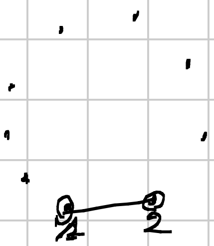
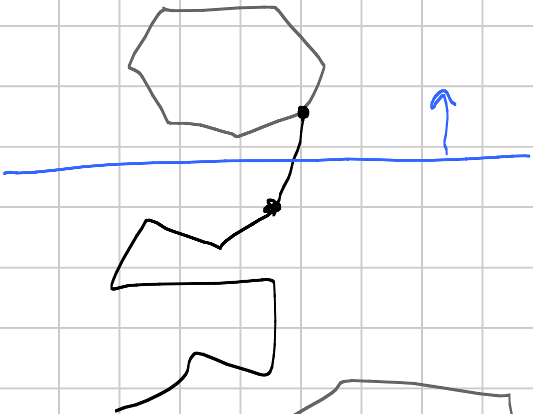
C2(G) ADVANCED

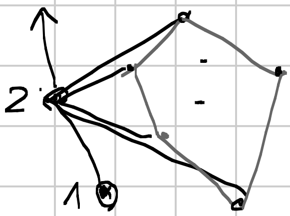
Note Title 07/09/2022

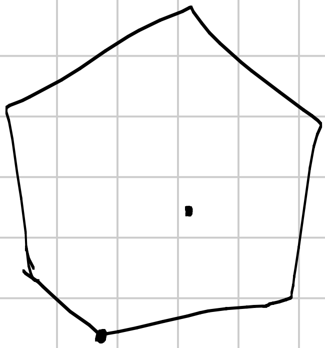
1. $n \cdot 2^{n-2} \cdot \frac{1}{2} = n \cdot 2^{n-3}$

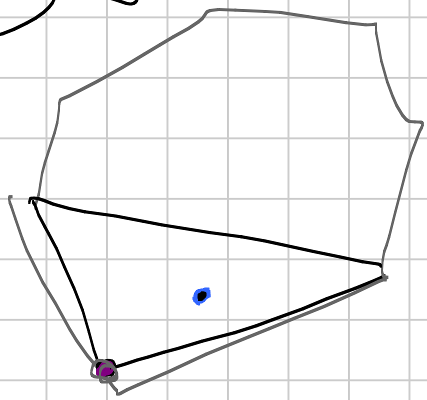




4 scelte







$\underline{m}$ — 1ª mossa

mosse: $\underbrace{2, \dots, n-1}$

mossa i: $(2 + a_i)$ possibilità

nª mossa forzata

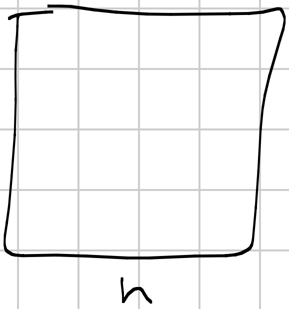
$\sum a_i = n - m$

Sul bordo del c.h. ho m punti inizialmente.

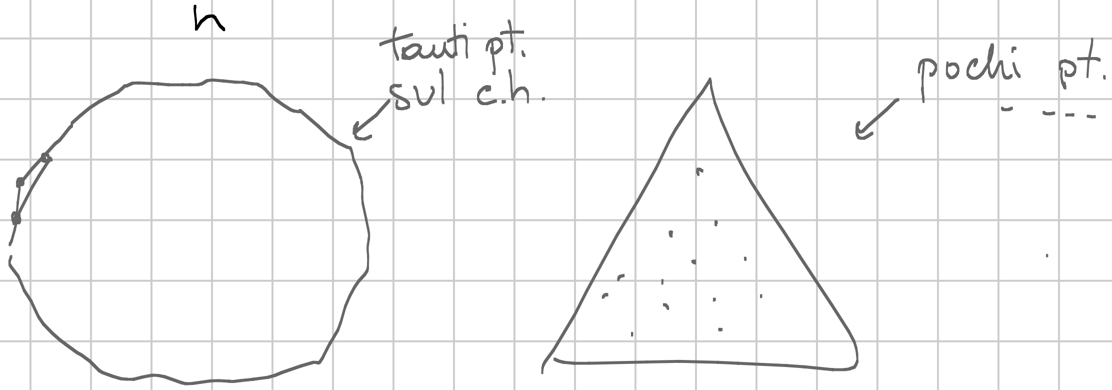
$$\frac{1}{2} m \cdot (2 + a_2) \dots (2 + a_{n-1}) \geq \frac{1}{2} m (2 + n - m) \cdot \cancel{2^{n-3}} > n \cdot \cancel{2^{n-3}}$$

$$\Longleftrightarrow \quad \underline{m(2+n-m)} > \underline{2 \cdot n} \qquad \square$$

2.



