

## INVOLUZIONI

$$\varphi \quad \varphi(\varphi(A)) = A \quad \forall A \in C$$

$\varphi$  SIA PROIETTIVA

$$(A, B; C, D) = (\varphi(A), \varphi(B); \dots)$$

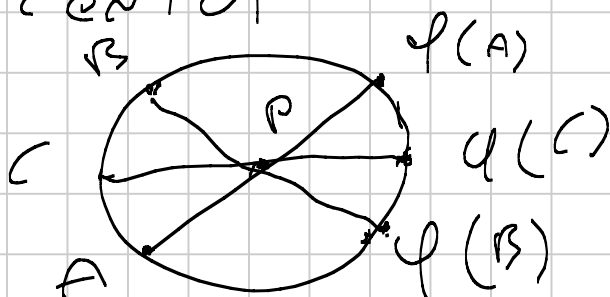
$$\varphi(\varphi(A)) = A$$

$$(A, \varphi(A); B, \varphi(B)) = (\varphi(A), \varphi(\varphi(A)); \varphi(B), \varphi(\varphi(B)))$$

$$= (A, \varphi(A); \varphi(\varphi(B)), \varphi(B))$$

$$\Rightarrow B = \varphi(\varphi(B))$$

5 cenica



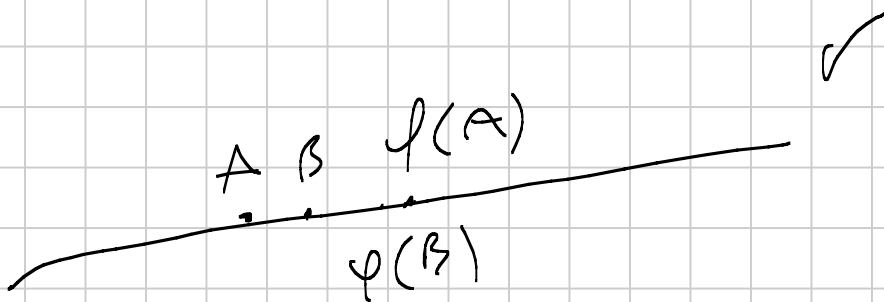
$X, \varphi(X)$  PASSA UN PUNTO FISSATO

$$\boxed{A, \varphi(A), B, \varphi(B)}$$

DATI UNA TRASFORMAZIONE

$$\underline{(A, B; C, X)} = \underline{(\varphi(A), \varphi(B); \varphi(C), \varphi(X))}$$

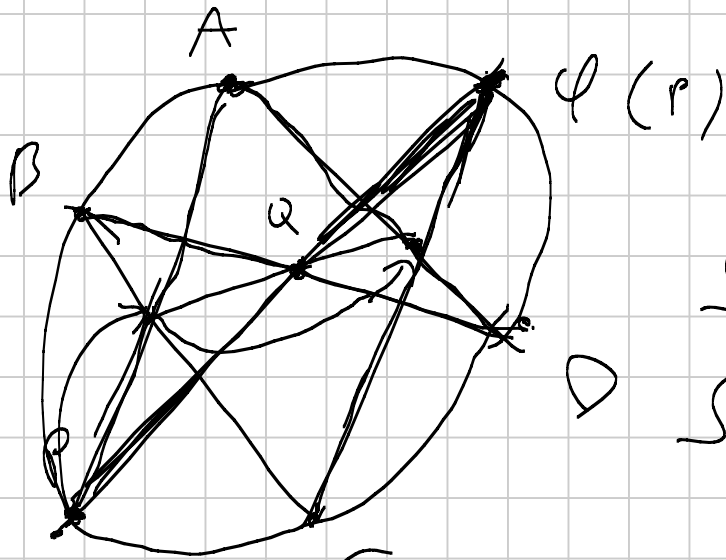
LEMMA 2



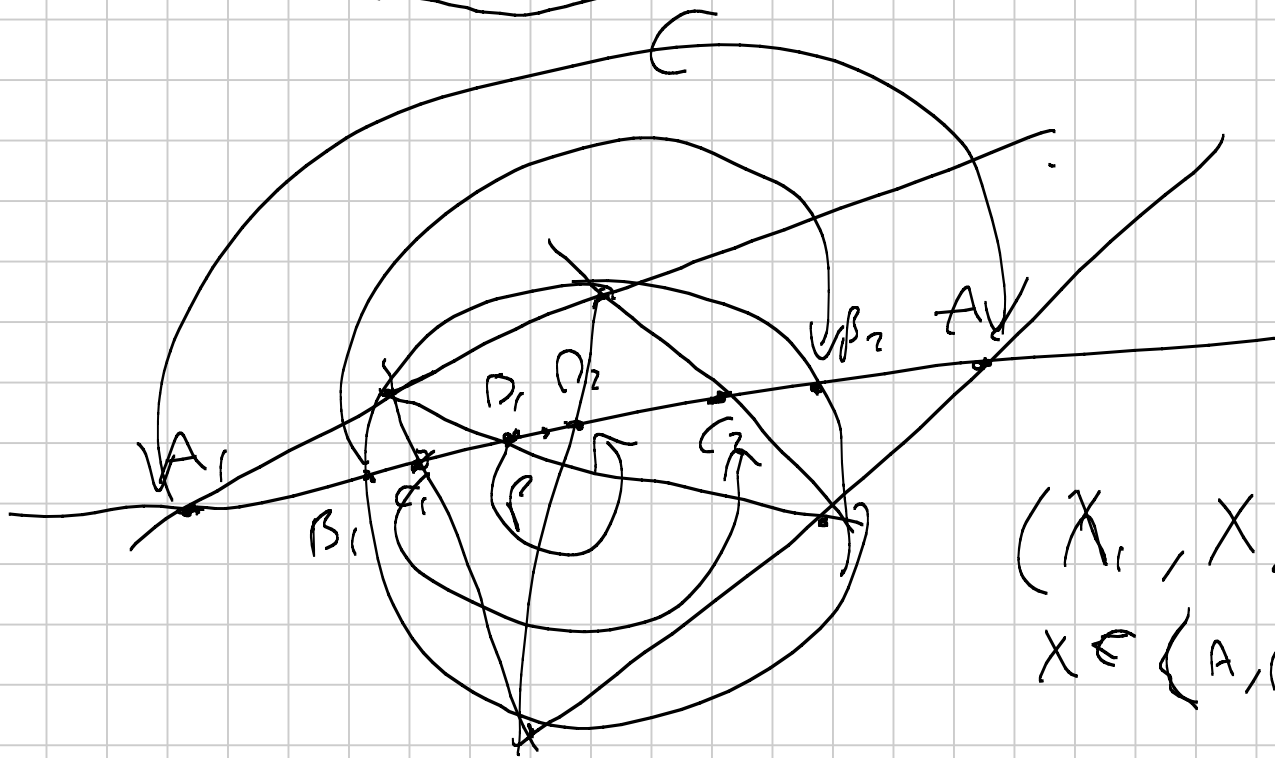
$$p_{00} \rightarrow \textcircled{A} \quad A, \sqrt{AB \cdot A\varphi(B)}$$
$$B \quad \varphi(B)$$

$$\boxed{\begin{array}{l} B \leftrightarrow \varphi(B) \\ A \leftrightarrow p_{00} \end{array}}$$

