WC 08 - GEOMETRIA 1

Titolo nota 24/01/2008

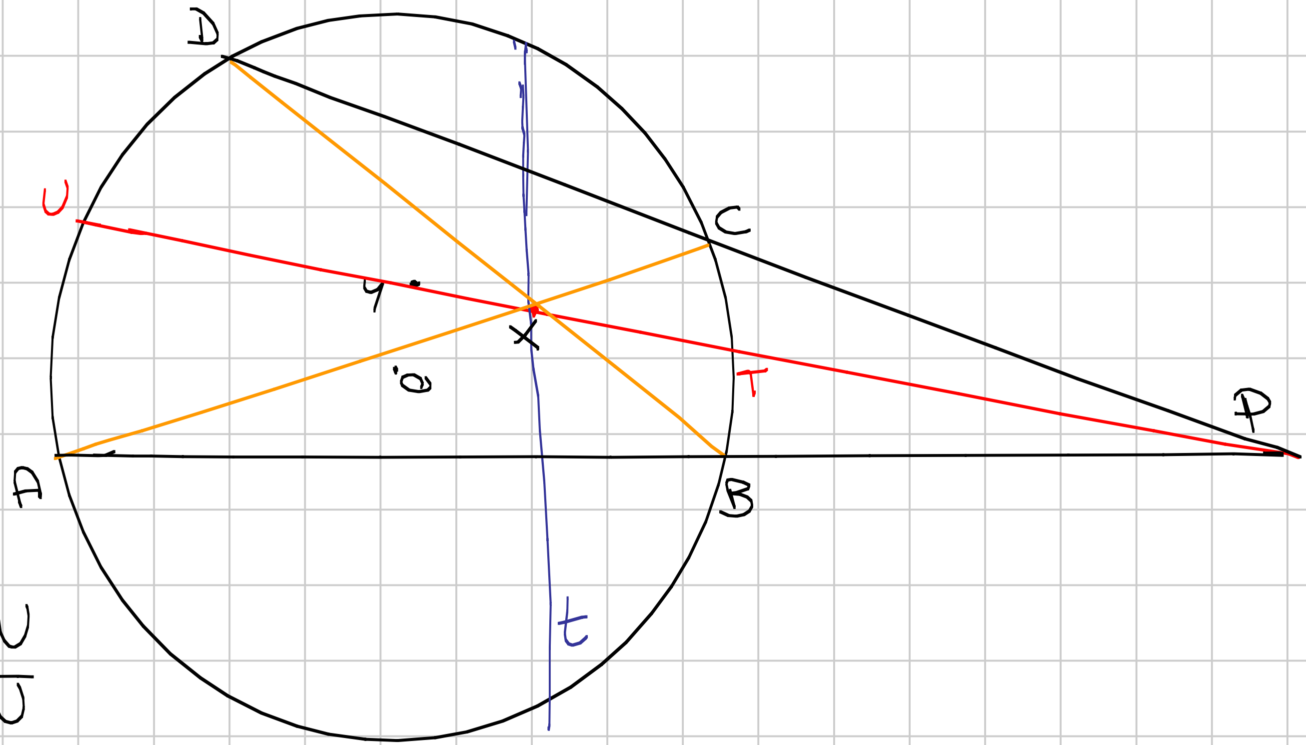
$OY \perp XY$

$Y \stackrel{?}{=}$ p.to medio UT

$$PY \stackrel{?}{=} \frac{PU + PT}{2}$$

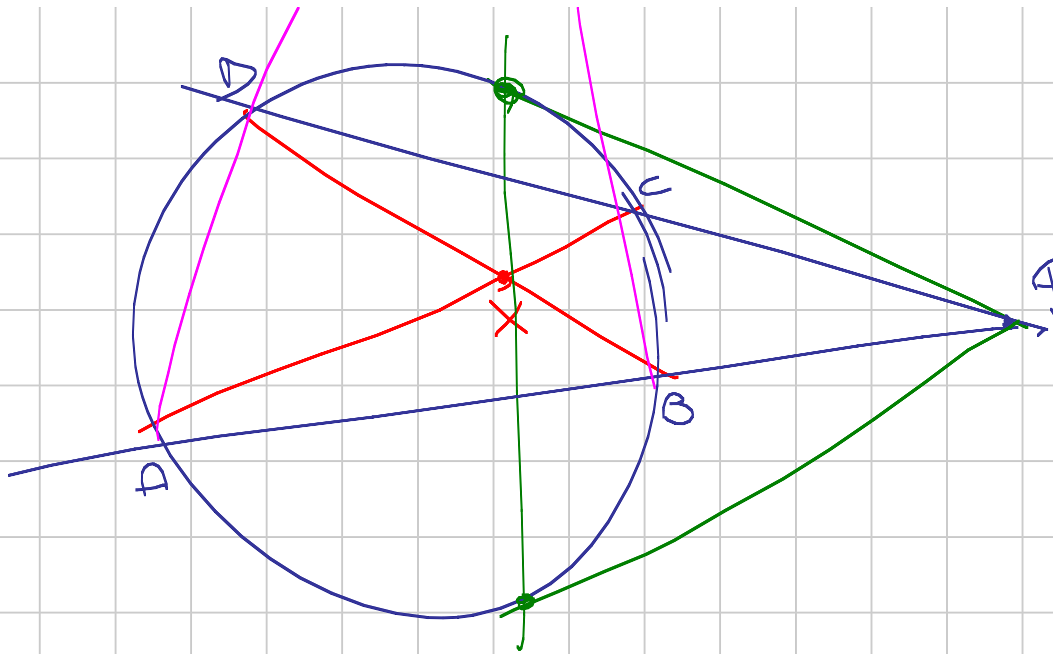
$$PX \cdot PY = PB \cdot PA =$$
$$= PT \cdot PU$$

$$PX = \frac{PT \cdot PU}{PY} \stackrel{?}{=} \frac{2PT \cdot PU}{PT + PU}$$
$$= \frac{2}{\frac{1}{PT} + \frac{1}{PU}}$$



