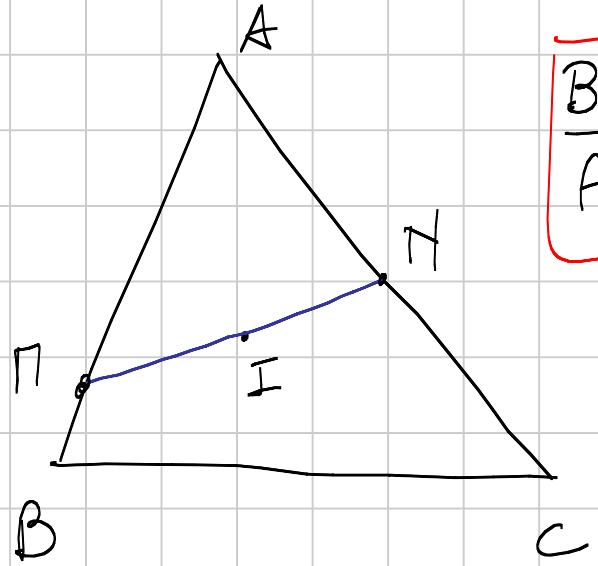
G3)

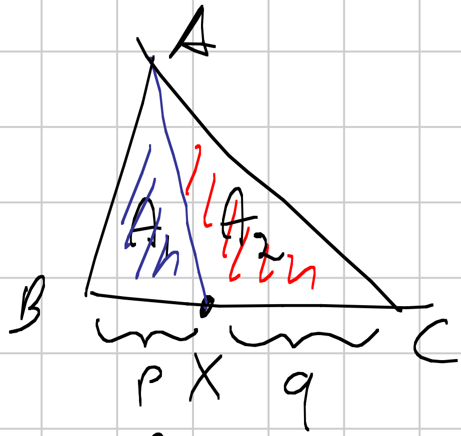


$$\boxed{\frac{B\Pi}{A\Pi}} \cdot \boxed{\frac{CN}{AN}} \le \frac{a^2}{4cb}$$

Coord. baricentriche di $P = [x : y : z]$

$$x : y : z = S_{PBC} : S_{PCA} : S_{PAB}$$

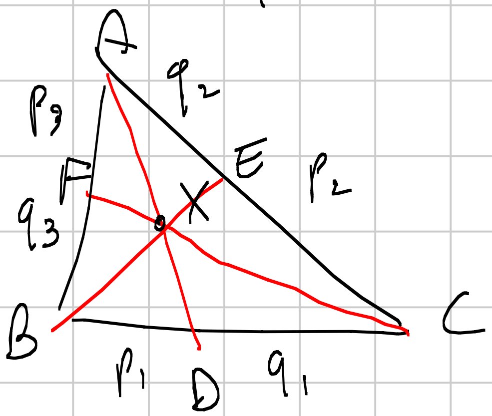
•)



$$\frac{A_1}{A_2} = \frac{p}{q}$$

$$X = [0 : q : p]$$

•)



$$X = [x : y : z]$$

$$S_{ABD} : S_{ADC} = p_1 : q_1$$

$$S_{XBD} : S_{XDC} = p_1 : q_1$$

$$\underbrace{\underset{S_{AXB}}{\underset{\shortparallel}{S_{ABD} - S_{XBD}}} : \underset{S_{AXC}}{\underset{\shortparallel}{S_{ADC} - S_{XDC}}}}_{z \,:\, y} = p_1 : q_1$$

$$z : y = p_1 : q_1$$

$\circ)$ $I = [a : b : c]$ $\qquad \alpha x + \beta y + \gamma z = 0$

retta

$$\alpha a + \beta b + \gamma c = 0$$

$\circ)$ $\Pi = [\underset{k}{\ } : \underset{(1-k)}{\ } : 0]$ $\qquad k \in (0,1)$

$$\begin{cases} \alpha k + \beta(1-k) = 0 \\ \alpha a + \beta b + \gamma c = 0 \end{cases}$$

$$\det\begin{pmatrix} i & j & k \\ a & b & c \\ k & 1-k & 0 \end{pmatrix} = i\,\underbrace{(k-1)c} + j\,\underbrace{ck} + k\,\underbrace{[a(1-k) - kb]}$$

$$\begin{cases} (k-1)c\,x + ck\,y + (a(1-k) - kb)\,z = 0 \\ y = 0 \end{cases}$$

$$(k-1)c\,x + (a(1-k) - kb)\,z = 0$$

$$[kb - a(1-k) : 0 : (k-1)c] = N$$

$$[k : (1-k) : 0] = M$$

$$\frac{B\Pi}{AM} \cdot \frac{CN}{AN} = \frac{k}{1-k} \cdot \frac{kb - a(1-k)}{(k-1)c} =$$

