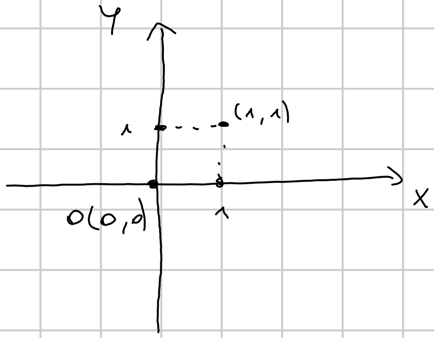
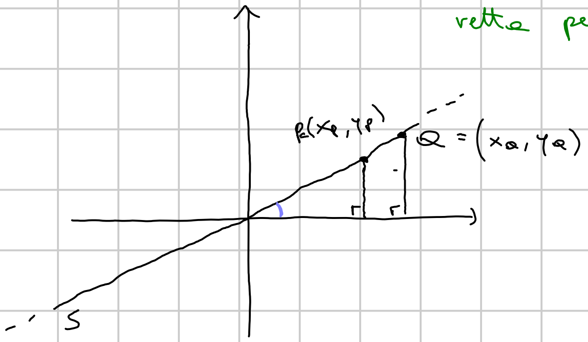
Geometria 1 basic ~ Tigyanda

Cartesiane



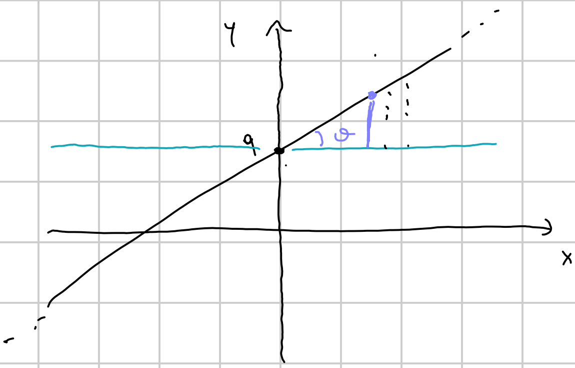


retta per l'origine

$$\frac{y_Q}{x_Q} = \frac{y_P}{x_P} =: m$$

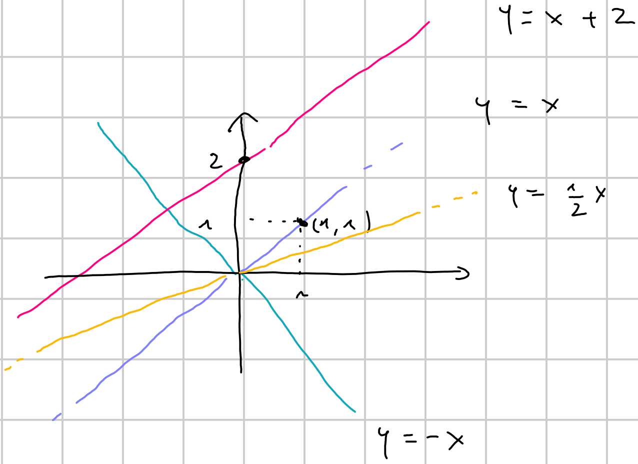
coeff angolare

$$s : y = mx$$

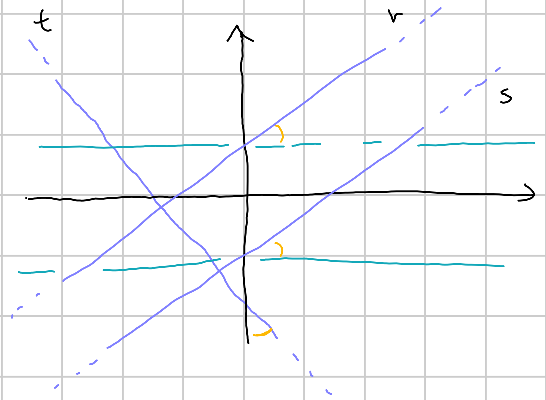


retta non per forza per l'origine

$$y = mx + q$$



rette parallele e perpendicolari



$r: y = m_1 x + q_1$

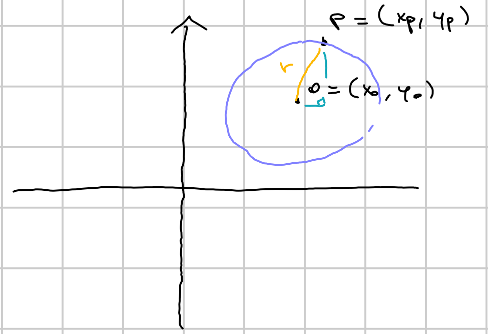
$s: y = m_2 x + q_2$

$t: y = m_3 x + q_3$

$m_1 = m_2$

$m_1 \cdot m_3 = -1$

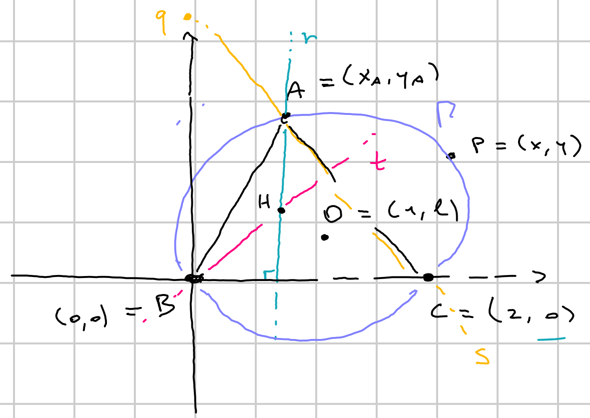
Circonferenze



$$(x_p - x_0)^2 + (y_p - y_0)^2 = r^2$$

Esercizio

Fissata una circonferenza Γ e due punti $B, C \in \Gamma$, trovare le coordinate dell'ortocentro H sul triangolo ABC al variare di A su Γ



