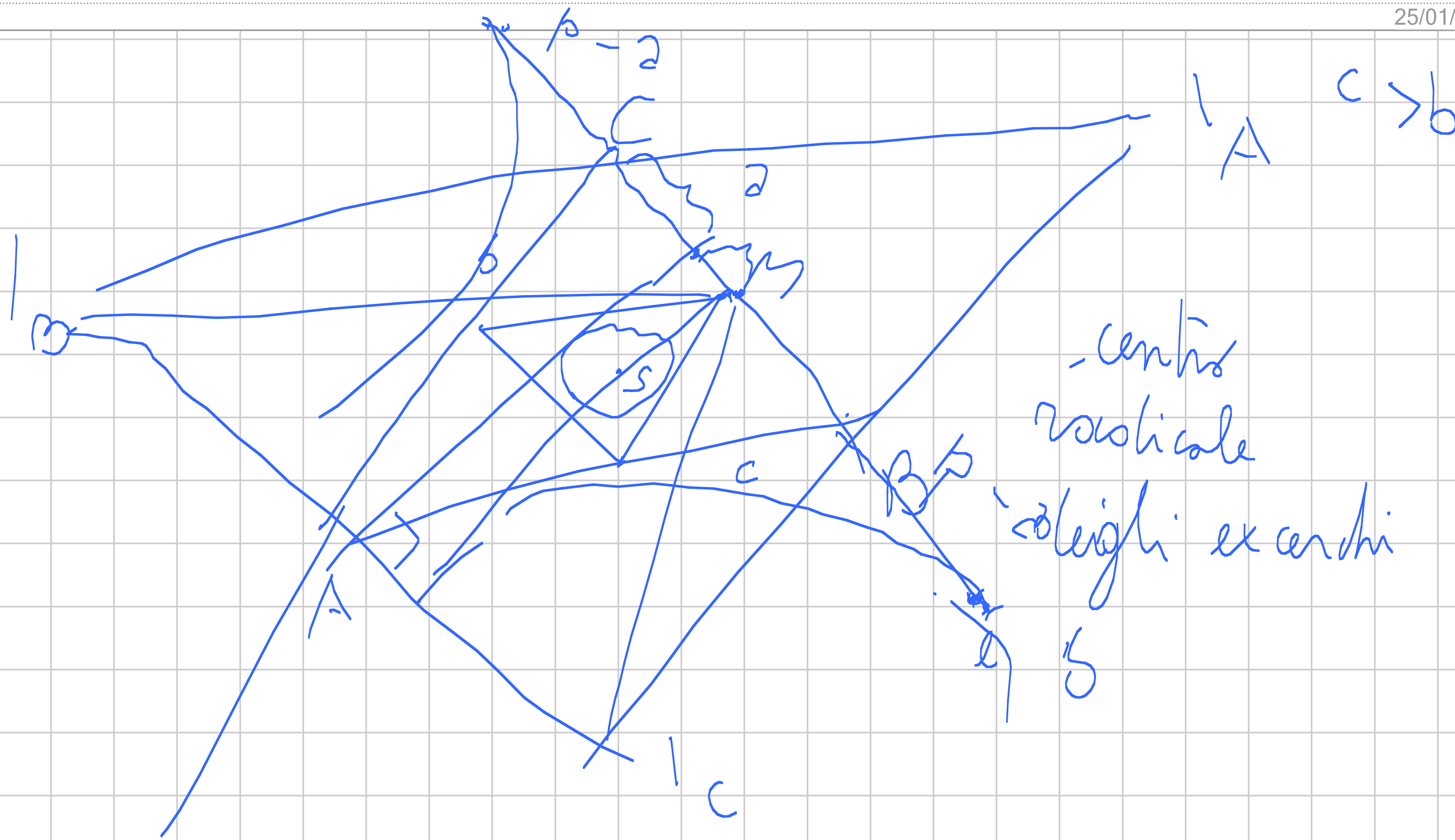
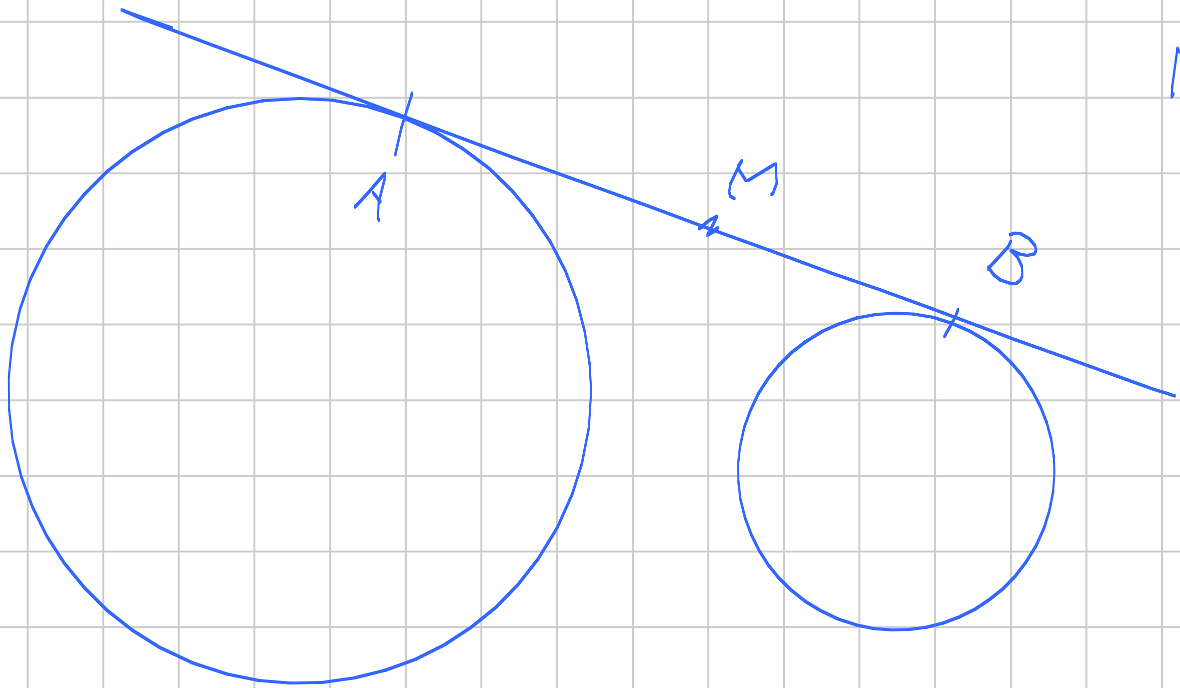