PIT

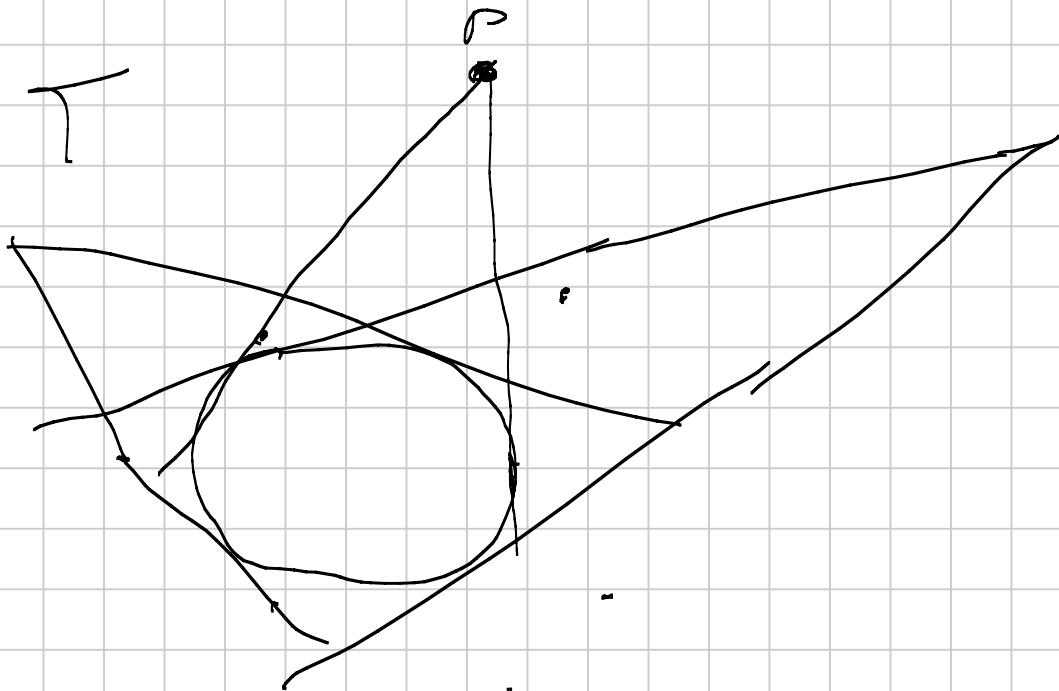


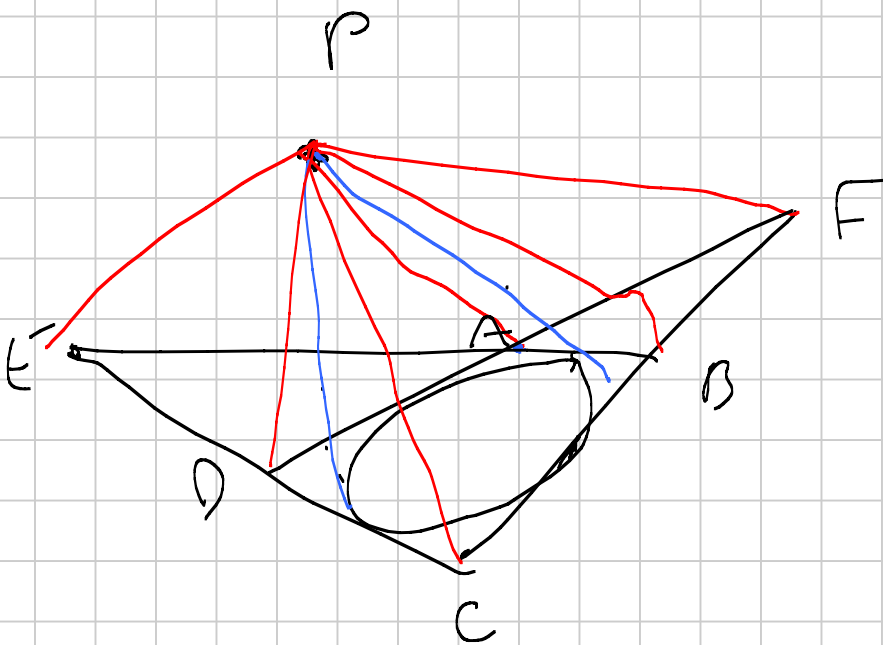
$\Omega_1(\cdot), \Omega_2(\cdot)$   
 $\Omega_3(\cdot)$



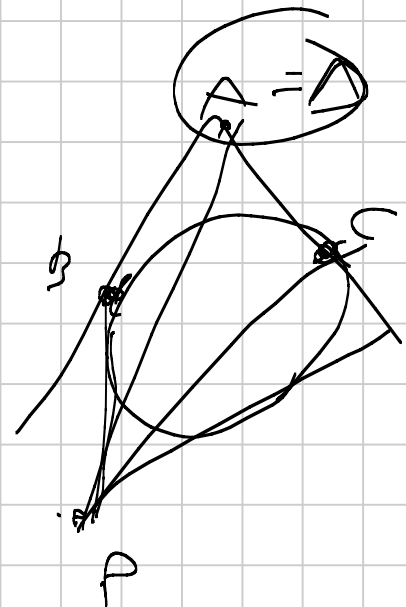
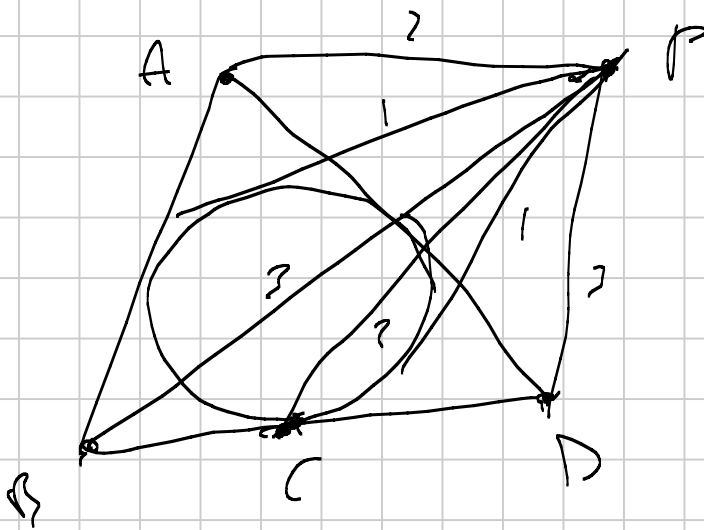
$(X_1, X_2)$   
 $X \in \{A, B, C, P\}$

