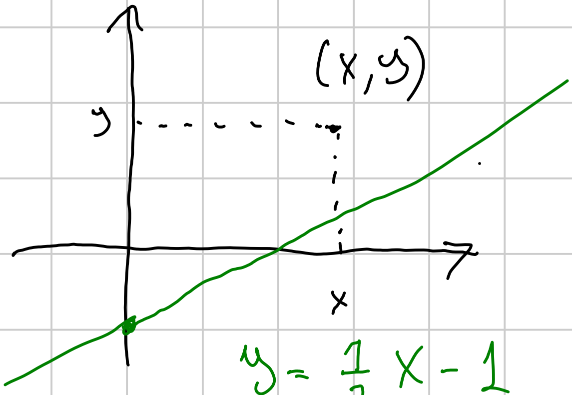
SENIOR 2019 − GEOMETRIA 1

Titolo nota 06/09/2019

- G1 Analitica, Vettori, complessi e trigon.
- G2 Trasformazioni sintetiche (Rot, omotetie, inversione)
- G3 Sintetica in generale.

Geometria Analitica



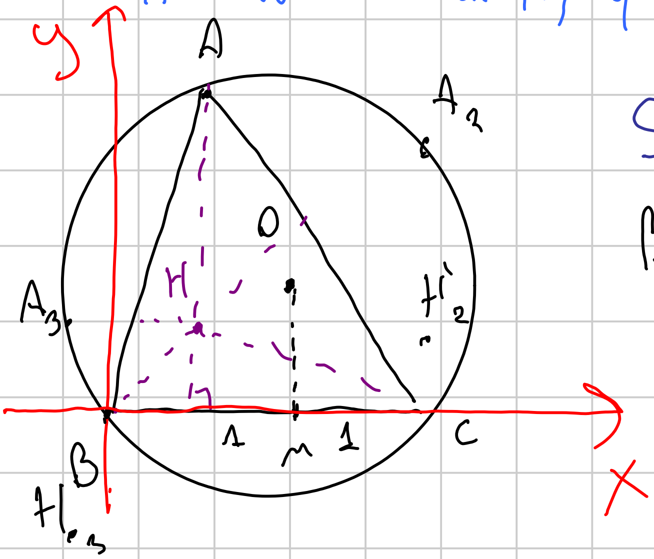
- Equazione della retta

$y = mx + k$

$y = \frac{1}{2}x - 1$ $(0,-1), (2,0)$

- Es: Abbiamo un cerchio Γ fissato, $B, C \in \Gamma$ fissati. Prendo A variabile su Γ, costruisco l'ortocentro H di ABC.

Al variare di A, qual è il luogo di punti di H?



Sol: Scelgo l'origine in B

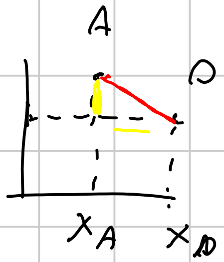
$B(0,0)$

$C(2,0)$

$O(1,\ell)$

Circ = punti equidistanti da O = 1

$r = \sqrt{1+\ell^2}$ Pitagora su BMO



$A(a,b)$ $|AO|^2 = (y_A - y_O)^2 + (x_A - x_O)^2$

$(x,y) \in \Gamma$ $(y-\ell)^2 + (x-1)^2 = r^2 = \ell^2 + 1$

$y^2 - 2y\ell + \ell^2 + x^2 - 2x + 1 = \ell^2 + 1$

$\boxed{y^2 - 2y\ell + x^2 - 2x = 0}$ equaz. di Γ

Voglio calcolare H

$AH: x = b$ $BH = ?$

$AC =$ $m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{0 - b}{2 - a}$ $\left(AC: y = \frac{b}{a-2} x + k \right)$

m_{BH} FATTO: due rette r e s sono $\perp$

$\iff m_r \cdot m_s = -1$

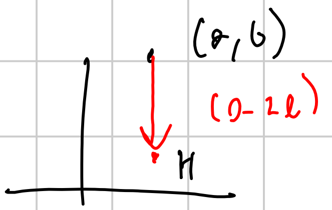
$m_{BH} = \frac{2-a}{b}$ BH) $y = m_{BH} \cdot x$ $y = \frac{2-a}{b} \cdot x$

$H = AH \cap BH = \begin{cases} x = a \\ y = \frac{2-a}{b} x \end{cases}$ $x = a,\ y = \frac{2a - a^2}{b}$

$H = \left(a, \frac{2a - a^2}{b} \right)$

$A \in \Gamma \Rightarrow y^2 - 2y\ell + x^2 - 2x = 0 \Rightarrow b^2 - 2b\ell = 2a - a^2$

$H = \left(a, \frac{b^2 - 2b\ell}{b} \right) = (a, b - 2\ell) = (a, b) + (0, -2\ell)$

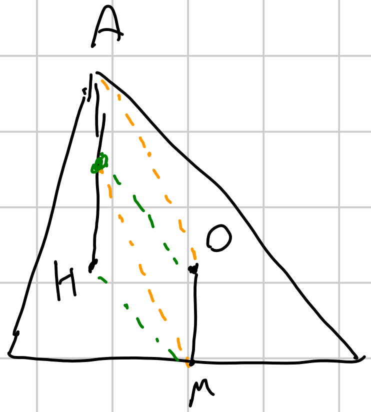


A si muove su Γ

$\Rightarrow$ H si muove su Γ spostato di -2ℓ

X Es: trovare l'equazione della circonferenza per H

OSS



$OM = l \quad AH = 2l$

$AH = 2OM$] $\rightarrow$ tanti parallelogrammi!

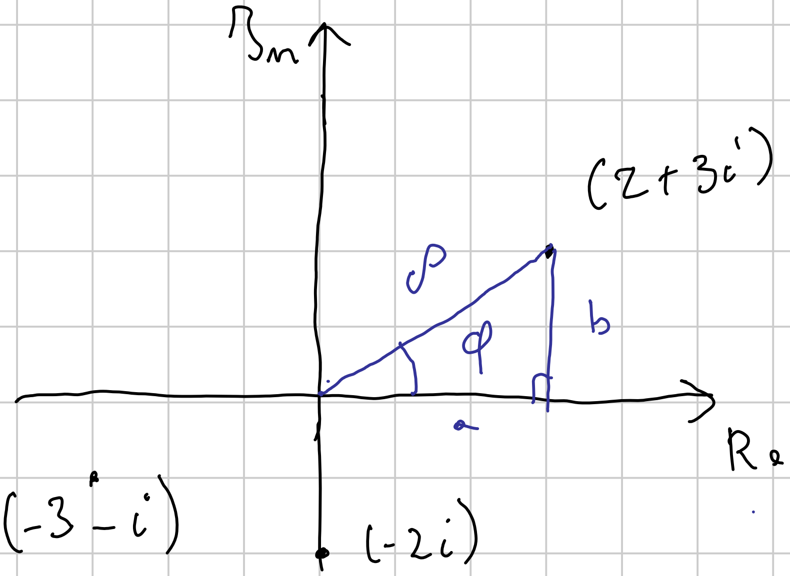
Numeri Complessi

$i^2 = -1 \quad \rightarrow \quad z = a + bi \quad a, b$ reali

$(a+bi) + (c+di) = (a+c) + i(b+d)$

$(a+bi)(c+di) = ac + adi + bci + bd\boxed{i^2}$ $= -1$

$= \underbrace{(ac - bd)}_{\text{Parte reale}} + i\underbrace{(ad + bc)}_{\text{Parte Immaginaria}}$