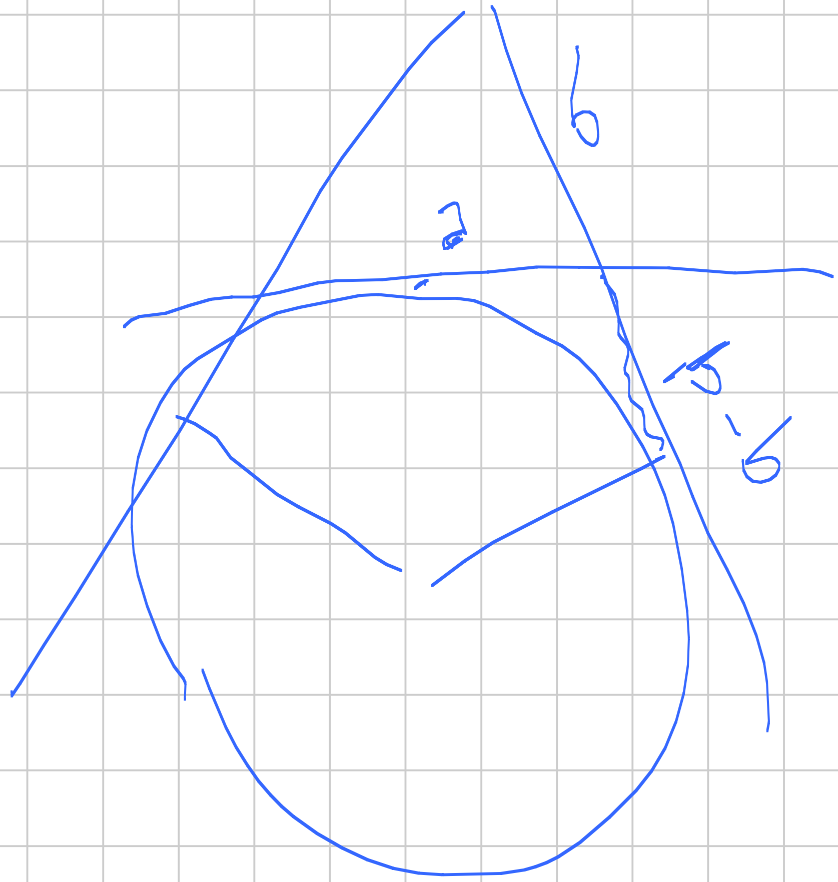
Titolo nota 25/01/2007