$\mathbb{C} \cup \mathbb{D} \cup \mathbb{I} \cup \mathbb{T}$

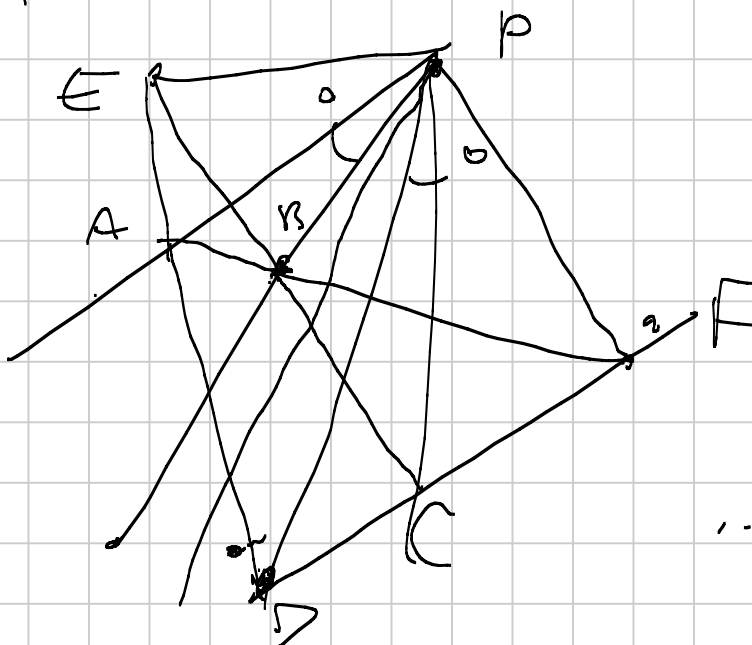




$(PE, PF), \dots$



150 EQUALITY LEMMA

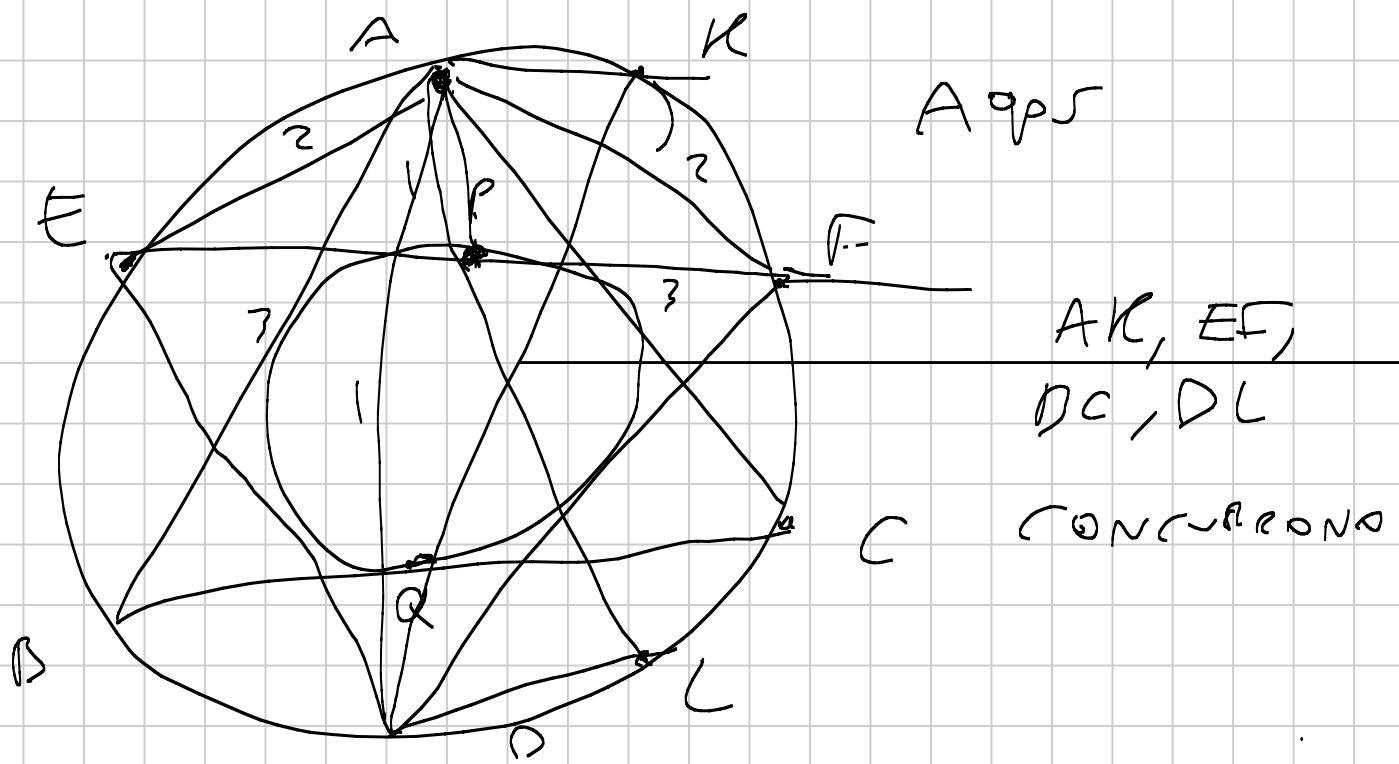


$P \in (C)$

$PA \leftrightarrow PC$   
 $PB \leftrightarrow PD$

$PE \leftrightarrow PF$