$\Gamma: (x-1)^2 + (y-\ell)^2 = 1^2 + \ell^2$

$x^2 - 2x + \cancel{1} + y^2 - 2\ell y + \cancel{\ell^2} = \cancel{1^2} + \cancel{\ell^2}$

$x^2 - 2x = 2\ell y - y^2$

$A \in \Gamma \Rightarrow x_A^2 - 2x_A = 2\ell y_A - y_A^2$

$r: x = x_A$

$m_s = \dfrac{y_A - y_C}{x_A - x_C} = \dfrac{y_A}{x_A - 2}$

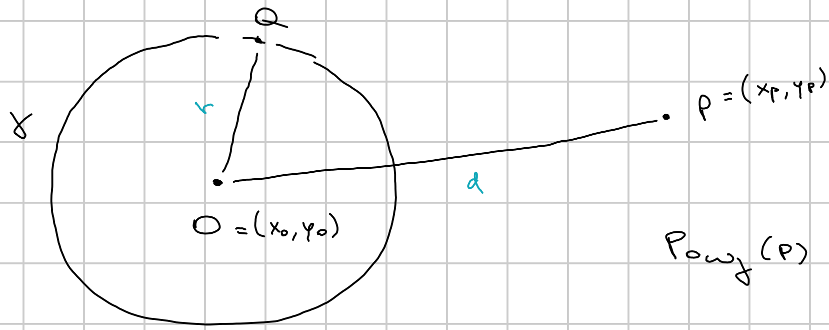
$m_s \cdot m_t = -1 \qquad m_t = \dfrac{2 - x_A}{y_A}$

$t: y = \dfrac{2 - x_A}{y_A} x$

$$\begin{cases} x_H = x_A \\ y_H = \dfrac{(2 - x_A)\, x_A}{y_A} = \dfrac{2x_A - x_A^2}{y_A} = \dfrac{y_A^2 - 2\ell y_A}{y_A} = y_A - 2\ell \end{cases}$$

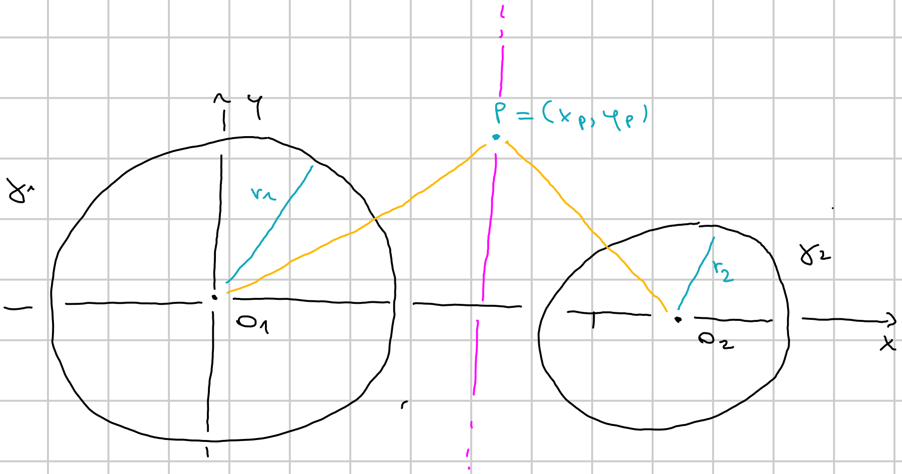
$H = (x_H, y_H) = (x_A, y_A - 2\ell) = (x_A, y_A) - (0, 2\ell)$

