WC 2008 - GEOMETRIA 2

Titolo nota 25/01/2008

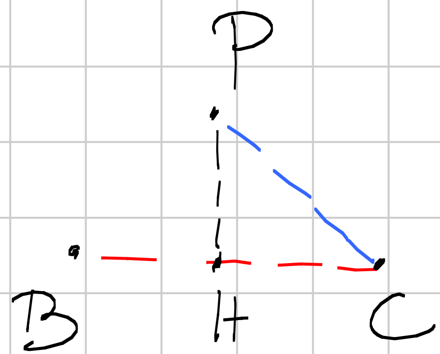
7) G bar. S = coniug isog di G

$\Rightarrow$ S è bari del proprio Tri pedale.

I) $[x : y : z] \longrightarrow \left[\frac{1}{x} : \frac{1}{y} : \frac{1}{z}\right]$

Coniug. isog.

II)

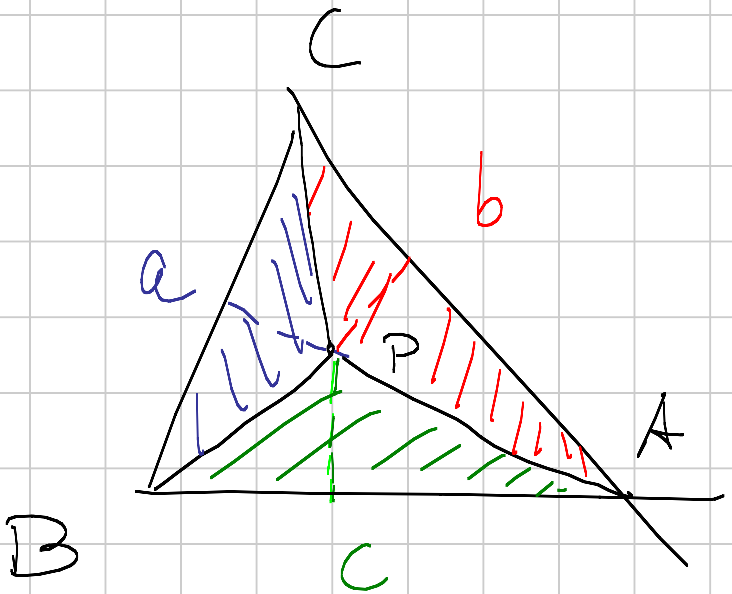


$$\vec{H} = \frac{(\vec{P}-\vec{C}, \vec{B}-\vec{C})}{\|\vec{B}-\vec{C}\|} \cdot \frac{\vec{B}-\vec{C}}{\|\vec{B}-\vec{C}\|} + \vec{C}$$

Proiez su un lato.

III) $\text{Bari}(\vec{X}, \vec{Y}, \vec{Z}) = \dfrac{\vec{X}+\vec{Y}+\vec{Z}}{3}$

IV)



$h_a \quad h_b \quad h_c$

$\lambda \vec{A} + (1-\lambda)\vec{B}$

$\lambda \vec{A} + \mu \vec{B} + (1-\lambda-\mu)\vec{C}$

$$\vec{P} = \frac{S \cdot \vec{A} + S \cdot \vec{C} + S\,\vec{B}}{S}$$

$S = \dfrac{a \cdot h_a}{2}$

$S = \dfrac{b \cdot h_b}{2}$

$S = \dfrac{c \cdot h_c}{2}$

se $P = [x : y : z]$ $\quad \vec{P} = \dfrac{ax\vec{A} + by\vec{B} + cz\vec{C}}{ax + by + cz}$