Notazione polare

$(\rho, \varphi) \longleftrightarrow (a+bi)$

$$\begin{cases} b = \rho \cdot \sin\varphi \\ a = \rho \cos\varphi \end{cases}$$

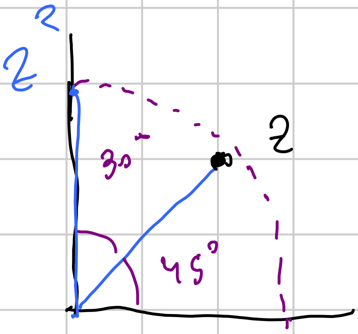
$$\begin{cases} \rho = \sqrt{a^2 + b^2} \\ \varphi = \arctan\frac{b}{a} \end{cases}$$

$z = \rho \cdot \underbrace{e^{i\varphi}}_{\cos\varphi + i\sin\varphi} \qquad w = r \cdot e^{i\theta}$

$$\boxed{z \cdot w = (\rho \cdot r) \cdot e^{i(\varphi + \theta)}}$$

Es. $e^{i(\varphi+\theta)} = e^{i\varphi}\cdot e^{i\theta}$ $\quad \cos(\theta+\varphi) + i\sin(\theta+\varphi) =$

$$= (\cos\theta + i\sin\theta)(\cos\varphi + i\sin\varphi)$$



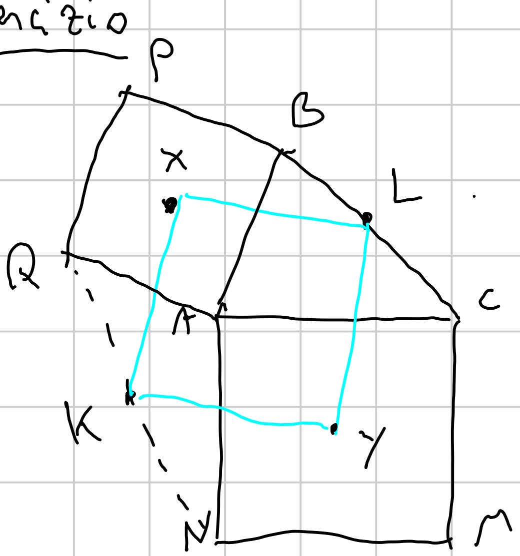
$$z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$$z^2 = \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{2\cdot i}{2} + \frac{i^2}{2}$$

$$= \frac{1}{2} + i - \frac{1}{2} = i$$

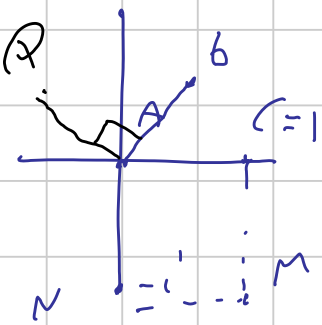
Es: $z = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ $\quad w = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$ $\quad z^2?$ $w^2?$ $w^3?$ $z^3?$ $z^6?$

• Esercizio



X, Y centri di ABPQ, ACMN
L, K punti medi di BC e NQ

Tesi: XLYK è un quadrato



Sol: Metto l'origine in $A = 0$ $\quad c = 1$

$N = -i$ $\quad M = 1 - i$

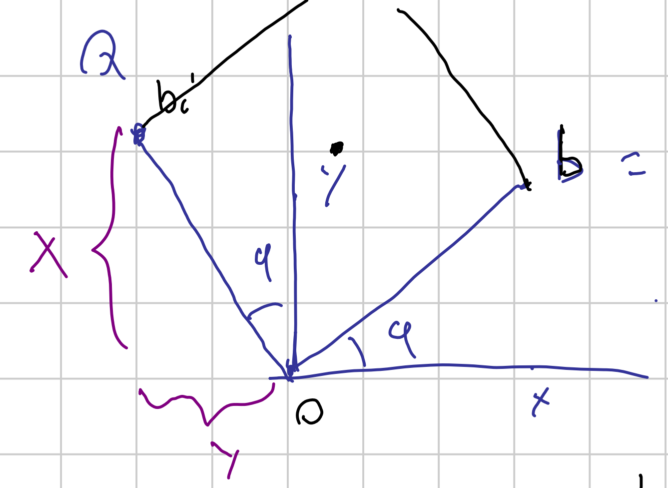
$$Y = \frac{c+n}{2} = \frac{1 + (-i)}{2} = \frac{1-i}{2}$$

B sarà dato da un parametro b

rotazione di 90° → moltiplicare per $e^{i\frac{\pi}{2}} = i$

$Q = bi$

$b + bi$

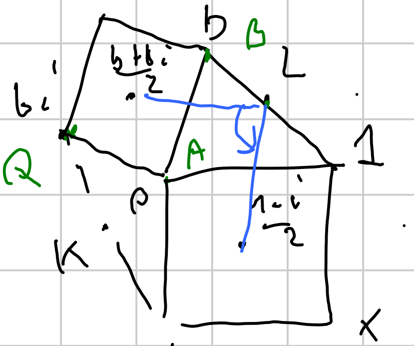


$b = x + iy$

$Q = -y + ix$

$bi = (x+iy)\cdot i = ix + i^2 y$

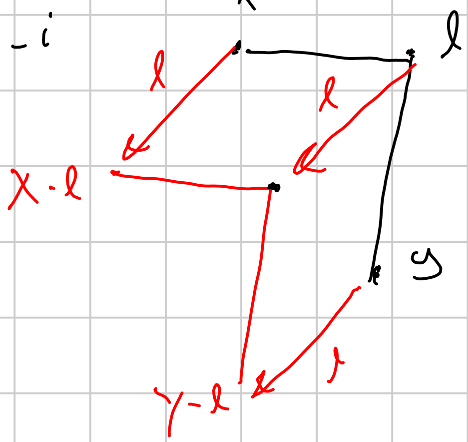
Centro: $\dfrac{b+bi}{2}$



$X = \dfrac{b+bi}{2}$ $\quad Y = \dfrac{1-i}{2}$

$L = \dfrac{b+1}{2}$ $\quad K = \dfrac{bi-i}{2}$

$L = 0$ $\quad XL \perp LY$? $\quad X\cdot i = Y$



$(X-l)\cdot i = Y - l$

$$\left(\frac{b+bi}{2} - \frac{b+1}{2}\right)\cdot i = \frac{1-i}{2} - \frac{b+1}{2}$$

$$\boxed{\frac{bi-1}{2}\cdot i} = \frac{1-i-b-1}{2} = \boxed{\frac{-b-i}{2}}$$

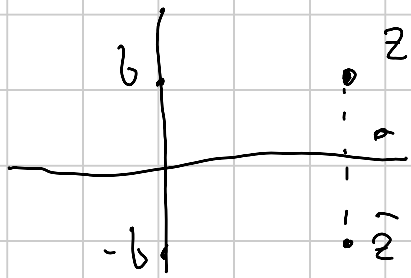
$$\frac{(bi-1)}{2} = \frac{bi^2-i}{2} = \frac{-b-i}{2}$$