Asse radicale



$$Pow_\gamma(P) = d^2 - r^2$$

$$Pow_\gamma(P) = (x_p - x_0)^2 + (y_p - y_0)^2 - r^2$$



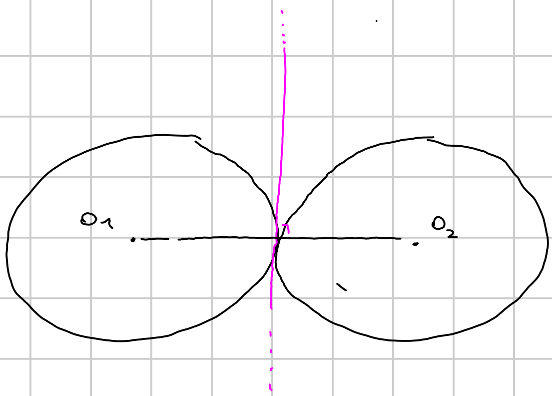
$O_1 = (0, 0)$

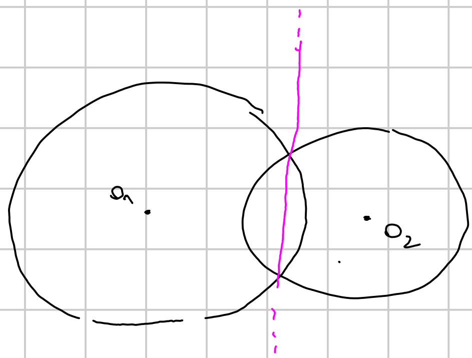
$O_2 = (a, 0)$

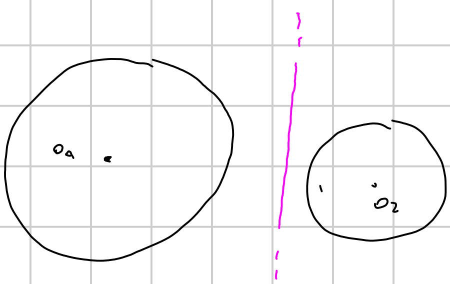
$$Pow_{\gamma_1}(P) = (x_p - x_{O_1})^2 + (y_p - y_{O_1})^2 - r_1^2 = x_p^2 + y_p^2 - r_1^2$$

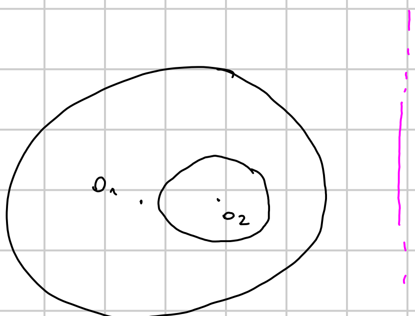
$$Pow_{\gamma_2}(P) = (x_p - \underset{a}{\underset{\parallel}{x_{O_2}}})^2 + (y_p - \underset{0}{\underset{\parallel}{y_{O_2}}})^2 - r_2^2 = x_p^2 - 2a x_p + a^2 + y_p^2 - r_2^2$$

$$\cancel{x_p^2} + \cancel{y_p^2} - r_1^2 = \cancel{x_p^2} - 2a x_p + a^2 + \cancel{y_p^2} - r_2^2 \qquad x_p = \frac{-r_2^2 + r_1^2 + a^2}{2a}$$

