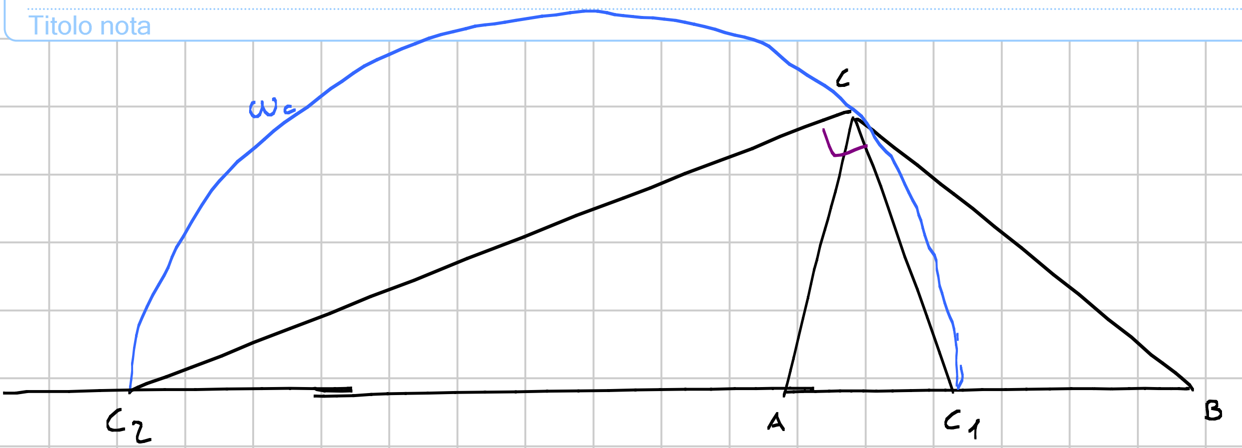
WC 2012 – GEOMETRIA SINTETICA

Titolo nota 25/01/2012



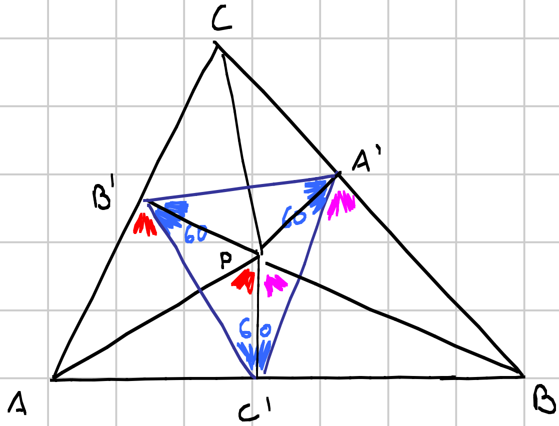
ω_c = circ con diam. $C_1 C_2$

$C \in \omega_c$ $\qquad \dfrac{AC_1}{BC_1} = \dfrac{AC_2}{BC_2} = \dfrac{b}{a}$

ω_c = luogo punti X t.c. $\dfrac{AX}{BX} = \dfrac{b}{a} \rightarrow \boxed{AX \cdot a = BX \cdot b}$

$P, Q = \omega_B \cap \omega_c$ $\qquad AP \cdot a = BP \cdot b = CP \cdot c$

[ω_A passa per P e Q]



idea da casa

A'B'C' triangolo pedale di P

$\dfrac{B'C'}{\sin\alpha} = AP$ (ciclicità AC'PB')

