Titolo nota

1 Dimostrare che $R(4,4) \leq 18$

(*) $R(4,4) = 18$

2 Siano dati r colori. Dimostrare che esiste $N(r)$ t.c. $\forall$ colorazione in r colori di $\{1, \dots, N(r)\}$, $\exists\, x, y, z$ dello stesso colore $| \; x + y = z$

IMO 78 P6
2.5 Dimostrare che va bene $N(6) = 1978$ [UGUALI OK]

3 Ci sono hp membri in un parlamento, $h \geq 1$, $p \geq 2$

Romania TST 2015 D.1/8
Alcune coppie di membri sono nemici tra di loro. Trovare il minimo numero di coppie di nemici per cui esiste una configurazione in cui, in ogni partizione del parlamento in h case da p membri ciascuna, 2 nella stessa casa sono nemici

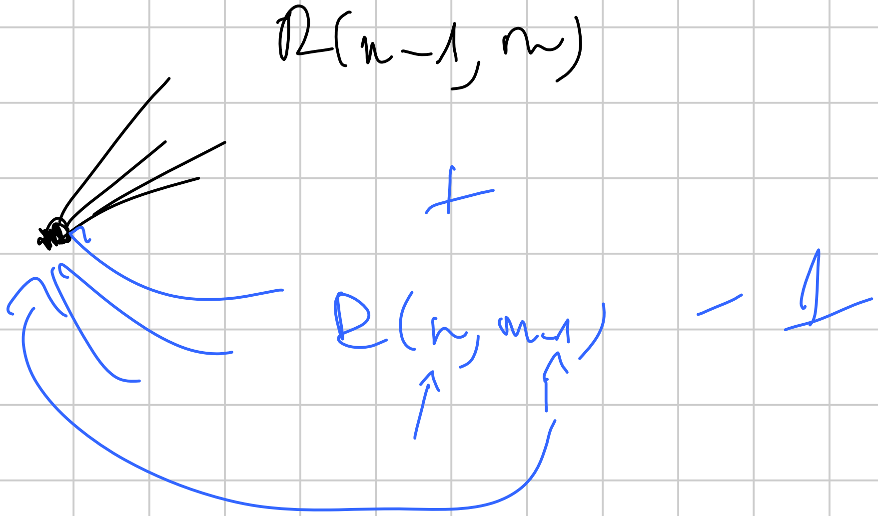
4
Romania TST 2015 D.4/8
Sia G un grafo completo di n vertici. Ogni arco ha un verso ed è colorato o blu o rosso. Dimostrare che $\exists$ un vertice v $| \; \forall u$ vertice esiste un cammino orientato monocolore da v a u.

TEORIA DI RAMSEY

$\forall n, \exists N(n) \mid \forall$ colorazione in rosso o in blu degli archi $K_{N(n)}$, $\exists K_n \subseteq K_{N(n)}$ monocolore

Se $n=3$, è il "party problem".

$R(n)$ $\quad R(n,m) =$ min di vertici $\mid \forall$ colorazione in rosso o in blu di $K_{R(n,m)}$ o K_n rosso o K_m blu



$$R(n,m) \leq \boxed{R(n-1,m) + R(n,m-1)} +$$

$$R(n,m) \leq \binom{n-1+m-1}{n-1}$$

$$R(n,n) \leq \binom{2(n-1)}{n-1} \simeq 4^n$$

$$n! \sim \left(\frac{n}{e}\right)^n \cdot \sqrt{2\pi n}$$

$$R(n,n) \gtrsim 2^{n/2}$$

$R(n)$

$$E[\# \text{ cricche monocolore}] = \binom{R}{n} \cdot \frac{1}{2^{\binom{n}{2}}} < 1$$

$$\binom{R}{n} < 2^{\frac{n^2 - n}{2}}$$

$$\binom{R}{n} \leq \frac{R^n}{n!} < 2^{\frac{n^2-n}{2}}$$

$$R < \frac{2^{\frac{n}{2}}}{\sqrt{2}} \cdot \sqrt[n]{n!} = \frac{2^{\frac{n}{2}}}{\sqrt{2}} \cdot \frac{n}{e} \left(\sqrt{2\pi n}\right)^{\frac{1}{n}}$$

