Titolo nota

1 Sia b intero ≥ 2. Sia $\{a_i\}_{i\geq 0}$ una succ. di interi $0 \leq a_i < b$, definitivamente periodica, frequentemente non nulla. $a_0 > 0$.

EGMO SA Sia S l'insieme degli n t.c. $n \mid (a_0 a_1 \ldots a_n)_b$ (base b).
Supponiamo $|S| = \infty$. Allora $\exists \infty$ primi che dividono elementi di S.

2 E Sia $a_1 = 2$, $a_{n+1} = \min\{m \in \mathbb{N} \mid m > a_n, \tau(m) > \tau(a_n)\}$
Dimostrare che $2a_{n+1} = 3a_n$ solo finite volte

3 Un intero n si dice d-ricopribile se $\forall S \subseteq \{0,1,\ldots,n-1\}$, $\exists\, p \in \mathbb{Z}[x]$, $\partial p \leq d$, $\mid p(\mathbb{Z}[n]) = S \pmod{n}$.
Per ogni n trovare, se esiste, il minimo $d \mid n$ è d-ricopribile

Sia $c \geq 1$.

4 Sia $a_1 = c$, $a_{n+1} = a_n^3 - 4ca_n^2 + 5c^2 a_n + c$.

Dimostrare che $\forall n \geq 2$, $\exists p$ t.c. $p \mid a_n$, $p \nmid a_i$ $\forall i < n$.

5 Siano $p, q \in \mathbb{N}^+$, $(p,q) = 1$. Un numero si dice bruttoecattivo se non si può scrivere come $n = px + qy$ con $x, y \geq 0$. Dimostrare che $\exists c \mid \forall p, q$ come sopra,
$(p-1)(q-1) \mid c \sum_{\substack{n \\ \text{bruttoe} \\ \text{cattivo}}} n^{2013}$

1 $a_0, a_1, \ldots$

$0 \leq a_i < b$

$a_0 \neq$

$n \mid (a_0, \ldots, a_n)_b$

$|S| = \infty \Rightarrow \exists \infty p \mid n \in S$

$c_n = (a_0, \ldots, a_n)_b$

$$\underbrace{c_{n+aT+t}}_{n} = b^{aT+T} c_n + \frac{b^{aT} - 1}{b^T - 1} b^{t+n} \tilde{c}_T + \tilde{c}_t$$

$\tilde{c}_0 = 0$

$$b^n \left(\frac{c_n}{b^n} \right) + b^n \left(\frac{1/b^{t+n} - 1}{b^T - 1} \right) b^t \tilde{c}_T + \tilde{c}_t$$

$p \mid n \quad p \mid b$

$$\nu_p(n) < \mathrm{Cont}$$

$$\tilde{C}_T - \frac{b^T \tilde{C}_T}{b^T - 1}$$

$$\frac{\tilde{C}_T}{b^T} - \frac{\tilde{C}_T}{b^T - 1} < \frac{\tilde{C}_T}{b^T} - \frac{\tilde{C}_T}{b^T}$$

$$\frac{\alpha_{n+1}}{b} + \dots + \frac{\alpha_{n+T}}{b^T} - \frac{\alpha_{2n+1}}{b} + \dots + \frac{\alpha_{2T}}{b^T}$$

$$< 0$$

$$b^n\left(\frac{c_n}{b^n} + \frac{1/b^{n+T} - 1}{b^T - 1}(b^T)\right) - \tilde{C}_T =$$