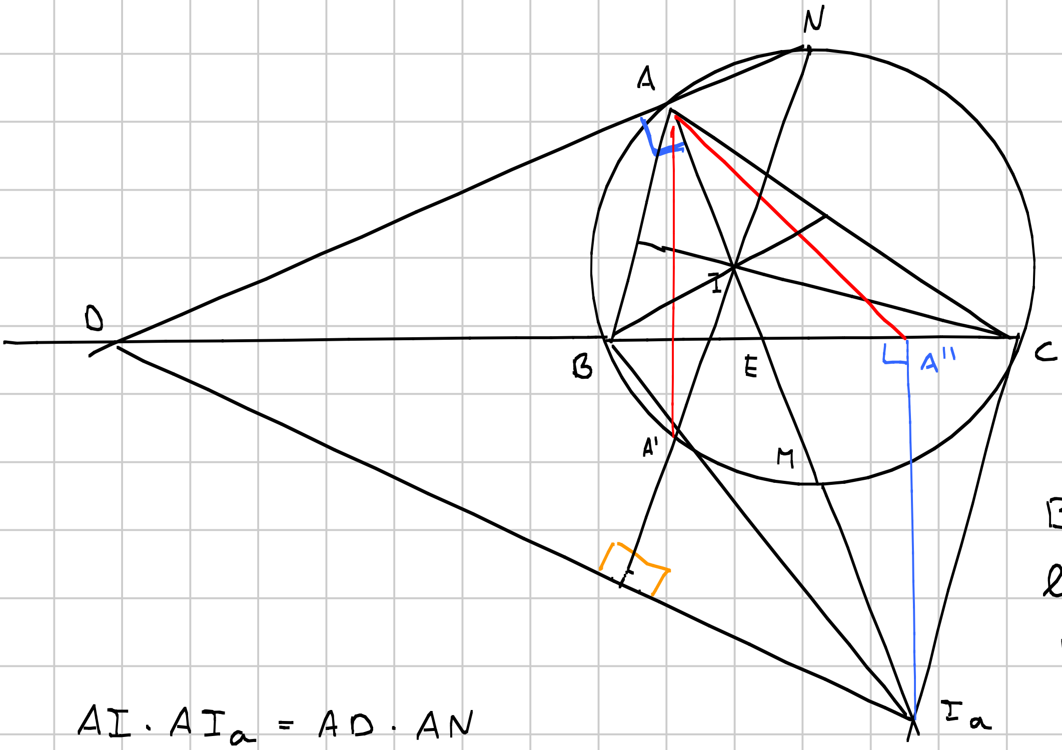
$B'C' = AP \cdot \sin\alpha = AP \cdot \dfrac{a}{2R}$

$A'B' = B'C' = C'A'$

$A\hat{P}B = A\hat{P}C' + C'\hat{P}B =$

$= A\hat{B'}C' + B\hat{A'}C' = (120 - CB'A') +$

$(120 - C\hat{A'}B') = 60 + \gamma$



inversione
centro A
raggio $\sqrt{bc}$
+
simmetria rispetto
a AI

$B \leftrightarrow C$

$\ell(BC) \leftrightarrow \Gamma(ABC)$

$E \leftrightarrow M$
$D \leftrightarrow N$ [pt medi $\overset{\frown}{BC}$]

$I \leftrightarrow I_a$
(similitudine $\overset{\Delta}{AIB} \sim \overset{\Delta}{ACI_a}$)

$AI \cdot AI_a = AD \cdot AN$

$$\frac{AI}{AN} = \frac{AD}{AI_a}$$

$\overset{\Delta}{AI_aD} \sim \overset{\Delta}{ANI}$

rotomotetia $I_a \to N$ e $D \to I$

$\Downarrow$

$NI \perp DI_a$

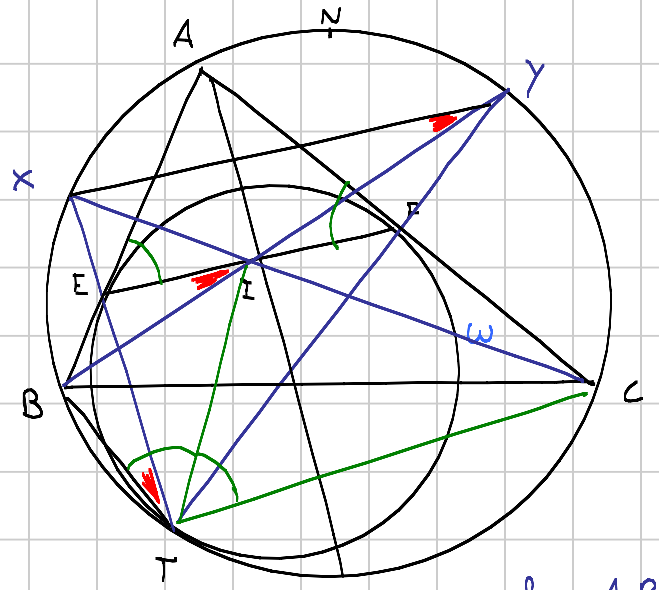
$\ell(NI) \leftrightarrow \Gamma(ADI_a)$