<u>Sol</u>: $G = \left[\frac{1}{a} : \frac{1}{b} : \frac{1}{c}\right] \longrightarrow S = [a : b : c]$

$$\longrightarrow \vec{S} = \frac{a^2\vec{A} + b^2\vec{B} + c^2\vec{C}}{a^2 + b^2 + c^2}$$

$$(\vec{S} - \vec{C}, \vec{B} - \vec{C}) \cdot \frac{\vec{B} - \vec{C}}{a^2} + \vec{C}$$

$$(\vec{S} - \vec{B}, \vec{A} - \vec{B}) \cdot \frac{\vec{A} - \vec{B}}{c^2} + \vec{B}$$

$$(\vec{S} - \vec{A}, \vec{C} - \vec{A}) \cdot \frac{\vec{C} - \vec{A}}{b^2} + \vec{A}$$

mettendo l'origine in $\vec{O}$ $\qquad (\vec{A}, \vec{B}) = R^2 - \frac{c^2}{2}$

$$\frac{(\vec{S} - \vec{C}, \vec{B} - \vec{C})}{a^2} = \frac{3b^2 + a^2 - c^2}{2(a^2 + b^2 + c^2)}$$

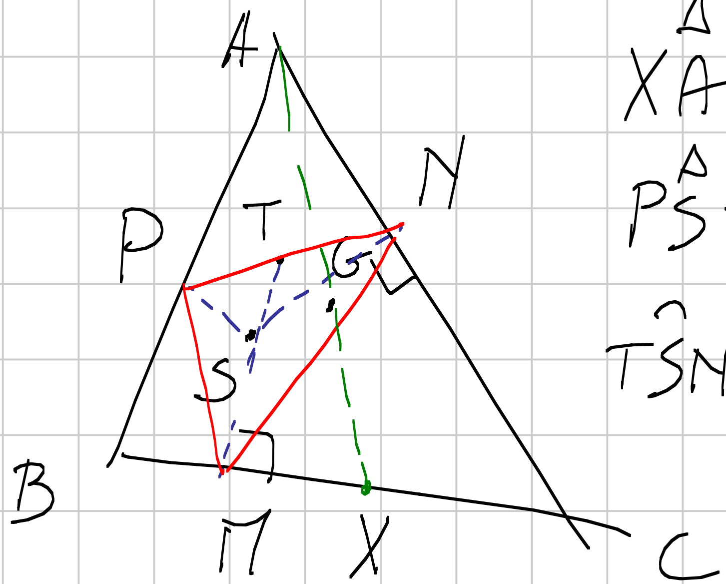
$$\vec{S} - \vec{C} = \frac{a^2 \vec{A} + b^2 \vec{B} - a^2 \vec{C} - b^2 \vec{C}}{a^2 + b^2 + c^2}$$

$$\frac{3b^2 + a^2 - c^2}{2(a^2 + b^2 + c^2)} (\vec{B} - \vec{C}) + \vec{C} + \frac{3c^2 + b^2 - a^2}{2(a^2 + b^2 + c^2)} (\vec{C} - \vec{A}) + \vec{A} +$$

$$+ \frac{3a^2 + c^2 - b^2}{2(a^2 + b^2 + c^2)} (\vec{A} - \vec{B}) + \vec{B}$$

$$\frac{\vec{A}\,(4a^2-2b^2-2c^2)}{2(\qquad)} + \ldots \qquad + \vec{A} + \vec{B} + \vec{C}$$