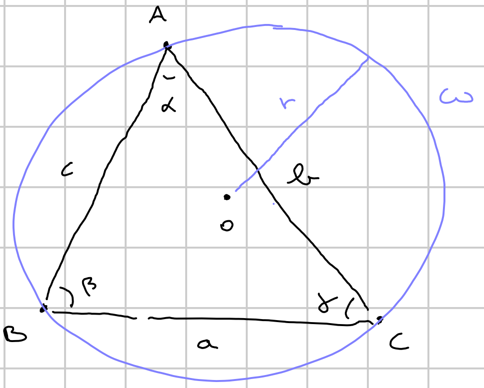








Trigonometria

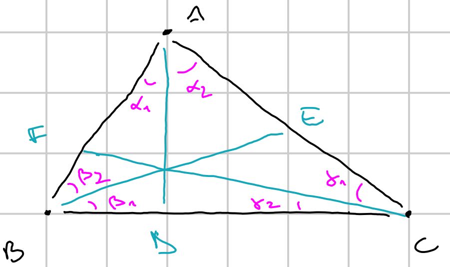


Teorema dei seni: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} = 2r$

Teorema del coseno: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

NB: se $\alpha = 90°$ è Pitagora

Teorema di Ceva

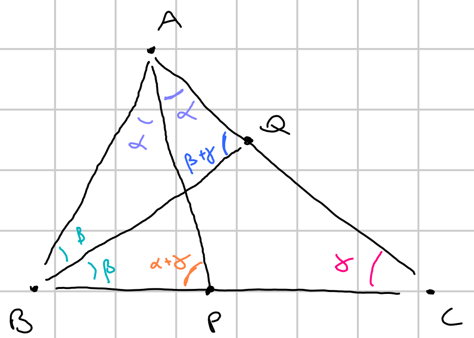


ABC triangolo, D, E, F su BC, AC, AB rsp.
Allora

AD, CF, BE concorrono $\iff \dfrac{AE}{EC} \cdot \dfrac{CD}{DB} \cdot \dfrac{BF}{FA} = 1$

Forma trigonometrica AD, CF, BE concorrono $\iff \dfrac{\sin\alpha_1}{\sin\alpha_2} \cdot \dfrac{\sin\gamma_1}{\sin\gamma_2} \cdot \dfrac{\sin\beta_1}{\sin\beta_2} = 1$

Esercizio Imo 2001/5



ABC triangolo, AP, BQ bisettrici:

$AP + BP = AQ + QB$

$\alpha = 30°$. Trovare β, γ

$2\alpha = 60°$ $2\beta + \gamma = 120°$

$\dfrac{AP}{\sin(2\beta)} = \dfrac{AB}{\sin(\alpha+\gamma)}$, $\dfrac{BP}{\sin\alpha} = \dfrac{AB}{\sin\alpha+\gamma}$, $\dfrac{AQ}{\sin\beta} = \dfrac{AB}{\sin(\beta+\gamma)}$, $\dfrac{QB}{\sin 2\alpha} = \dfrac{AB}{\sin(\beta+\gamma)}$

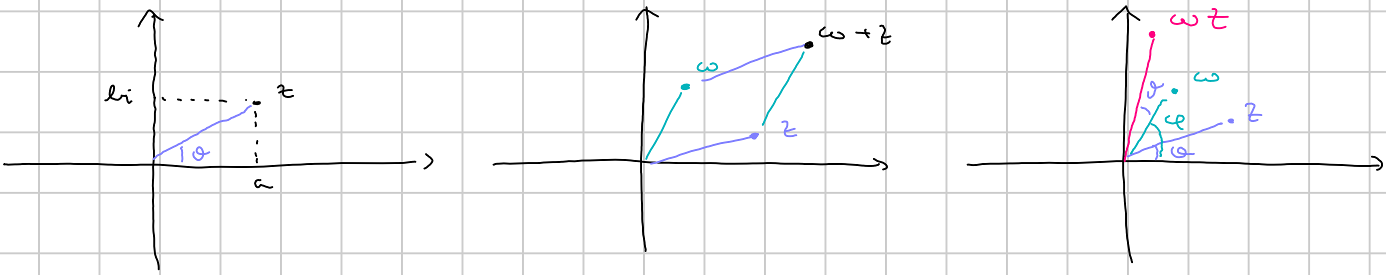
$$AP + BP = AQ + QB$$

$$\frac{\cancel{AB}}{\sin(\alpha+\gamma)} \cdot \sin(2\beta) + \frac{\cancel{AB}}{\sin \alpha+\gamma} \cdot \sin\alpha = \frac{\cancel{AB}}{\sin(\beta+\gamma)} \cdot \sin\beta + \frac{\cancel{AB}}{\sin(\beta+\gamma)} \cdot \sin 2\alpha$$

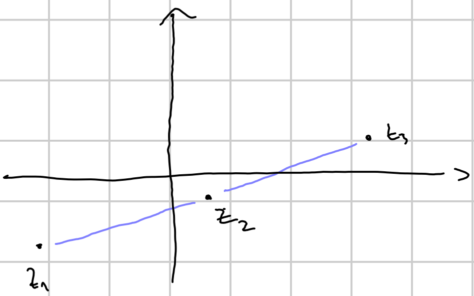
Complessi

$$\mathbb{C} = \{ a + bi \mid a, b \in \mathbb{R},\ i^2 = -1 \}$$

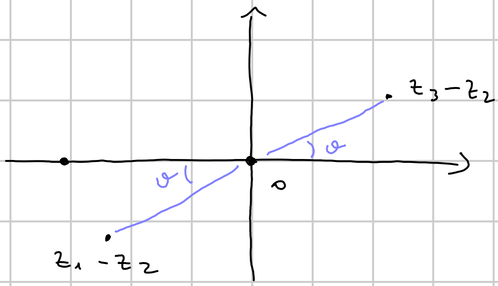
$$z = a + bi = \sqrt{a^2+b^2}\,(\cos\theta + i \sin\theta) = \sqrt{a^2+b^2}\, e^{i\theta}$$



Collinearità

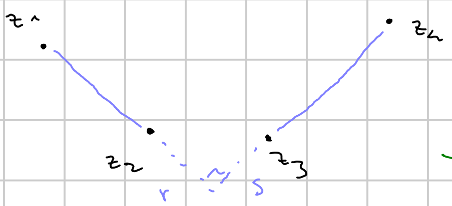


Quando z_1, z_2, z_3 sono allineati?