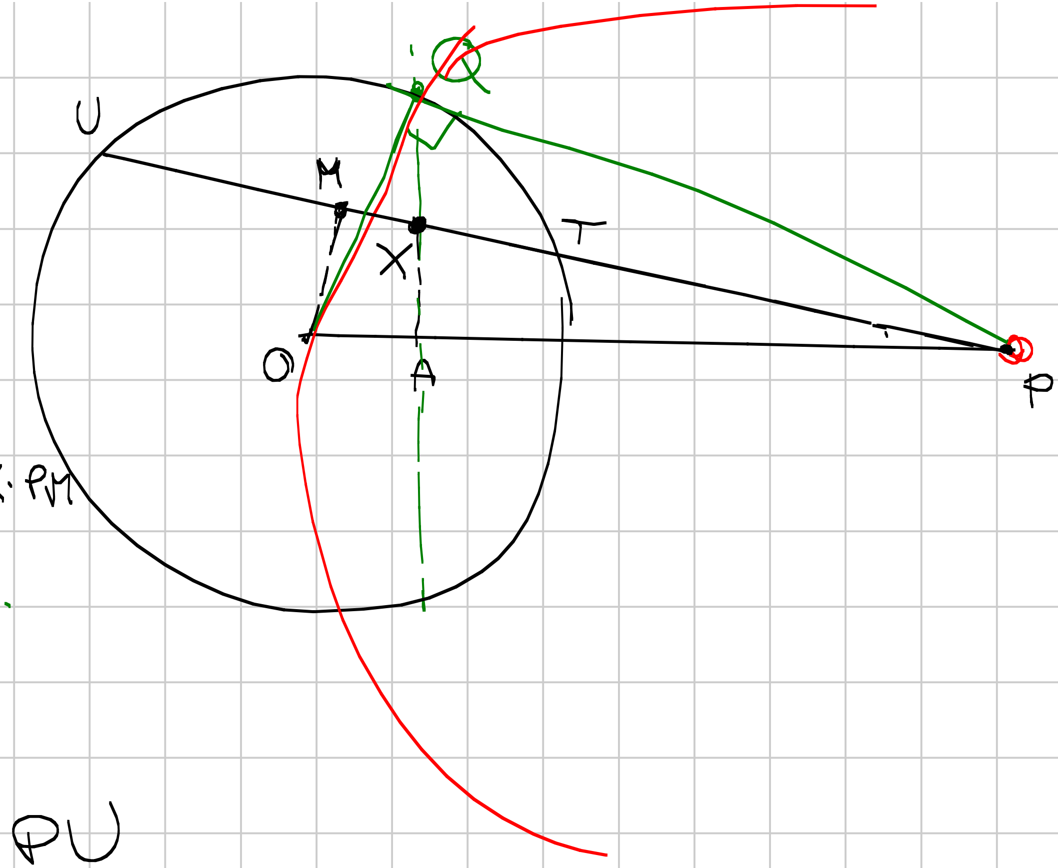
$t = pol(P)$

$X \in t$ per il LEMMA DEL LA POLARE



Lemma

$$PX = \frac{2}{\frac{1}{PT} + \frac{1}{PU}}$$



