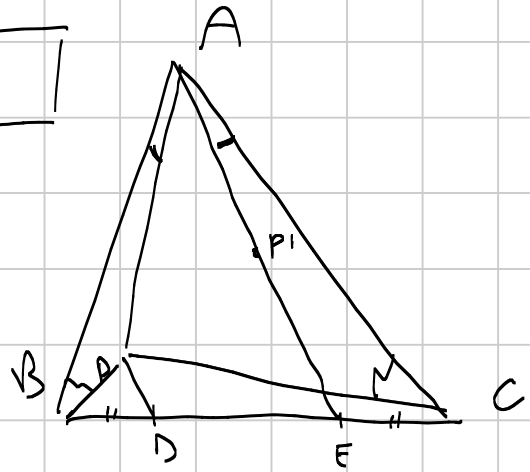
Senior 2019 - G1 Advanced

Titolo nota 06/09/2019

ELMO 2012



$AB \times AC$

Tesi: $\angle PBA = \angle ACP$

$\angle CBP' = \angle PBA$, $\angle P'CB = \angle PCA$

$\Rightarrow$ Tesi, P' sta sull' asse di BC

$D = [0, m, n]$ $E = [0, n, m]$, $n + m = 1$

Parallelismo di PD e AE $\rightarrow$ $[PDE] = [PDA]$

$P = [\alpha, \beta, \gamma]$

$$\frac{[PDE]}{[ABC]} = \begin{vmatrix} \alpha & \beta & \gamma \\ 0 & m & n \\ 0 & n & m \end{vmatrix} = \alpha (m^2 - n^2)$$

$$\frac{[PDA]}{[ABC]} = \begin{vmatrix} \alpha & \beta & \gamma \\ 0 & m & n \\ 1 & 0 & 0 \end{vmatrix} = \beta n - \gamma m$$

$$\Rightarrow \boxed{\alpha (m^2 - n^2) = \beta n - \gamma m}$$

AE : $-ym + zn = 0$

$Q \in AE$, $Q = [\text{qualcosa}, n, m] = \left[\frac{a^2}{t}, n, m\right]$

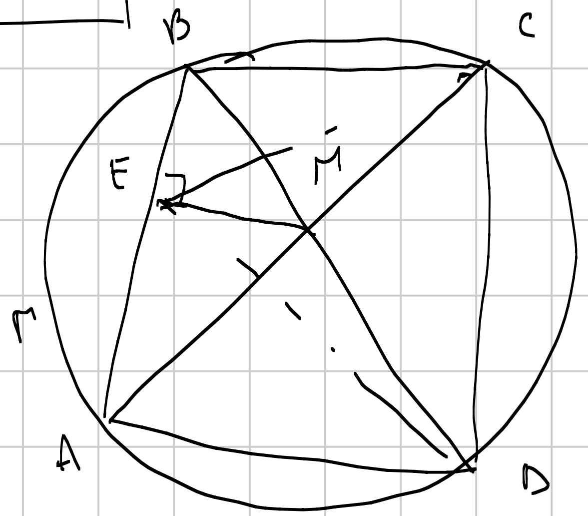
$$P' = \left[\frac{a^2}{t}, u, w\right] \Rightarrow P = \left[t, \frac{b^2}{u}, \frac{c^2}{w}\right]$$

$$t(w^2 - u^2) = b^2 - c^2$$
$$t(w+u)(w-u) = b^2 - c^2$$
$$w+u=1 \Rightarrow t = \frac{b^2-c^2}{w-u}$$

$$P = \left[\frac{b^2-c^2}{w-u}, \frac{b^2}{u}, \frac{c^2}{w}\right]$$

$$P' = \left[\frac{a^2(w-u)}{b^2-c^2}, u, w\right]$$

$\boxed{\text{BMO 2018-1}}$



$\angle DEM = \angle MEC$

Tesi: AB diametro di Γ

$AB > CD$ AB no parallelo CD

$E = [1,0,0]$, $C = [0,1,0]$, $D = [0,0,1]$
$A = [\lambda, -b, c]$ $B = [\mu, -b, c]$