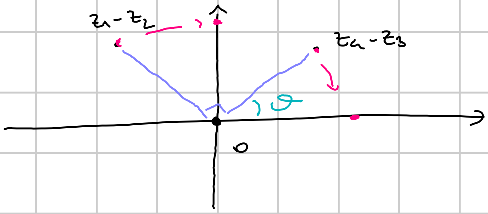


$$\underbrace{\frac{z_1-z_2}{z_3-z_2}}_{\in \mathbb{R}} = \overline{\left(\frac{z_1-z_2}{z_3-z_2}\right)} = \frac{\overline{z_1}-\overline{z_2}}{\overline{z_3}-\overline{z_2}}$$

Perpendicolarità



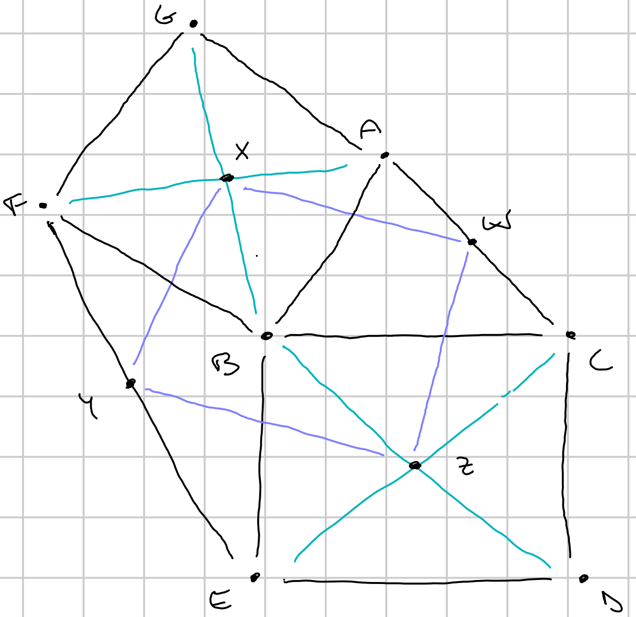
Voglio vedere se $r \perp s$



$$\frac{z_1 - z_2}{z_4 - z_3} = -\left(\overline{\frac{z_1 - z_2}{z_4 - z_3}}\right) = -\frac{\overline{z_1} - \overline{z_2}}{\overline{z_4} - \overline{z_3}}$$

Esercizio

Noto



ABC triangolo

ABFG, BCED quadrati

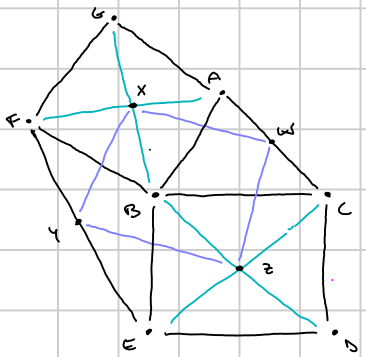
Y punto medio di EF,

W punto medio di AC,

X centro di ABFG

Z centro di BEDC

Tesi: WXYZ è un quadrato



$B = 0 \qquad C = 2 \qquad A = 2a \in \mathbb{C}$

$D = 2 - 2i$

$E = -2i$

$F = 2ai$

$Z = 1 - i$

$X = \frac{A + F}{2} = \frac{2a + 2ai}{2} = a + ai$

$Y = \frac{F + E}{2} = \frac{2ai - 2i}{2} = ai - i$

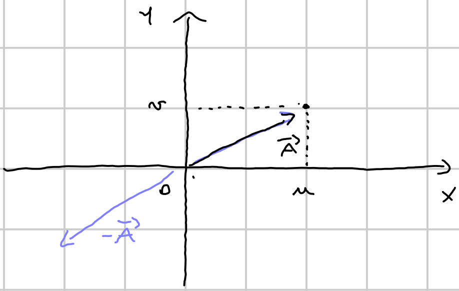
$W = a + 1$

$(W - Z)i = (Y - Z)$

$(a + \cancel{1} - \cancel{1} + i)i = (ai - \cancel{i} - 1 + \cancel{i})$

$ai - 1 = ai - 1 \quad \checkmark$

Vettori

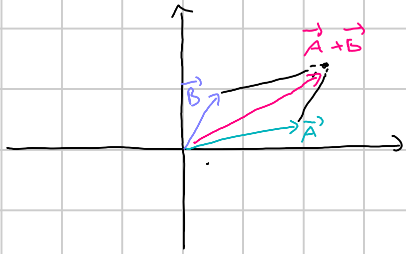


modulo $\sqrt{u^2 + v^2}$

direzione (angolo con l'asse delle x)

verso (segno)

+ !