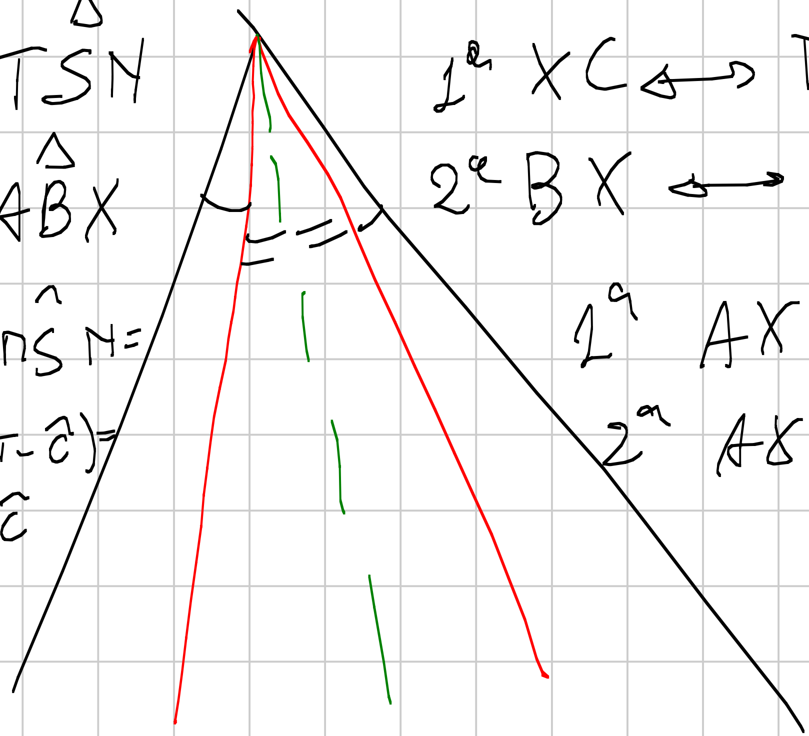
$$\frac{(6\vec{A}a^2 + 6\vec{B}b^2 + 6\vec{C}c^2)}{2\,(a^2+b^2+c^2)} = 3\vec{S}$$



