A1 ADVANCED

Note Title 03/09/2022

1) $f(n) \mid g(n)$ per infiniti n

$\Rightarrow f(x) \mid g(x)$ in $\mathbb{Z}[x]$.

$$g(x) = f(x)\cdot h(x) + r(x)$$

$$\frac{g}{f} = \underbrace{h(x) + \boxed{\frac{r(x)}{f(x)}}}$$

$$\frac{r(n)}{f(n)} \in \mathbb{Z}$$

$\Rightarrow r(x) = 0$

2)

3) $(a_n)_{n\geq 1}$ $\qquad \lim_{n\to\infty} \frac{a_n}{n} = 0$

$\underbrace{\left(\frac{n}{a_n}\right)_{n\geq 1}}$ assume ogni valore di $\mathbb{Z}^+$

$\frac{n}{a_n} < \frac{n+1}{a_{n+1}}$, $\quad a_{n+1} = a_n$

Se per assurdo m non compare nella forma $\frac{n}{a_n}$

Ci sarà un momento in cui $\frac{n}{a_n} < m$ ma $\frac{n+1}{a_{n+1}} \geq m$ vale =

$\frac{n+1}{a_{n+1}} \geq m > \frac{n}{a_n}$ $\quad \frac{n+1}{a_{n+1}} > \frac{n}{a_n}$ $\quad a_n = a_{n+1}$

$a_n m > n \quad n+1 \geq a_n m$

---

5) $f\left(\frac{10^n - 1}{9}\right) = \frac{10^{a_n} - 1}{9}$

f grado m

$m \quad A. \quad f = A x^m + \dots$

$\approx A \cdot \left(\frac{10^n - 1}{9}\right)^m \approx \boxed{A \cdot \frac{10^{nm}}{9^m}}$

$\approx \frac{10^{a_n} - 1}{9}$

$10^{a_n - nm} \approx C$ $\quad a_n = n \cdot m + k$

$n > M$

$f\left(\frac{10^n - 1}{9}\right) = \frac{10^{n \cdot m + k} - 1}{9}$

$10^k = 9x + 1$ $\quad \frac{(10^n)^m \cdot 10^k - 1}{9}$

$\frac{(9x+1)^m \cdot 10^k - 1}{9}$ vero per infiniti

$$\Rightarrow \quad f(x) = \frac{(9x+1)^m 10^k - 1}{9}$$

5) $a, b > 1$

FATTO IMPORTANTE: la somma dei reciproci dei numeri scritti in base b senza la cifra k converge (é finita)

$$\frac{1}{x} \leq \frac{1}{y} \qquad y \leq x$$

Consideriamo i numeri con m cifre
sono tutti $\geq b^{m-1}$

$$\sum_{\substack{x \text{ m cifre} \\ \text{base } b, \text{ no cifra } k}} \frac{1}{x} \leq \frac{(b-1)^m}{b^{m-1}}$$

$$\sum_{m=1}^{\infty} b \frac{(b-1)^m}{b^m} = b \underbrace{\sum_{m=1}^{\infty} \left(1-\frac{1}{b}\right)^m}_{< 1 < \infty} \qquad \left(\sum x^n = \frac{1}{1-x} \quad |x| < 1 \right)$$

$$\sum_{k=1}^{\infty} \frac{1}{ka} = \infty \qquad \left(\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverge} \right)$$

6) $x_n = a_n - a_{n-1}$
$y_n = b_n - b_{n-1}$

$$x_n(x_n+x_{n-1})+y_n(y_n+y_{n-1})=0$$

$$2\cdot \boxed{x_n^2+x_nx_{n-1}+y_n^2+y_ny_{n-1}=0}$$

$$x_n^2+(x_n+x_{n-1})^2+y_n^2+(y_n+y_{n-1})^2=x_{n-1}^2+y_{n-1}^2$$

$$\Rightarrow \boxed{x_{n-1}^2+y_{n-1}^2}\geq x_n^2+y_n^2$$

$=\iff \boxed{\begin{matrix} x_n+x_{n-1}=0 \\ \wedge\ y_n+y_{n-1}=0 \end{matrix}} \Rightarrow a_n=a_{n+2}\ \forall n>M$

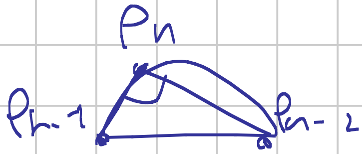
e

$(a_n=a_n+2022)$

[(inutile oops)
(cinofto)

$P_n=(a_n,b_n)$

Condizione $\iff$ $P_n\in (P_{n-1}P_{n-2})$

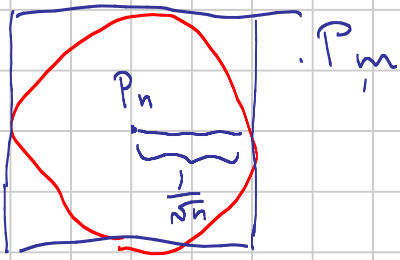


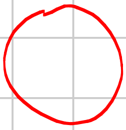
---

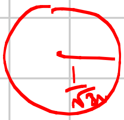
7) Esistono $a_1,a_2\dots$ e $b_1,b_2\dots$ sequenze limitate di reali tali che $\forall\ m>n\in\mathbb{N}^2$ almeno una delle due $|a_m-a_n|>\frac{1}{\sqrt{n}}$ e $|b_m-b_n|>\frac{1}{\sqrt{n}}$ è rispettata?

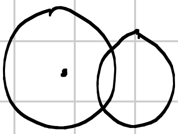
$P_n(a_n,b_n)$

P_i limitati: stanno in una regione limitata del piano

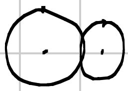






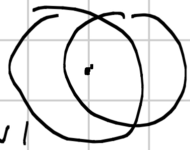


NON E' COMODO



dimezziamo il raggio

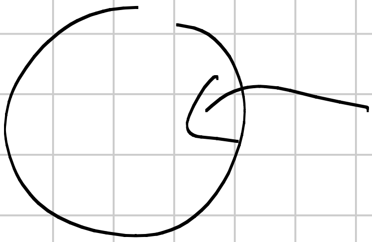
×2



$\frac{1}{2\sqrt{n}}$ la condizione diventa NO ~~OVERLAP~~ INTERSEZIONI

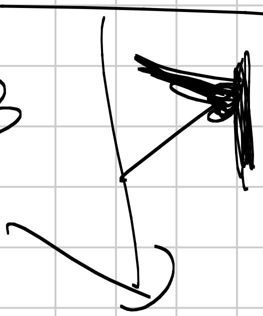
e se $R = \frac{1}{2\sqrt{n}}$

$\sum_{n=1}^{\infty} \frac{\alpha}{n}$ esplode



NON CI STANNO

- IBEROAMERICAN 2021/3
- USA TSTST 8 2021
- USAMO 6 2015



(AoPS)

Per il 3)

(consider $\left\{ n : \frac{a_{mn}}{mn} \geq \frac{1}{m} \right\}$

$1 \in$ (prendi il minimo)