$A' = NI \cap \Gamma(ABC)$

$\updownarrow$

$A'' = \Gamma(ADI_a) \cap BC$

AA'' BB'' CC'' → concorrono in Nagel



ω = circonf. mistilinea

$DI_a \perp NI$

$A' = NI \cap \Gamma(ABC)$

idea da casa → N, I, T allineati

pascal su ABYTXC → $AB \cap TX = E$
$BY \cap CX = I$
$YT \cap AC = F$

BTIE ciclico   CFIT ciclico

BTI = AEZ = AFZ = CTZ

A' ≡ T (pt. tangenza)

teorema di Monge

γ = circonf. inscritta $\gamma \to \omega$ (omotetia di centro A)

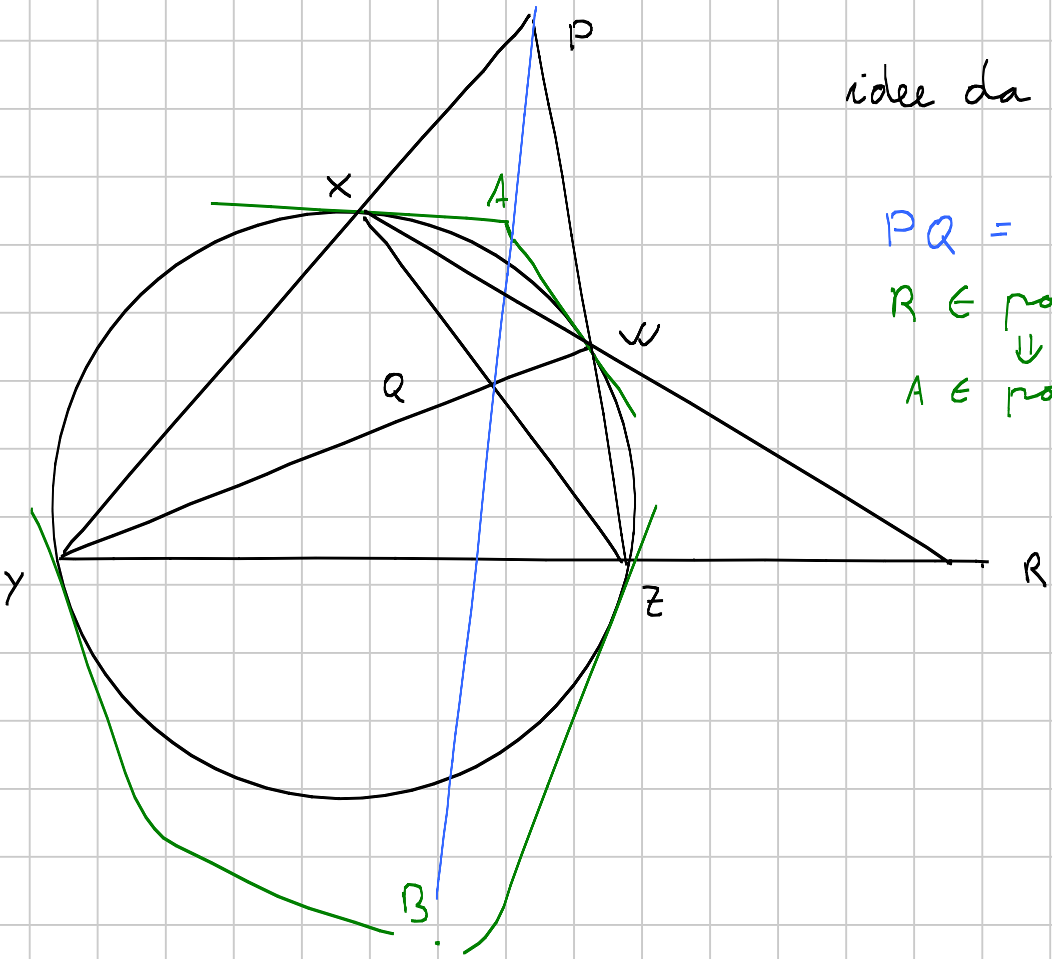
Γ = circ. circoscritta $\omega \to \Gamma$ (" " " A')