$\overset{\triangle}{XAC} \simeq \overset{\triangle}{TSN}$

$\overset{\triangle}{PST} \simeq \overset{\triangle}{ABX}$

$T\hat{S}N = \pi - M\hat{S}N = \pi - (\pi - \hat{C}) = \hat{C}$

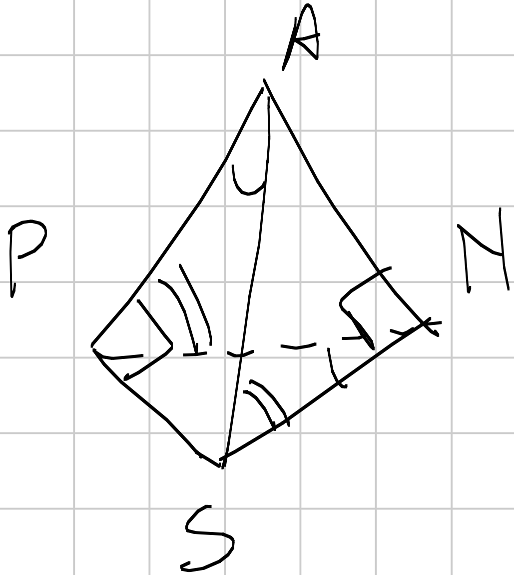


1^a $XC \longleftrightarrow TS$

2^a $BX \longleftrightarrow TS$

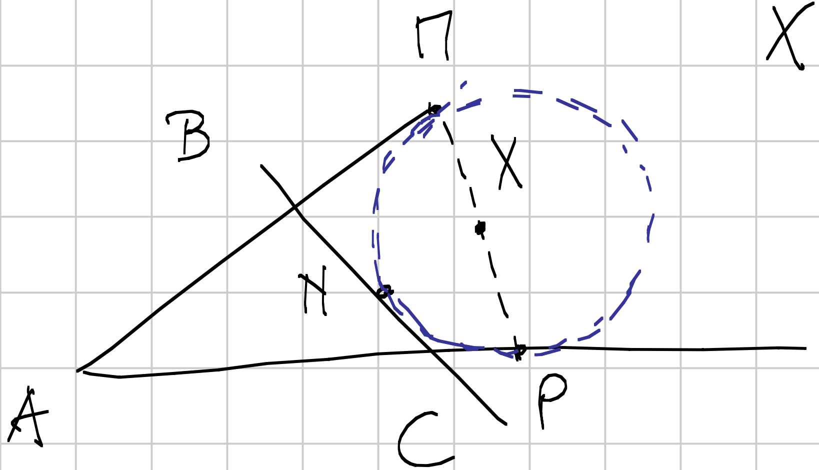
1^a $AX \to MT$

2^a $AX \to PT$



$P\hat{N}S = P\hat{A}S = X\hat{A}C$

8) <u>Trilineari</u>



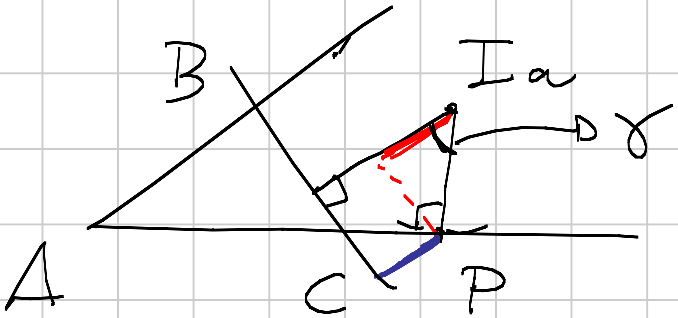
$X \in \Gamma_{ABC} \Rightarrow I, O, H$ allineati

I) $\{ [x:y:z] \mid ayz + bxz + cxy = 0 \} = \Gamma_{ABC}$

$I = [1:1:1]$

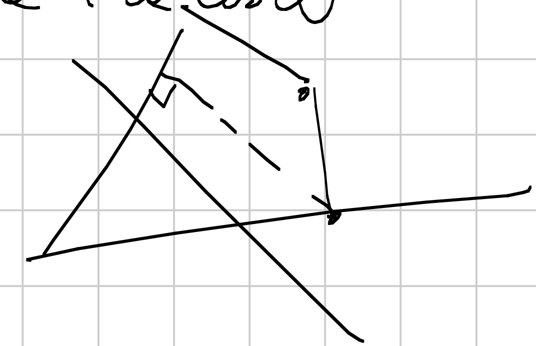
$O = [\cos\alpha : \cos\beta : \cos\gamma]$

II)



$r_a - r_a \cos\gamma$

$r_a + r_a \cos\alpha$



$$P = \left[-\cancel{r_a}(1-\cos\gamma) : 0 : \cancel{r_a}(1+\cos\alpha)\right]$$

$$M = \left[-\cancel{r_a}(1-\cos\beta) : \cancel{r_a}(1+\cos\alpha) : 0\right]$$

$$N = \left[\ 0 : 1-\cos\gamma : 1-\cos\beta\right]$$

$$X = \left[-(2-\cos\gamma-\cos\beta) : 1+\cos\alpha : 1+\cos\alpha\right]$$

$X \in \Pi_{ABC}$

$$a\cancel{(1+\cos\alpha)}(1+\cos\alpha) - b\cancel{(1+\cos\alpha)}(2-\cos\gamma-\cos\beta) -$$

$$- c\cancel{(1+\cos\alpha)}(2-\cos\gamma-\cos\beta) = 0$$

$$a(1+\cos\alpha) - (b+c)(2-\cos\gamma-\cos\beta) = 0$$

$$a\left(\frac{2bc+b^2+c^2-a^2}{2bc}\right) - (b+c)\left(\frac{2ab-a^2-b^2+c^2}{2ab} + \frac{2ac-a^2-c^2+b^2}{2ac}\right)$$

$$a^2\left(\underline{(b+c)^2-a^2}\right) - (b+c)\left(c\left(\underline{c^2-(a-b)^2}\right) + b\left(\underline{b^2-(a-c)^2}\right)\right)$$

Tesi I, O, N allineati

$$\det\begin{bmatrix} 1 & 1 & 1 \\ \cos\alpha & \cos\beta & \cos\gamma \\ 0 & 1-\cos\gamma & 1-\cos\beta \end{bmatrix} = 0$$