ho dimostrato che $XL = LY$ e $\angle XLY = 90°$

Coniugato

$z = a + ib$

$\bar{z} = a - ib$



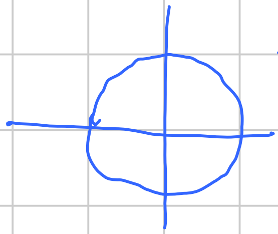
$\overline{(z+w)} = \bar{z} + \bar{w}$

$\overline{(z\cdot w)} = \bar{z}\cdot\bar{w}$

$\overline{\left(\dfrac{z}{w}\right)} = \dfrac{\bar{z}}{\bar{w}}$

$$\frac{(x+iy)}{a+ib} = \frac{(x+iy)}{(a+ib)}\,\frac{(a-ib)}{(a-ib)} = \frac{(ax+by)+i(ay-bx)}{a^2-(-ib)^2 = a^2+b^2}$$

$z\cdot\bar z = a^2+b^2 = \rho^2$



$e^{i\theta}$

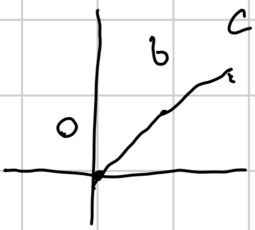
- $|z| = 1$ z sta sulla circonferenza unitaria

$\Rightarrow \rho = 1 \qquad z\cdot\bar z = 1 \qquad \bar z = \frac{1}{z}$

- $z, w \in$ Circ. Unit, $\overline{(z+w)} = \frac{1}{z}+\frac{1}{w} = \frac{z+w}{zw}$
- z non sta sull'unitaria $\bar z = \frac{\rho^2}{z}$

Cartesiano $(x,y) \rightarrow z = x+iy \qquad z\ e\ \bar z$

Allineamento e perpendicolarità



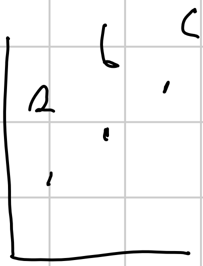
b, c, origine allineati $\iff \varphi_1 = \varphi_2$

$b = \rho_1 \cdot e^{i\varphi_1}$

$c = \rho_2 \cdot e^{i\varphi_2}$

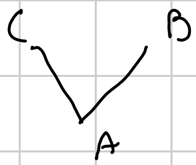
$$\frac{b}{c} = \frac{\rho_1}{\rho_2}\, e^{i(\varphi_1-\varphi_2)} = \frac{\rho_1}{\rho_2} \in \mathbb{R}$$

b, c, o allineati $\iff \frac{b}{c} \in \mathbb{R} \iff \frac{b}{c} = \overline{\left(\frac{b}{c}\right)}$



$$\boxed{\frac{b-a}{c-a} = \overline{\left(\frac{b-a}{c-a}\right)} = \frac{\bar b-\bar a}{\bar c-\bar a}}$$ formula allineamento.

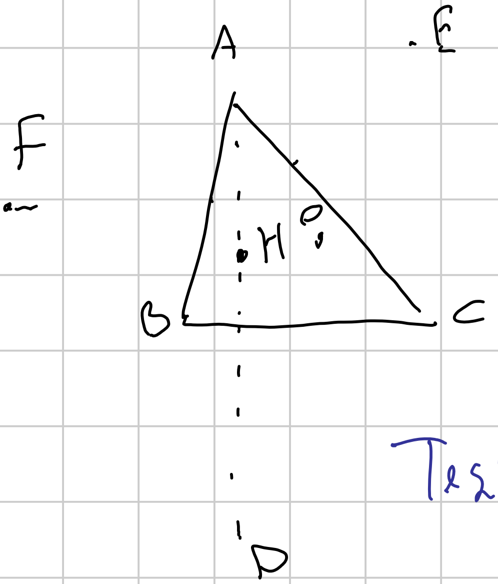
Perpendicolari $CA \perp AB$



$$\boxed{\frac{b-a}{c-a} = -\left(\frac{\bar b-\bar a}{\bar c-\bar a}\right)}$$ formula perpendicolarità

$\rho\cdot e^{i\theta} = \rho\cdot(\cos 90 + i\sin 90) = \rho i$

IMOSL1998 - G5



D è simm di A rispetto a BC

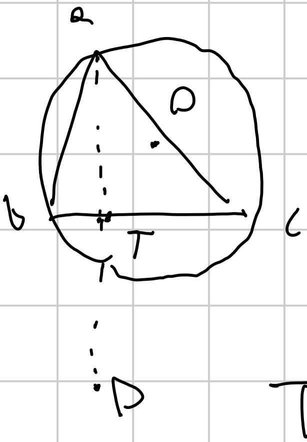
E " B " AC

F " C " AB

O circocentro, H ortocentro.

Tesi: D, E, F allineati $\Leftrightarrow$ $OH = 2R$

Circocentro = circonferenza unitaria



$G = \frac{a+b+c}{3}$ O, G, H allineati, $\vec{OG} = \frac{1}{3}\vec{OH}$

$h = 3 \cdot g = a+b+c$

a, b, c $\quad a \cdot \bar{a} = 1, \quad b \cdot \bar{b} = 1 \quad c \cdot \bar{c} = 1$

T = il piede dell'altezza da A

$T \in BC$, $T \in AH$ (oppure $T \in BC$, $AT \perp TC$)

$$\frac{t-b}{c-b} = \frac{\bar{t}-\bar{b}}{\bar{c}-\bar{b}} \qquad (t-b)(\bar{c}-\bar{b}) = (\bar{t}-\bar{b})(c-b)$$

$$(t-b)\left(\frac{\cancel{b-c}}{bc}\right) = \left(\bar{t}-\frac{1}{b}\right)(\cancel{c-b}) \quad -1$$

$$\boxed{t-b} = bc\left(\frac{1}{b}-\bar{t}\right) = \boxed{c-bc\bar{t}}$$ BCT allineati

$$\frac{t-a}{h-a} = \frac{\bar{t}-\bar{a}}{\bar{h}-\bar{a}} \qquad h-a = b+c \qquad \bar{h}-\bar{a} = \frac{1}{b}+\frac{1}{c} = \frac{b+c}{bc}$$

$$\frac{t-a}{b+c} = \frac{\bar{t}-\frac{1}{a}}{\frac{b+c}{bc}} \rightarrow \boxed{t-a = bc\left(\bar{t}-\frac{1}{a}\right)}$$

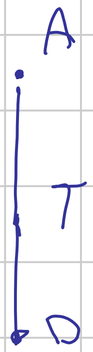
$$\begin{cases} bc\bar t - \frac{bc}{a} = t - a \\ -bct + c = t - b \end{cases}$$

