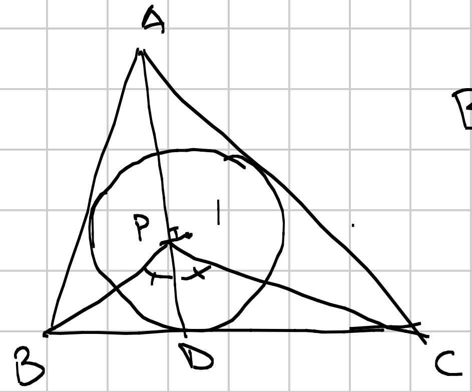
S23 ADV G2

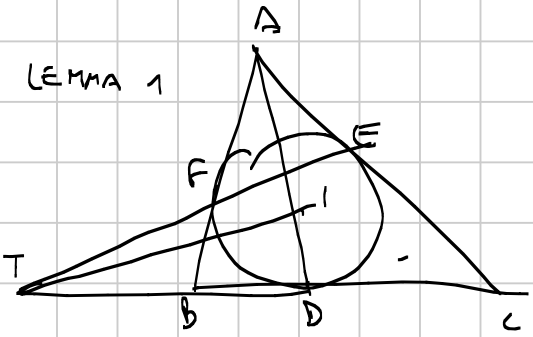
Note Title 06/09/2023

1)