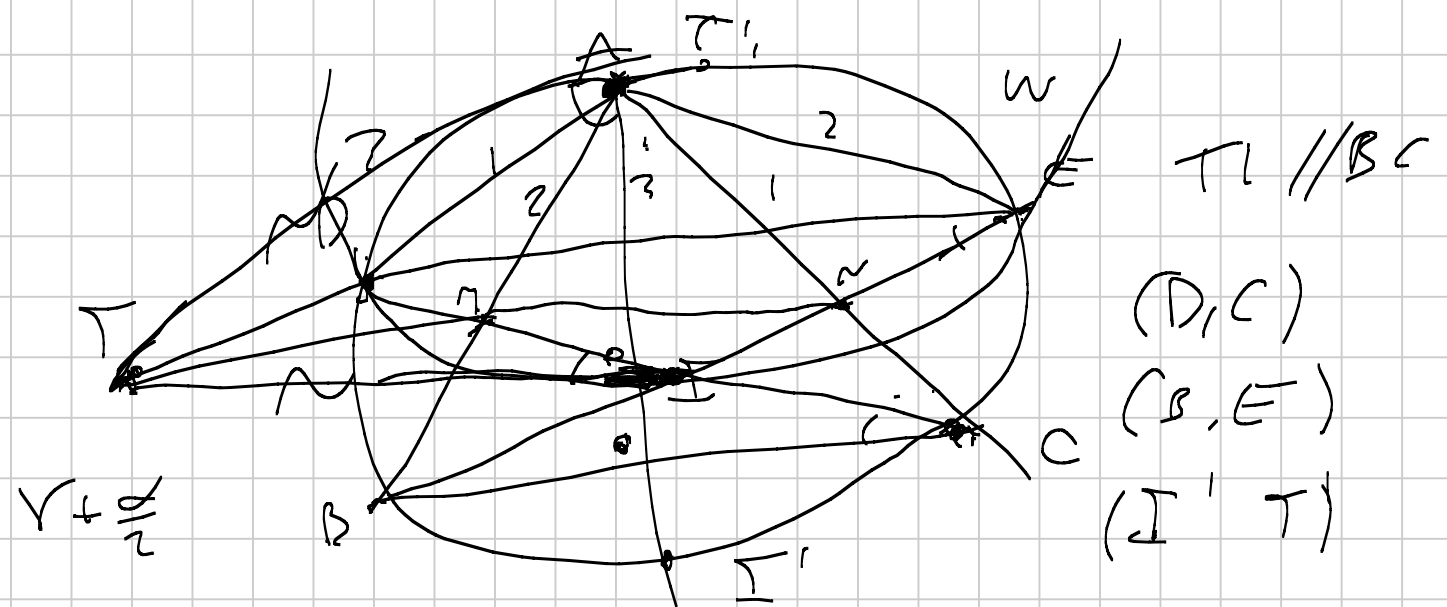
$\dots$



A

$\varphi$   
 $\Omega$   
 $(\Omega^{-1} \circ \varphi \circ \Omega)$

$(E, F), (B, C), (D, L)$   
 sono coppie di INVOLUZIONE



$(AT, AI), (AN, AM), (AE, AD)$

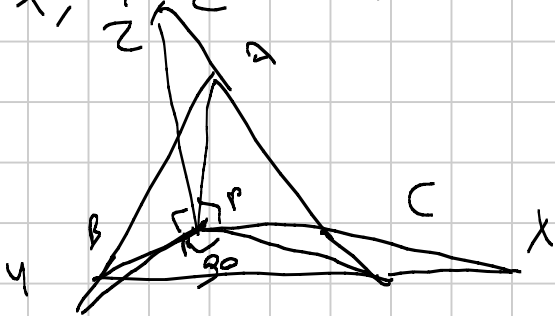
SIA  $P$  UN PUNTO,  $\triangle ABC$  TRIANGOLO,

