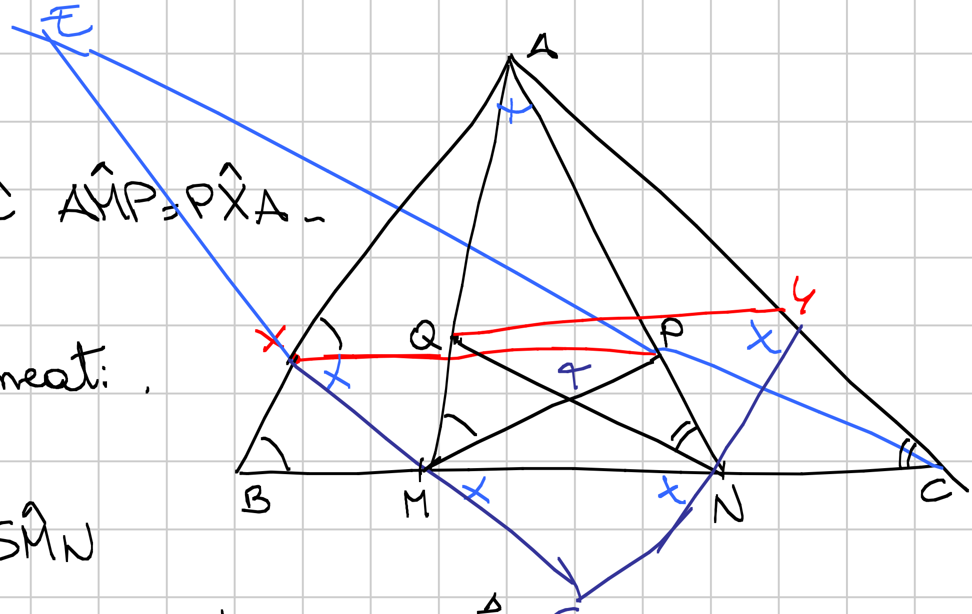
WC 2011 — GEOMETRIA 1

$X = \odot AMP \cap AB$

$PX \parallel BC$ perchè $A\hat{M}P = P\hat{X}A$.

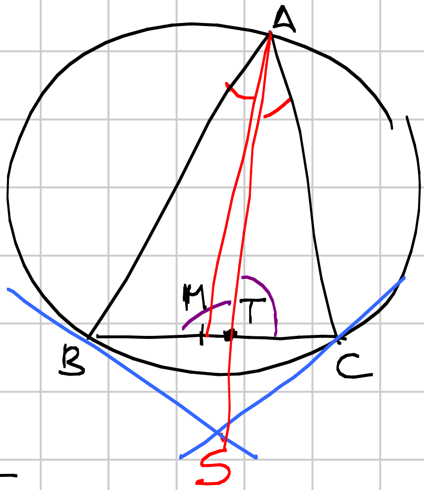


Tesi: A, S, T allineati.

$M\hat{N}S = M\hat{A}N = S\hat{M}N$

$\Rightarrow S$ sta sulla simmediana di $A\hat{M}N$

Lemma



$B\hat{A}M = S\hat{A}C$

$$\frac{BT}{TC} = \frac{AB^2}{AC^2}$$

$$\frac{BT}{TC} = \frac{\sin B\hat{A}T \cdot \frac{AB}{\cancel{\sin B\hat{T}A}}}{\sin T\hat{A}C \cdot \frac{AC}{\cancel{\sin A\hat{T}C}}} =$$

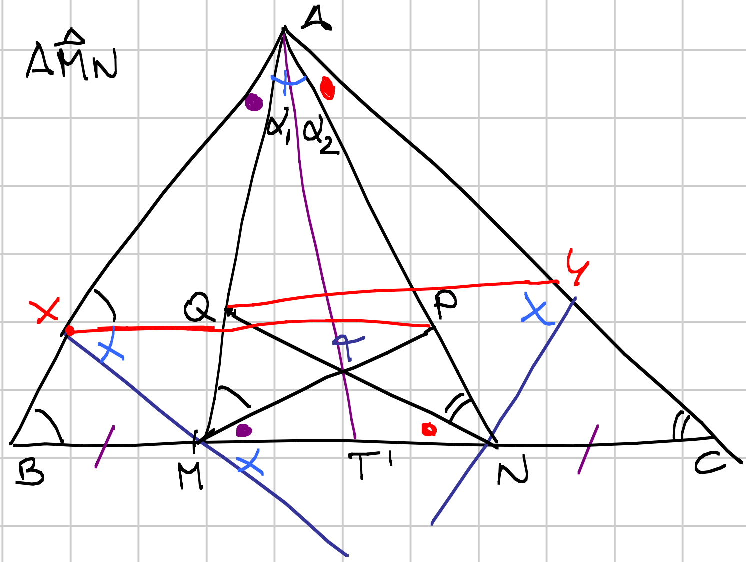
teo seni su ABT...