$B\hat{P}D = D\hat{P}C$

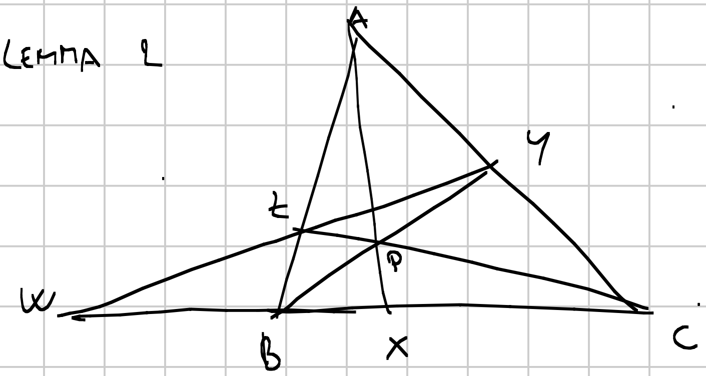
LEMMA 1



$T = BC \cap EF$

$IT \perp AD$

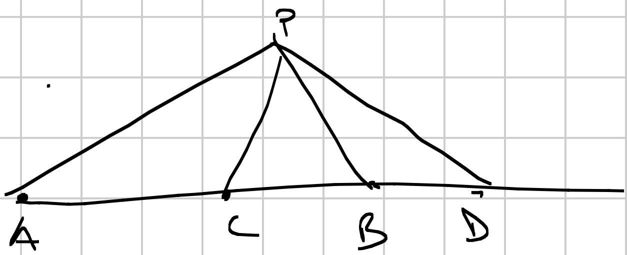
LEMMA 2



$W = BC \cap YZ$

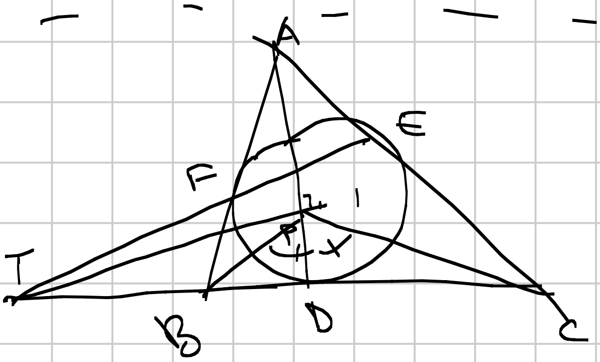
$(W, X; B, C) = -1$

LEMMA 3



- $(A, B; C, D) = -1$
- $AP \perp PB$
- $A\hat{P}C = C\hat{P}B$

Due implicano l'altra

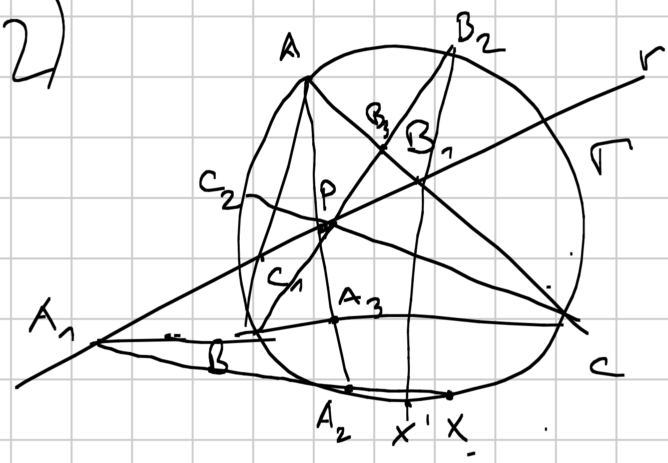


USANDO I LEMMI

$(T, D; B, C) = -1$ e $PT \perp PD$

IMPLICANO LA TESI

2)



A_1A_2, B_1B_2 e C_1C_2 CONCORRONO

CLAIM: CONCORRONO SULLA CIRCOSCRITTA

$X = A_1A_2 \cap \Gamma$ $\quad$ $X' = B_1B_2 \cap \Gamma$ $\quad$ se $X \equiv X'$ FINE

$A_3 = AP \cap BC$ $\quad$ $B_3 = BP \cap AC$

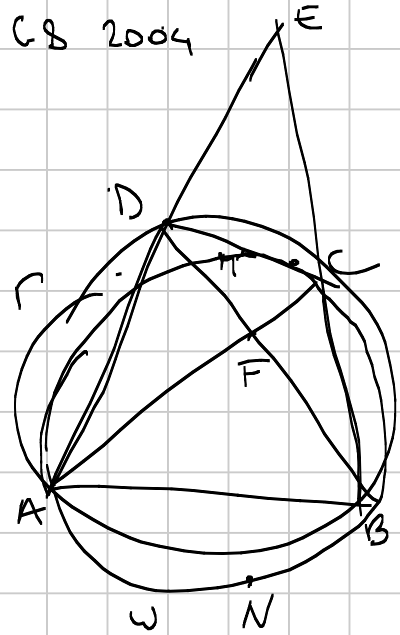
$$(X, A; B, C) \overset{A_2}{=} (A_1, A_3; B, C) \overset{P}{=} (B_1, A; B_3, C)$$
$$\overset{B_2}{=} (X', A; B, C)$$

QUINDI $(X, A; B, C) = (X', A; B, C)$

BIRAPPORTO è BIETTIVO $\longrightarrow$ $X \equiv X'$ FINE

IMO SL G8 2004

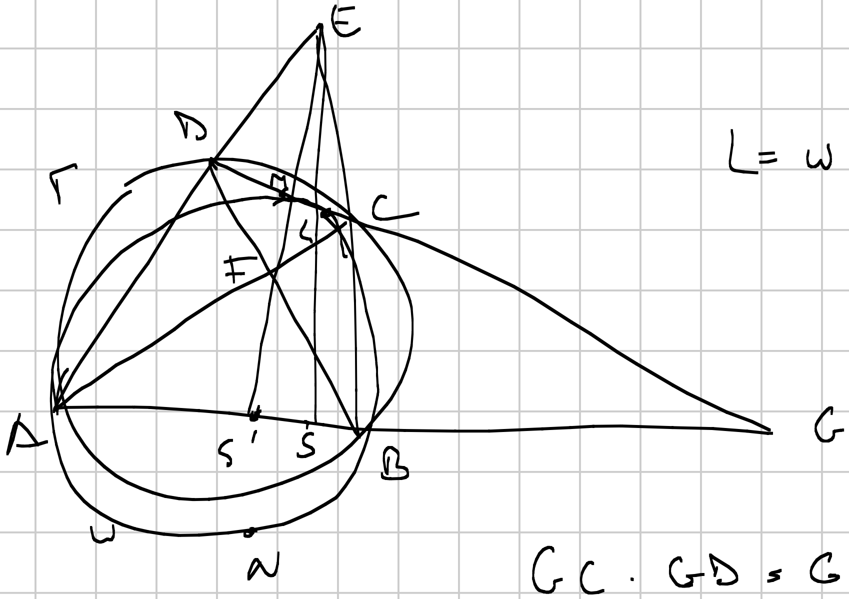
3)



$\frac{AN}{BN} = \frac{AM}{BM}$, $N \in \omega$

E, F, N ALLINEATI

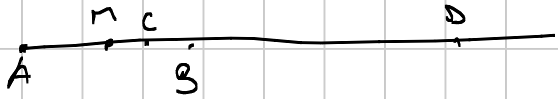
ANBM è ARMONICO
$\longrightarrow (A, B; N, M) = -1$



