Relazione A3

Titolo nota 29/01/2009

$a_1, a_2, \dots, a_{2009} \in [0;1]$

Determinare $MAX\left(\sum_{i=1}^{2009} \sqrt{|a_i - a_{i-1}|}\right)$ $\quad i$ inteso mod 2009

$\exists\ a_{i-1}, a_i, a_{i+1}$ t.c. $\quad a_{i-1} \leq a_i \leq a_{i+1}$

$\qquad\qquad\qquad\qquad\qquad a_{i-1} \geq a_i \geq a_{i+1}$

$a_i = a_{i+1}$ PER ASSURDO

wlog $\quad a_1 > a_2 \implies a_2 < a_3 \quad \dots \quad a_{2009} > a_1$

$a_{2009} > a_1 > a_2$ ASSURDO

$|a_i - a_{i-1}| + |a_{i+1} - a_i| \leq 1$

$a_{i-1} \leq a_i \leq a_{i+1} \qquad \cancel{a_i} - a_{i-1} + a_{i+1} - \cancel{a_i} = a_{i+1} - a_{i-1} \leq 1$

$a_{i-1} \geq a_i \geq a_{i+1} \qquad -\cancel{a_i} + a_{i-1} - a_{i+1} + \cancel{a_i} = a_{i-1} - a_{i+1} \leq 1$

AM-QM $\longrightarrow \dfrac{\sqrt{|a_i - a_{i-1}|} + \sqrt{|a_{i+1} - a_i|}}{2} \leq \sqrt{\dfrac{|a_i - a_{i-1}| + |a_{i+1} - a_i|}{2}}$

$\sqrt{|a_i - a_{i-1}|} + \sqrt{|a_{i+1} - a_i|} \leq \sqrt{2} \cdot \sqrt{|a_i - a_{i-1}| + |a_{i+1} - a_i|}$

$\leq \sqrt{2}$

$\sqrt{|a_j - a_{j-1}|} \leq 1$

$\frac{\sqrt{2}}{2}$ $\quad 1 \quad 1 \quad \frac{\sqrt{2}}{2}$

$[0, 1, 0, 1, \dots, 1, \frac{1}{2}]$

$\sum \sqrt{|a_j - a_{j-1}|} \leq 2007 + \sqrt{2}$

D, S

$D = \{\forall i \text{ t.c. } a_i \geq a_{i-1}\}$

$S = \{\forall i \text{ t.c. } a_i < a_{i-1}\}$

$|D| + |S| = 2009$

$||D| - |S|| \geq 1$

Se $\exists i, i+1 \in D$ $\quad a_{i+1} \geq a_i \geq a_{i-1}$

$\not\exists$ $\quad \in S$ $\quad a_{i+1} < a_i < a_{i-1}$

$[0; \frac{1}{2}]$ $\quad [\frac{1}{2}; 1]$

Ci saranno $i, i+1$ t.c. $a_i, a_{i+1} \in [0; \frac{1}{2})$ o $(\frac{1}{2}; 1]$

wlog $a_i, a_{i+1} \in [0; \frac{1}{2}]$

1. a_{i-1}, a_{i+2} : se uno qualsiasi di questi cade in $[0; \frac{1}{2}]$

$|a_i - a_{i+1}| \leq \frac{1}{2}$

wlog $a_{i+2} \in [0; \frac{1}{2}]$: $|a_{i+2} - a_i| \leq \frac{1}{2}$

$|a_{i+2} - a_{i+1}| + |a_{i+1} - a_i| \leq 1$

② $a_{i-1}, a_{i+2} \in [\frac{1}{2}; 1]$

wlog $a_{i+1} \geq a_i$ nell'altro considereremo a_{i-1}

$a_{i+2} \geq a_{i+1}$ perché ricade in $[\frac{1}{2}; 1]$ a differenza di a_{i+1}

$a_{i+1} \geq a_i$

Abbiamo 3 consecutivi ordinati