$$\frac{BT}{\sin B\hat{A}T} = \frac{AB}{\sin B\hat{T}A}$$

$$= \frac{\sin MAC}{\sin MAB} \cdot \frac{AB}{AC} = \frac{AB^2}{AC^2}$$

Tesi $\iff T \in$ simmediana di $A\hat{M}N$

$$\iff \frac{MT'}{T'N} = \frac{AM^2}{AN^2}$$



$$\frac{MT'}{T'N} = \frac{\sin\alpha_1 \cdot \dfrac{AM}{\cancel{\sin AT'M}}}{\sin\alpha_2 \cdot \dfrac{AN}{\cancel{\sin AT'N}}}$$

teo seni su $AT'M$ e ___

$$| = \frac{\sin\alpha_1}{\sin\alpha_2} \cdot \frac{\sin ANQ}{\sin QNM} \cdot \frac{\sin NMP}{\sin PMA}$$

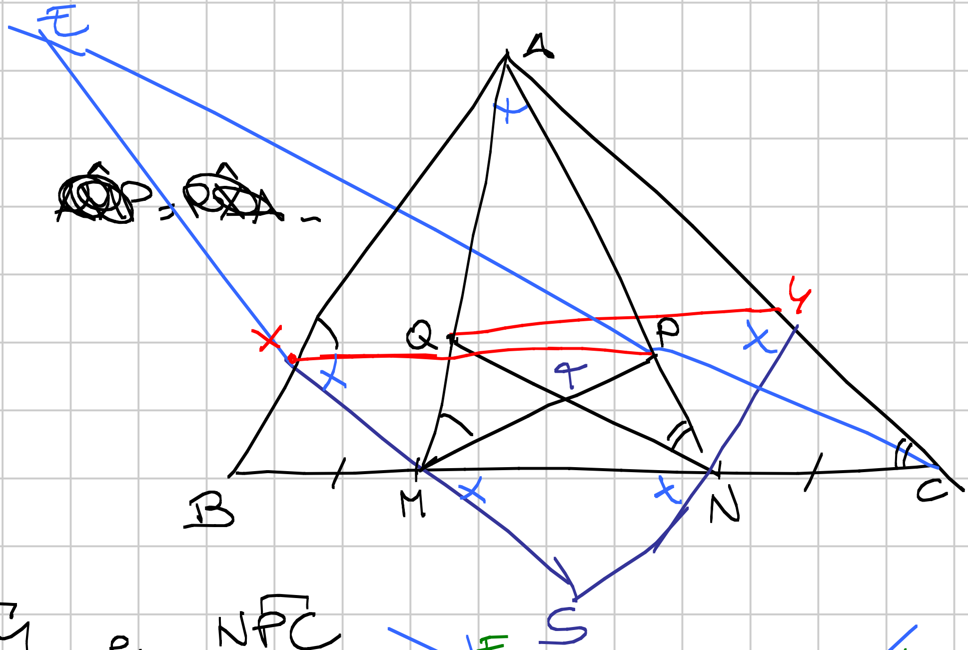
$$= \frac{\sin Q\hat{N}M}{\sin A\hat{N}Q} \cdot \frac{\sin P\hat{M}A}{\sin N\hat{M}P} \cdot \frac{AM}{AN} =$$

$$= \frac{\sin N\hat{A}C}{\sin A\hat{C}N} \cdot \frac{\sin M\hat{B}A}{\sin B\hat{A}M} \cdot \frac{AM}{AN}$$