$L = \omega \cap CD$

$GC \cdot GD = GA \cdot GB = GL \cdot GM$

LEMMA



$$(A,B;C,D) = -1 \iff DA \cdot DB = DC \cdot DM$$

QUINDI PER IL LEMMA HO $(D,C;L,G) = -1$

$EL \cap AB = S$ $\quad$ $EF \cap AB = S'$

1. E, L, F ALLINEATI

$(A,B;S',G) = -1$, $(A,B;S,G) \overset{E}{=} (D,C;L,G) = -1$

$\rightarrow S = S'$ FINE

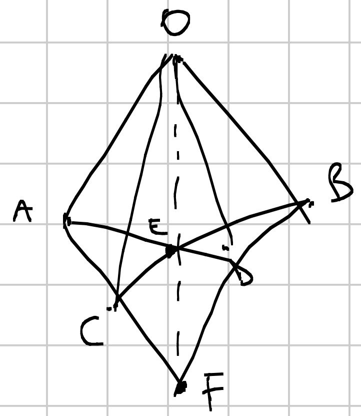
QUINDI BASTA E, L, N ALLINEATI

$(LA, LB; LN, LM) = -1 \rightarrow (LA, LB; LN, LG) = -1$ (1)

$(LA, LB; LS, LG) = -1 \rightarrow (LA, LB; LE, LG) = -1$ (2)

(1) e (2) $\rightarrow$ LA RETTA LE COINCIDE CON LA RETTA LN $\rightarrow$ E, L, N ALLINEATI $\rightarrow$ E, N, F ALLINEATI

ISOGONALITY LEMMA



BISETTRICE DI $A\hat{O}B$

=

BISETTRICE DI $C\hat{O}D$

$E = AD \cap BC$ $\quad$ $F = AC \cap BD$

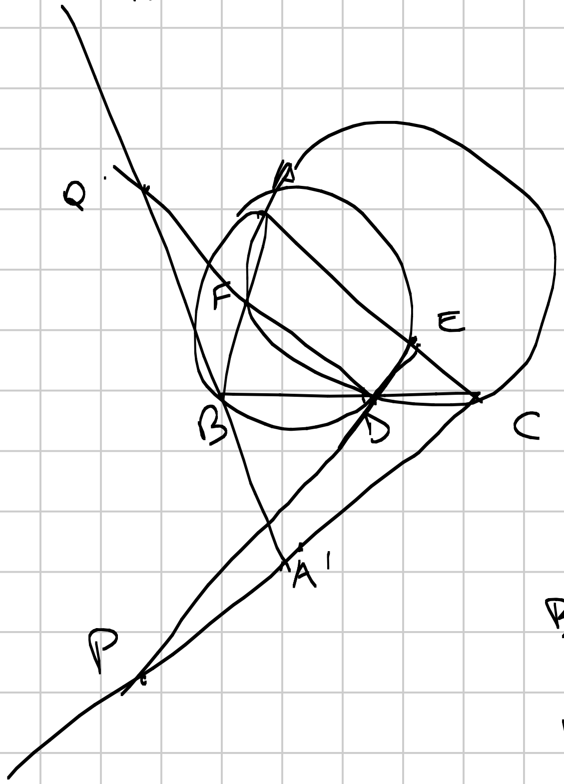
TESI: BISETTRICE DI $A\hat{O}B$

=

BISETTRICE DI $E\hat{O}F$

RMM 1 2016

4)



AD, CQ, BP CONCORRONO

LEMMA CON $O \equiv D$

BISETTRICE DI DB e DC

=

BISETTRICE DI DP e DQ

$X = BQ \cap CP$ $\quad$ e $\quad$ $Y = BP \cap CQ$

$\equiv$

A'