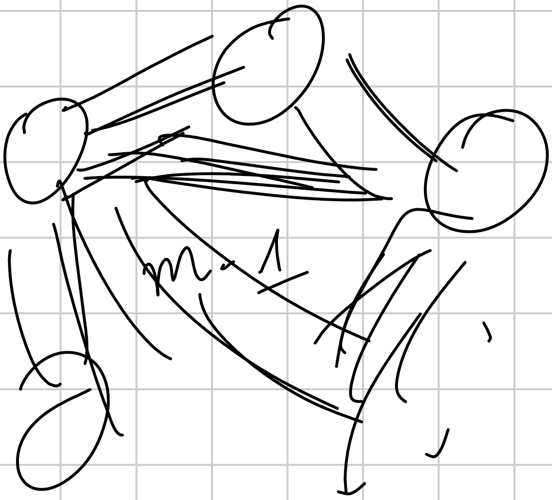
$$\frac{\sqrt{2}\, 2^{\frac{n}{2}}\, n}{e}$$

$$\left(4 - \frac{1}{20}\right)^n$$

TURAN

$ex(n, K_m)^{(n \geq m)}$ = min # archi che colorare rossi in K_n per avere un K_m rosso

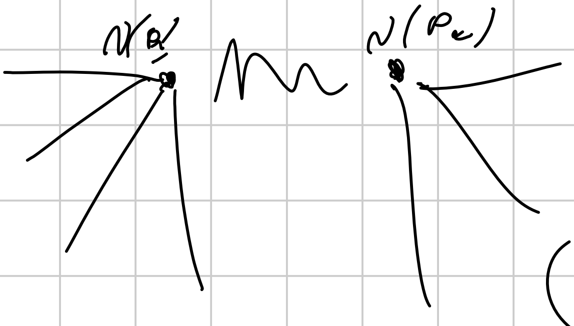
$\sum \nu(\ell)$



$\forall p \in V(G),\ \nu(p)$

$\forall \ell \in E(G),\ \nu(\ell) = \nu(p_1)\,\nu(p_2)$ $\qquad p_1 \overset{\ell}{—} p_2$

All'inizio, $\sum \nu(\ell) = |E|$.

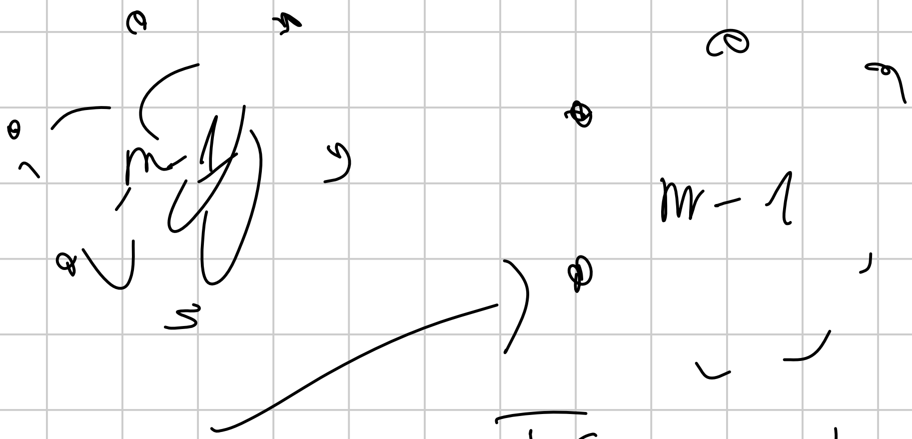


$\nu(p_i)\left(\sum_{p \text{ adiacenti}} \nu(p)\right)$

$(\nu(p_1)-t)\left(\sum_{p \text{ adiac.}}\right) + (\nu(p_2)-t)\left(\sum\right)$

Alla fine del processo, ho trovato una cricca, diciamo di $K < m$

$T(n, m-1)$ $\sum v(v) \le n$



$\sum x(e) = E \le \max \le T(n, m-1)$

$|E(T(n,m-1))| = ex(n,m) - 1$

IMO 92 P3

Ho 9 punti su un piano.

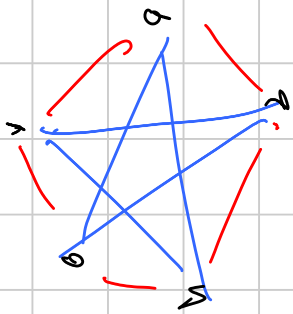
n_9, n lati colorati rosso o blu.