$bc\bar t = t + \frac{bc}{a} - a$

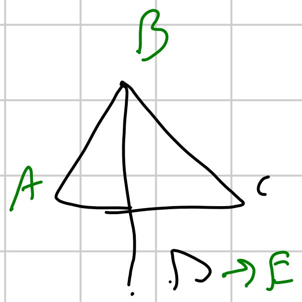
Sostituisco

$-\left(t + \frac{bc}{a} - a\right) + c = t - b$

$2t = b + c + a - \frac{bc}{a} \Rightarrow$ ho T



t pt medio di $AD \Rightarrow t = \frac{a+d}{2}$



$d = 2t - a = b + c - \frac{bc}{a}$

$e = a + c - \frac{ac}{b}$

$f = a + b - \frac{ab}{c}$

$$\frac{d-e}{f-e} = \frac{\bar d - \bar e}{\bar f - \bar e}$$

$d - e = \left(b + c - \frac{bc}{a}\right) - \left(a + c - \frac{ac}{b}\right) = b - a + \frac{ac}{b} - \frac{bc}{a} = (b-a)\left(1 + \frac{c(a+b)}{ab}\right)$

$\bar d - \bar e = \left(\frac{1}{b} - \frac{1}{a}\right)\left(1 + \frac{1}{c}\left(\frac{1}{a} + \frac{1}{b}\right)\frac{1}{\frac{1}{ab}}\right)$

$\Rightarrow$ conti: $(a-c)\left(\boxed{a^2b + ba^2 + a^2c + ac^2 + bc^2 + b^2c - abc}\right) = 0$

$\Updownarrow$

D, E, F allineati

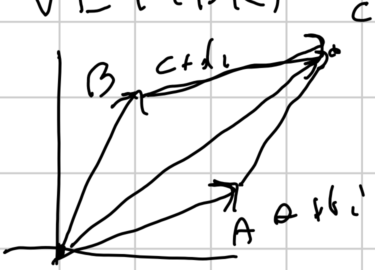
- $OH = 2R$ $\quad O = 0,\ h = a+b+c \quad R = 1$

$|h - o| = |h| \qquad |h|^2 = h \cdot \bar h = (a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) =$

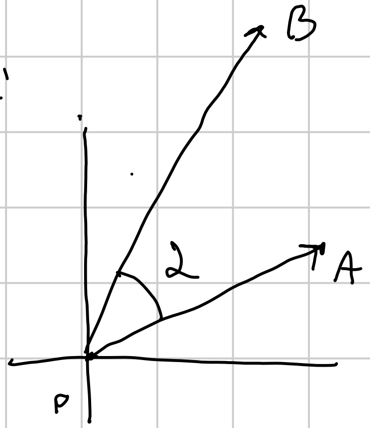
$= \frac{(a+b+c)(ab+ac+bc)}{abc} = 4$

$\overset{1}{\cancel{4}}abc = a^2b + a^2c + \cancel{abc} + ab^2 + \cancel{abc} + b^2c + \cancel{abc} + ac^2 + bc^2$

VETTORI



$$\vec{A} + \vec{B} = \vec{C}$$



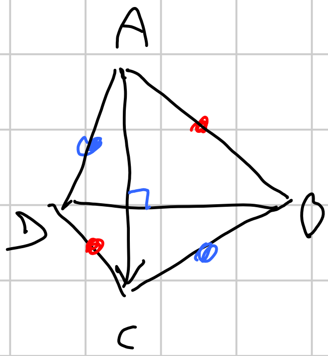
$$\vec{A} \cdot \vec{B} = |A| \cdot |B| \cdot \cos\alpha$$

$\vec{A} = (x, y)$ $\quad \vec{B} = (z, w)$

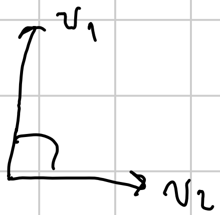
$$\vec{A} \cdot \vec{B} = (xz + yw)$$

Fatto: ABCD quadrilatero, $AC \perp BD \iff$

$$\iff AB^2 + CD^2 = AD^2 + BC^2$$



Sol: $AC \perp BD$



$v_1 \cdot v_2 = 0$

$\vec{AC} = \vec{C} - \vec{A}$

$\vec{BD} = \vec{D} - \vec{B}$

$$(\vec{C} - \vec{A})(\vec{D} - \vec{B}) = 0$$

$$AC \perp BD \iff \boxed{\vec{C} \cdot \vec{D} - \vec{C} \cdot \vec{B} - \vec{A} \cdot \vec{D} + \vec{A} \cdot \vec{B} = 0}$$

$\vec{v}_1 \cdot \vec{v}_1 = |v_1|^2 \cdot \cos 0 = |v|^2$ $\qquad |AB|^2 = (\vec{B} - \vec{A})^2$

$\vec{AB} = \vec{B} - \vec{A}$

$\vec{A} + \vec{AB} = \vec{B}$

$$AB^2 + CD^2 = AD^2 + BC^2$$

$$(\vec{B} - \vec{A})^2 + (C - D)^2 = (A - D)^2 + (B - C)^2$$

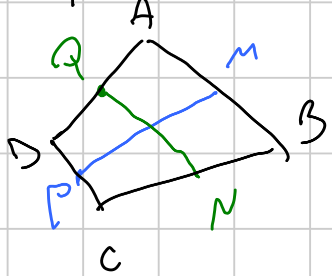
$$\cancel{\vec{B} \cdot \vec{B}} - 2A \cdot B + \cancel{A \cdot A} + \cancel{C^2} - 2C \cdot D + \cancel{D^2} = \cancel{A^2} - 2AD + \cancel{D^2} + \cancel{B^2} - 2BC + \cancel{C^2}$$

$$\iff \boxed{\vec{A} \cdot \vec{D} + \vec{B} \cdot \vec{C} - \vec{A} \cdot \vec{B} - \vec{C} \cdot \vec{D} = 0}$$

ESER:

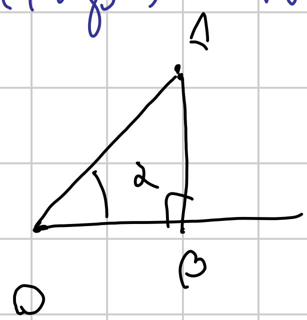
- $AC \perp BD \iff M, N, P, Q$ pt medi di AB, BC, CD, DA

$|MP| = |NQ|$



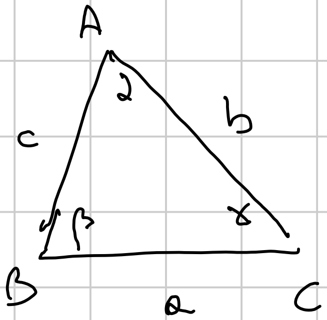
- ABCD quadrilatero, M, N, P, Q come sopra
SEMPRE, MNPQ è parallelogramma.

Trigonometria.