$X, Y, Z$  PUNTI SULLE RETTE

$BC, AC, AB$  T.C.  $\angle APX = \angle BPY = \angle CPZ = 90^\circ$

DIMOSTRA

$X, Y, Z$  ALLINEATI



SIA  $\omega$  LA CIRCOSCRITTA AD

$\triangle ABC$ ,  $\omega_A$  L'A-EXSCRITTA, LE

TANGENTI COMUNI DI  $\omega, \omega_A$

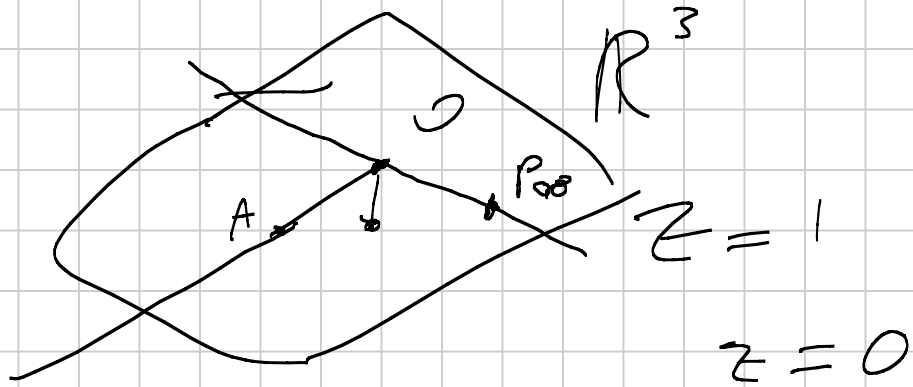
INTERSECANO  $BC$  IN  $P, Q$

DIMOSTRA  $\angle PAB = \angle CAQ$

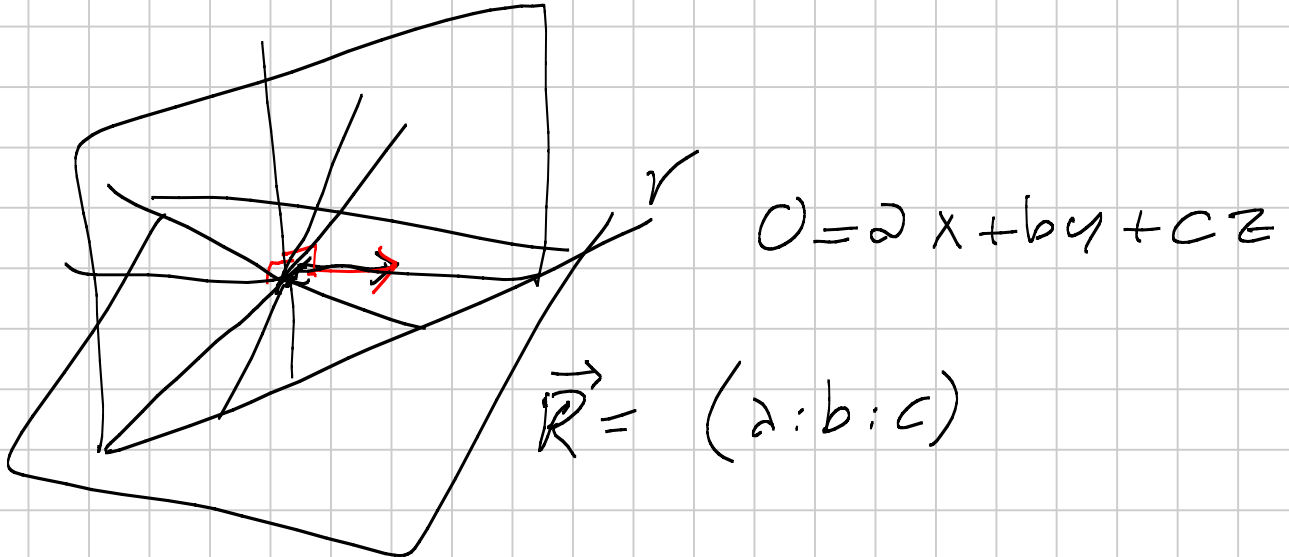
# PIANO PROIETTIVO

$$\mathbb{R}^2, \mathbb{R}^2 + \{\infty\}$$

$$\mathbb{P}^2 \mathbb{R}$$

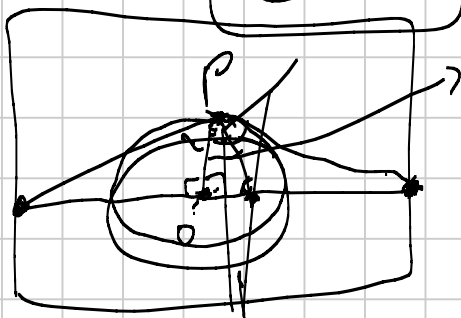


$$A = (a : b : c) = \lambda (a : b : c)$$



$$\vec{A} \times \vec{B}$$

$$\vec{C} \times \vec{D}$$



$\mathbb{P}^2 \subset \mathbb{C}$   
 $\in \mathbb{C}$

6 DIM D1  $\mathbb{C}^3$   
4 DIM D1  $\mathbb{P}^2 \subset \mathbb{C}$

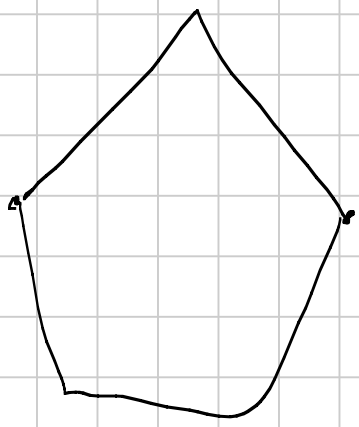
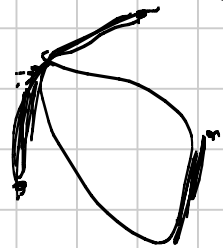
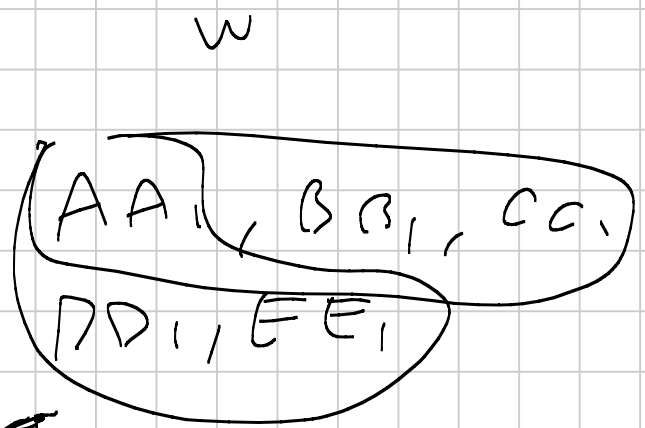
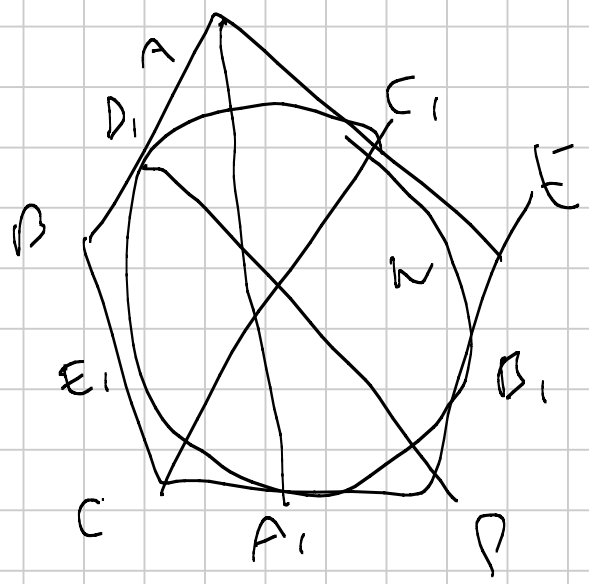
$\lambda(a:b:c)$