$$= \frac{\alpha b^n + C_T}{\sim}$$

$$v_p(a b^n + c) \quad \sqrt{\mathrm{LOG}(a, p)}$$

$$(c, p) = 1$$

$$v_p(c^{p-1} - 1) = e$$

$$c \equiv g^{x p^i} \quad (p^M) \quad M \geq e$$

$$c^{p-1} - 1 \equiv (g^{x p^i})^{p-1} - 1$$

$$v_p(\;''\;) = v_p(\;''\;)$$

$$\shortparallel$$

$$e$$

$$v_p(g^{p-1}) + i = 1 + i$$

$\Rightarrow$ i costante

$$a b^n + c$$

$$b^n \equiv c/\alpha \equiv g^{t p^e}$$

$$p^{\alpha} \mid n \mid b^n - g^{\alpha p^e}$$

$$g^{\operatorname{ord}(g)/n} \equiv g^{k p^e} \pmod{p^M}$$

$\nu_p(n) \to \infty$

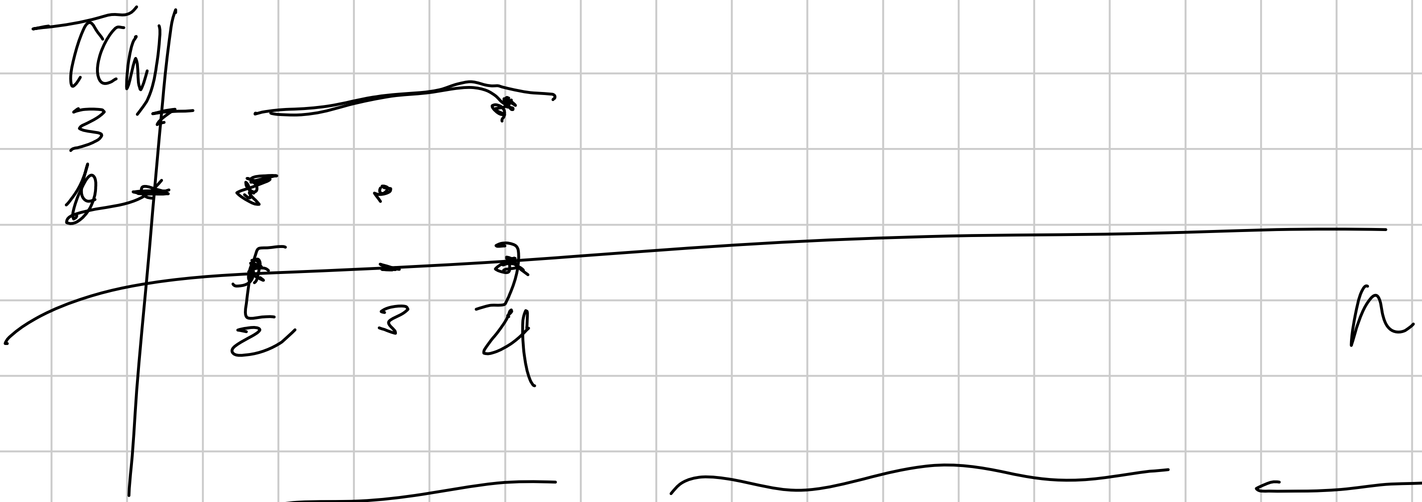
cont

2 $a_1 = 2$ BALKAN MO P4 2020

$$a_{n+1} = \min\{m \mid m > a_n, \tau(m) > \tau(a_n)\}$$

$a_{n+1} = 3a_n$ infinitely often

$a_{n+1} = \frac{3}{2} a_n$



$$a_n = p_1^{e_1} p_2^{e_2} \cdots p_n^{e_n}$$

$e_i = 0$ OK

$e_i < e_{i+1}$

$\frac{\alpha_n}{p_i+1}$ p_i°

$e_i \geq e_{i+1}$

$$\alpha_{n+1} = \frac{3}{2}\alpha_n \quad \text{assurdo}$$

(1) $\underline{\tau(\alpha_n) \geq \tau\left(\frac{4}{3}\alpha_n\right)}$
$<$ ~~assurdo~~

(2) $\tau(\alpha_n) \geq \tau\left(\frac{f}{8}\alpha_n\right)$

$\tau(\alpha_n) = \prod(e_i + 1)$ $\quad \underline{e_i' = e_i + 1}$

$\alpha_n = \prod p_i^{e_i}$

(1) $(e_2')(e_3') \geq (e_2' + 2)(e_3' - 1)$

$\quad$ " $\quad$ " $\geq (e_2' - 3)(e_3' + 2)$

$$e_2' + e_3' \geq e_2 e_3' + 2e_3' - e_2' - 2$$

$$'' \quad '' \geq e_2' e_3' - 3e_3' + 2e_2' - 6$$

$$2e_3' - 2e_2' - 4 \leq 0$$
$$-3e_3' + 2e_2'' - 6 \leq 0$$

$$e_3' - 10 \leq 0$$

$$e_3' \leq 10$$

$$v_p(\alpha_n) \to +\infty$$

$$\begin{array}{cccc} & c & c & c \\ & \vee\!/ & \vee\!/ & \vee\!/ \\ 2^{+\infty} \geq & 3^{a} \geq & 5^{a} \geq & 7^{e} \end{array}$$

$$(e_2+1)(e_3+1)($$

$$N_2(\alpha_n) \to \infty$$

$$\alpha_n = \prod p_i^{e_i} \qquad e_i < C$$

$$(p_1, \ldots, p_r)$$

$$\boxed{\left(\frac{C+1}{C}\right)^S > 2}$$

$$p_N \gg (p_S)^S$$

$$\tilde{\alpha}_n \leftarrow \alpha_n$$

$$\underline{\tau(\tilde{\alpha}_n) > \tau(\alpha_n)}$$

$$v_{p_N}(\alpha_n) \geq 1$$

$$\tilde{\alpha}_n = \frac{\alpha_n \cdot p_{N+1}}{p_N}$$

$$\frac{p_{N+1}}{p_N} < 1 + \varepsilon < \frac{3}{2}$$

$$(e_{p_N} + 1) \to 1$$

$$(e_{p_N})^{\wedge} \cdot 2$$

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3 n d-ricopribile $\rightsquigarrow$