$\sin\alpha = \frac{BA}{OA}$ $\cos\alpha = \frac{OB}{OA}$

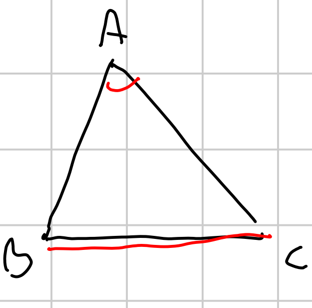
Teorema dei Seni



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$$

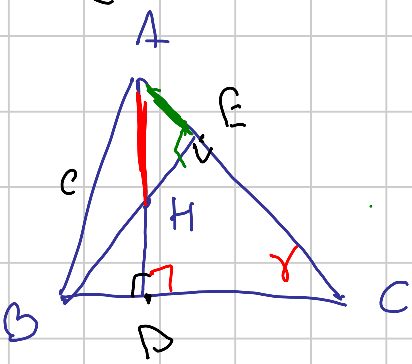
R: raggio della circ. circoscritta.

Teorema del Coseno

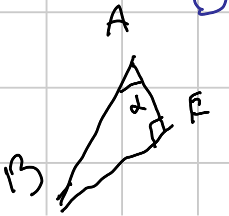


$$a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha$$

Es:



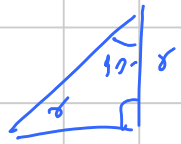
H ortocentro.
Trovare $|AH|$ in termini di $a, b, c, \alpha, \beta, \gamma$



$\cos\alpha = \frac{AE}{AB} \Rightarrow AE = AB\cdot\cos\alpha$

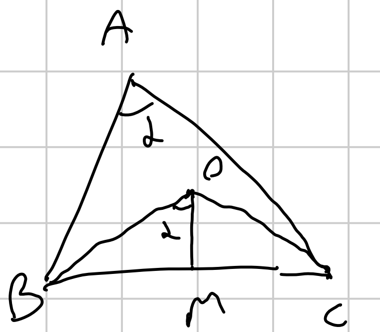
$\cos \widehat{HAE} = \frac{AE}{AH}$

$\widehat{HAE} = 90-\gamma$



$$AH = \frac{AE}{\cos(90-\gamma)} = \frac{AB \cdot \cos\alpha}{\sin\gamma} = \frac{c}{\sin\gamma} \cdot \cos\alpha = \frac{a}{\sin\alpha} \cdot \cos\alpha$$

$$AH = \frac{a}{tg\,\alpha}$$



$BO = R \qquad \angle BOC = 2\alpha$

$OM = R \cdot \cos\alpha$

$= \frac{a \cdot \cos\alpha}{2 \sin\alpha}$

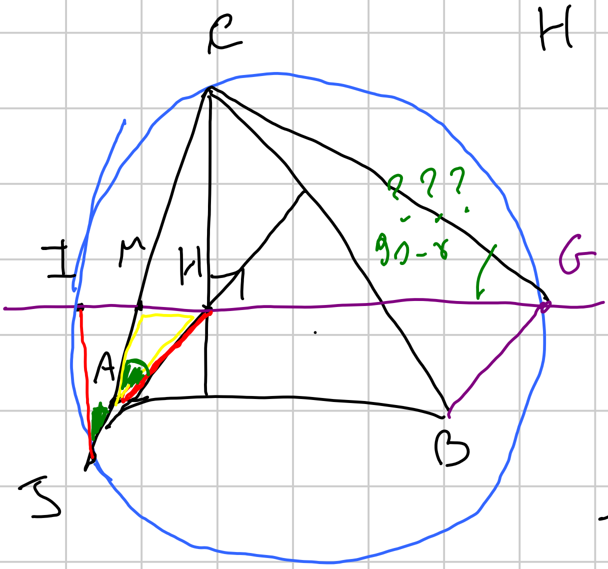
Teorema di Seni

$\frac{\sin\alpha}{a} = \frac{1}{2R}$

$R = \frac{a}{2} \cdot \frac{1}{\sin\alpha}$

$AH = 2\,OM$

IMO SL 2015 G1



H ortocentro   G pt tale che ABGH è parallelogramma

$HA \cap AC = M$

I = simmetrico di H rispetto M

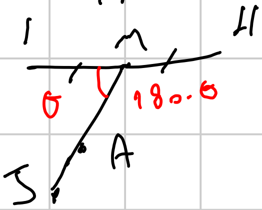
Γ: circonferenza GCI

$J = \Gamma \cap AC$ Tesi: $IJ = AH$

teo seni su ΔIJM

$$\frac{\sin \widehat{IMJ}}{IJ} = \frac{\sin \widehat{IJM}}{IM}$$

$IM = MH$



$\sin IMJ = \quad = \quad \sin HMJ$

sin

$\sin\theta = \sin 180-\theta$

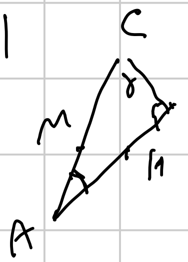
teo seni MAH $\quad \dfrac{\sin AMH}{AH} = \dfrac{\sin MAH}{MH}$

$$\underbrace{IJ}_{\text{Tesi}} \cdot \sin IJM = IM \cdot \sin IMJ = MH \cdot \sin HMA = \underbrace{AH}_{\text{Tesi}} \cdot \sin MAH$$

Nuova Tesi: $\sin \widehat{IJM} = \sin \widehat{MAH}$

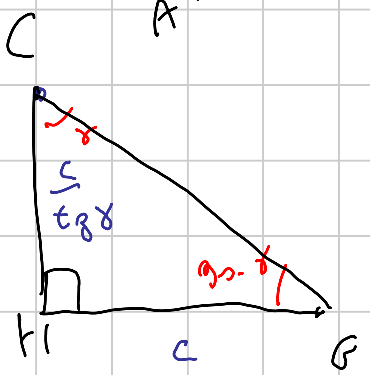
$\angle IJM = \angle IGC$ perché $GCIJ$ ciclico

$\angle MAH$



$\angle MAH = 90 - \gamma$

$$\boxed{\text{Tesi}: \angle IGC = 90 - \gamma}$$



$\left.\begin{array}{l} HG \parallel AB \\ CH \perp AB \end{array}\right\} \Rightarrow CH \perp HG$

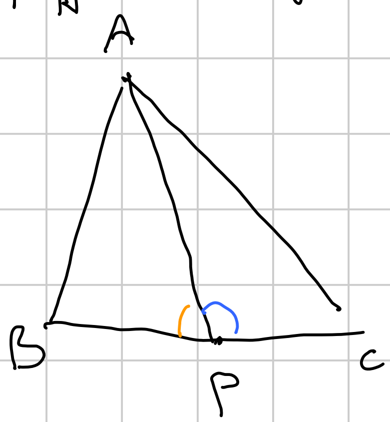
$HG = AB = c$

$CH = \dfrac{c}{tg\gamma}$

$AH = CH \cdot tg(ACH) = \qquad c = \dfrac{c}{tg\gamma} \cdot tg(ACH)$

$tg(ACH) = tg\gamma \Rightarrow \angle ACH = \gamma$

Applic trigo: Teorema di Stewart.