# OMOGRAFIA

È UNA TRASFORMAZIONE PROIETTIVA

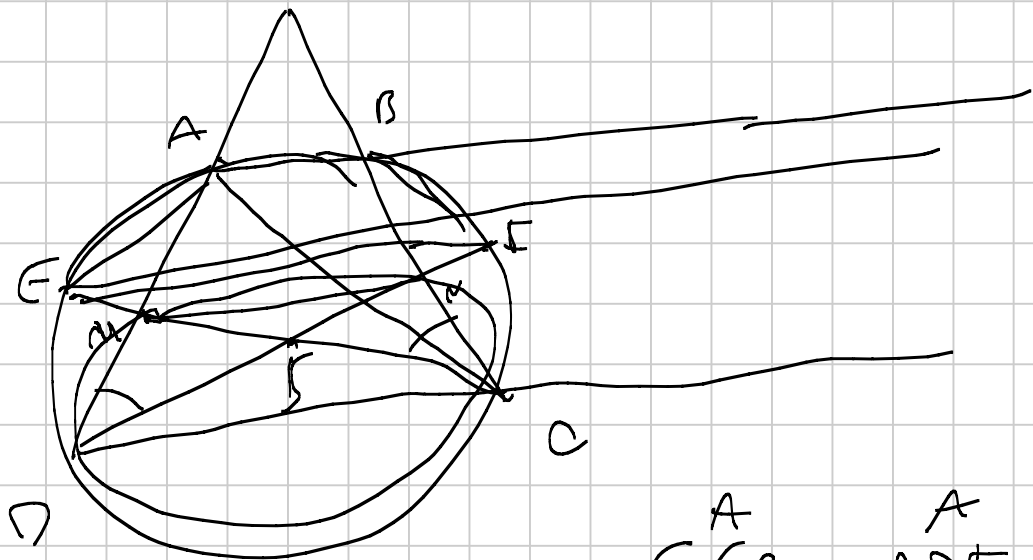
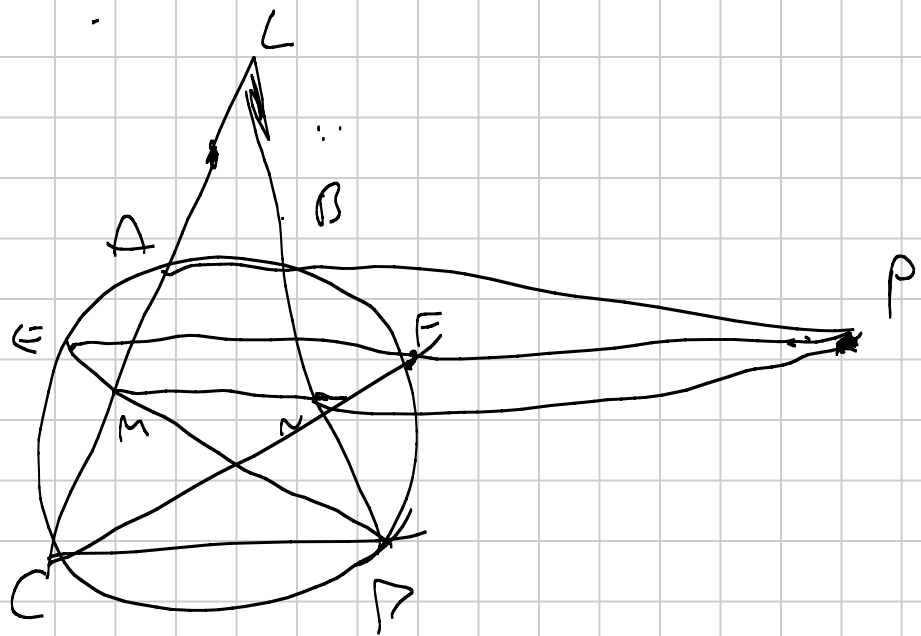
$\mathbb{P}^2 \subset \mathbb{C}^3$

$$(x : y : z) \rightarrow (ax+by+cz : d+ex+fy : gx+hy+iz)$$



1° IL PUNTO DENTRO  
LA CIRCONFERENZA  
IN QUALUNSI PUNTO DENTRO  
LA CIRCONFERENZA È LA  
CIRCONFERENZA IN SE STESSA





$$E \overset{A}{C} B = A \overset{A}{D} F$$



# MOVING POINTS

$$\text{obj}(A(x) : B(x) : C(x)) = \max(\text{obj } A, \text{obj } B, \text{obj } C)$$

in  $P^2$ .

$$\left( \frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} : 1 \right) \in \mathbb{R}$$