A = centro est. di sim. tra γ e ω

A' = " " " " " ω e Γ

X = " " " " " γ e Γ

allineati!



idee da casa

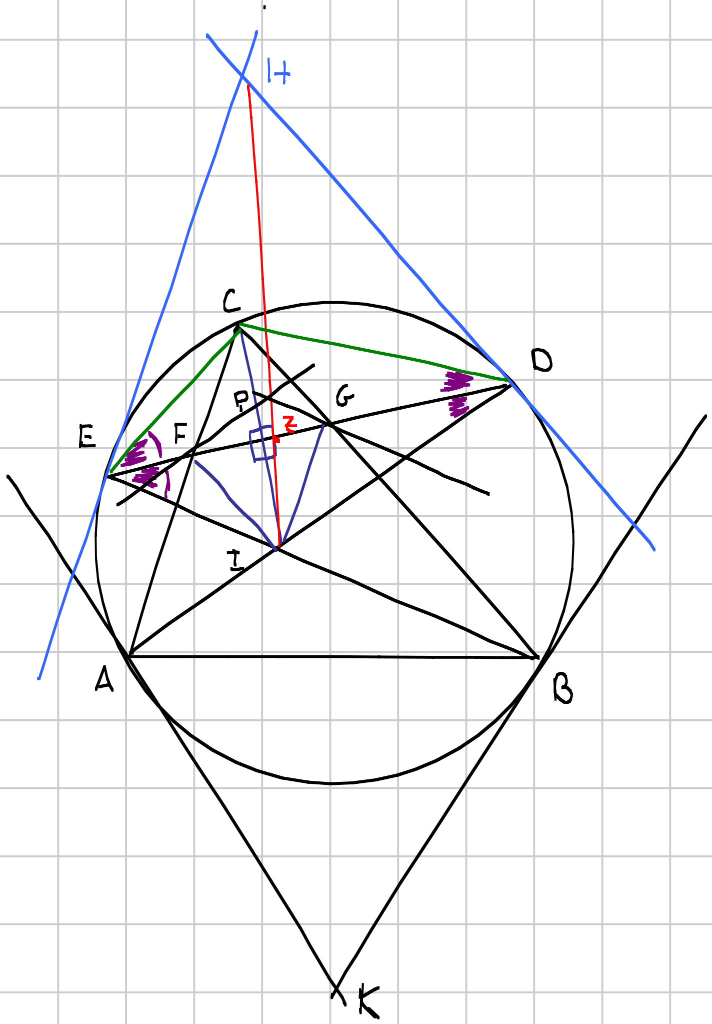
PQ = pol (R)

R ∈ pol (A)

⇓

A ∈ pol (R)