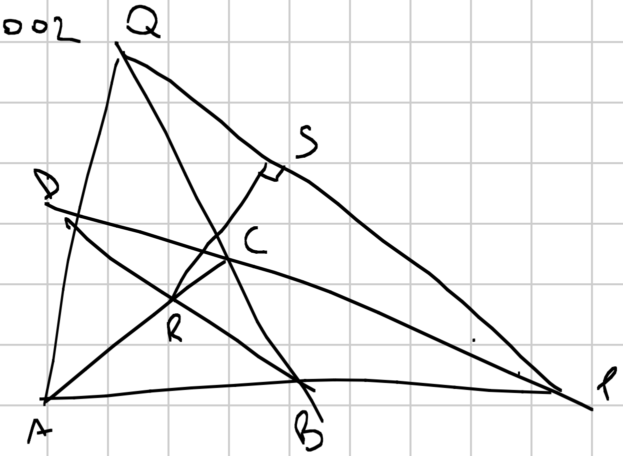
//

BISETTRICE DI $A'D$ e DY

$\longrightarrow$ $A \in DY$ $\longrightarrow$ A, D e Y ALLINEATI

CHINA TST 2002

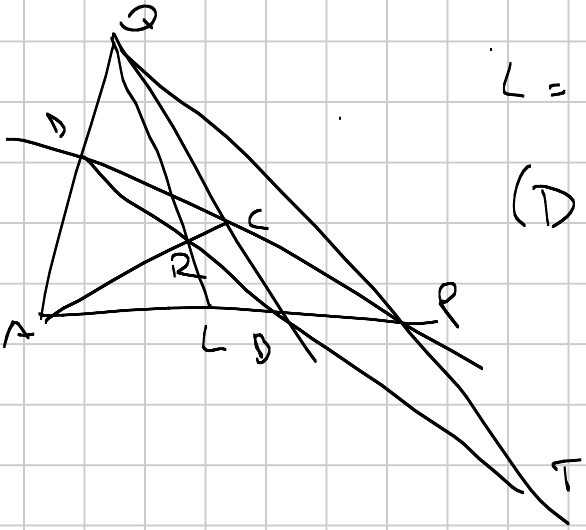
5)



$D\hat{S}A = C\hat{S}B$

SR biseca $D\hat{S}B$:

$T = DB \cap PQ$, voglio $(T, R; D, B) = -1$ poi ho chiuso per il lemma



$L = QR \cap AP \rightarrow (A, B; L, P) = -1$

$(D, B; R, T) \overset{Q}{=} (A, B; L, P) = -1$

BISETTRICE $D\hat{S}B$ = BISETTRICE DI $Q\hat{S}P$

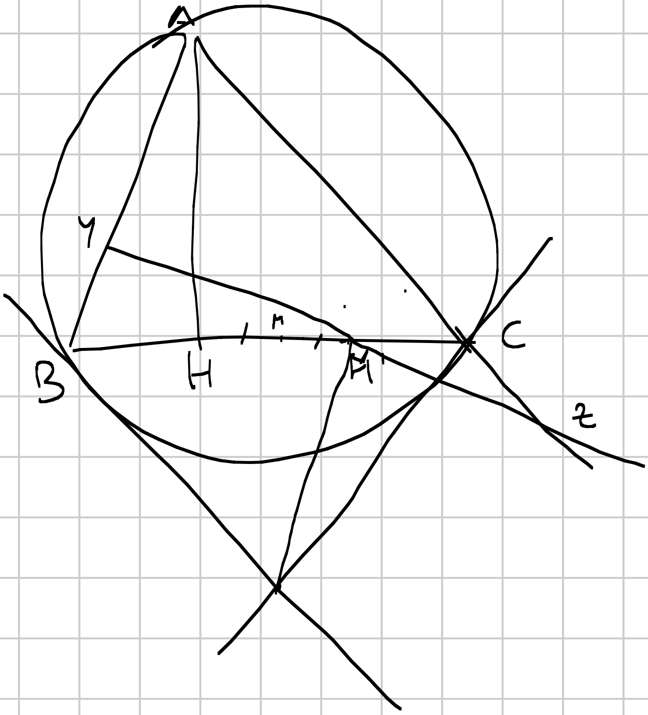
QUINDI (SA, SC) e (SB, SD) ISOGONALI

E QUINDI LA TESI È OVVIA.