$(n+1)^2$ punti all'interno



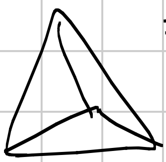
Sia k il n° di pt. sul c.h.

$k \geq 4n$ $\qquad a_1, a_2, \dots, a_k \qquad \sum a_i \leq 4n$

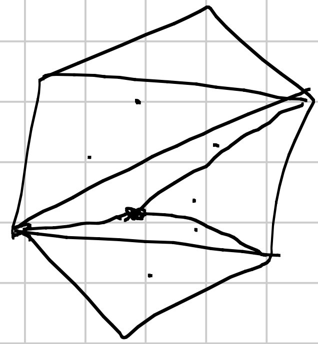
$$\sum_{cyc.} \frac{a_i + a_{i+1}}{2} \leq 4n$$

$$\exists i \quad \frac{a_i + a_{i+1}}{2} \leq \frac{4n}{k} \leq 1 \longrightarrow \frac{a_i \cdot a_{i+1} \cdot \sin \angle(a_i, a_{i+1})}{2} \leq \frac{1}{2}$$

$k < 4n$ $\qquad$ Triangolo il k-agono $\rightarrow$ $k-2$ triangoli



$$\# tr = k-2 + 2\left((n+1)^2 - k\right)$$
$$= 2(n+1)^2 - k - 2$$
$$= 2n^2 + 4n - k$$

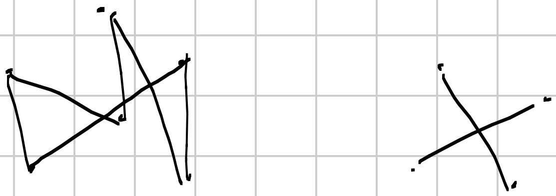


Want: $\# tr \cdot \frac{1}{2} > n^2$

$$\cancel{n^2} + 2n - \frac{k}{2} > \cancel{n^2} \qquad 4n > k$$

$\square$

3. $n \geq 4$ punti, n segmenti, ogni punto estremo di 2 segmenti

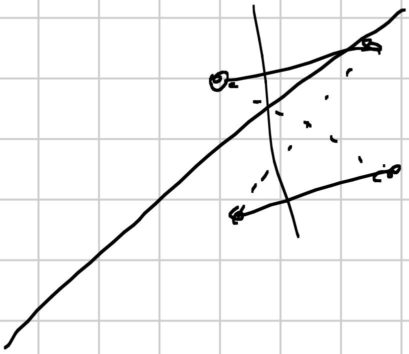


Max $n^2/4$ mosse

Mosse finite: la lunghezza totale diminuisce

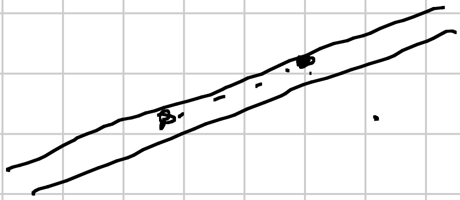
Cerchiamo monovariante "quantitativo", circa n^3

Considerare intersezioni tra segmenti e rette.