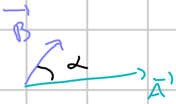


$\vec A = (u, v)$ $\qquad$ $\vec A + \vec B = (u + w, v + x)$

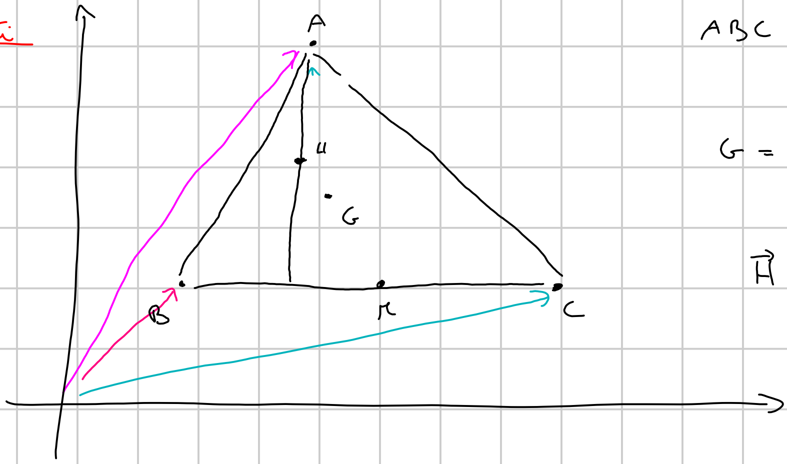
$\vec B = (w, x)$

· : prodotto scalare

$$\vec A \cdot \vec B = |\vec A||\vec B| \cos\alpha =$$
$$= uw + vx$$
$$\vec A \cdot \vec A = u^2 + v^2 = |\vec A|^2$$



Fatti

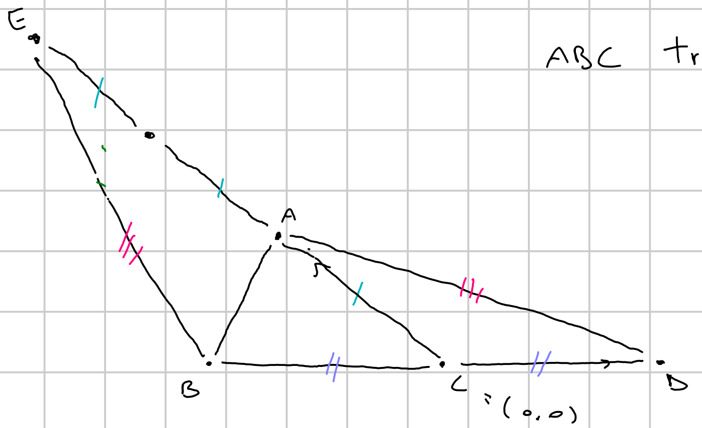


ABC triangolo

$G = \dfrac{\vec A + \vec B + \vec C}{3}$ $\qquad$ G baricentro

$\vec H$ ortocentro, $\vec H = \vec A + \vec B + \vec C$

Problema $\qquad$ Esame 2013/1



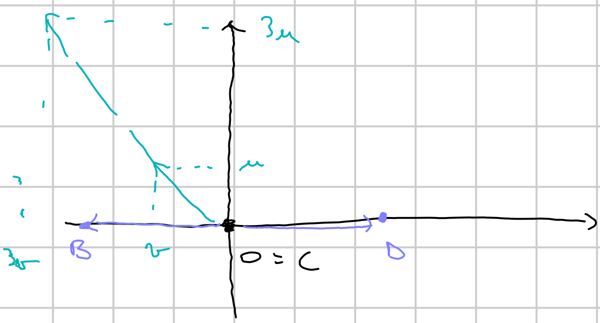
ABC triangolo

CD = CB

AE = 2AC

Ipotesi AD = BE

Tesi $B\hat{A}C = 90^\circ$



$\vec D = -\vec B$ $\qquad$ $\vec E = 3\vec A$

AD = BE

$$|\vec D - \vec A|^2 = |\vec E - \vec B|^2$$
$$(\vec D - \vec A)(\vec D - \vec A) = (\vec E - \vec B)\cdot(\vec E - \vec B)$$
$$\cancel{|\vec B|^2} + |\vec A|^2 + 2\vec B\,\vec A = \underbrace{|3\vec A|^2}_{9|\vec A|^2} + \cancel{|\vec B|^2} - 6\vec A \cdot \vec B$$