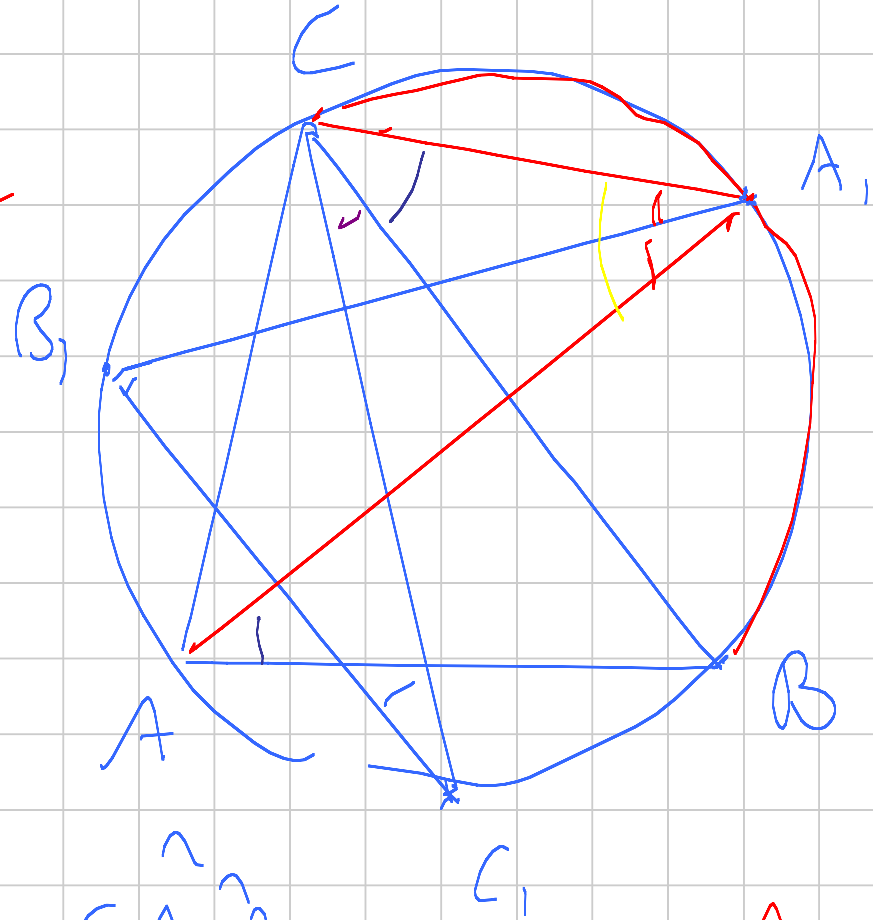




$MA^2 = MB^2$


a
b

$C\hat{C_1}B_1 = \beta/2$



$CB_1A_1C_1$

$CC_1 \perp A_1B_1$

$C_1\hat{B_1}A_1$

$C\hat{C_1}B_1$

$C\hat{C_1}B_1 = C\hat{A_1}B_1 =$

$C\hat{A_1}A = \beta$

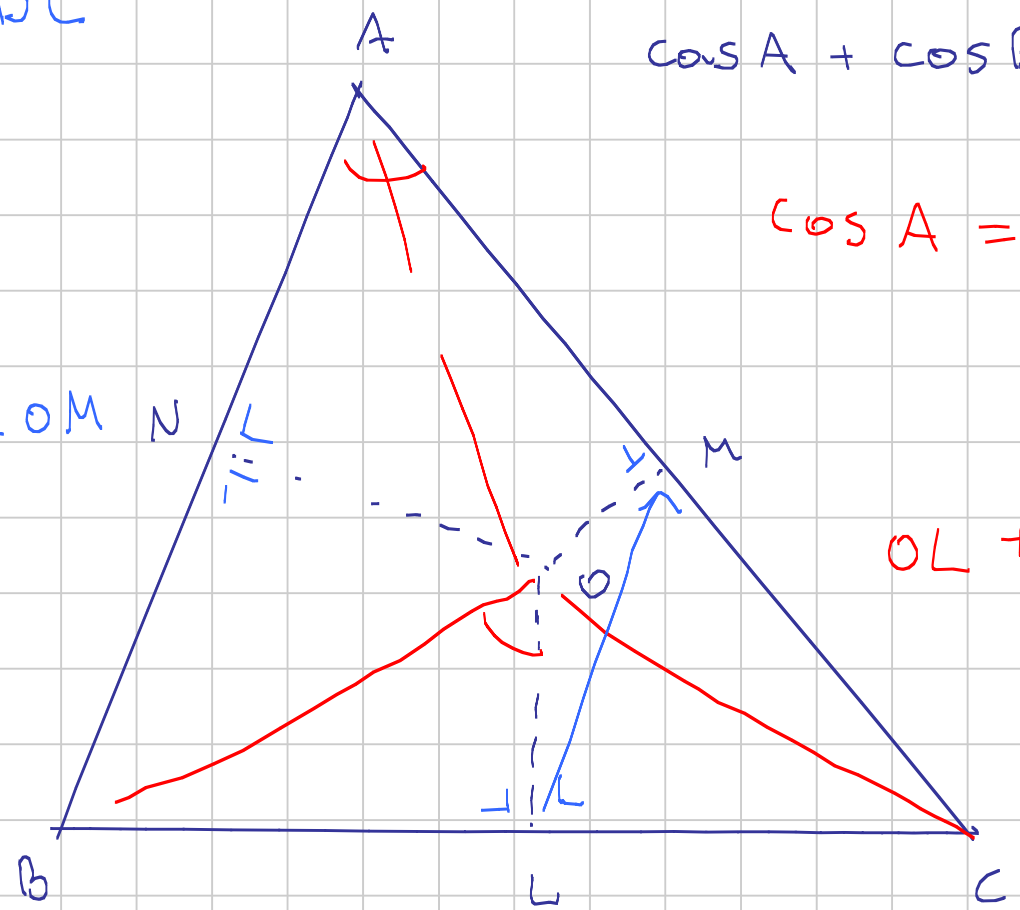
$A_1\hat{B_1}C_1 = C_1\hat{C}A_1$

$S = \text{area } ABC$

$S = (OBC)$
$+ (OCA)$
$+ (OAB)$

$= \frac{a}{2} \cdot OL + \frac{b}{2} \cdot OM + \frac{c}{2} \cdot ON$

$= p \cdot r$



$$\cos A + \cos B + \cos C = 1 + \frac{r}{R}$$

$$\cos A = \frac{OL}{R}$$

$$OL + OM + ON = R + r$$

OLCM ciclico

TOLOMEO ci dice che $OL \cdot CM + LC \cdot OM = OC \cdot LM$

$$OL \cdot \frac{b}{2} + OM \cdot \frac{a}{2} = R \cdot \frac{c}{2}$$

$$OM \cdot \frac{c}{2} + ON \cdot \frac{b}{2} = R \cdot \frac{a}{2}$$