$P \in BC$

$$AB^2 \cdot PC + AC^2 \cdot BP = AP^2 \cdot BC + BC \cdot BP \cdot PC$$
$$c^2 \cdot PC + b^2 \cdot BP = AP^2 \cdot a + a \cdot BP \cdot PC$$

Teorema del coseno su $\triangle APC$

$AC^2 = AP^2 + PC^2 - 2\,AP\cdot PC\cdot \boxed{\cos(\widehat{APC})}$ K

$\cos\alpha = -\cos(180-\alpha)$

$\cos(\widehat{APC}) = -\cos(\widehat{BPA})$

$AB^2 = AP^2 + BP^2 - 2\cdot BP\cdot PA\cdot \boxed{\cos(\widehat{BPA})}$ $-K$

Moltiplico la 1ª eq per BP, la 2ª per PC

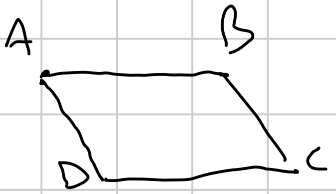
$AC^2\cdot BP = AP^2\cdot BP + PC^2\cdot BP - 2\,AP\cdot PC\cdot BP\cdot K$

$AB^2\cdot CP = AP^2\cdot CP + BP^2\cdot PC + 2\,BP\cdot PC\cdot AC\ (+K)$

$AC^2\cdot BP + AB^2\cdot CP = AP^2(BP+PC) + BP\cdot PC\,(PC+BP) + 0$

$= AP^2\cdot BC + BP\cdot PC\cdot BC$

ESER x CASA

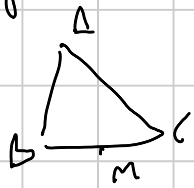


ABCD parallelogramma

dimostra che $2(AC^2+BD^2) = AB^2+BC^2+CD^2+AD^2$

1) trigonometria 2) Vettori

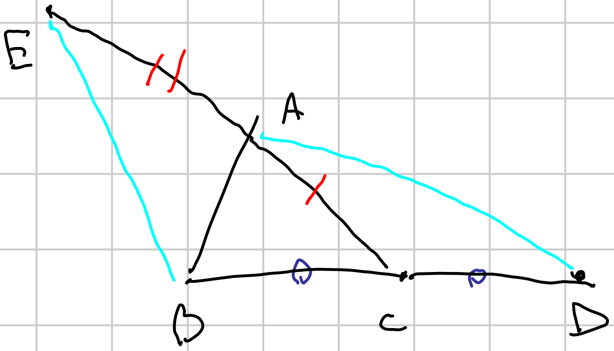
Hint:



AM mediana. Quanto è lungo? a, b, c

Problemi:

- EGMO 2013 - 1

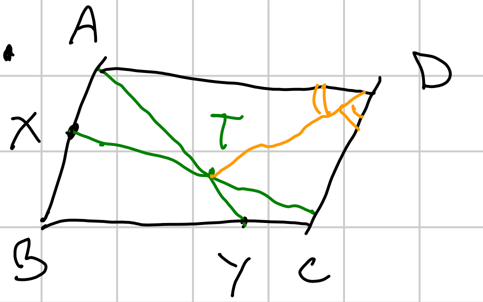


$D \in BC$ $BC = CD$

$E \in AB$ t.c. $EA = 2AC$

Tesi: se $\boxed{AD = BE}$

$\Rightarrow ABC$ è rettangolo

- 

$X \in AB$, $Y \in BC$ $\quad AX = CY$

$AY \cap CX = T$.

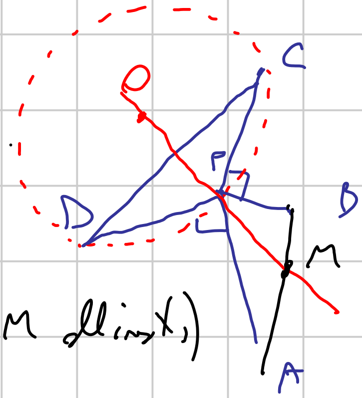
Dimostrare che DT è bisettrice di $\angle ADC$

- ITA TST 2016-1

$ABCD$ quadrilatero, P dentro t.c. ① $\angle APD = \angle BPC = 90$

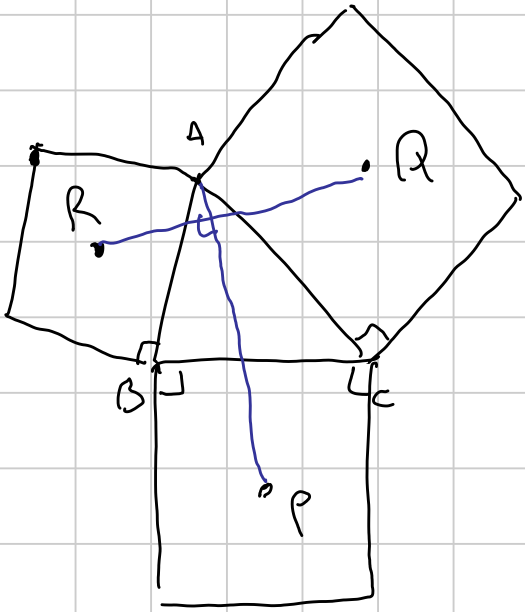
② $PA \cdot PD = PB \cdot PC$

O circocentro di $\triangle CPD$

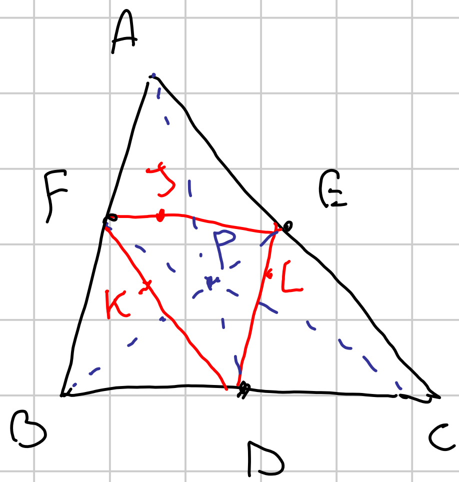


Tesi: PO passa per il pt medio di AB

(O, P, M allineati)



Tesi: $AP \perp RQ$

- 

D, E, F sui lati BC, AC, AB

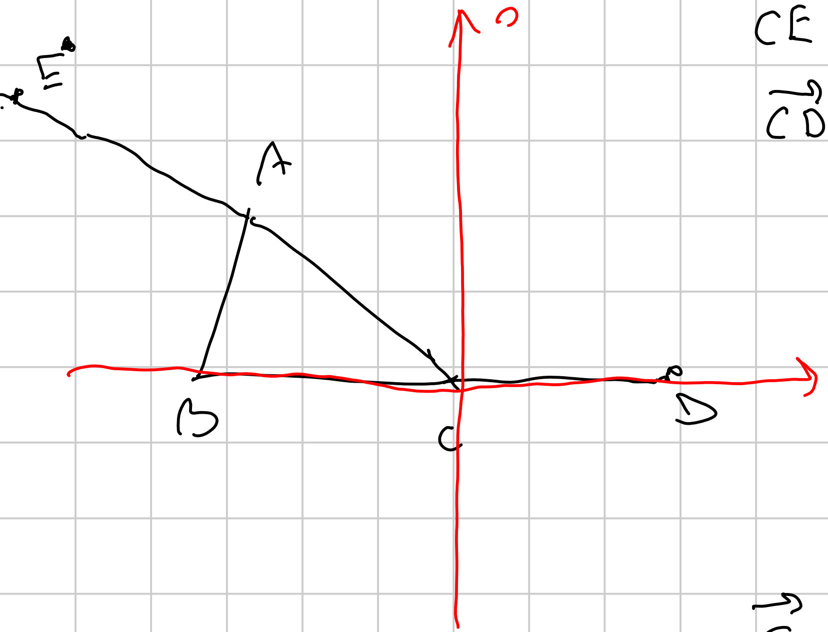
t.c. AD, BE, CF concorrono in P

J, K, L su EF, FD, ED

t.c. DJ, EK, FL concorrono

Tesi: allora anche AJ, BK, CL concorrono.