PQAB allineati



$X = AE \cap BD$

Th $\leftrightarrow$ K P X allineati

H = intersez. tangenti in D e E

K I H X allineati

Th $\leftrightarrow$ H I P allineati

$DH \parallel BC$

$EH \parallel AC$

$I\hat{D}E = E\hat{D}C$

$I\hat{E}D = D\hat{E}C$

$\Downarrow$

I = simm (C) rispetto a DE

$\Downarrow$

CFIG rombo

$FI \parallel BC \parallel DH$

$GI \parallel AC \parallel EH$

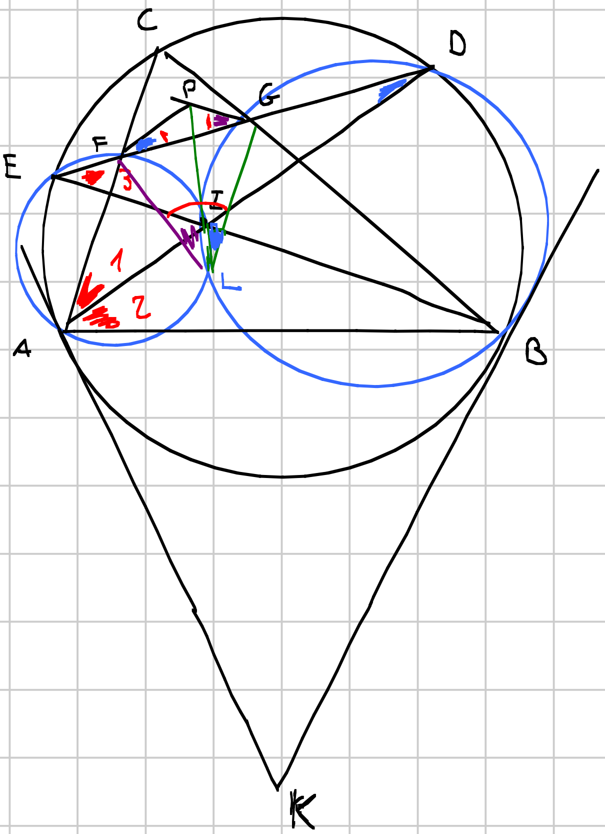
$Z = ED \cap HI$

omotetia centro Z che manda H in I

D → F ⇒ retta DI → retta FP

E → G ⇒ retta EI → retta GP

QUINDI → I va in P! → FINE!



$X = AE \cap BD$

$Th \Leftrightarrow P \in IX$

$AIFE$ ciclico (ω_1)

$I\hat{A}F = D\hat{A}C = B\hat{A}D =$

$= B\hat{E}D = I\hat{E}F$

analog. $\rightarrow$ $BDGI$ ciclico (ω_2)