$$= (1-t^2 : 2t : 1+t^2)$$

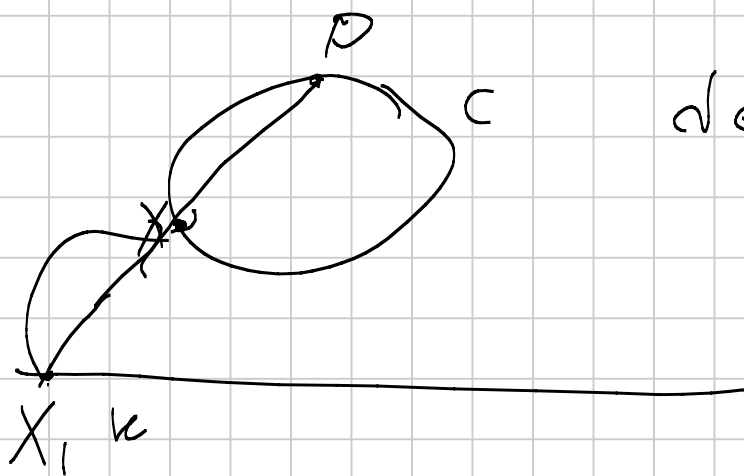
deg A    deg B    A x B

$$\deg(AB) \leq \deg A + \deg B - k$$

k COSTA IL NUMERO DI VOLTE A=B

$$\vec{A} \times \vec{B} = \vec{0} \quad (x-t) \mid 1$$

CONIC DOUBLING



$$\deg(x_i) = \frac{\deg(x)}{2}$$

$$\mathbb{P}^1 : (x : y : z) \rightarrow (xz : yz : x^2 + y^2)$$

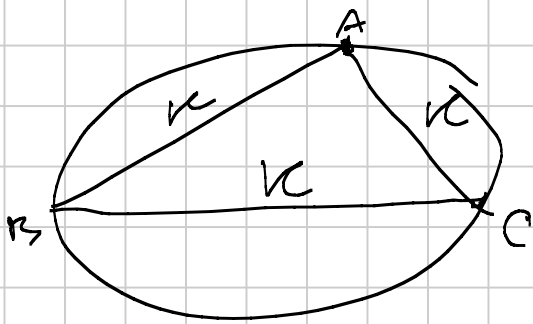
SE CENTRATA NELL'ORIGINE

k

p

$$ax + by + cz = 0$$

di GRADO k



$$\frac{k + k - k}{2} = \frac{k}{2}$$

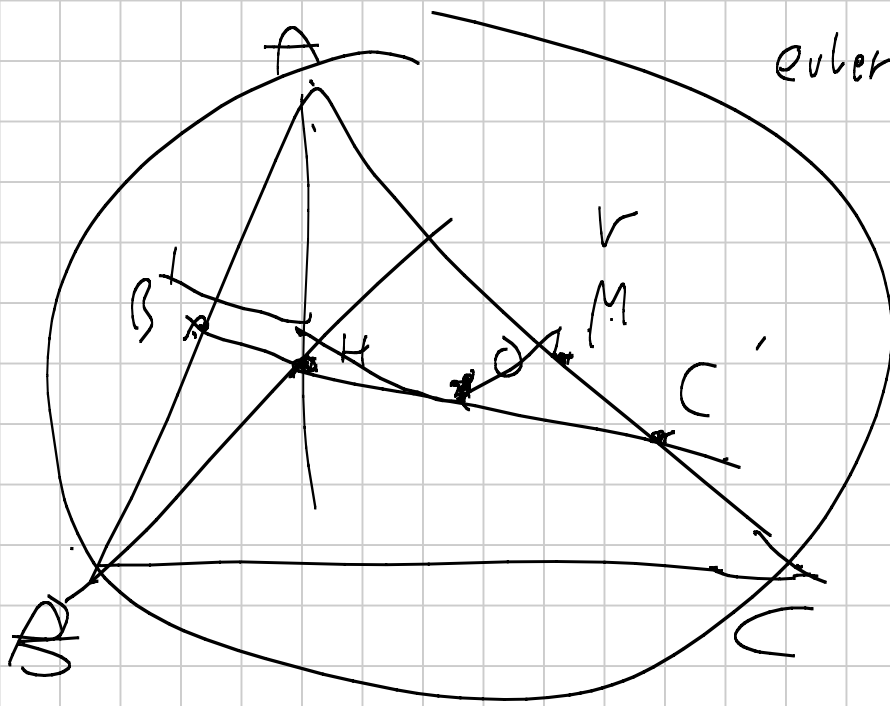
PA tra GRADO

$$k - \frac{k}{2} + 0 = \frac{k}{2}$$

3 punti, 3 rette

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} = 0$$

$$A + B + C + 1$$



Euler's  $A'B'C' \parallel BC$

$$\deg C = \deg l$$

$$\deg H = 1$$

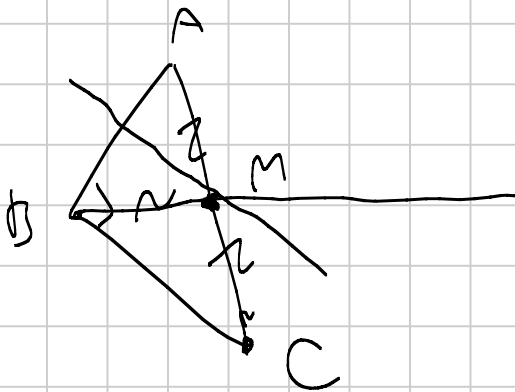
$$\deg O = 1$$

$$\deg OH = 2$$

$$\deg(OH \times \{\infty\}) = 1$$

$$C \rightarrow \infty$$

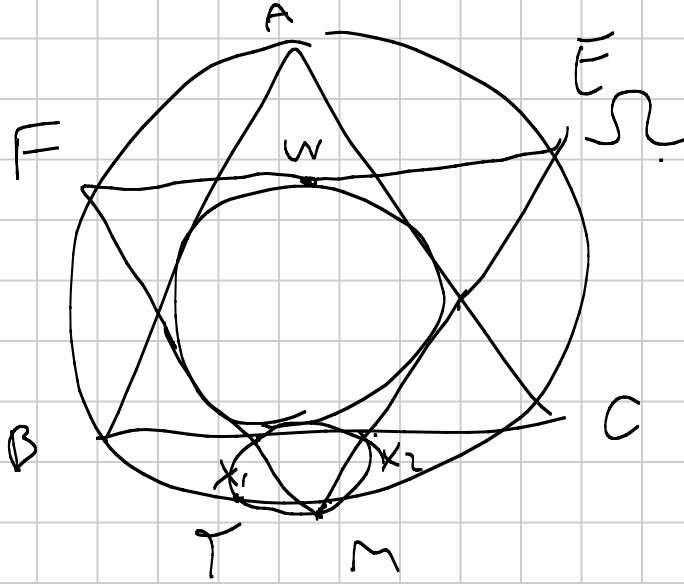
$$\deg(BC \cap \{\infty\}) = 1$$



$\varphi$

$$\varphi(\varphi(P_\infty)) = P_\infty$$

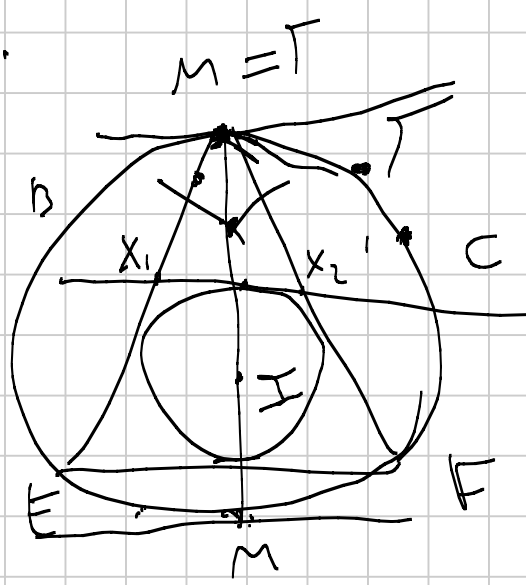

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$M, X_1, X_2, T$   
 $C \in C \in C$   
 (con  $T$  punto  
 di costante  
 mistilinea