$$= \frac{\cancel{NC}}{AN} \cdot \frac{AM}{\cancel{BM}} \cdot \frac{AM}{AN}$$

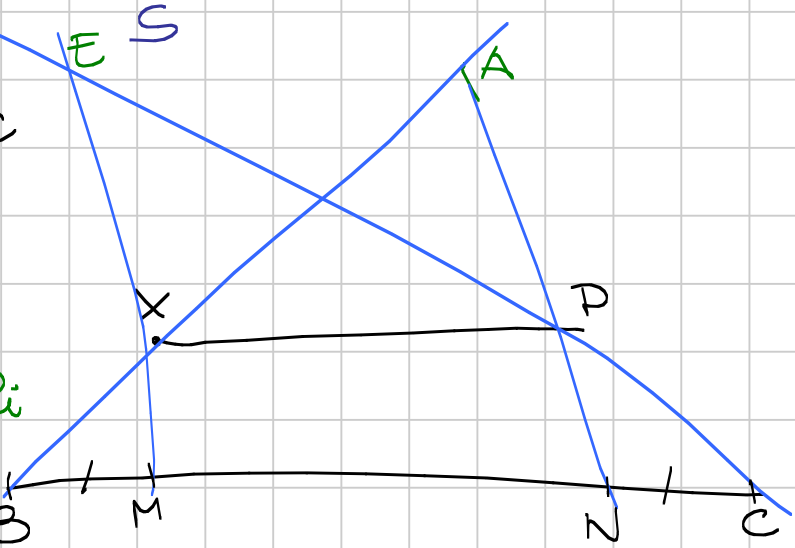
$$= \frac{AM^2}{AN^2}$$

Mostriamo $AE \parallel BC$.



BXM e NPC

$BX \cap PN = A \quad XM \cap PC$



$EA \parallel BC$

$\Longrightarrow E\hat{A}P$ e $C\hat{N}P$ simili

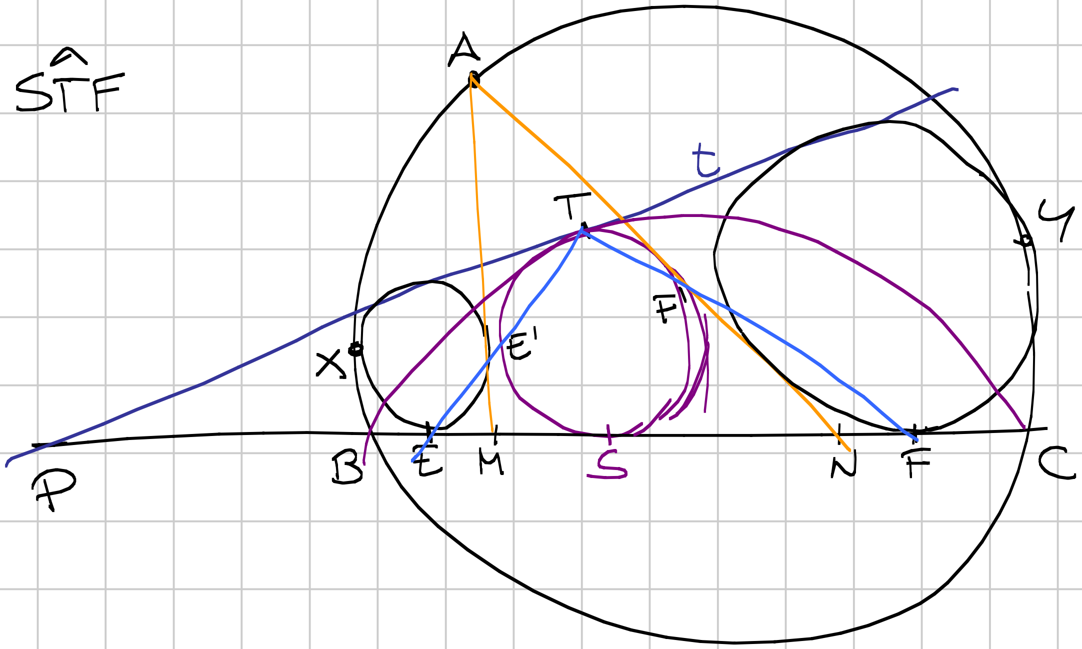
$\Longrightarrow \dfrac{PN}{PA} = \dfrac{PC}{PE}$

