RELAZIONE A2

Titolo nota 29/01/2009

$$\frac{a_1}{1-a_1}\cdot \ldots \cdot \frac{a_n}{1-a_n} \le \frac{1}{n^{n+1}} \cdot \frac{1-a_1-\ldots-a_n}{a_1+a_2+\ldots+a_n}$$

$$\sum_{i=1}^{n} a_i < 1$$

$$a_{n+1} = 1-\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n+1} = 1$$

$$\prod \frac{a_i}{1-a_i} \le \frac{1}{n^{n+1}}$$

AM-GM

$$\prod \frac{a_i}{\sum_{j\ne i} a_j} \le \prod_{i=1}^{n+1} \frac{a_i}{n \sqrt[n]{\prod_{j\ne i} a_i}} = \frac{1}{n^{n+1}}$$

$$a_i > \frac{1}{n+1} \qquad a_j < \frac{1}{n+1}$$

$$a_i \leftarrow \frac{1}{n+1} \qquad a_j \leftarrow a_i + a_j - \frac{1}{n+1}$$

$$\frac{a_i}{1-a_i}\cdot\frac{a_j}{1-a_j} < \frac{\frac{1}{n+1}}{\frac{n}{n+1}} \cdot \frac{a_i+a_j-\frac{1}{n+1}}{\frac{n+2}{n+1}-a_j-a_i}$$

$$\left(a_i+a_j-1\right)\left(a_i-\frac{1}{n+1}\right)\left(a_j-\frac{1}{n+1}\right) > 0$$

$$\sum_{i=1}^{n+1} \log \frac{a_i}{1-a_i} \le \log \frac{1}{n^{n+1}}$$

$$f(x) = \log \frac{x}{1-x} \qquad \text{concava in } \left[0; \frac{1}{2}\right]$$

$$a_i < \frac{1}{2}$$

$$\sum f(a_i) \le (n+1)\, f\left(\frac{\sum a_i}{n+1}\right) = \frac{1}{n^{n+1}}$$

wlog $a_1 > \frac{1}{2}$ $\qquad s = \sum_{i=2}^{n+1} a_i$

$$f(a_1) + \sum_{i=2}^{n+1} f(a_i) \le f(a_1) + n\, f\left(\frac{s}{n}\right) \le \log \frac{1}{n^{n+1}}$$

$$\frac{s^{n-1}(1-s)}{(n-s)^n} \le \frac{1}{n^{n+1}} \qquad s \le \frac{1}{2}$$

$$s(1-s) \le \frac{1}{4}$$

$$\text{LHS} \le \frac{1}{4} \frac{s^{n-2}}{(n-s)^n} \le \frac{1}{4} \frac{\frac{1}{2}^{n-2}}{\left(n-\frac{1}{2}\right)^n} \le \frac{1}{(2n-1)^n} \le \frac{1}{n^{n+1}}$$

$$(2n-1)^n \ge n^{n+1} \iff \left(\frac{2n-1}{n}\right)^n \ge n$$

$$\left(1 + \frac{n-1}{n}\right)^n \ge 1 + \frac{(n-1)}{n} n \qquad \text{BERNOULLI}$$