$$M = [\lambda\mu, -\lambda b, c\mu]$$

Impongo $\frac{-\lambda b}{c\mu} = \frac{b}{c} \Rightarrow \lambda = -\mu$

$$B = [-\lambda, -b, c]$$

Equazione di Γ : $a^2yz + b^2xz + c^2xy - (x+y+z)(ux+vy+cz) = 0$

$C \in \Gamma \Rightarrow v = 0$

$D \in \Gamma \Rightarrow w = 0$

$A \in \Gamma \Rightarrow u = \dfrac{-a^2bc + \lambda b^2c - \lambda bc^2}{(\lambda - b + c)\lambda}$ (1)

$B \in \Gamma \Rightarrow u = \dfrac{a^2bc + \lambda b^2c - \lambda bc^2}{\lambda(-\lambda - b + c)}$ (2)

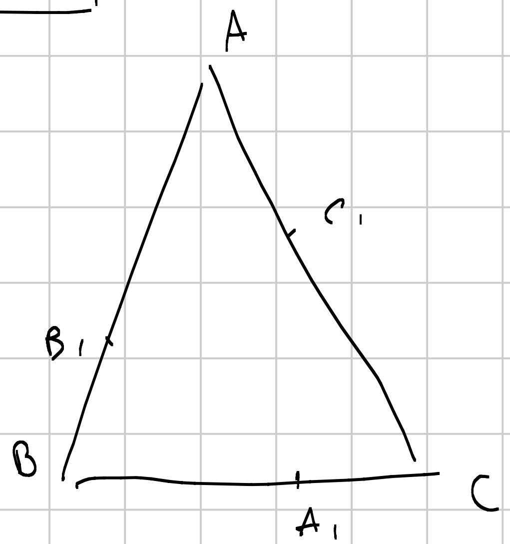
Imponiamo: RHS di (1) e (2) uguali

$$(b-c)(a-\lambda)(a+\lambda) = 0$$

$b \neq c \Rightarrow \lambda = a$ o $\lambda = -a$

A, B excentri di ΔCED, chiude.

$\boxed{\text{IMO 2013 - 3}}$

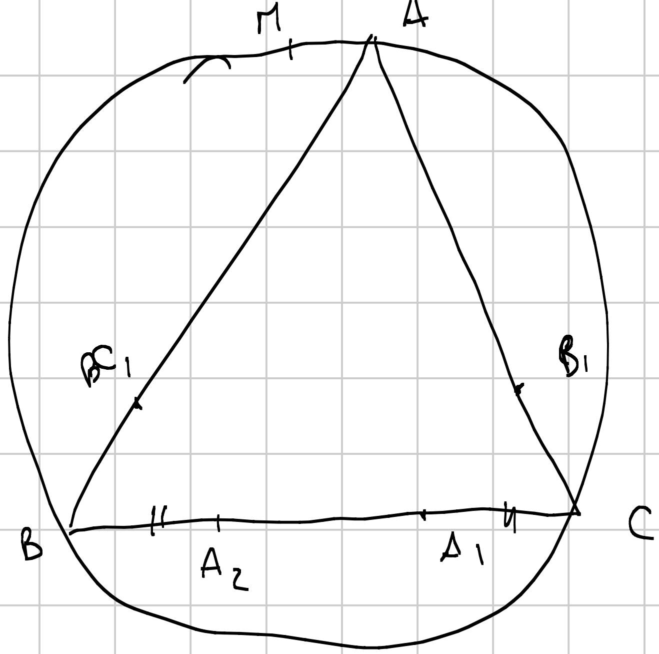


circocentro di $\Delta A_1B_1C_1$ sta su $\odot ABC$

Tesi: ΔABC rettangolo

O_1 è circocentro di $\Delta A_1B_1C_1$

$\Delta A_1B_1C_1$ è ottusangolo. $\angle B_1A_1C_1 > 90°$



$BM = CM$, $\angle C_1BM = \angle B_1CM$

A_2 pto di tangenza dell'inscritta

$$C_1B = BA_2 = CA_1 = CB_1$$

$\Rightarrow \Delta BC_1M = \Delta CB_1M \qquad M \in B_1C_1 \Rightarrow M \equiv O_1$!