$$ON \cdot \frac{a}{2} + OL \cdot \frac{c}{2} = R \cdot \frac{b}{2}$$

$$\frac{b}{2}(OL+ON) + \frac{c}{2}(OM+OL) + \frac{a}{2}(OM+ON)$$

$$= R \cdot p$$

$$\frac{b}{2}(OL+ON+OM) + \frac{c}{2}(OM+OL+ON)$$

$$+ \frac{a}{2}(OM+ON+OL) = R \cdot p + r \cdot p$$

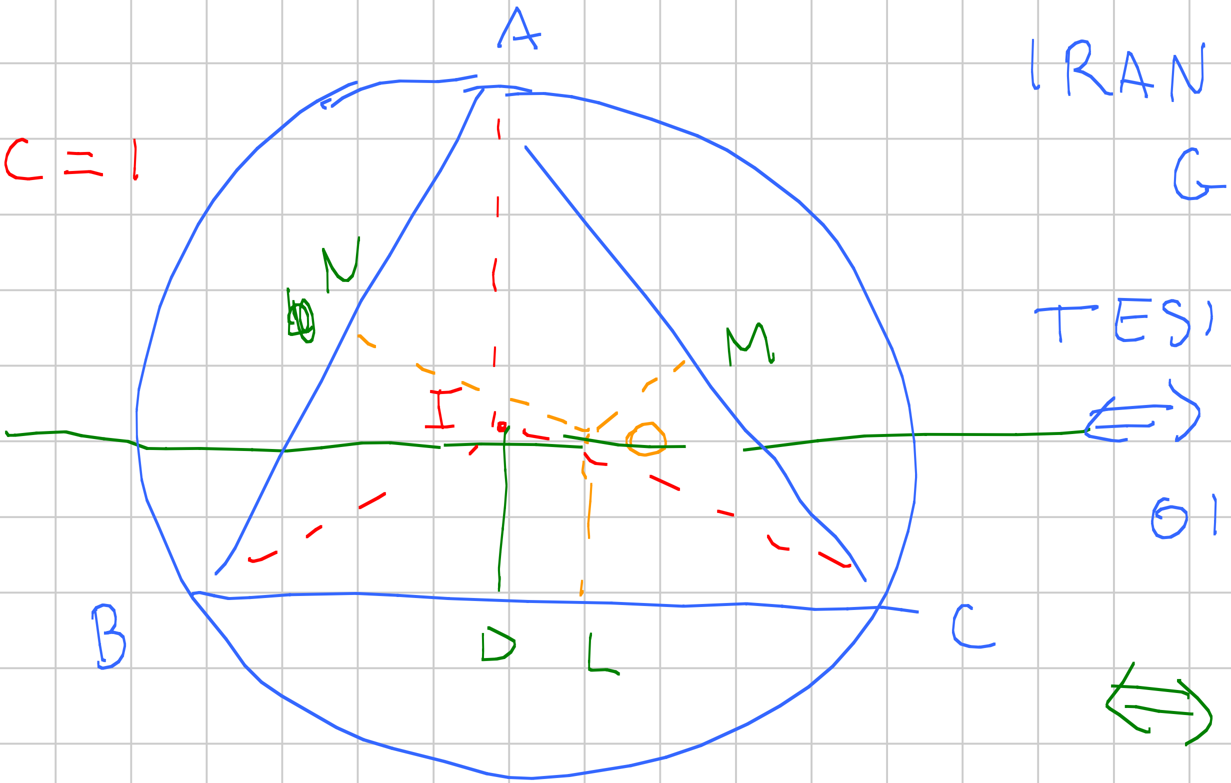
$$OL + ON + OM = r + R$$

IPOTESI:

$\cos B + \cos C = 1$

IRAN 2006

G 5



TESI

$\Longleftrightarrow$ $OI \parallel BC$

$\Longleftrightarrow$ $ID = OL$

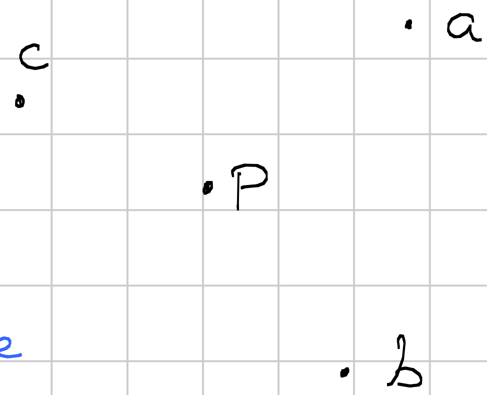
$OL + OM + ON = R + r$ $\Longleftrightarrow$ $OL = r$

$\Longleftrightarrow$ $OM + ON = R$

$\Longleftrightarrow$ $\cos B + \cos C = 1$

$$BC \cdot PC \cdot PB + AC \cdot PA \cdot PC + AB \cdot PA \cdot PB \geq AB \cdot BC \cdot CA$$