$AXP \sim ABN \Longrightarrow \boxed{\dfrac{AP}{AN}} = \dfrac{XP}{BN}$

$= \dfrac{XP}{MC} = \boxed{\dfrac{EP}{EC}}$

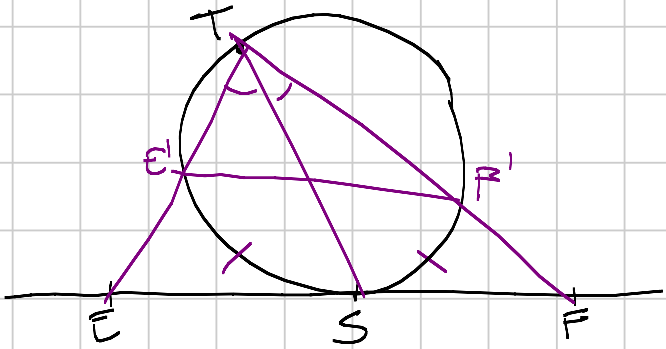
WC 2

Tesi: $E\hat{T}S = S\hat{T}F$



Oss 1: Tesi $\iff E'F' \parallel EF$.

$\iff \overset{\frown}{E'S} = \overset{\frown}{F'S}$



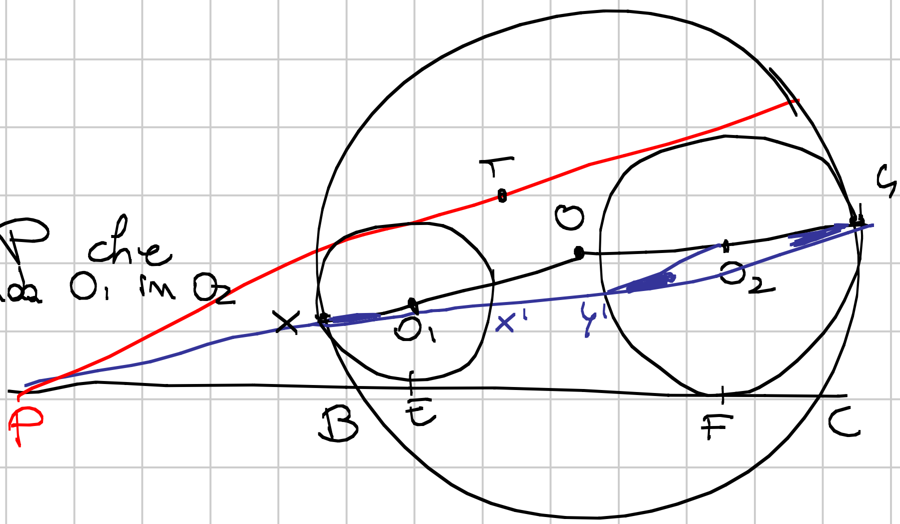
Oss 2: P, X, Y sono all.

1° modo:

$X'\hat{X}O_1 =$

Omotetia di centro P che manda O_1 in O_2

$X \longrightarrow Y'$



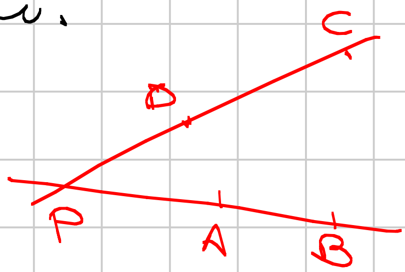
$XO_1 \parallel Y'O_2$

Oss 3: X, Y, F, E sono conciclici.