$PXA \sim POM$

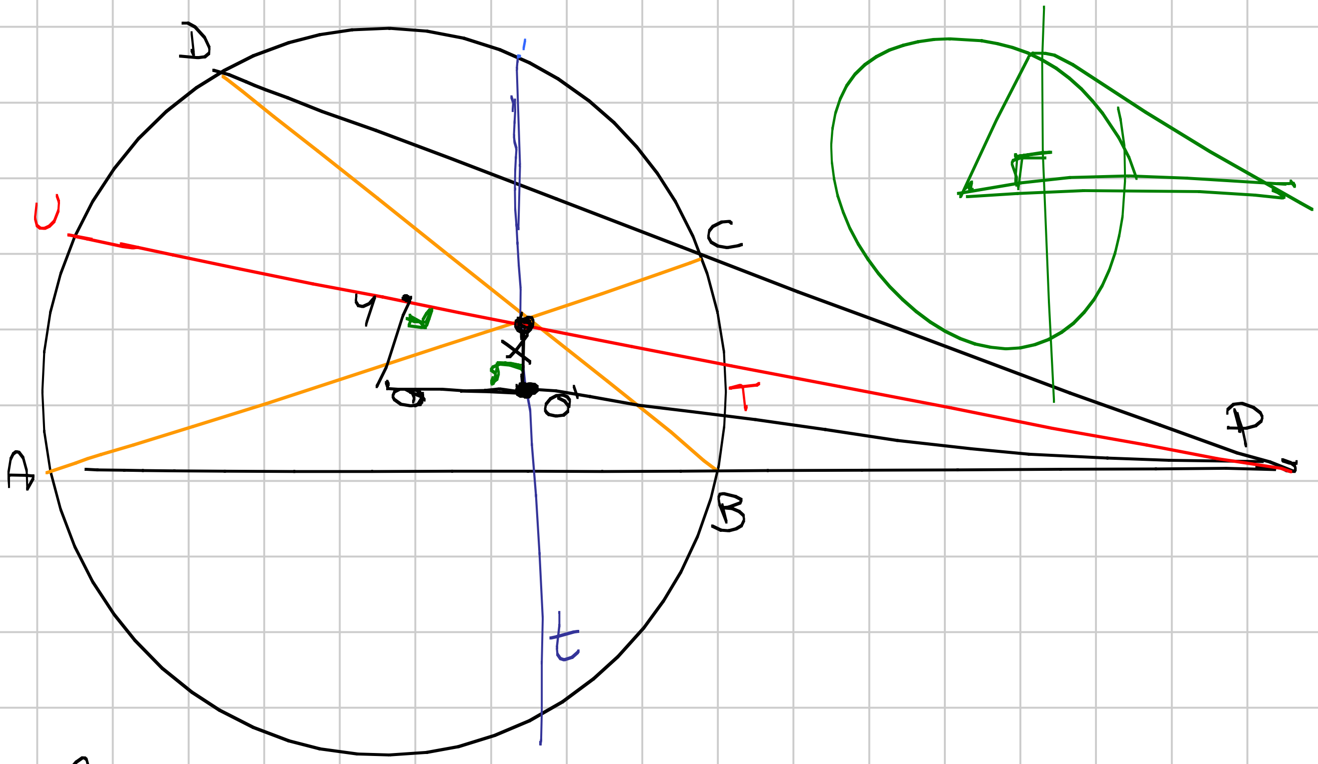
$$\frac{PX}{PO} = \frac{PA}{PM} \Longrightarrow PA \cdot PO = PX \cdot PM$$