Numeri complessi wlog $P = 0$ $A = a$, $B = b$, $C = c$



$$|b-c| \cdot |c| \cdot |b|$$

$$\sum_{cyc} |bc(b-c)| \geq |(a-b)(b-c)(c-a)|$$

DISUG. TRIANG. Hope

$$\sum_{cyc} |bc(b-c)| \geq \left| \sum_{cyc} bc(b-c) \right| \geq |(a-b)(b-c)(c-a)|$$

identità algebrica

$$\sum_{cyc} bc(b-c) = (a-b)(b-c)(c-a)$$

Tolomeo

$A = a$ $\quad B = b$

$C = c$

$D = d$

$$AB \cdot DC + AD \cdot BC \geq AC \cdot BD$$ Numeri complessi !!!

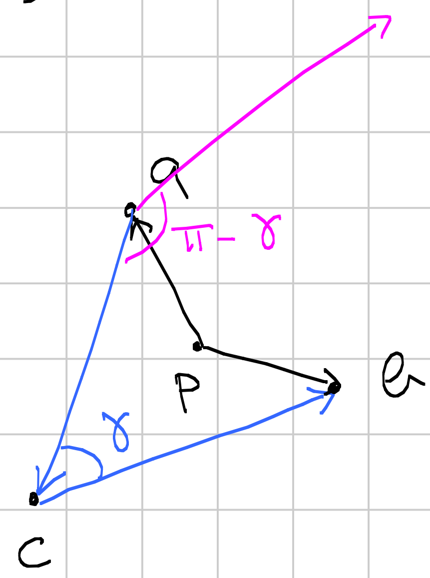
$$|(b-a)\cdot(d-c)| + |(a-d)(c-b)| \geq |(c-a)(d-b)|$$

$$|\ \cdot\ | + |\ \ | \underset{\Delta}{\geq} |\ \cdot + \cdot\ | \underset{\text{identità}}{=} |(c-a)(d-b)|$$

Nel problema dato uguaglianza $\Leftrightarrow$

$$\frac{b\cancel{c}(b-c)}{\cancel{c}a(c-a)} = \lambda \in \mathbb{R}_{>0} \qquad \frac{c\cancel{a}(c-a)}{\cancel{a}b(a-b)} = \mu \in \mathbb{R}_{>0}$$

vuol dire $\theta = 0$



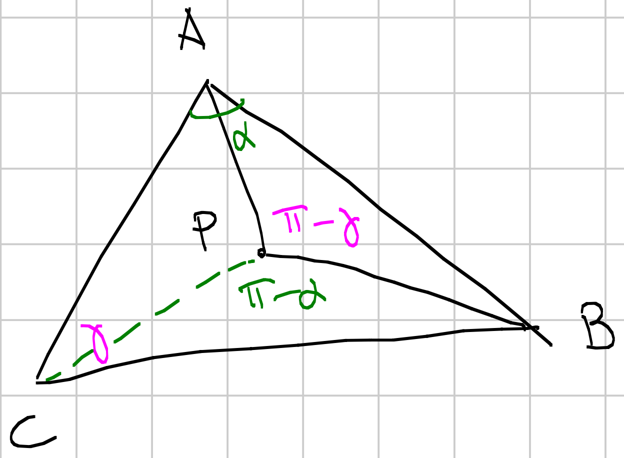
$$\frac{\frac{b}{a}}{\frac{c-a}{b-c}}$$

Angolo θ

stesso angolo θ

occhio al caso $a=0$, $b=0$, $c=0$

P = ortocentro



Vale anche per ottusangolo?

$$|(b-a)\cdot(d-c)| + |(a-d)(c-b)| \geq |(c-a)(d-b)|$$

Uguagliuanza in Tolomeo $\Leftrightarrow$

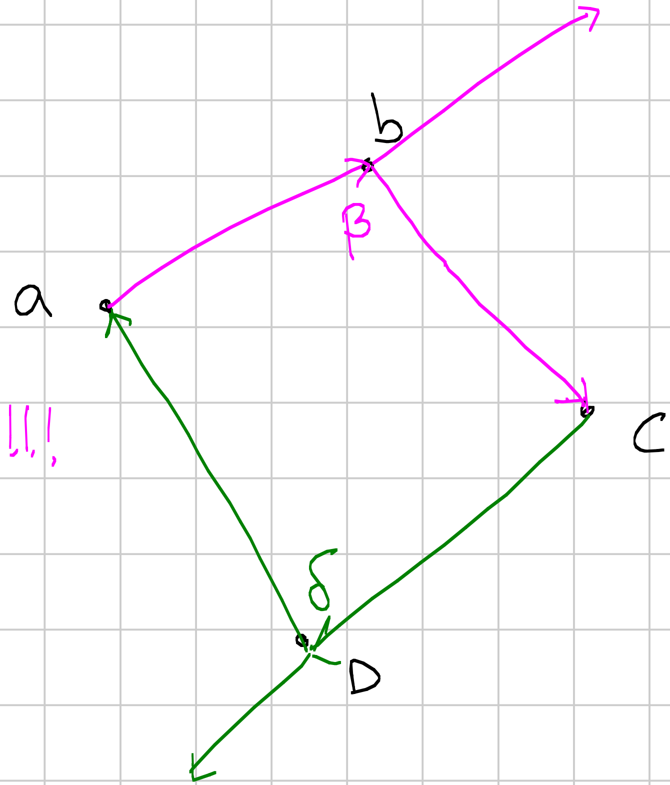
$$\frac{(b-a)(d-c)}{(a-d)(c-b)} = \lambda \in \mathbb{R}_{>0}$$

$$\frac{\left.\frac{(b-a)}{(c-b)}\right\} \theta}{\left.\frac{(a-d)}{(d-c)}\right\} \theta}$$

$\pi - \beta$

errore di segno!!!