Correzione



$$\vec{CE} = 3\vec{CA}$$
$$\vec{CD} = -\vec{CB}$$
Metto l'origine in C

$$\vec{CA} = \vec{A}$$
$$\vec{CB} = \vec{B}$$

$$\vec{D} = -\vec{B}$$
$$\vec{E} = 3\vec{A}$$

Hp: $|AD| = |EB|$

$$|AD|^2 = (\vec{AD})\cdot(\vec{AD}) = (\vec{D}-\vec{A})(\vec{D}-\vec{A}) = \vec{D}\cdot\vec{D} + \vec{A}\cdot\vec{A} - 2\underbrace{\vec{D}}_{-\vec{B}}\cdot\vec{A}$$

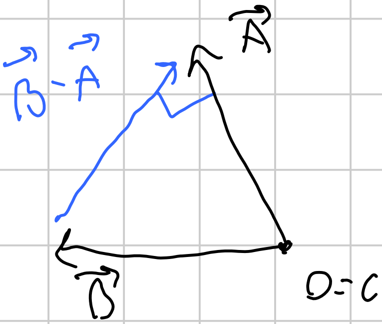
$$\boxed{\vec{D} = -\vec{B}} \Rightarrow \vec{B}^2 + \vec{A}^2 + 2\vec{A}\cdot\vec{B}$$

$$|BE|^2 = (\vec{B}-\vec{E})(\vec{B}-\vec{E}) = (\vec{B}-3\vec{A})(\vec{B}-3\vec{A}) = \vec{B}^2 + 9\vec{A}^2 - 6\vec{A}\cdot\vec{B}$$

$$\cancel{\vec{B}^2} + \cancel{\vec{A}^2} + \cancel{2}\vec{A}\cdot\vec{B} = \cancel{\vec{B}^2} + \overset{8}{\cancel{9}}\vec{A}^2 - \overset{8}{\cancel{6}}\vec{A}\cdot\vec{B}$$

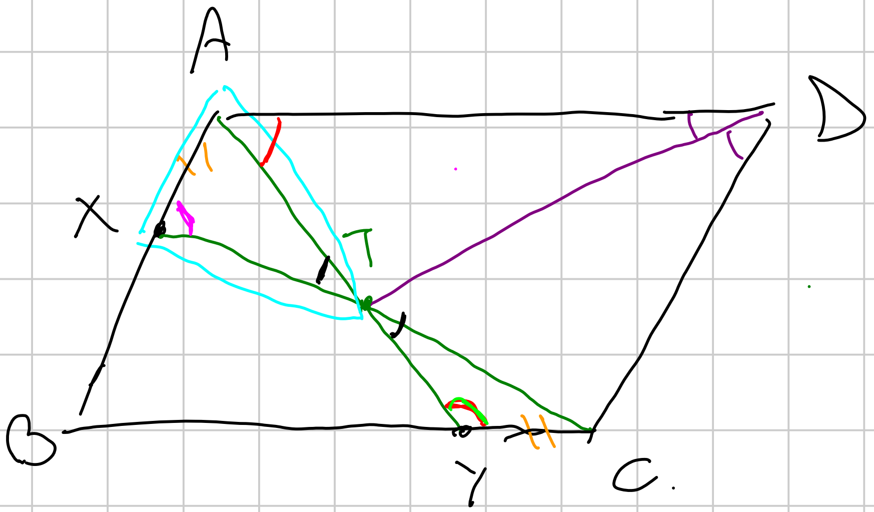
$$8\vec{A}^2 - 8\vec{A}\cdot\vec{B} = 0 \qquad 8\vec{A}(\vec{A}-\vec{B}) = 0$$

$\vec{A} = 0$
$\vec{A} - \vec{B} = 0$ } CASI DEGENERI
$\boxed{\vec{A} \perp \vec{A} - \vec{B}}$



$$\Rightarrow \widehat{BAC} = 30°$$

2



AX = CY

Tesi: DT bisettrice

$$\frac{AT}{\sin A\hat{X}T} = \frac{AX}{\sin XTA} = \frac{YC}{\sin YTC} = \frac{TC}{\sin T\hat{Y}C} \to \frac{AT}{TC} = \frac{\sin AXT}{\sin TYC}$$

Th Seni $\triangle AXT$

th Seni su $\triangle TYC$

Teo Seni su $\triangle ATD$, $\triangle CTD$

$$\frac{AT}{\sin ADT} = \frac{TD}{\sin TAD} \qquad \frac{TC}{\sin TDC} = \frac{TD}{\sin TCD}$$

$$\frac{AT}{TC} \cdot \boxed{\frac{\sin TDC}{\sin ADT}} = \cancel{\frac{TD}{TD}} \cdot \frac{\sin TCD}{\sin TAD}$$

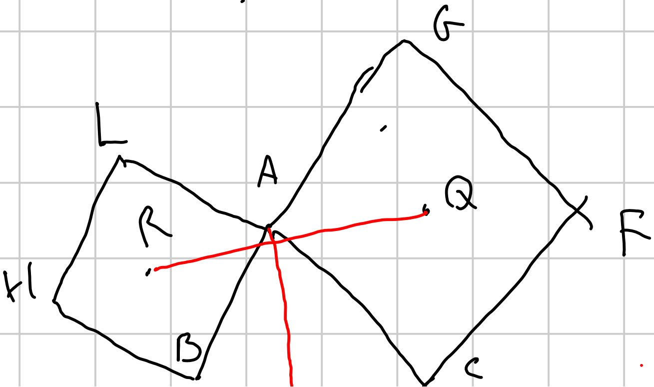
Hope = 1

$$\frac{\sin TDC}{\sin ADT} = \frac{TC}{TA} \cdot \frac{\sin TCD}{\sin TAD} = \frac{\cancel{\sin}\ \boxed{TYC}}{\sin AXT} \cdot \frac{\sin TCD}{\cancel{\sin}\ \boxed{TAD}} = 1$$

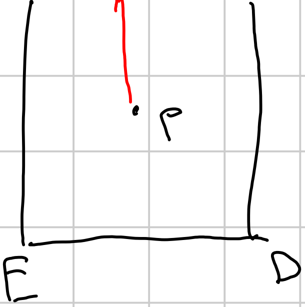
$\widehat{TYC} + \widehat{TAD} = 180° \Rightarrow$ i seni sono uguali

$\widehat{TCD} + \widehat{AXT} = 180° \Rightarrow$ i SENI SONO uguali

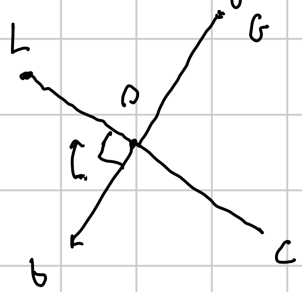
(3)