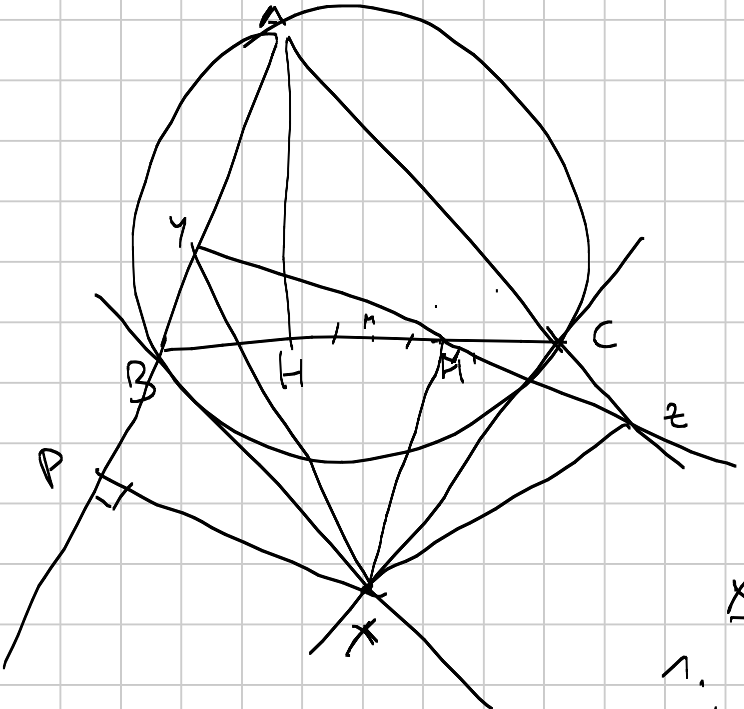
x

IRAN TST 2015

6)



$B\hat{X}Y = C\hat{X}Z$



APPLICATO SU X

PER ISOGONALITY BASTA

$C\hat{X}A = H'XY$

P t.c. $XP \perp AB$

$(XPYH)$

$\Delta H'BP \sim \Delta ACX$:

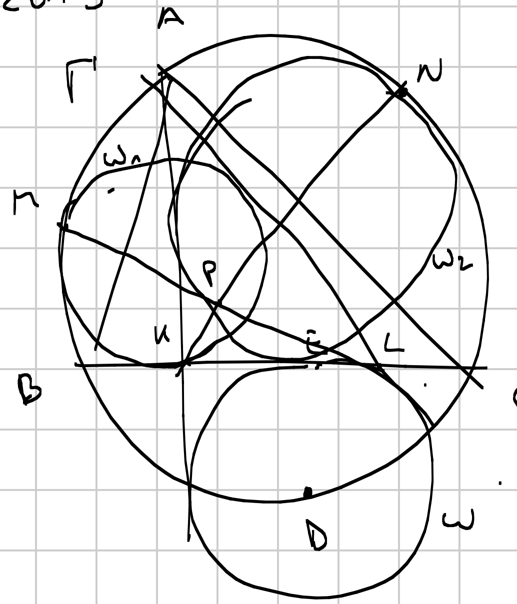
1. $H'\hat{B}P = A\hat{C}X$
2. $\frac{AC}{CX} = \frac{AC \cdot \cos\gamma}{BX \cos\gamma} = \frac{BH'}{BP}$

$C\hat{X}A = B\hat{P}H' = H'\hat{X}Y$

$\rightarrow (XH', XA)$ ISOGONALE A $(XC, XY) \rightarrow$ TESI

RMMSL G4 2019

7)



D PUNTO MEDIO ARCO $\widehat{BC}$
ω CENTRO IN D E TANGE BC in E
AK e AL TANGENTI A ω
ω_1 TANGE BC, AL e Γ in M
W_2 TANGE BC, AK e Γ in N
$P = ML \cap KN$

TESI: $K\hat{A}P = E\hat{A}L$

MONGE: DATE TRE CIRCONFERENZE SI HA CHE I TRE EXSIMILCENTER SONO ALLINEATI E GLI INSIMILCENTER DI DUE COPPIE SONO ALLINEATI CON L'EXSIMILCENTER DELLA TERZA.

VOGLIAMO AP SIMMEDIANA
TANGENTI IN B e C ALLA CIRCOSCRITTA CHE SI INTERSECANO IN T SI HA CHE ω È L'INCERCHIO DI TBC ⟶ T È L'EXSIMILCENTER TRA Γ, ω.