$\pi - \delta$



$$\sum \left| \frac{(p-b)(p-c)\cancel{(c-b)}}{(a-b)(a-c)\cancel{(c-b)}} \right| \geq 1$$

$$\vee|$$

$$\left| \sum \frac{(p-b)(p-c)}{(a-b)(a-c)} \right| \overset{?}{\geq} 1$$

$$GA + GB + GC + 3\frac{OG^2}{R} \geq 3R$$

$$R \cdot GA + R \cdot GB + R \cdot GC + 3OG^2 \geq 3R^2$$

$$|\vec{OA}| \cdot |\vec{GA}| + |\vec{OB}| \cdot |\vec{GB}| + |\vec{OC}| \cdot |\vec{GC}| + 3|\vec{OG}|^2 \geq 3R^2$$

$$\sum |\vec{OA}| \cdot |\vec{GA}| \geq \sum \langle \vec{OA}, \vec{GA} \rangle =$$

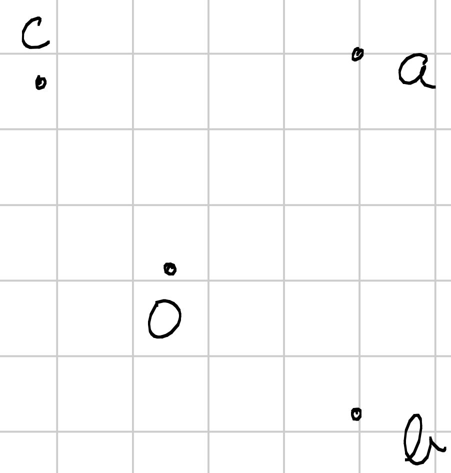
$$= \sum \langle \vec{OA}, \vec{OA} - \vec{OG} \rangle = \sum |\vec{OA}|^2 - \sum \langle OA, OG \rangle =$$

$$= 3R^2 - \langle \sum OA, OG \rangle = 3R^2 - \langle OH, OG \rangle =$$

$$|\vec{OA}|\cdot|\vec{GA}| + |\vec{OB}|\cdot|\vec{GB}| + |\vec{OC}|\cdot|\vec{GC}| + 3|\vec{OG}|^2 \geq 3R^2$$

$$\sum_{cyc} |a|\cdot|a-g| + 3|g|^2 \geq 3$$

$\boxed{R=1}$



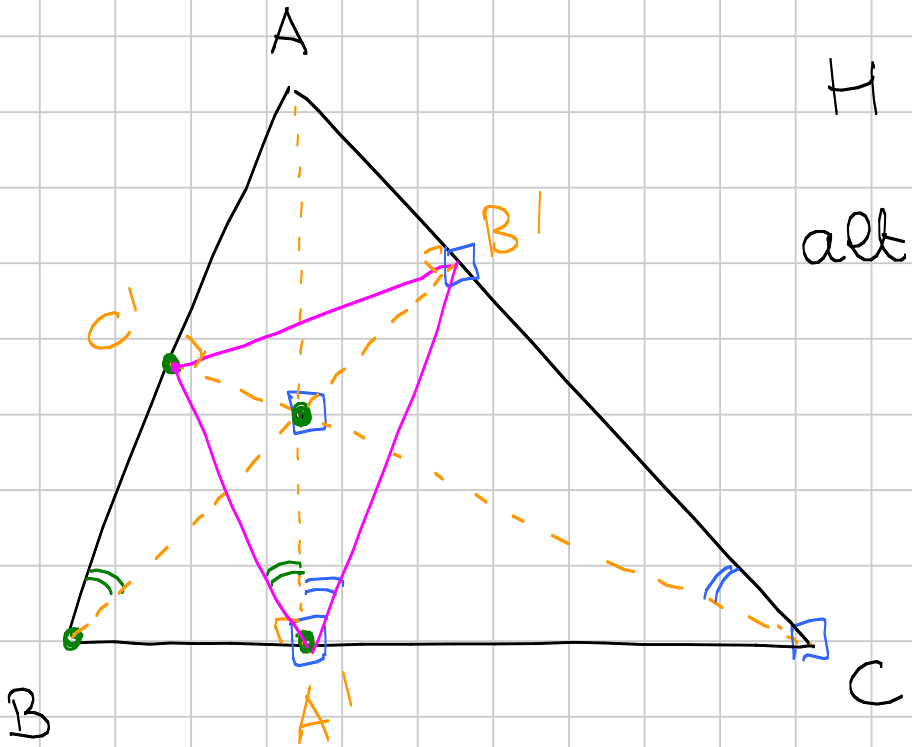
$$\sum_{cyc} a\cdot(a-g) + 3|g|^2 \geq 3$$

$$\underbrace{\sum_{cyc}|a|^2}_{\cancel{3}} - \left(\sum_{cyc} a\right)\cdot g + 3|g|^2 \geq \cancel{3}$$

$$3|g|^2 \geq \underbrace{\sum_{cyc} a}_{3g} \cdot g$$

$$3g \cdot g$$

Lemma 1 (TRIANGOLO ORTICO)