$x_1 \rightarrow \infty$

$\rho$



$\deg Q = 2$

$\deg N Q (1)$

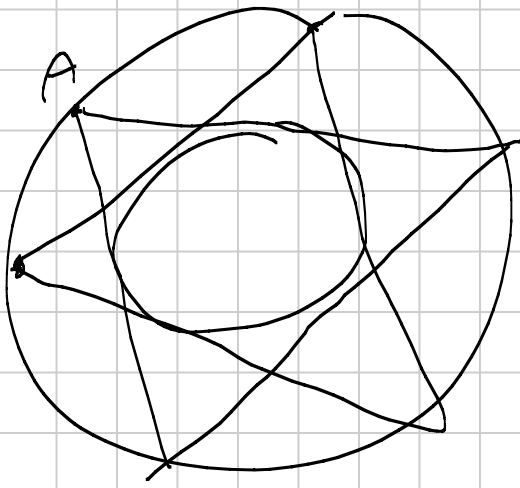
$x_2, x_1$  piano  $\deg = 1$

$\deg T = 2$

$\deg MT = 1$

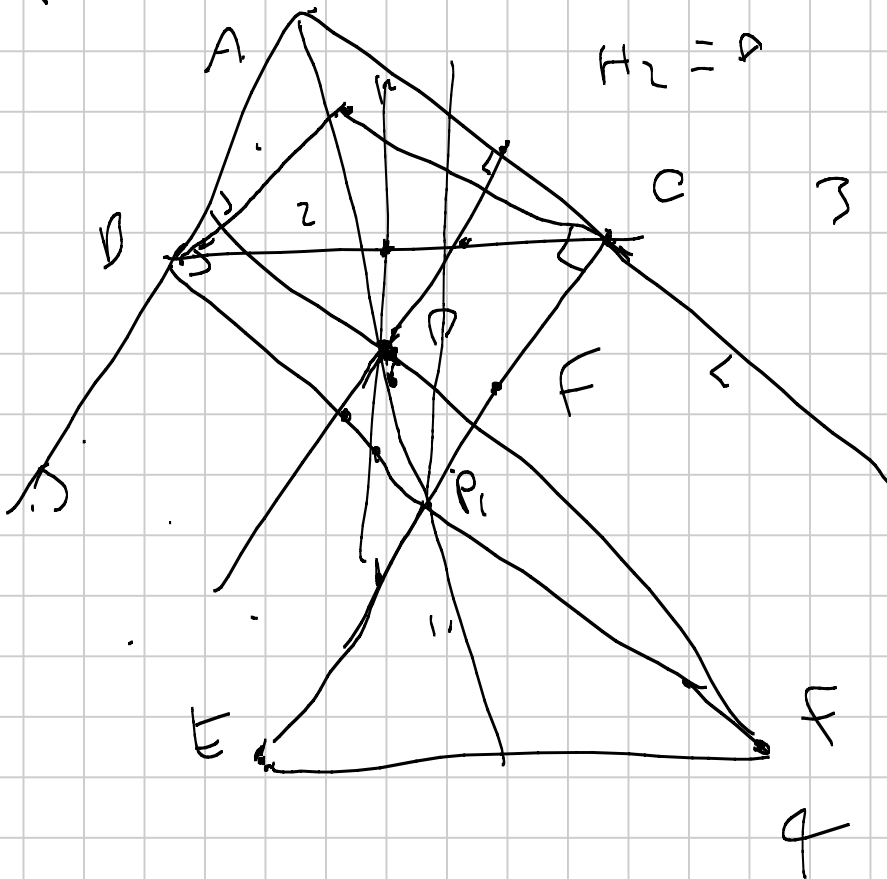
$E' \deg T' \rho \text{ di } MT$

$1 + 1 + 1 + 1 = 4 \quad \text{CASI}$



SUNOAT

$\triangle ABC, \triangle DEF$  SONO ORTOGONALI  
 E PROSPETTICI ALLORA I  
 DUE CENTRI DI ORTOGONIA E IL  
 PERSPETTORE SONO ALLINEATI



$H_2 = A$

$\triangle ABC$

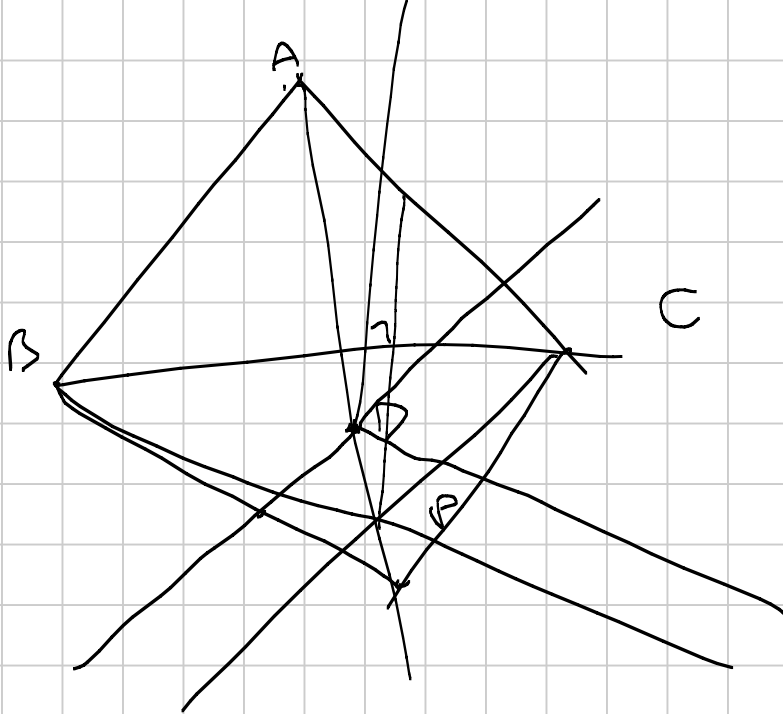
3 RETTE PER  $\triangle ABC$

D

$\deg PE = 1$

$(\deg H_2) = 2$

$\deg H_1 = 1$



• A, B che

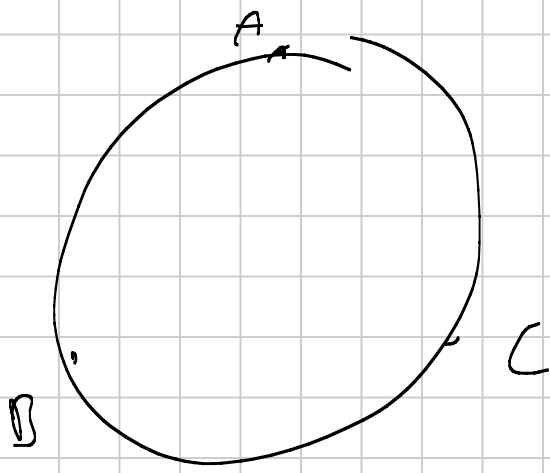
$M \in A, B$

$MA \neq MB$

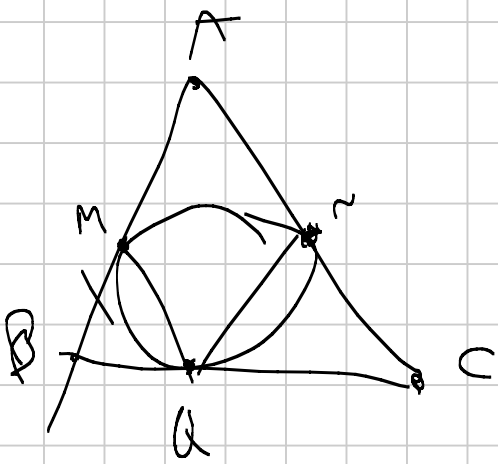
deg A + d. di B = K.

AB LA RETTA ALL'INFINITO





deg 2



$P^2 C$  SI MUOVE SU UNA NUMA DI  
 GRADO  $\omega$  CON GRADO  $n \cdot \omega$

$P \in X \rightarrow Y \quad n \rightarrow 1$

GS 2016

Gh 2021

$\triangle ABC$

$I \rightarrow$  INCENTRO

$I$  È IL PUNTO A, B, C EXCENTRO

$N$  È BIC  $X(5)$ ,  $NBC$ ,  $X(5)$   $N_1$

$NN_1 \parallel$  EULERO  $\triangle ABC$