DISEGNIAMO L'INSCRITTA DI AKL γ E USIAMO MONGE

Γ, ω_1, γ: EXSIMILCENTER TRA Γ, γ È SU ML

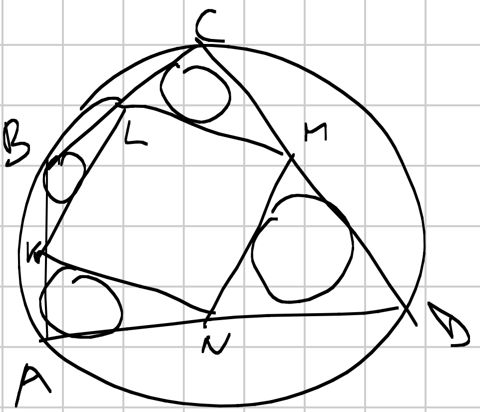
Γ, ω_2, γ: // Γ, γ È SU NK

⟶ SARÀ $ML \cap NK = P$

Γ, γ, ω: A, P, T SONO ALLINEATI ⟶ FINE

IMO SL G5 2020

8)



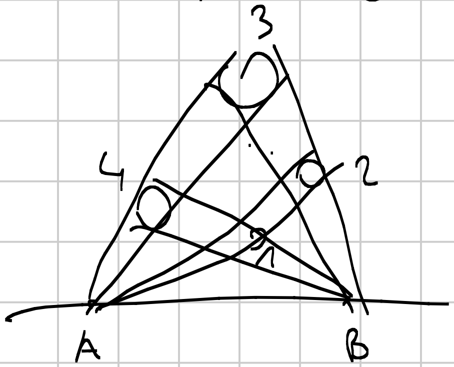
KLMN ROMBO, $KL \parallel AC$, $KN \parallel BD$

ω_1 INCERCHIO ANK

$\omega_2, \omega_3, \omega_4$

TANGENTI INTERNE A ω_1 e ω_3 e TANGENTI INTERNE A ω_2 e ω_4 CONCORRONO

LEMMA: 4 CIRCONFERENZE, TUTTE DALLO STESSO LATO DI AB E TALI CHE



$O^+_{14} = O^+_{23} = B$ e $O^+_{12} = O^+_{34} = A$

$\longrightarrow O^-_{13} = O^-_{24}$

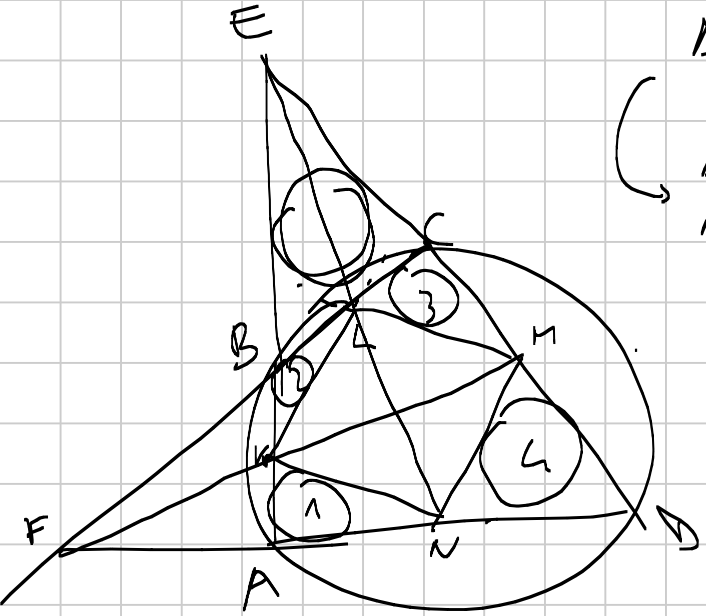
$1, 2, 4$: A, O^-_{14}, O^-_{24} ALLINEATI

$1, 3, 4$: A, O^-_{14}, O^-_{13} ALLINEATI

O^-_{13} e O^-_{24} ESISTONO SU AO^-_{14}

ANALOGAMENTE O^-_{13} e O^-_{24} ESISTONO SU BO^-_{34}

$\longrightarrow O^-_{13} = O^-_{24}$



$\triangle ECA \sim \triangle EBD$, $KN \parallel BD$, $MN \parallel AC$

$$\frac{AN}{ND} = \frac{AN}{AD}\cdot\frac{AD}{ND} = \frac{KN}{BD}\cdot\frac{AC}{NM} = \frac{AC}{BD} = \frac{AE}{ED}$$

→ $N \in$ BISETTRICE DI $A\hat{E}D$

ANALOGAMENTE L e K e M $\in$ BISETTRICE DI $D\hat{F}C$

EKLM È UN DELTOIDE

→ ESISTE UNA CIRCONFERENZA INSCRITTA ω IN EKLM

$\omega_1, \omega_2, \omega_3$: $O^+_{23} \in KM \to O^+_{23} = O^+_{41} = F$

e $O^+_{12} = O^+_{34} = E$

→ $O^-_{13} = O^-_{24}$ FINE.