H = incentro di $A'B'C'$

altezze di ABC =

bisettrici di $A'B'C'$

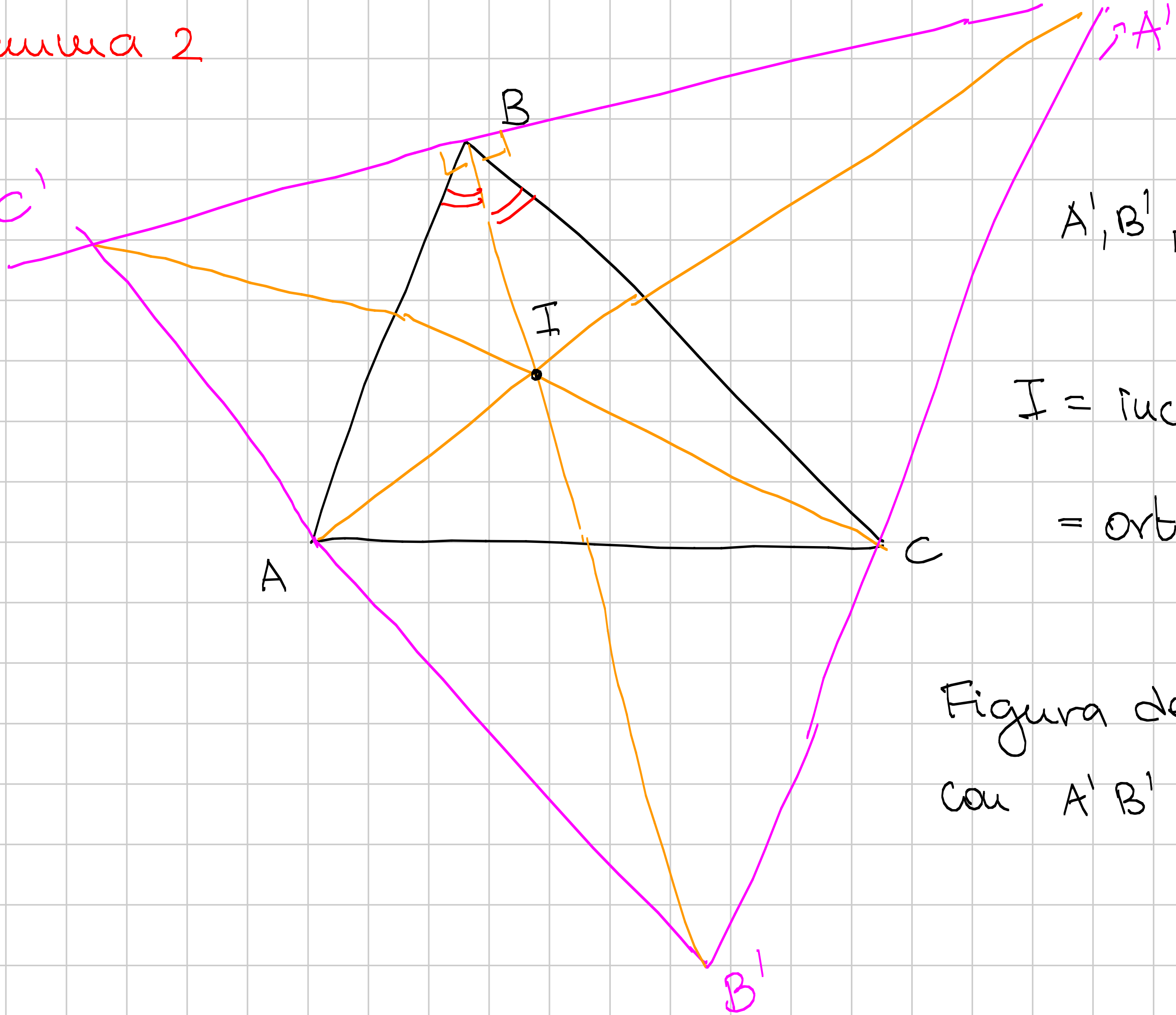
$)) =)) = 90 - \alpha$

A, B, C sono ex-centri di $A'B'C'$.

Lati ABC = bisettrici angoli esterni di $A'B'C'$

Circoscritto di $A'B'C'$ = Feuerbach di ABC

Lemma 2



A', B', C' = ex-centri di ABC

I = incentro di ABC

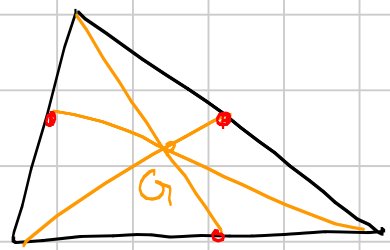
= ortocentro di $A'B'C'$

Figura del Lemma 1 con $A'B'C' \leftrightarrow ABC$

Prendiamo la figura del Lemma 2 e facciamo

omotetia $-\frac{1}{2}$ risp. a G

$A'B'C'$ diventa XYZ.