$$8\overbrace{|\vec A|^2}^{=\vec A\cdot\vec A} - 8\vec A \cdot \vec B = 0$$

$8\vec{A}(\vec{A}-\vec{B})=0$

$\vec{A}\cdot\vec{A} = |\vec{A}|^2$

$\begin{cases} \vec{A}=0 & A=C \text{ triangolo degenere} \\ \vec{A}=\vec{B} & A=B \\ \vec{A}\perp\vec{A}-\vec{B} & B\hat{A}C=90° \end{cases}$

Esercizio 1

Calcolare $\sum_{n=0}^{90} \sin^2(n)$

$\sin(n) = \cos(90-n)$

$\sin^2(n)+\cos^2(n) = 1$

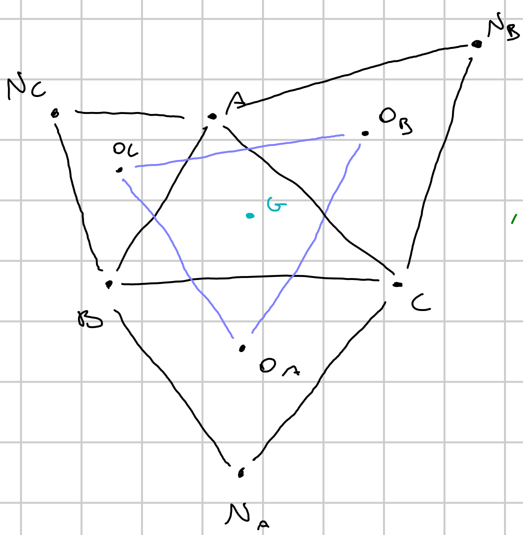
$\sin^2(0) + \sin^2(1) + \sin^2(2) + \dots + \sin^2(45) + \dots + \sin^2(88) + \sin^2(89) + \sin^2(90)$

$\cos^2(44) + \cos^2(43) + \dots + \cos^2(0)$

$45 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{91}{2}$

$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

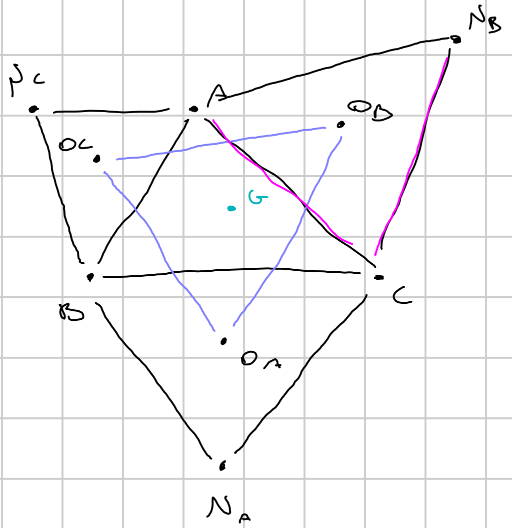
Esercizio 2 Teorema di Napoleone



ABC triangolo, N_CBA, AN_BC, BCN_A equilateri.

