\substack{\text{si} \\ 0}}{\dfrac{M}{M+M^2}} + \underset{\substack{\text{si} \\ 1}}{\dfrac{M^2}{M^2+1}} \simeq 1 \qquad M \gg 1$

$- (M^2, M, 1) \to \quad \frac{M^2}{M^2+M} + \frac{M}{M+1} + \frac{1}{1+M^2} \simeq 2$

$\qquad\qquad\qquad\qquad \overset{\simeq}{1} \qquad \overset{\simeq}{1} \qquad \overset{\simeq}{0}$

dim: $\sum \frac{a}{a+b} \geq 1$

$$\sum \frac{a^2}{a^2+ab} \geq \frac{(\sum a)^2}{\sum a^2+ab} = \frac{\sum a^2 + 2\sum ab}{\sum a^2 + \sum ab} \geq 1$$

$\Rightarrow C_1 = 1$

il testo è $\sum \frac{a}{a+b}$, noi prendiamo $\sum \frac{b}{a+b}$

$$= \frac{b}{b+a} + \frac{c}{c+b} + \frac{a}{a+c}$$

$$C_1 \leq \sum \frac{b}{a+b} \leq C_2 \qquad C_1 \leq \sum \frac{a}{a+b} \leq C_2$$

$$\sum \frac{a}{a+b} + \sum \frac{b}{a+b} = 3$$

$$C_2 \geq \underset{\underset{C_1}{\vee\!\vert}}{\sum \frac{a}{a+b}} = \underset{\underset{3-C_2}{\vee\!\vert}}{3 -} \sum \frac{b}{a+b} \leq 3 - C_1$$

$C_1 \leq \frac{3}{2}$

$C_2 \geq \frac{3}{2}$

$C_1 + C_2 = 3$

ricaviamo $C_2 = 2$ (perché ci avviciniamo vicini con $(M^2, M, 1)$)

• trovare la più piccola costante K t.c. $(4x+5y+3z)^2 \le K(3x^2+4y^2+5z^2)$

$$\left(\sqrt{3}x \quad 2y \quad \sqrt{5}z\right)\cdot\left(\frac{4}{\sqrt{3}} \quad \frac{5}{2} \quad \frac{3}{\sqrt{5}}\right) = 4x+5y+3z$$

$$(4x+5y+3z)^2 \le (3x^2+4y^2+5z^2)\underbrace{\left(\left(\frac{4}{\sqrt{3}}\right)^2+\left(\frac{5}{2}\right)^2+\left(\frac{3}{\sqrt{5}}\right)^2\right)}_{=K}$$

• $a^3b^6+b^3c^6+c^3a^6+3a^3b^3c^3 \ge abc(a^3b^3+b^3c^3+c^3a^3)+a^2b^2c^2(a^3+b^3+c^3)$

divido per $a^3b^3c^3$

$$\left(\frac{b}{c}\right)^3+\left(\frac{c}{a}\right)^3+\left(\frac{a}{b}\right)^3+3 \ge \frac{1}{a^2b^2c^2}(\quad)+\frac{1}{abc}(\quad)$$

oss: $\frac{b}{c}\cdot\frac{c}{a}\cdot\frac{a}{b}=1$

magari è Schur su $\left(\frac{a}{b}, \frac{b}{c}, \frac{c}{a}\right)$?

fa il conto ed è VERO

funziona anche $x=a^2b$ e cicliche

faccio Schur + altro