$\iff PX \cdot PY = PE \cdot PF$

(1) $PE^2 = PX \cdot PX'$

$PF^2 = PY \cdot PY'$



$PE^2 \cdot PF^2 = PX \cdot PX' \cdot PY \cdot PY' = PX^2 \cdot PY^2$

$$\frac{PX}{PY'} = \frac{PX'}{PY}$$

(2) $X\hat{E}P \overset{?}{=} X\hat{Y}F$

$X\hat{E}P = Y'\hat{F}P = Y'\hat{Y}F$

(3) Inversione di centro P, rapporto $PT^2 = PB \cdot PC$

$\Gamma \longrightarrow \Gamma$

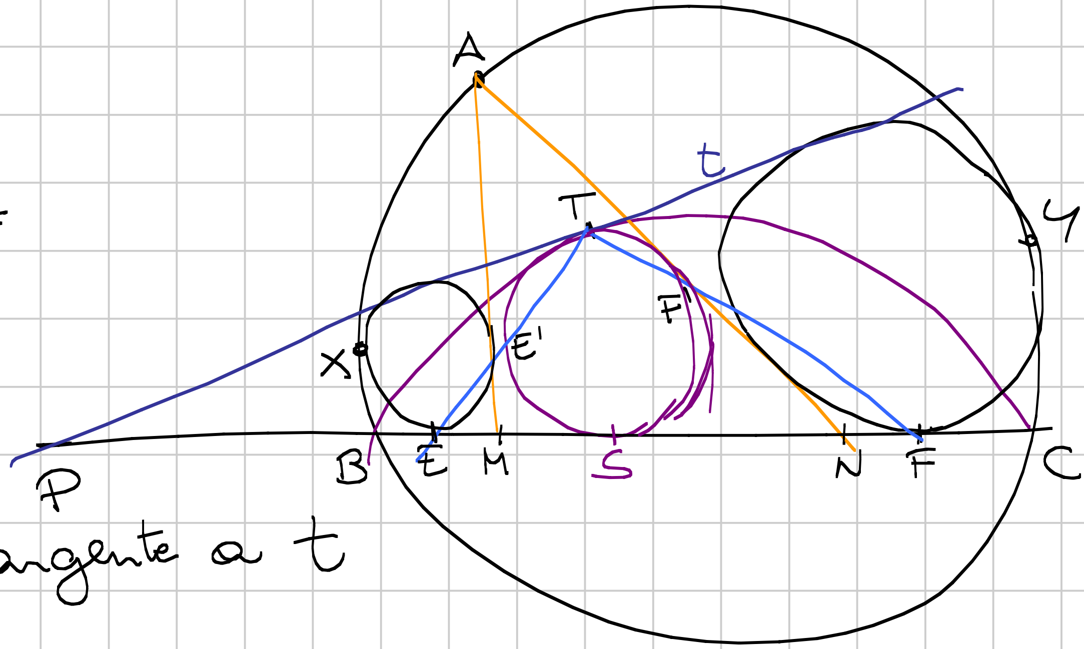
$PE \cdot PF = PB \cdot PC = PX \cdot PY$.

<u>Sol:</u>

$PX \cdot PY = PE \cdot PF$

$ \shortparallel$

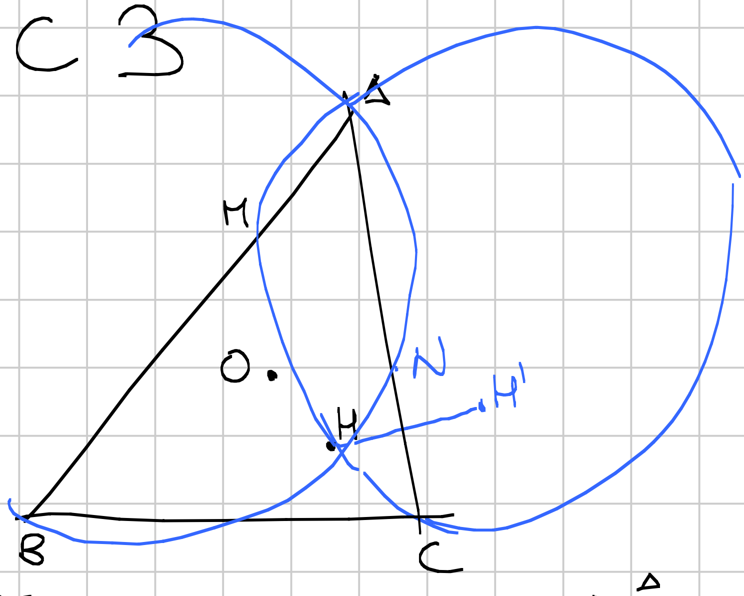
$PB \cdot PC = PT^2$



$\odot EFT$ è tangente a t in T.