O_B, O_C, O_A centri dei triangoli

Tesi: G baricentro di $O_AO_BO_C$ è baricentro di ABC



$B = 0 \qquad C = 1 \qquad \omega = e^{i\frac{\pi}{3}}$

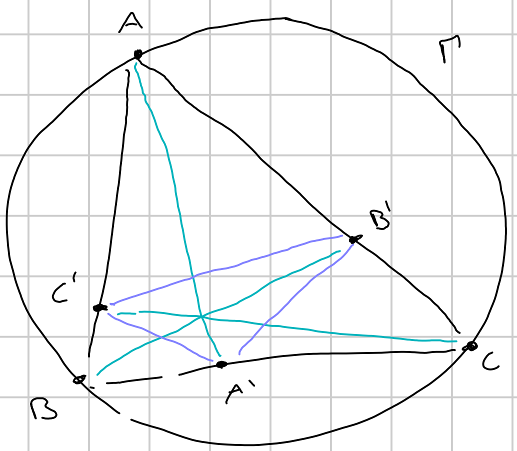
$A = a$

G_0 baric di ABC $\qquad G_0 := \frac{a+1}{3}$

$X = O_C - B \qquad x = (A-B)\omega$

$(N_B - c)\omega = c - a \qquad \frac{O_C + O_B + O_A}{3} = 0 = G$

Problema 3 $\qquad$ Noto



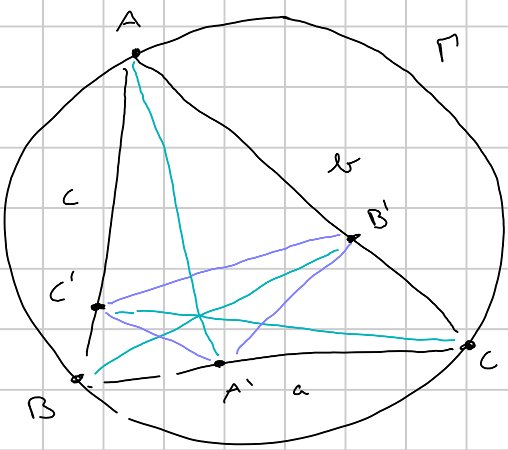
ABC triangolo

$\Gamma = (ABC)$

A', B', C' su BC, CA, AB: CC', AA', BB' concorrono, allora

$S' = [A'B'C']$ $\quad d$ diametro di Γ

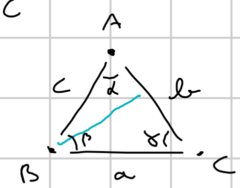
Tesi: $d \cdot S' = AB' \cdot BC' \cdot CA'$



$$\frac{AB'}{B'C} \cdot \frac{CA'}{A'B} \cdot \frac{BC'}{C'A} = 1$$

$$S' = [ABC] - [AB'C'] - [A'C'B] - [A'B'C]$$

$$[ABC] = \frac{bc \sin\alpha}{2} = \frac{ab}{2}\sin\gamma = \frac{ca \sin\beta}{2}$$



$$= \frac{cab}{2d}$$

$$[AB'C'] = \frac{AC' \cdot AB'}{2} \sin\alpha = \frac{AC' \cdot AB'}{bc} [ABC]$$

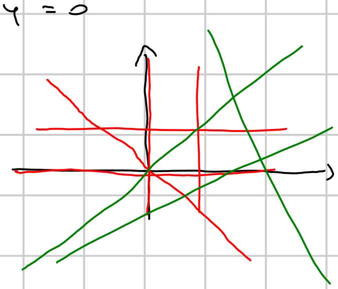
$$[CA'B'] = \frac{A'C \cdot CB'}{2} \sin\gamma = \frac{A'C \cdot CB'}{ab} [ABC]$$

$$[BA'C'] = \frac{BA' \cdot BC'}{2} \sin\beta = \frac{A'B \cdot BC'}{ca} [ABC]$$

Finite i conti!

Problema 1 Imo 2025/1

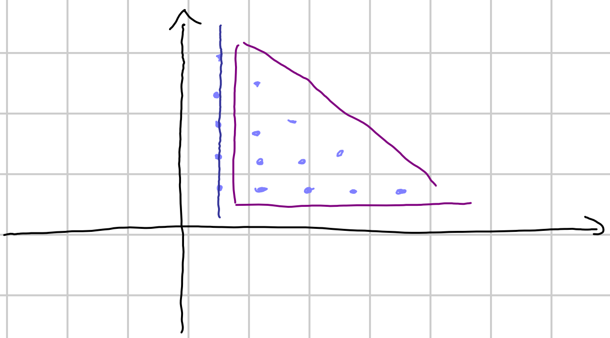
Una retta nel piano è "soleggiata" se non è parallela all'asse x, y o alla retta $r: x + y = 0$

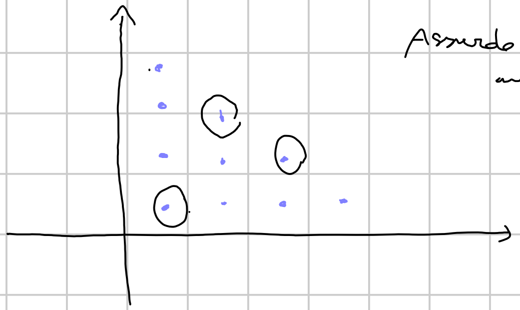


Sia $n \geq 3$ un intero fissato. Determinare tutti gli interi non negativi k: esistono n rette distinte nel piano che soddisfano

a) per ogni intero positivo a e b: $a + b \leq n + 1$, (a, b) è su almeno una delle n rette

b) Esattamente k rette sono soleggiate





Assurdo: non esiste una retta non soleggiata

Se abbiamo sempre una retta non soleggiata, $n \to n-1$

Mostriamo che ne abbiamo sempre una