Trovare il minimo n per cui $\exists T$ monocolore

$n = 33$

$ex(9,6) = E(T(9,6)) + 1$

$\frac{8 + 8 \cdot 7}{2} = 32 + 1 = 33$



$\{G_1, \dots, G_m\}$

$R(G_1, \dots, G_m) \quad K_R$

$\forall\, i\; G_i = K_{n_i}$

$R(R(n,n), n)$

$K_{R(n,n)}$ rouge $\qquad$ bleu

1

$R(4,4) \leq 2R(3,4)$ ≤ 18 ?

$R(3,4) \leq 9$

blu rosso

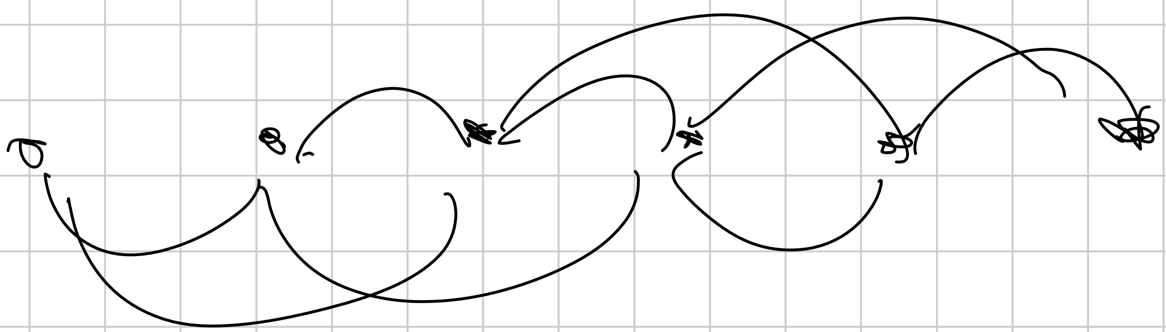
r_y

5 3 ≤ 9.5

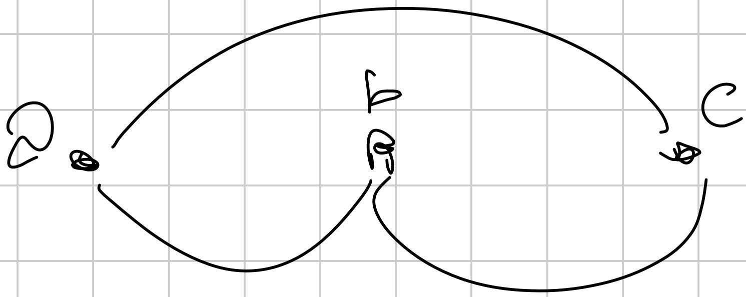
2 (Lemma di Schur)

$\forall$ r-colorazione degli interi, $\exists N(r)$ |

$\exists$ x, y, z monocolore | $x+y=z$



$a-b + b-c = a-c$



$c - b + b - a = c - a$

$R(3,3,3) \leq 17$

$3 \cdot 5 + 1 + 1 = 17$

$R(nT) := a_n$

$R(nT) \leq n(R((n-1)T) - 1) + 1 + 1$

$a_n \leq n a_{n-1} + 2$

$b_n := a_n - 1$

e_{n-1}

$b_n = n(b_{n-1}) + 1$ $\qquad b_1 = a_1 - 1$

$=$

$b_n = n\, b_{n-1} + 1 = n(n-1)\, b_{n-1} + n + 1$

$= n!_e \left(\frac{1}{n!} + \frac{1}{(n-1)!} + \dots + \frac{b_n}{(n-1)!} \right) =$

$= n! \left(\sum_{n=0}^{n} \frac{1}{n!} \right) = e\, n!,$

$a_n = b_n + 1 = \lfloor e n! \rfloor + 1;$

3 np

$\binom{n+1}{2}$ K_{n+1}

$p(n-1)+1$

Irom: $< \frac{n(n+1)}{2}$

$\left\lfloor \frac{np}{p-1} \right\rfloor$

n

$\leq$

$T(np, p-1) <$

$\frac{\left(np - \left\lfloor \frac{np}{p-1} \right\rfloor\right) np}{2} < \frac{n^2p^2 - np}{2} - \frac{n(n+1)}{2} + 1$