$\forall S \neq \emptyset, \; S \subseteq \{0, 1, \dots, n-1\}$

$\exists\, p^{\in \mathbb{Z}[x]}$ $\deg(p) \leq d$ tale che

$\{p(0), p(1), \dots, p(n-1)\} = S$

$n = p$

$d = p - 1$

$$\underbrace{x^{p-1} - 1}$$

$$\underline{p(x) = \sum_{r=0}^{p-1} \left((x-r)^{p-1} - 1 \right) (-\alpha_r)}$$

$f : \{0, \dots, p-1\} \to \{0, \dots, p-1\}$

è rapp con un pol di grado $\leq d$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad || \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad p-1$

OSS $\forall f$, $\exists !\, p$ che la realizza

$d < p - 1$

Per s specifico

$S = \{\alpha, \beta\}$

$$r(x) = \sum_{r=0}^{p-1} \left((x-r)^{p-1} - 1\right)(-\alpha_r) =$$

$$x^{p-1}(-\beta_r \{\# \{t \in \{0, \dots, p-1\} \mid f(t) = \beta\}\}$$

$$-\beta_r \# \neq 0 \ (p)$$

$$x^{\varphi(n)} - 1$$

$p, q \mid n$ $\qquad$ $n \mid m$

$n = pq$

$\alpha = \beta \ (p)$

$p(\alpha) = p(\beta) \quad (p)$

$\#_r \qquad \#_q$

$\#_p \#_q \qquad \#_p = p$

$\#_q = q$

$p<q$ $|\#_p \#_q| \leq (\#_p - 1)(\#_q \quad) \leq \#_p \#_q - 2$

$pq - 1$

$p' \quad \{0, \dots, p-1\}$

$\underline{I} = \{ \tilde{x} \mid p'(x) = 0 \}$

$\tilde{x} = x(p),\ x \in \underline{I}$

$p(\tilde{x}) \rightarrow$ unico valore nel p^n

$\tilde{x} = x \notin \underline{I}$

$p(\tilde{x})$ al variare di $\tilde{x} = \cancel{x}(p)$

P^{n-1}

$|S| = p^n \quad I = \emptyset$

$I = \emptyset \qquad |S| = |p(x)|$

$\cancel{\neq}$

$\|$

$r(p)$

$= p^{n-1}(u)$

$1 \leq k \leq p$

or $n \geq p$

$|S| \leq p^{n-1}(p-1)$

$2, 3$

$\leq p^{n-1}(p - |I|) + |I| \leq$

$\leq p^{n-1}(p-1) + 1$

or p^n or $p^n - p^{n-1} + 1 \leq p^n - 1$

$-p^{n-1} + 1 \leq -1 \qquad n \geq 2$

$2 \leq p^{n-1}$

$n = 4$ unico caso patologico

$p'(x) = 0 \quad (2)$

$p'(x) \neq 0$

$x^2 + ax + b$

$\cancel{2x} + d$

$d \nmid 3$

$x^3 \qquad (4)$

$d = p - 1 \qquad$ (p primo)

$d = 3 \qquad (n = 4)$

d non esiste altrimenti

4 $a_1 = c$

$$a_{n+1} = a_n^3 - 4ca_n^2 + 5c^2a_n + c$$

Ter: $\exists p \mid a_n$ $\forall n \geq 2$, $p \nmid a_i$ $\forall i < n$

$$b_n = \frac{a_n}{c}$$

$$c\,b_{n+1} = c^3 b_n^3 - c^3 4 b_n^2 + 5c^3 b_n + c$$

$b_1 = 1$

$$\underbrace{b_{n+1} = c^2(b_n^3 - 4b_n^2 + 5b_n) + 1}$$

mod b_n $\quad b_{n+1} = 1 = b_1$

$p \mid b_n \Rightarrow b_{2n}$

$$\underbrace{b_{n+1} = 5c^2 b_n + 1} \qquad (b_n^2$$

$$b_{n+2} = c^2\left(5c^2 b_n(3 - 8 + 5) + 1^3 - 4\cdot 1^2 + 5\cdot 1\right) + 1$$

$= 2c^2 + 1 = b_2$

$\min n \mid p \mid b_n \quad p^\alpha \| b_2$

$p^\alpha \| b_{2n}$

$p \mid b_m \iff m = kn$

$p^\alpha \| b_m$

$\phi_m \quad \phi_{pm}$