$\omega_1, \omega_2, \Gamma(ABC)$

AE = asse radicale di Γ e ω_1

BD = " " " Γ e ω_2

$X \rightarrow$ centro radicale

$I = \omega_1 \cap \omega_2 \Rightarrow IX$ = asse radicale

$L = \omega_1 \cap \omega_2 \quad (L \neq I)$

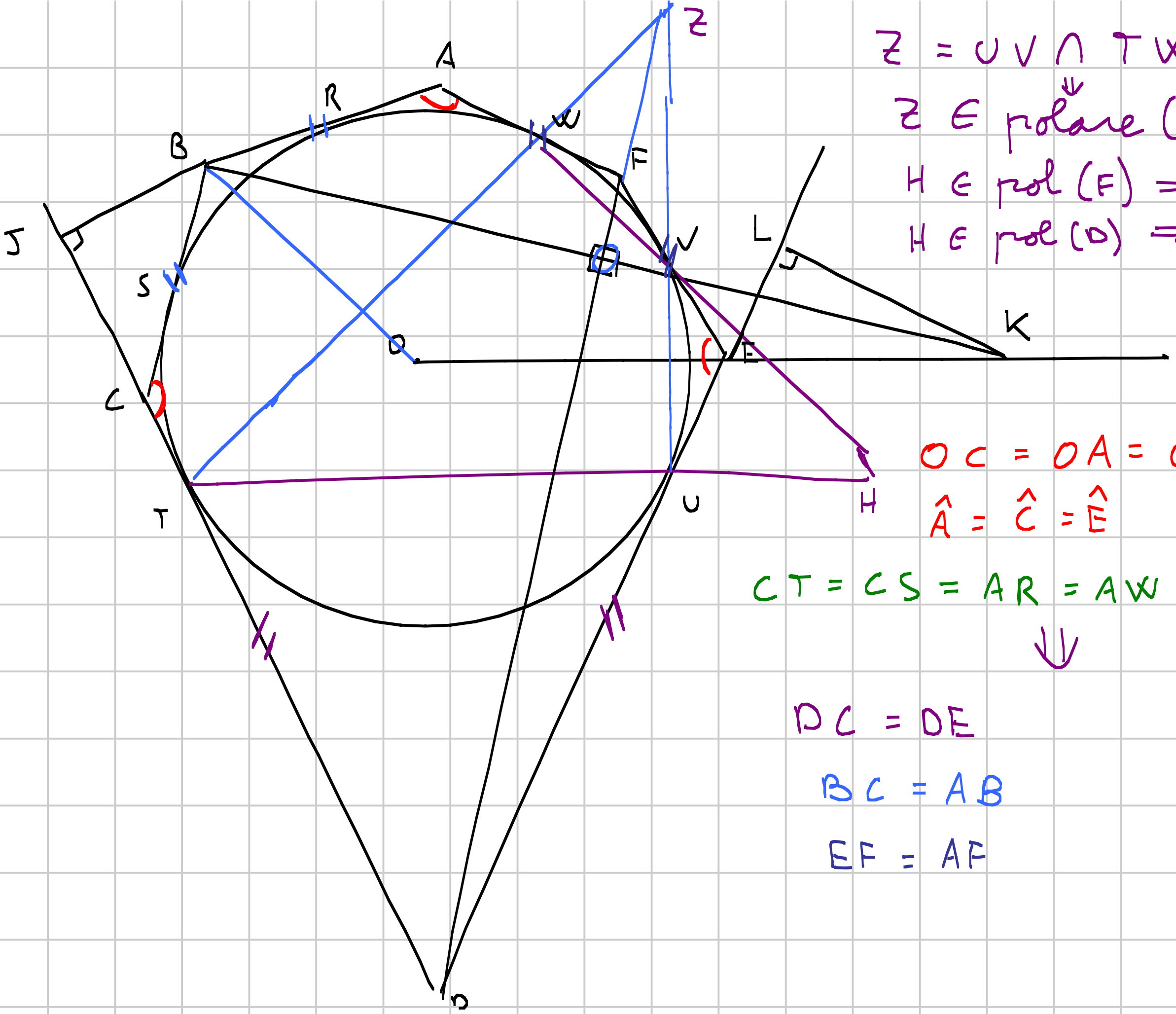
$Th \rightarrow PIL$ allineati $\rightsquigarrow G\hat{L}I = G\hat{L}P$

$I\hat{L}G = I\hat{D}G = A\hat{D}E = P\hat{F}G$

$Th \Leftrightarrow P\hat{F}G = P\hat{L}G \Leftrightarrow FLGP$ ciclico

dall'altra parte $\rightarrow F\hat{G}P = F\hat{L}I$

QUINDI $F\hat{L}G = F\hat{L}I + I\hat{L}G = F\hat{G}P + P\hat{F}G = 180 - F\hat{P}G$



$Z = UV \cap TW$
$\Downarrow$
$Z \in$ polare (H)
$H \in$ pol $(F) \Rightarrow F \in$ pol(H)
$H \in$ pol $(D) \Rightarrow D \in$ pol(H)

$OC = OA = OE$
$\hat{A} = \hat{C} = \hat{E}$

$CT = CS = AR = AW = EV = EU$
$\Downarrow$

$DC = DE$
$BC = AB$
$EF = AF$

$DL = DJ$ $\qquad DL \cdot DU = DJ \cdot DT$ $\qquad \vec{DK} \cdot \vec{DU} = \vec{DB} \cdot \vec{DJ}$

origine in O $\qquad (K-D)(U-D) = (B-D)(T-D)$

so che $D(U-D) = D(T-D)$
mi serve $K(U-D) = B(T-D)$
X pt qualsiasi del piano
$K(U-X) + K(X-D) = B(T-X) + B(X-D)$