$PA \cdot PO = PQ^2 = PT \cdot PU$

$PX \cdot PM = PT \cdot PU$

$$PX \cdot \frac{PT + PU}{2} = PT \cdot PU$$

$O\hat{Y}X = 90^\circ$



$OO'XY$ ciclico

$\iff PO \cdot PO' = PX \cdot PY$

$PQ^2 = pot(P) =$

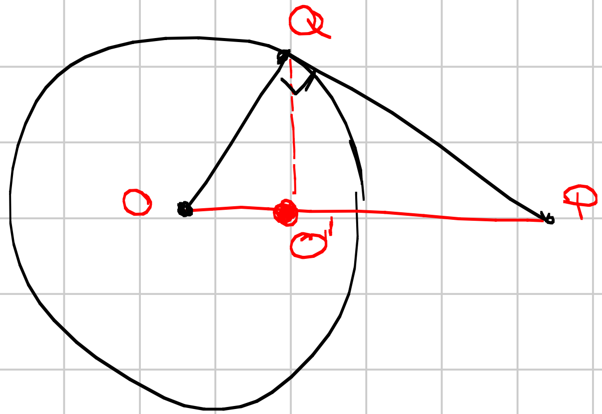
$= PX \cdot PY$

Inv. centro P che lascia fissa la circ

$O \to O' \in pol(P)$

$PO \cdot PO' = PQ^2$

$X \in pol(P)$