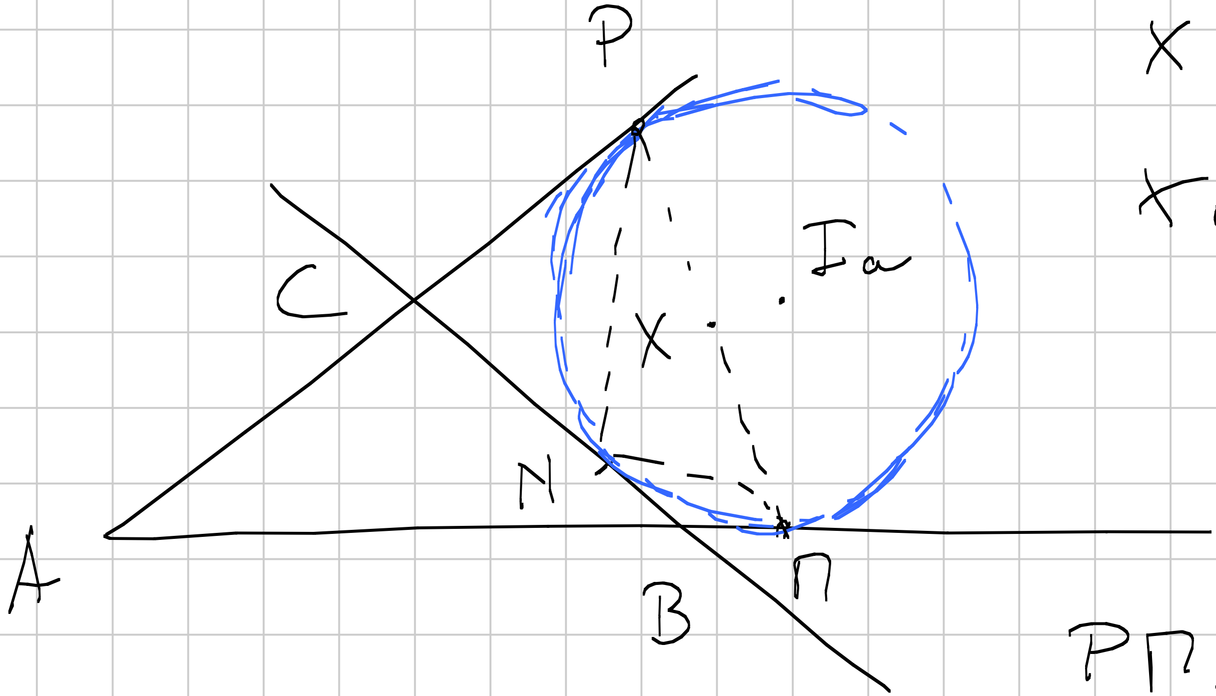
$$0 = \cos\beta - \cos^2\beta - \cos\gamma + \cos^2\gamma - \cancel{\cos\alpha} + \cos\alpha\cos\beta$$

$$+ \cancel{\cos\alpha} - \cos\alpha\cos\gamma = (\cos\beta - \cos\gamma)(1 - \cos\gamma - \cos\beta + \cos\alpha)$$

$$1 - \cos\gamma + 1 - \cos\beta - (1 - \cos\alpha) =$$

$$= \frac{2ab - a^2 - b^2 + c^2}{2abc} c + \frac{2ac - a^2 - c^2 + b^2}{2acb} b - \frac{2bc - b^2 - c^2 + a^2}{2bca} a$$

