WC 2007

Titolo nota 24/01/2007

$$\frac{1+a}{1-a} = \frac{2}{1-a} - 1 = \frac{2}{b+c} - 1$$

$$LHS = \frac{\cancel{2}}{a+b} + \frac{\cancel{2}}{a+c} + \frac{\cancel{2}}{b+c} - \frac{3}{2} \overset{\#}{\le} \cancel{2}\left(\frac{b}{a} + \frac{c}{b} + \frac{a}{c}\right)$$

$$LHS = \frac{c}{a+b} + \frac{b}{a+c} + \frac{a}{b+c} + \frac{3}{2}$$

$$Th \iff \frac{3}{2} \le \frac{bc}{\underline{a(a+c)}} + \frac{ab}{c(b+c)} + \frac{ac}{b(a+b)}$$

$$RHS = 2\sum_{cyc} b^2c^2(b+c)(a+c) = 2\sum_{cyc} \underline{ab^3c^2} + \underline{b^4c^2} + \underline{ab^2c^3} + b^3c^3 =$$

$$= 2\cancel{\sum_{sym} a^3b^2c} + \sum_{sym} a^3b^3 + 2\sum_{cyc} a^4b^2$$

$$LHS = 3abc(a+b)(b+c)(a+c) = \cancel{3}\sum_{sym} a^3b^2c + \sum_{sym} a^2b^2c^2$$

$$\sum a^3b \geq \sum a^3b^2c \qquad 2\sum_{cyc} a^4b^2 \geq 2\cancel{\sum}\, 6a^2b^2c^2$$

rearrang o AM $\geq$ GM

$$\frac{3}{4} \leq \sum \frac{bc}{a(a+c)}$$

$$\frac{bc}{a} \qquad \boxed{\frac{1}{a+c} \qquad \frac{ac}{b}}$$

$$\sum \frac{ac}{b(a+c)}$$

$$\sum \frac{x}{y+z} \geq \frac{3}{2} \qquad x = \frac{1}{e}$$

$$\sum a_i = 1 \qquad \sum b_i^2 = 1$$

$$a_1(b_1 + a_2) + a_2(b_2 + a_3) \ldots\ldots < 1$$

$$(a_1 a_2 + a_2 a_3 + \ldots + a_n a_1) + \sum a_i b_i <$$

$$< \sum_{sym} a_i a_j + \sum a_i b_i$$

$$\left(\sum a_i b_i\right)^2 \le \left(\sum a_i^2\right)\left(\sum b_i^2\right)$$

$$\left(\sum a_i b_i\right) \le \underline{\sqrt{\sum a_i^2}}$$

$$\sqrt{\sum a_i^2} = t$$

$$\overline{\sum_{sym} a_i a_j = \frac{1-t^2}{2}}$$

$$\frac{1-t^2}{2} + t < 1 \implies (t-1)^2 > 0$$

$$\sum a_i a_{i+1} \leq \frac{1}{4} \quad \text{se} \quad \sum a_i = 1 \qquad a_i \geq 0$$

$\boxed{n \text{ pari}}$

$$a_1 a_2 + a_2 a_3 + \dots + a_{2k-1} a_{2k} + a_{2k} a_1 \leq$$

$$\leq (a_1 + a_3 + \dots + a_{2k-1}) \cdot (a_2 + a_4 + \dots + a_{2k}) \leq$$

$$\leq \left(\frac{\sum a_i}{2} \right)^2 = \frac{1}{4}$$

$$\boxed{\begin{aligned} a_2 &= \tfrac{1}{2} \\ a_3 + a_1 &= \tfrac{1}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} a_1 &= \tfrac{1}{2} \\ a_2 &= \tfrac{1}{2} \end{aligned}}$$

$\boxed{n \text{ generico}}$ $\quad a_1, a_2, \dots, a_n \rightsquigarrow a_1 + a_2, a_3, \dots, a_n$

$$a_1 a_2 + a_2 a_3 + \boxed{\dots + a_{n-1} a_n} + a_n a_1 \overset{?}{\leq} (a_1 + a_2) a_3 + \boxed{a_3 a_4 + \dots +} \; a_n (a_1 + a_2)$$

$$a_1 a_2 + \cancel{a_2 a_3} + \cancel{a_n a_1} \overset{?}{\leq} a_1 a_3 + \cancel{a_2 a_3} + \cancel{a_n a_1} + a_n a_2$$