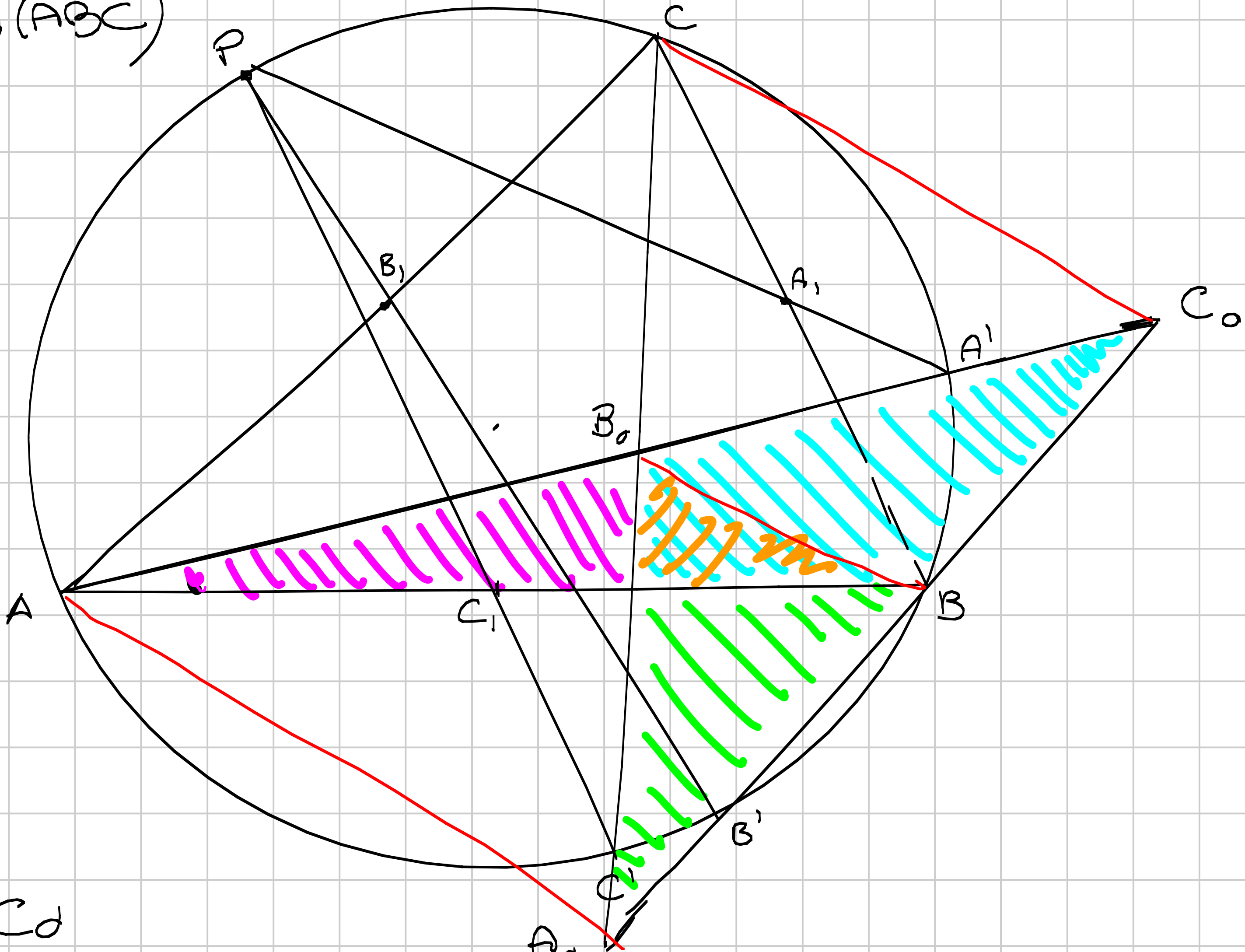


$A(A_0B_0C_0) \leq \frac{1}{2} A(ABC)$

1

B_1, A_1, C_0 sono allineati

P wlog $\in \overset{\frown}{AC}$



ABC_0

$A(A_0BB_0) =$
$\leq A(ABB_0)$

2

$AA_0 // BB_0 // CC_0$

TEOREMA DI PASCAL

A C B B' P A'

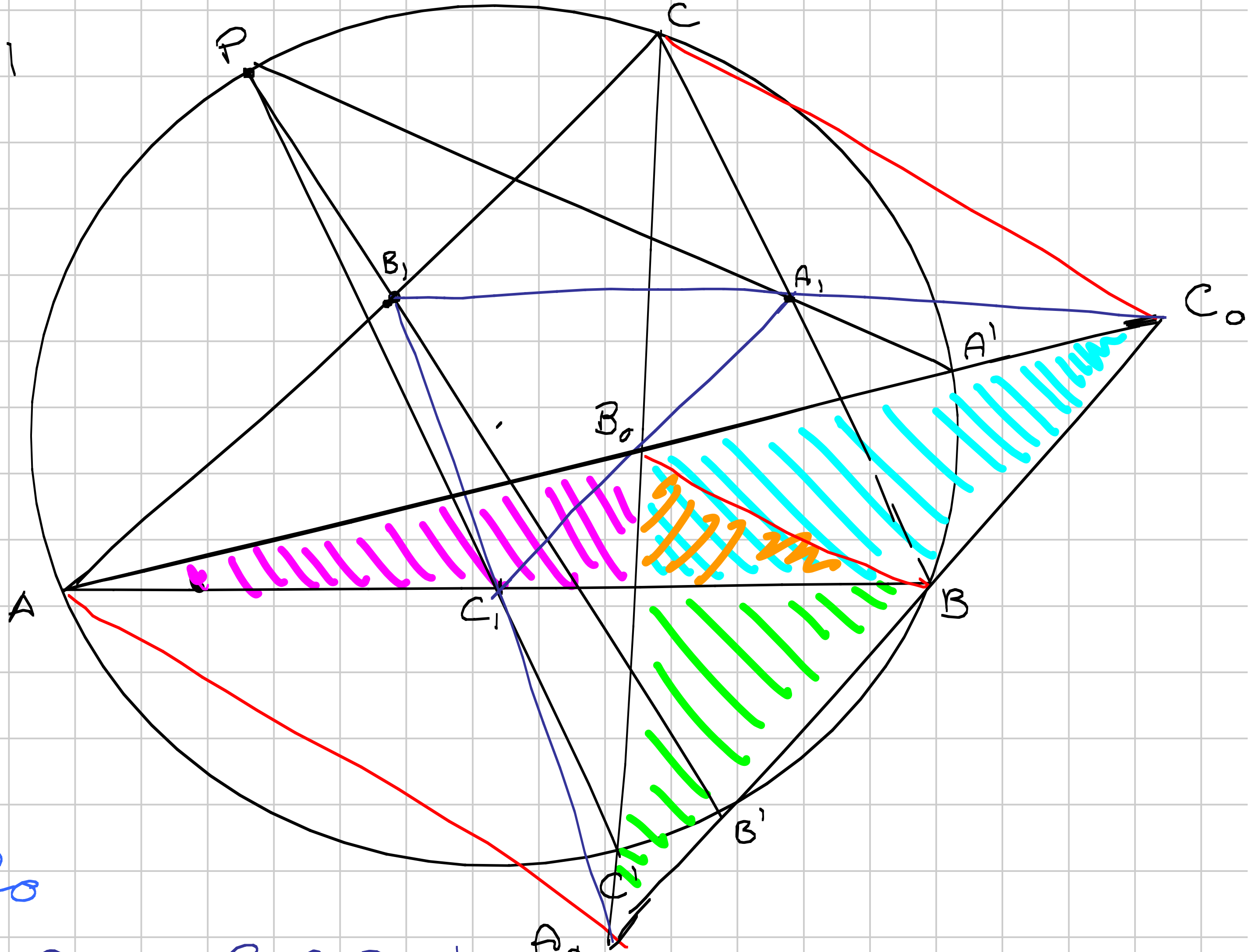
$AC \cap PB' = B_1$

$CB \cap PA' = A_1$

$BB' \cap A'A = C_0$

A B C C' P A'