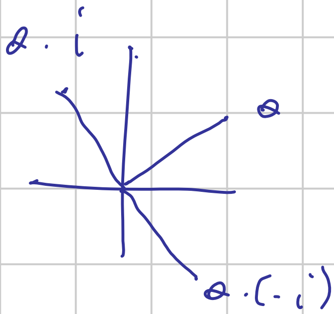


RQ ⊥ AP



Metto l'origine in $A = o$



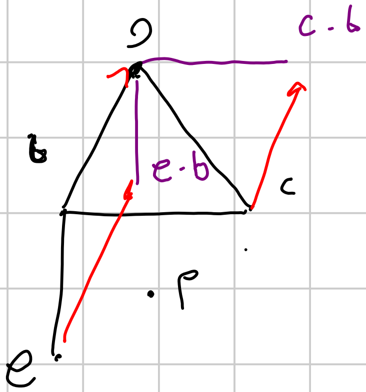


$l = b \cdot (-i)$

$g = c \cdot i$

R pt medio di BL $\Rightarrow r = \frac{b+l}{2} = \frac{b - bi}{2}$

Q " " $CG = q = \frac{c + ci}{2}$



$c - b$ ruotato di $90°$ ↷ fa $e - b$

$(c-b)(-i) = e - b$ → $\frac{c-b}{e-b} = -i$

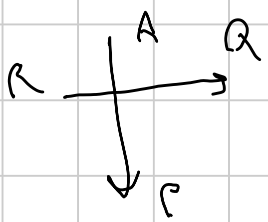
$\sim \frac{c-b}{e-b} = -(\overline{\quad})$

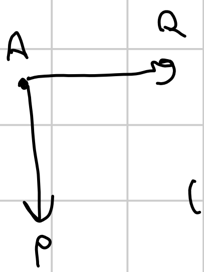
$e = bi + b - ic$

$p = \frac{e+c}{2} = \frac{bi + b - ic + c}{2}$

$\vec{AP} = p - o = p =$

$\vec{RQ} = r - q = \frac{b - bi - c - ci}{2}$



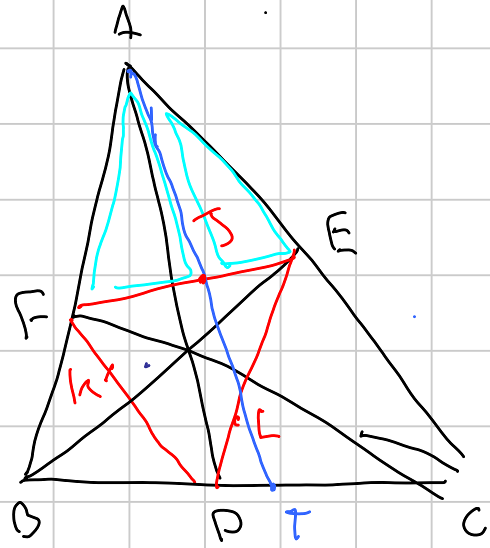


$(r - q) \cdot i \cdot \lambda = p$

$\lambda = 1$

$(r-q) \cdot i = \frac{bi - bi^2 - ci - ci^2}{2} = \frac{bi + b - ci + c}{2} = p$

$AP \perp RQ$, ma anche $|AP| = |RQ|$!



AD, BE, CF concorrono

Per teorema di Ceva, è vero che

(1) $\frac{BD}{DC}\cdot\frac{CE}{EA}\cdot\frac{AF}{FD}=1$

(2) $\frac{FJ}{JE}\cdot\frac{EL}{LD}\cdot\frac{DK}{KF}=1$ (Sempre Ceva)

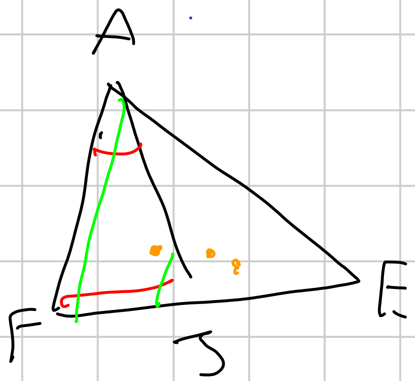
Voglio dimostrare AJ, BK, CL concorrono.

$\Updownarrow$

$\boxed{\frac{\sin BAJ}{\sin JAE}}\cdot\frac{\sin ECL}{\sin LCD}\cdot\frac{\sin DBK}{\sin KBF}=1$

Ceva $\qquad$ Ceva trig.

$\frac{BD}{DC}\rightarrow\frac{\sin BAD}{\sin DAC}$



Teo seni su ΔAFJ, AJE

$\frac{\sin BAJ}{FJ}=\frac{\boxed{\sin AJF}}{AF}$

SONO UGUALI

$\frac{\sin JAE}{JE}=\frac{\boxed{\sin AJE}}{AE}$

$\frac{\sin BAJ}{\sin JAE}\cdot\frac{JE}{FJ}=\frac{AE}{AF}$ su ΔAFE

$\frac{\sin DBK}{\sin KBF}\cdot\frac{DK}{KF}=\frac{DF}{BD}$ Δ

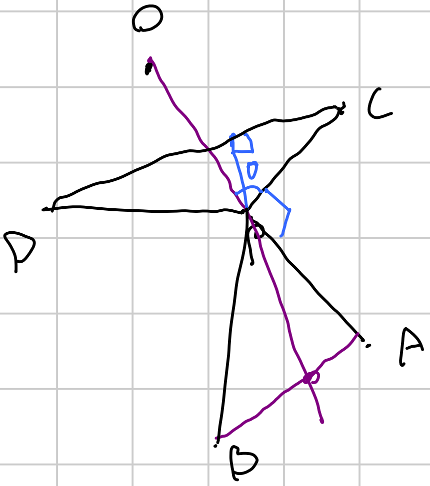
$\frac{\sin}{\sin}$ $\qquad$ — $\qquad$ —

Ceva su $\qquad$ Ceva su AD, BE, CF

$\Rightarrow\prod\frac{\sin BAJ}{\sin JAE}=1$

$\rightarrow$ Ceva trig fine

BJ, BK, CL

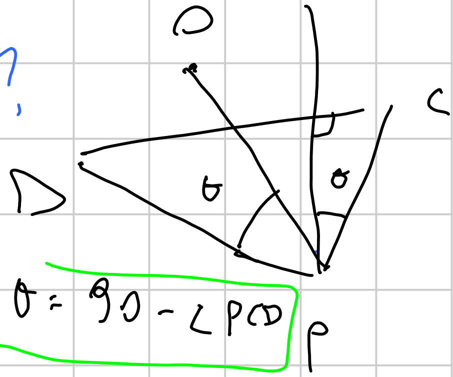


$\angle APM = 90 - \theta$?

$\cos\theta$, $\sin\theta$ in $\triangle DPC$

$\Downarrow$

APPLY CAPW in $\triangle PAB$



$\boxed{\theta = 90 - \angle PCD}$