$$a^2 (b + c + a) - (b + c) \left(b (b + a - c) + c (c + a - b) \right)$$



X pt med di MP

$X \in \Gamma_{ABC}$

$PM \perp$ bisett da A

bisett da $A \perp I_b I_c$

$PM \parallel I_b I_c$

$I I_c \parallel PN$

$I I_b \parallel MN$

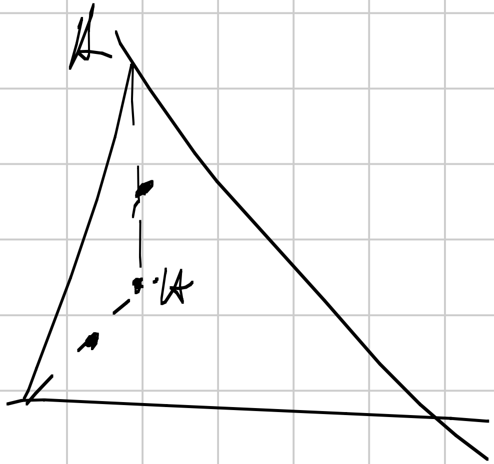
$\overset{\triangle}{MNP} \simeq I_b I I_c$

Γ_{ABC} è il Feuerbach di $I I_b I_c$

Γ_{ABC} è F. di $I_a I_b I_c$

I è ortocentro di $I_a I_b I_c$

$\Rightarrow \Gamma_{ABC}$ è F. di $I I_b I_c$



(Le bisett. di ABC sono altezze in $I_a I_b I_c$)

Invertendo nella circ. exinscritta opposta ad A

$ABC \longrightarrow$ Tri mediale di MNP.

$X \in \Gamma_{ABC}$

$A \in \Gamma_{ABC}$ $\Rightarrow \Gamma_{ABC}$ "sta ferma"

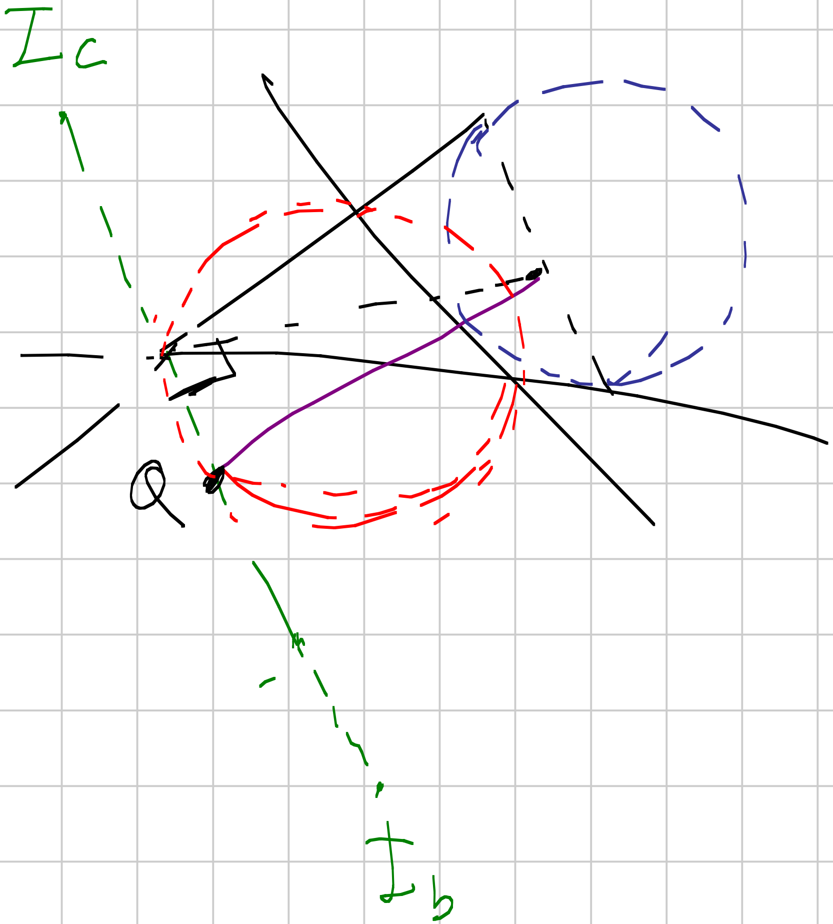
Γ_{ABC} = l'inverso di Γ_{ABC} = Feuerbach di MNP