$AB \cap$



$AA_0 // BB_0 // CC_0$

$C_0A_1B_0 \sim C_0B_1A$

$C_0A_1B \sim C_0B_1A_0$

$$\frac{B_0 C_0}{A C_0} = \frac{A_1 C_0}{B_1 C_0} \qquad \frac{B_0 C_0}{A C_0} = \frac{B C_0}{A_0 C_0}$$

$$\frac{A_1 C_0}{B_1 C_0} = \frac{B C_0}{A_0 C_0}$$

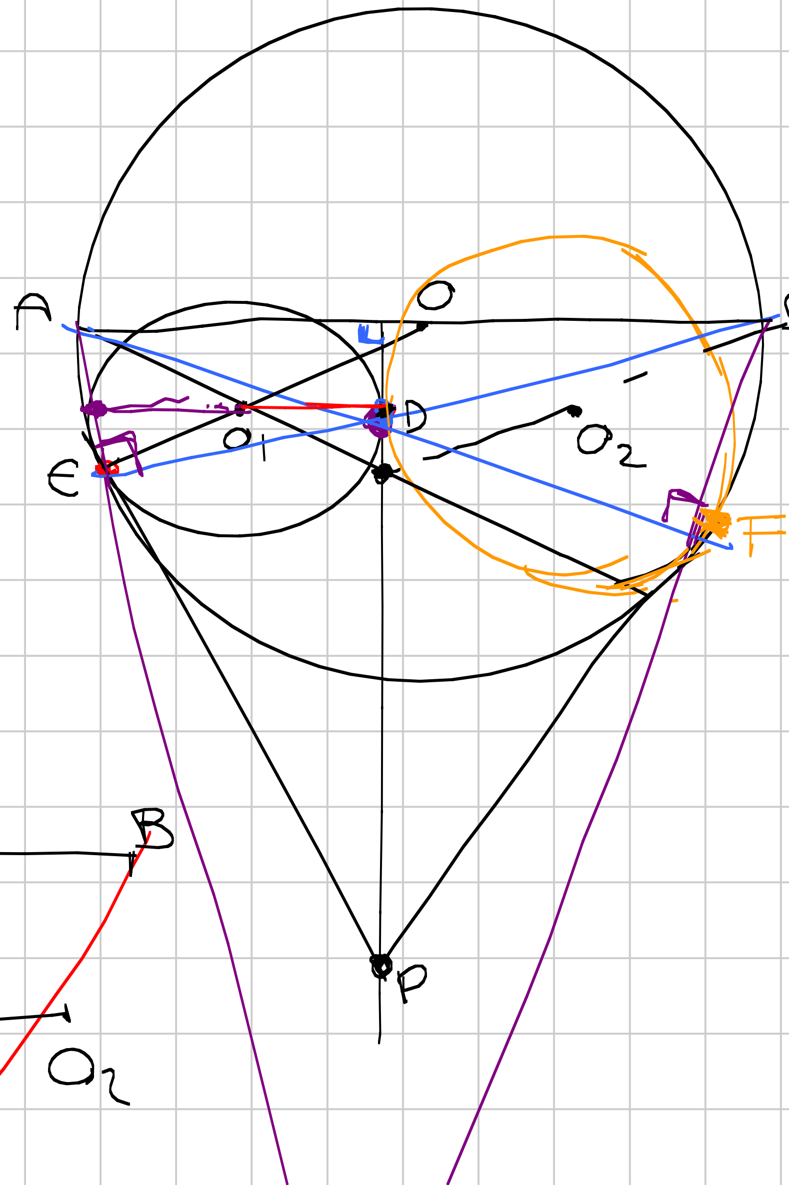
ADF sono allineati:

Lemma 1

EDB $\qquad O_1D \parallel AB$

Per omotetia centro E e

rapp. $\frac{R}{r}$



Lemma 2:

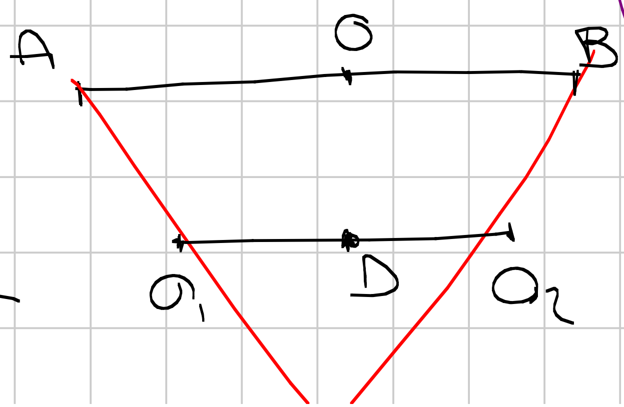
PE = PD = PF

EDF ha circocentro P.

Lemma 3:

PAPPO

AOB $\qquad O_1DO_2$



$AO_1 \cap BO_2 = Q$

$AD \cap OO_2 = F$

$BD \cap OO_1 = E$

Q, F, E

$\angle 1$: ADF $\quad DX = TX - SO_1 = SO_1\left(\frac{AT}{AS} - 1\right)$

$\angle 2$: EDF ~~ka~~ circocentro P

AO_1, PD, EF concorrono

$X = AO_1 \cap PT$

E, X, F allineati?

EF = asse radicale

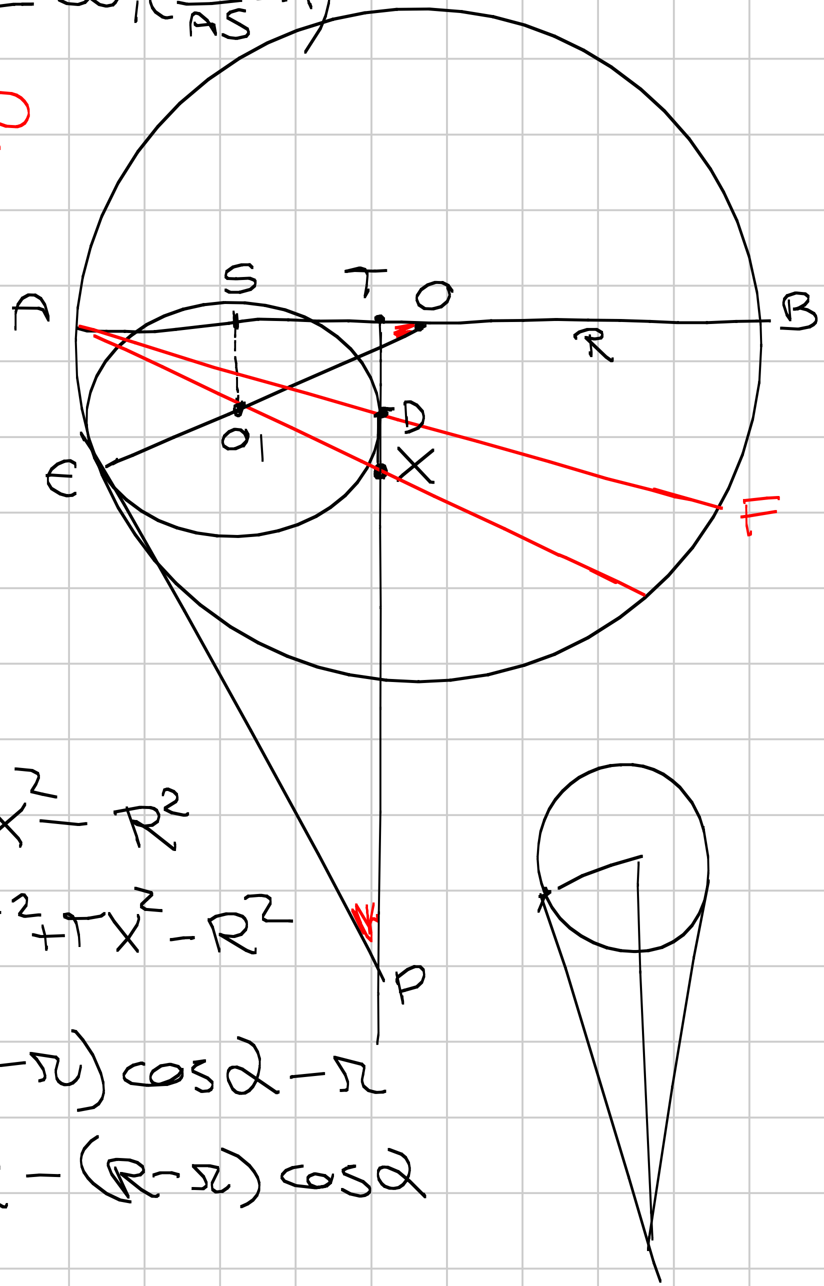
$$X \overset{?}{\in} EF \iff PX^2 - PE^2 = OX^2 - R^2$$