Omotetia di centro T e rapporto giusto (che manda $\odot EFT \rightsquigarrow$ circo per T e S)

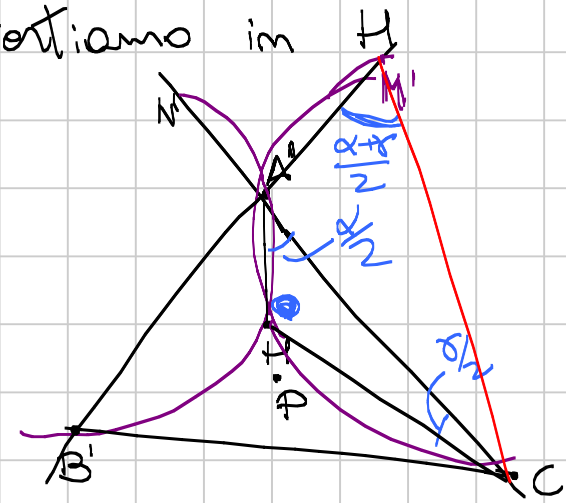
WC 3



I cerchi blu hanno lo stesso raggio (sono $= \Gamma_{ABC}$, segue da $H' \in \Gamma_{ABC}$)

Tesi: circocentro di $M\hat{N}H$ sta sulla retta OH.

Invertiamo in H



Oss: H è incentro di A'B'C'

$B\hat{M}'C' = 180 - A'\hat{H}C'$

$= \frac{\alpha+\gamma}{2}$

$180 - B - \frac{\alpha+\gamma}{2} = \frac{\alpha+\gamma}{2}$

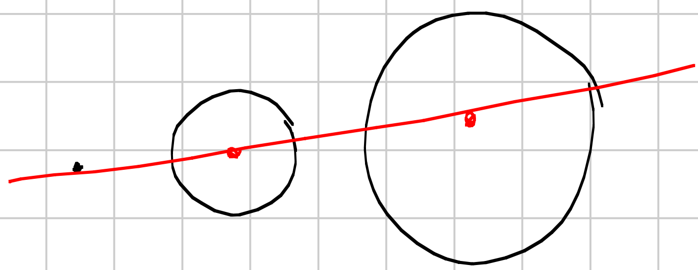
Tesi $\iff$ OH è asse di simm per Γ_{MNH}.

$OH \perp \Gamma_{MNH}$

$\iff O'H \perp M'N'$

O' NON è circocentro di A'B'C'

O', P, H sono allineati.

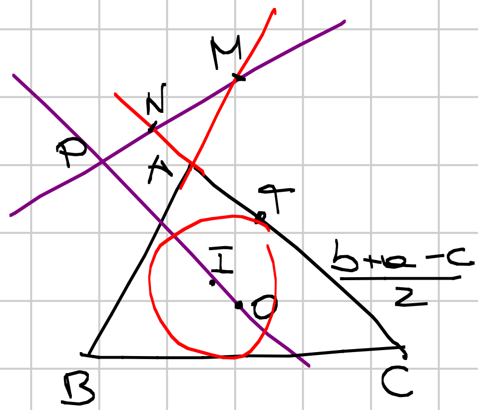


Tesi $\iff HP \perp M'N'$.

Vale $M'B' = B'C'$ (angoli)

$BM = CN = BC$

Tesi $OI \perp MN$



1° modo: complessi

$$M = B + (A-B)\frac{a}{c} \qquad N = C + (A-C)\cdot\frac{a}{b}$$

$$I = \frac{aA + bB + cC}{a+b+c}$$

$$(M-N)\cdot I = 0$$

2° modo $MN \perp OI \iff$

$$ON^2 + IM^2 = OM^2 + IN^2$$

$\iff$ sottraggo $r^2 + R^2$

$$\mathrm{pow}_{\Gamma_{ABC}} N + \mathrm{pow}_{\gamma_{ABC}} M = \mathrm{pow}_{\Gamma_{ABC}} M + \mathrm{pow}_{\gamma_{ABC}} N$$

circonf iscritta

$\mathrm{pow}_{\Gamma_{ABC}} N - \mathrm{pow}_{\gamma_{ABC}} N \overset{?}{=}$ espressione simmetrica in b e c

$$= NA \cdot NC - NT^2$$

$$= a(a-b) - \left(a - \frac{b+a-c}{2}\right)^2$$

$$= a(a-b) - \left(\frac{a+c-b}{2}\right)^2$$

$$= \frac{1}{4}\left(-3a^2 + b^2 + c^2 + 2ab + 2ac - 2bc\right)$$