$\shortparallel$

Feuer di II_bI_c

$II_bI_c \simeq MNP$

$\Rightarrow II_bI_c \equiv MNP$ $(2R = r_a)$



$X \in I I_a$

X pt medio $\widehat{BC}$

$Q =$ pt medio di $I_b I_c$

O

$X =$ pt medio di MP

sono allineati

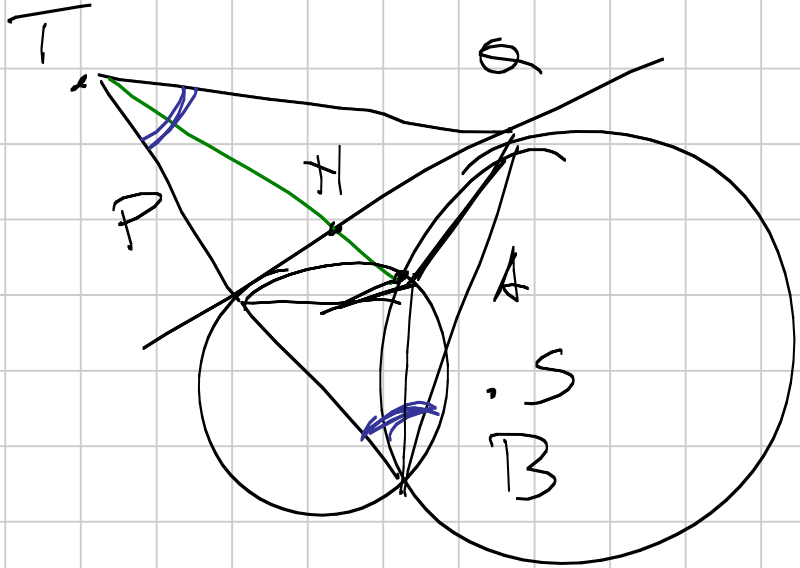
e $OQ = OX$.

O, I, M allineati

$OI = OM$.

$\Gamma = \quad I I_b I_c \longrightarrow MNP$

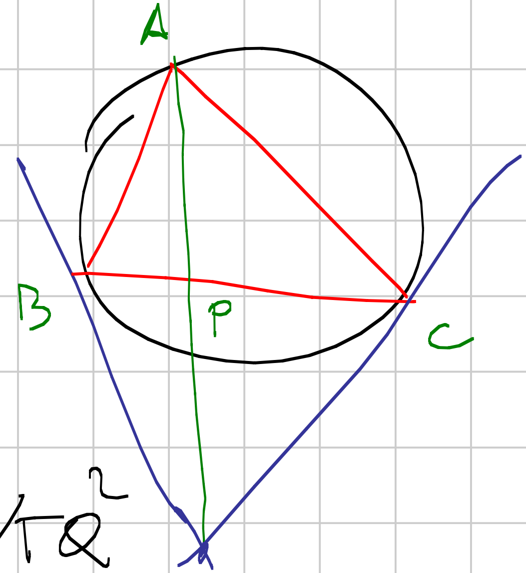
g)



S = int delle tg a Γ_{PQA} in P, Q

T = simm di B risp a PQ

<u>TS</u>: T, S, A allineati



$$\frac{BP}{PC} = \frac{AB^2}{AC^2}$$

$\boxed{\text{TPAQ ciclico}}$

$\Downarrow$

TS è simmed di TPQ

Voglio dim che $PH/HQ = TP^2/TQ^2$

$$\frac{PN}{NQ} = \frac{[PAT]}{[QAT]} = \frac{TP \cdot PA}{TQ \cdot QA}$$

Voglio
$$\frac{PA}{QA} \overset{?}{=} \frac{TP}{TQ} \overset{!}{=} \frac{BP}{BQ}$$

$$\frac{PA}{BP} \overset{?}{=} \frac{QA}{BQ}$$

$$\frac{PA}{BP} = \frac{KP}{\cancel{KB}} \qquad \frac{QA}{QB} = \frac{KQ}{\cancel{KB}}$$

BA mediane in APQ BPQ