$$PX^2 - PE^2 = OT^2 + TX^2 - R^2$$

$$OT = R\cos\alpha - r\cos\alpha - r = (R-r)\cos\alpha - r$$

$$AS = R - SO = R - (r + OT) = R - (R-r)\cos\alpha$$

$$SO_1 = TD = (R-r)\sin\alpha$$

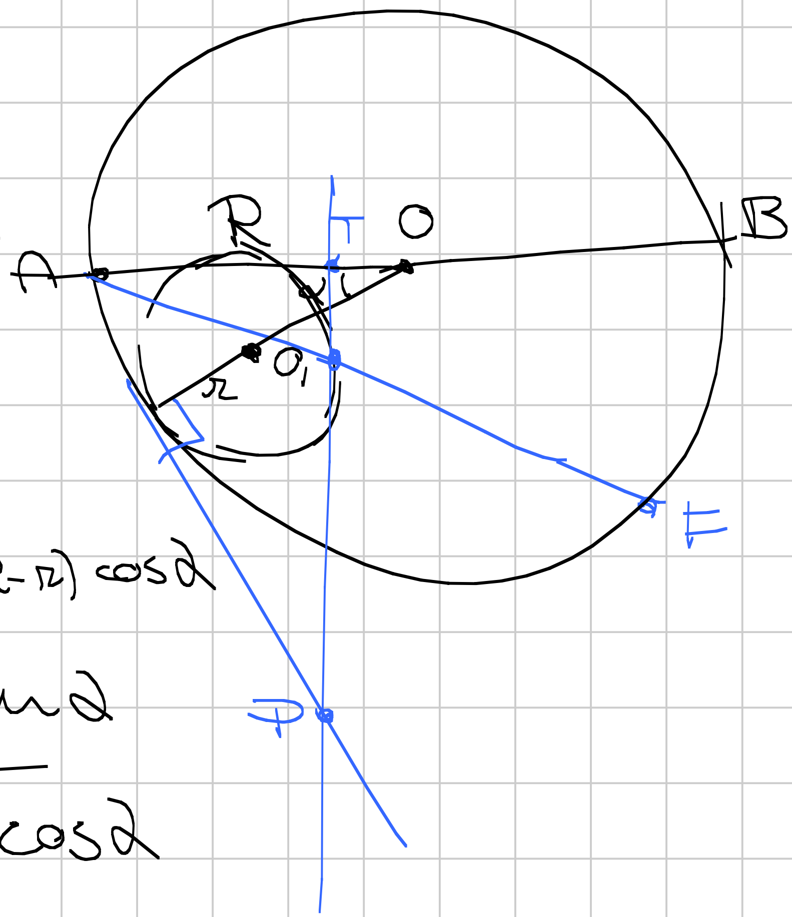


$$AT = R + r - (R - r)\cos\alpha$$

$$PE = \frac{EO_1}{\operatorname{tg}\frac{\alpha}{2}} = r\,\frac{1+\cos\alpha}{\sin\alpha}$$

$$TX = \frac{SO_1}{AS}\cdot AT \qquad R,\ r,\ \alpha$$

$$(PD - DX)^2 - PE^2 \overset{?}{=} OT^2 + (SO_1 + DX)^2 - R^2$$



$$\left(-\cancel{2}r\,\frac{1+\cos\alpha}{\sin\alpha} - \cancel{2}(R-r)\cos\alpha\right) DX \overset{?}{=} \cancel{2}r^2 - \cancel{2}Rr - \cancel{2}r(R-r)\cos\alpha$$

$$DX = SO_1\left(\frac{AT}{AS} - 1\right) = \frac{(R-r)\,r\sin\alpha}{R-(R-r)\cos\alpha}$$