$\frac{(np - n)\, np}{2} < \frac{n^2p^2 - np}{2} - \frac{n(n+1)}{2} + 1$

$$\frac{n(n+1)}{2} < \frac{n^2 p - np}{2} + 1$$

$$n+1 < np - p + \frac{2}{n}$$

$$n+1 < p(n-1) + \frac{2}{n}$$

$$n+1 < 2(n-1) + 2/n$$

$$3 < n + 2/n$$

$$p \geq n$$

$$< \frac{n(n+1)}{2}$$

C

$$< \frac{n(n+n)}{2}$$

$$n-1+\binom{n-1}{2}=\binom{n}{2}$$

n

$$\binom{n}{2} < E \leq \binom{n+1}{2}$$

$$E \leq \binom{n+1}{2}$$

$$p(n-1)$$

$$p(n-1)+1 \leq \frac{n(n+1)}{2}$$

$$p \leq \frac{n}{2}\left(\frac{n+1}{n-1}\right) - \frac{1}{n-1} =$$

$$= \frac{n}{2}\left(1+\frac{2}{n-1}\right) - \frac{1}{n-1} =$$

$$= \frac{n}{2}+1$$

$$2p-2 \leq n$$

$$E \leq p(h-1) \, .$$

$$E \leq \frac{h(h+1)}{2} \qquad h < p$$

$$E \leq \frac{(h-1)h}{2}$$

$$h = p \qquad p(h-1) = \frac{h(h+1)}{2}$$

$$p \leq h \leq 2p-2$$

t, $h = t+1$

m = max grado uscente di p-anticatena

$$E - m \geq \binom{h}{2}$$

$E \geq \binom{n}{2} + m$

$\frac{n(n-1)}{2} + m$

$E < \binom{n+1}{2}$

$\sum dg(v) \geq n(n-1) + 2m$

$(n-1)$ contenute

e 1 quasi contenuto

$m \geq \frac{n(n-1) + 2m}{N}$

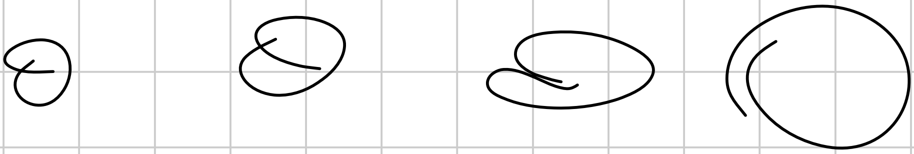
$\Rightarrow m \geq n$

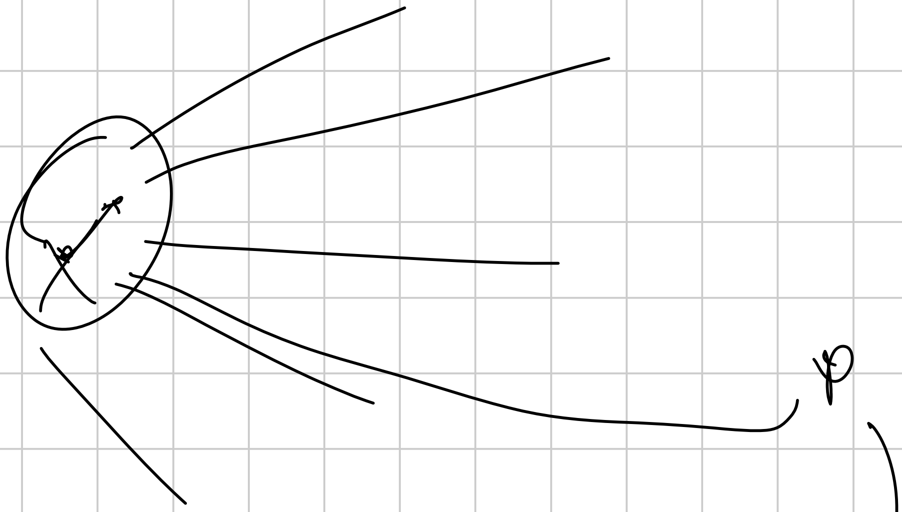
$N = n+1$

$(n+1)m \geq n(n-1) + 2m$

$$(n-1)m \geq n(n-1)$$

$$(n-1)(m-n) \geq 0$$





$$\underbrace{np - p + 1}$$