$\Rightarrow$ X, Y, Z = excentri del triangolo "mediale".

$DF \parallel AI$

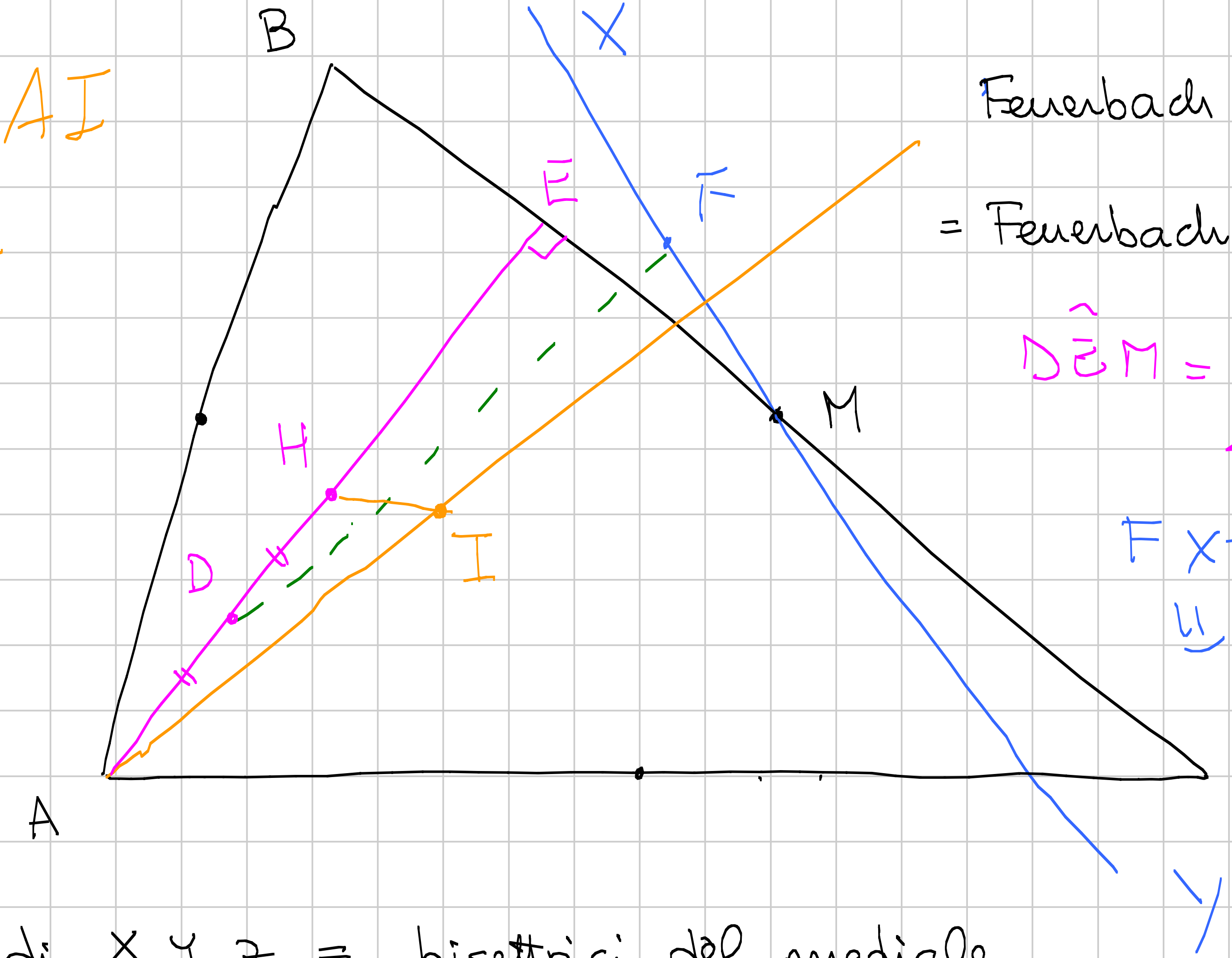
$\Uparrow$

AHI

Feuerbach ABC =

= Feuerbach XYZ

$D\hat{E}M = \frac{\pi}{2}$



$FX = FY$

$\Downarrow$

$F \in$ Feuerb.

$\Downarrow$

$D\hat{F}M = \frac{\pi}{2}$

$\Rightarrow DF \perp XY$

Altezze di X, Y, Z = bisettrici del mediale

Piedi " = vertici del mediale

ABC Tri. D, E, F ex-centri

$$H_{DEF} = I_{ABC} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c} = I$$

$$F_{DEF} = O_{ABC} = O$$

O_{DEF} = simm di H_{DEF} in F_{DEF} $= -I$

Omot. di O_{DEF} in G_{ABC} di fattore $-\frac{1}{2}$

$$-\frac{1}{2}\left(O_{DEF} - G_{ABC}\right) + G_{ABC} = -\frac{O_{DEF}}{2} + \frac{3}{2} G_{ABC} =$$

$$= \frac{1}{2} I + \frac{3}{2} G = \frac{1}{2}(I + H)$$