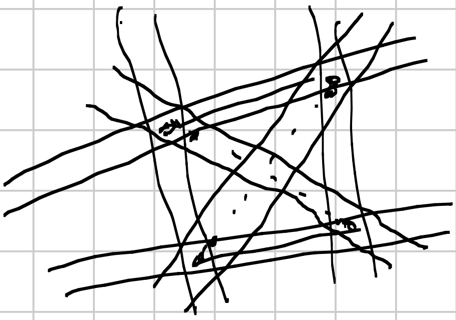


Il n° di intersezioni non aumenta

Vogliamo che diminuisca ad ogni mossa



Per ogni coppia di punti

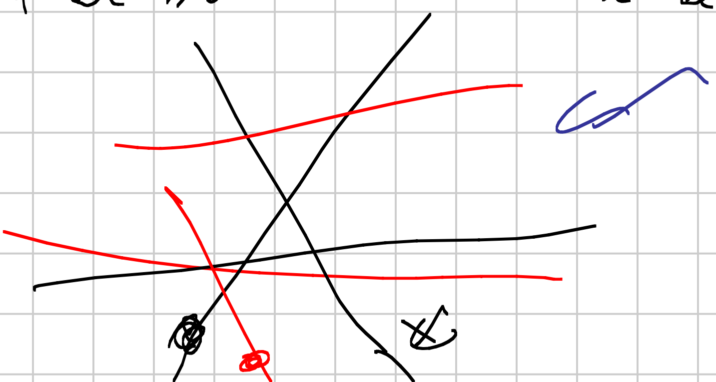


Rette parallele ai segmenti nuovi: -4 intersezioni

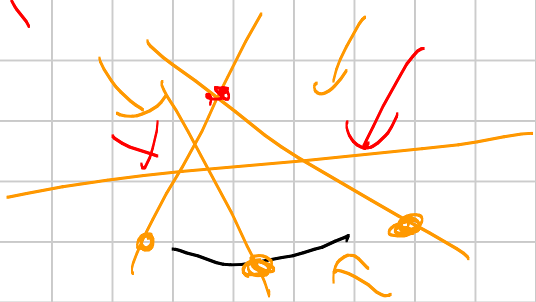
Valore iniziale: al più $n \cdot \frac{n \cdot (n-1)}{2} \cdot 2 = n^2(n-1)$

finale ≥ 0 mosse al più $\frac{n^2(n-1)}{4}$.

9



(più n) ok.



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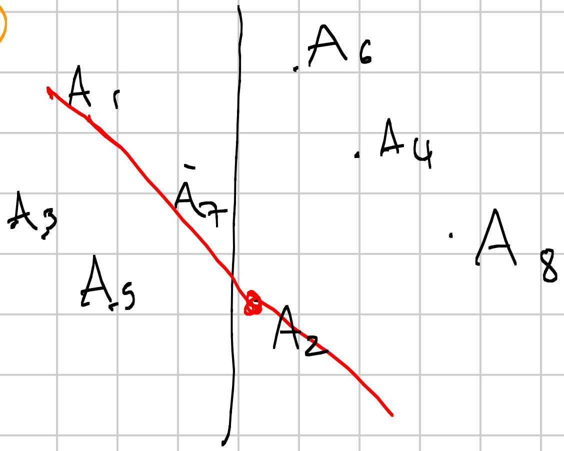
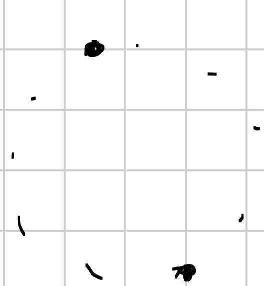


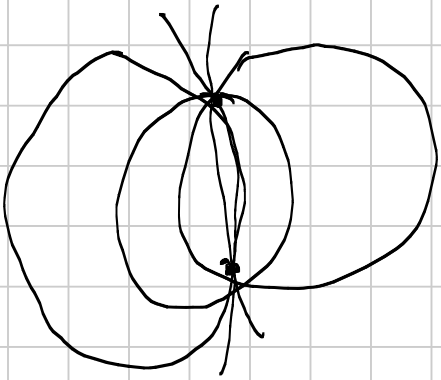
Somma con segno degli angoli è 0.

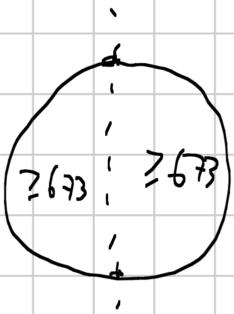
6



Converso

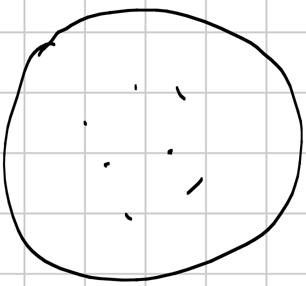
Z punti, ogni circonferenza ne contiene almeno $\sim \frac{n}{3}$

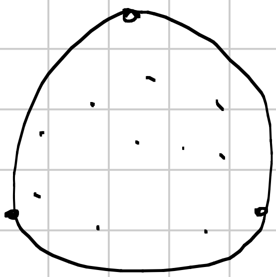


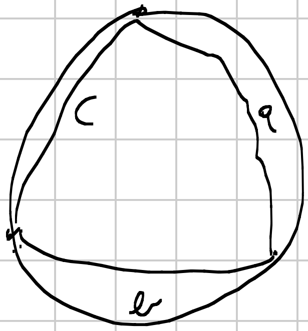


allora ho vinto

Cerco circonferenza grande, che contiene tutti i punti

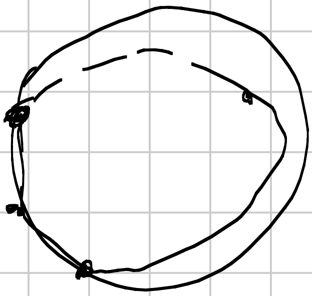






$a+b+c = 2018$, almeno uno è ≥ 673

$a, b, c \leq 1009$



7) ERDOS (2e)

n^2+1 numeri in fila, esiste (sotto) successione crescente lunga almeno $n+1$ o una decrescente lunga almeno $n+1$.

X CASA (se non avete studiato...)

$P_1 \dots$ $\qquad$ P_{n^2+1}