M sta sull'asse di $A_1A_2 \Rightarrow A_1M = A_2M$

$A_2 \in \odot A_1B_1C_1$

Baricentriche su ΔABC !

$$A_1 = [0, a+c-b, a+b-c]$$
$$A_2 = [0, a+b-c, a+c-b]$$
$$B_1 = [b+c-a, 0, b+a-c]$$
$$C_1 = [b+c-a, c+a-b, 0]$$

$$\sum a^2xy = (x+y+z)(ux+vy+wz)$$

Passaggio di A_1 e A_2

$$v(a+c-b) + w(a+b-c) = v(a+b-c) + w(a+c-b)$$

$\boxed{\text{Se } b \neq c} \Rightarrow \quad v = w$

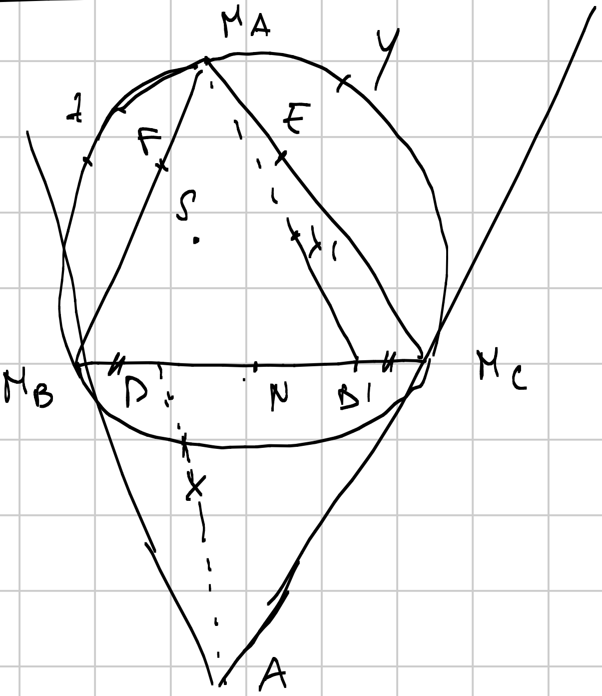
$$v = w = \frac{(a+c-b)(a+b-c)}{4}$$

B_1 : $\quad b^2 (b+c-a)(a+b-c) = 2b\left[c(b+c-a) + \frac{(a+b+c)^2(a+c-b)}{4}\right]$

$$(b-c)(b^2+c^2-a^2) = 0 \quad \Rightarrow \quad a^2 = b^2+c^2 \quad \Rightarrow \text{Teo}$$

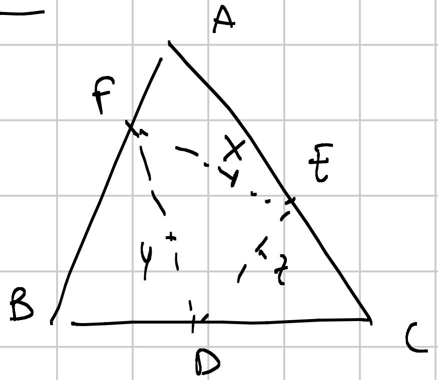
Se $b = c$ invece $M \equiv A$ e si conclude

USA TST 2015 - 6



Tesi: AX, BY, CZ concorrono

Teorema (Cevian Nest):



Due qualsiasi delle seguenti implicano la terza

i) AD, BE, CF concorrono

ii) DX, EY, FZ concorrono

iii) AX, BY, CZ concorrono

Quindi per il Cevian Nest la nostra tesi è equivalente alla concorrenza di $M_A D$, $M_B E$, $M_C F$

La tesi è equivalente a:

$M_A X'$, $M_B Y'$, $M_C Z'$ concorrono

Baricentriche su $\Delta M_A M_B M_C$

$$S = (a^2 S_A + t,\ b^2 S_B + t,\ c^2 S_C + t)$$