$a_2 \leq a_3$ ← prima o poi succede

$n=3$ si fa a mano !!!

$$\frac{x_1}{1+x_1^2} + \dots + \frac{x_n}{1+\dots+x_n^2} < \sqrt{n}$$

$$a_i = \frac{x_i}{1+\dots+x_i^2} \qquad b_i = 1$$

$$\left(\sum a_i\right)^2 \leq n \cdot \left(\sum a_i^2\right)$$

$$\sum a_i \leq \sqrt{n} \cdot \sqrt{\sum a_i^2}$$

$$\text{Th} \Longleftarrow \sum a_i^2 < 1$$

$$\frac{y_1}{(1+y_1)^2} + \frac{y_2}{(1+y_1+y_2)^2} + \dots < 1 \qquad y_i = x_i^2$$

$$\leq 1 - \frac{1}{1+y_1+\dots+y_n}$$

$$\underbrace{\frac{y_1}{(1+y_1)^2} + \dots \dots}_{n \text{ termini}} + \frac{y_{n+1}}{(1+y_1+\dots+y_{n+1})^2} \leq 1 - \frac{1}{(1+y_1+\dots)_{-}}$$

$$\leq 1 - \frac{1}{1+y_1+\dots+y_n} \leq 1 - \frac{1}{(1+y_1+\dots+y_{n+1})}$$

$$\le (1 + \dots + y_{n+1})(1 + \dots + y_n)$$

$$f_n(a) = \sup \left\{ \frac{x_1}{a + x_1^2} + \frac{x_2}{a + x_1^2 + x_2^2} + \dots + \frac{x_n}{a + x_1^2 + \dots + x_n^2} \right\}$$

$f_n(a)$ e $f_n(1)$

$$\frac{1}{a} \frac{x_1}{1 + \frac{x_1^2}{a}} + \dots = \frac{1}{a} \sqrt{a} \frac{\frac{x_1}{\sqrt{a}}}{1 + \left(\frac{x_1}{\sqrt{a}}\right)^2} + \dots$$

$$\frac{x_i}{\sqrt{a}} = y_i$$

$$= \frac{1}{\sqrt{a}} \left(\frac{y_1}{1 + y_1^2} + \dots \right)$$

$$\boxed{f_n(a) = \frac{1}{\sqrt{a}} f_n(1)}$$

$$f_n(1) = \frac{x_1}{1+x_1^2} + \boxed{\frac{x_2}{1+x_1^2+x_2^2} + \dots + \frac{x_n}{1+x_1^2+x_2^2+\dots+x_n^2}}$$

$$\leq f_{n-1}(1+x_1^2) = \frac{f_{n-1}(1)}{\sqrt{1+x_1^2}} < \frac{\sqrt{n-1}}{\sqrt{1+x_1^2}}$$

$x_1 = z$

$$\frac{z}{1+z^2} + \frac{\sqrt{n-1}}{\sqrt{1+z^2}} < \sqrt{n} \qquad \text{BASTA}$$

si fanno i conti

$$1 + x_1^2 + x_2^2 + \dots + x_n^2 = y_n$$

$$\frac{\sqrt{y_1 - 1}}{y_1} + \frac{\sqrt{y_2 - y_1}}{y_2} + \frac{\sqrt{y_3 - y_2}}{y_3} + \dots + \frac{\sqrt{y_n - y_{n-1}}}{y_n} < \sqrt{n}$$

$$\frac{1}{y_1} < \frac{1}{\sqrt{y_1}}$$

$$\frac{1}{y_2} < \frac{1}{\sqrt{y_1 y_2}}$$

$$\frac{1}{y_i} < \frac{1}{\sqrt{y_{i-1} y_i}}$$

$$\sqrt{1-\frac{1}{y_1}}+\sqrt{\frac{1}{y_2}-\frac{1}{y_2}}+\sqrt{\frac{1}{y_2}-\frac{1}{y_3}}+\dots+\sqrt{\frac{1}{y_{n-1}}-\frac{1}{y_n}}<\sqrt{n}$$

$M_{\frac{1}{2}} < AM$

$$\text{LHS} < \sqrt{n}\left(\sqrt{1-\frac{1}{y_n}}\right) < \sqrt{n}$$