$K(U-X) = B(T-X) + (B-K)(X-D)$
① ② ③

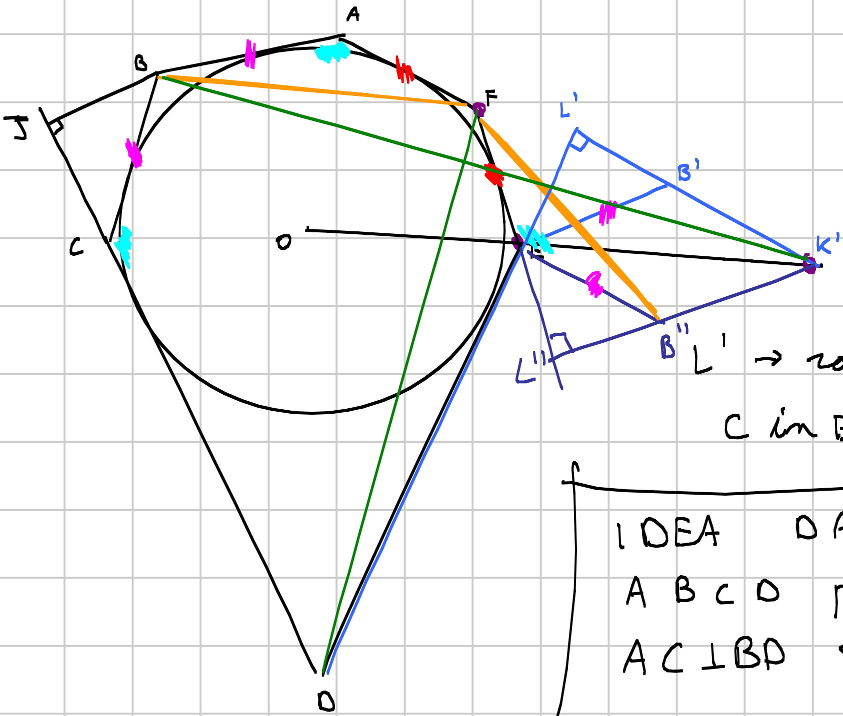
HOPE → posso scegliere X in modo che ①, ②, ③ si annullino?

① $= 0 \iff UX \perp OK \iff X \in UV$
② $= 0 \iff TX \perp OB \iff X \in TW$
③ $= 0 \iff BK \perp OX \iff X \in DF$

se UV, TW e DF concorrono, ho finito!

$F\hat{A}B = B\hat{C}D = D\hat{E}B' = F\hat{E}B''$

(rotazione) (simmetria)



L' t.c. $L' \in DE$ e $DL' = DJ$

K' t.c. $K' \in OE$ e $K'L' \perp DE$

Th $\Leftrightarrow$ $K'B \perp DF$

$L' \to$ rotazione centro D che manda C in E, J in L' e B in B'

IDEA DA CASA

$A\ B\ C\ D$ punti del piano

$AC \perp BD \iff AB^2 - CB^2 = AD^2 - CD^2$

$BK' \perp DF \iff BD^2 - K'D^2 = BF^2 - K'F^2$

so che $B'K' \perp DE \Rightarrow B'D^2 - K'D^2 = B'E^2 - K'E^2$

$B'D^2 = BD^2$

Th $\iff BF^2 - K'F^2 = B'E^2 - K'E^2 \iff K'E^2 - K'F^2 = B'E^2 - BF^2$

LHS

$K'E^2 - K'F^2 = XE^2 - XF^2 \quad \forall X \in$ perpendicolare da K' a EF

$\parallel$

simmetrica di $K'L'$

$X \to B''$

Th $\leftrightarrow B''E^2 - B''F^2 = B'E^2 - BF^2$

$B'E = B''E$ per simmetria

Th $\leftrightarrow BF = B''F$ (HOPE)

Th $\Leftarrow F\hat{E}B'' = F\hat{A}B \to$ vedi sopra!