$$= (k^2 b + k^2 a - ak) \cdot \frac{1}{(-c)(1-k)^2} =$$

$$= \frac{ak - k^2(a+b)}{c(1-k)^2} \le \frac{a^2}{4bc}$$

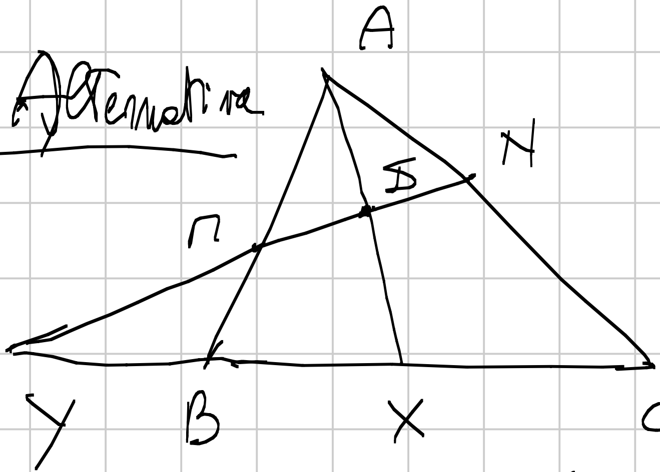
$$4ab\cancel{c}k - 4k^2b\cancel{c}(a+b) \le a^2\cancel{c}(1-k)^2$$

$$4k^2b(a+b) + a^2(1-k)^2 - 4abk \ge 0$$

$$k^2(4ab+4b^2+a^2) - 2k(2ab+a^2) + a^2 \ge 0$$

$$\left(k(a+2b)\right)^2 - 2ak(a+2b) + a^2 \ge 0$$

Alternativa

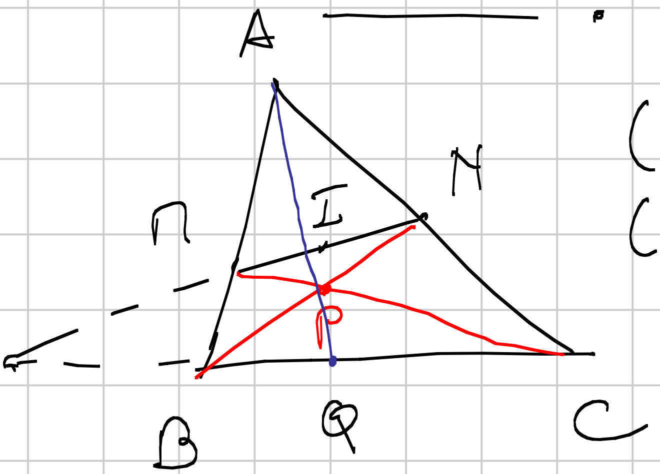


$$\frac{AN}{NC}\cdot\frac{CY}{YX}\cdot\frac{XI}{IA} = +1$$

$$\frac{AM}{MB}\cdot\frac{BY}{YX}\cdot\frac{XI}{IA} = 1$$

$\frac{BX}{XC} = \underline{\text{noto}}$. $\quad BX = \ldots$

$CX = \ldots$



$(NB, NC, NQ, NM) = -1$

$(NP, NA, NQ, NM) = -1$

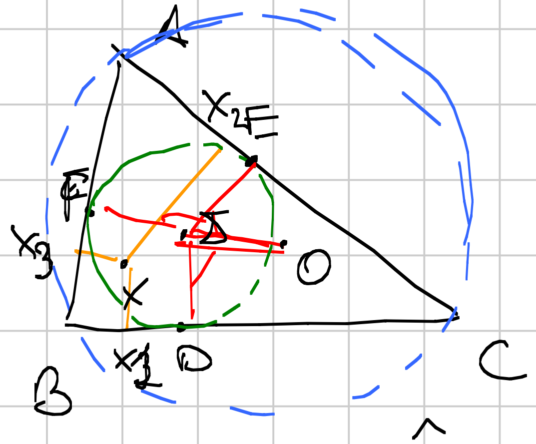
$\mathcal{C}$ = conica per A, B, C e tangente a BI, CI

$\Rightarrow P \in \mathcal{C}$ $\left[\Leftarrow MI = MN = \mathrm{pol}_{\mathcal{C}}(Q)\right]$

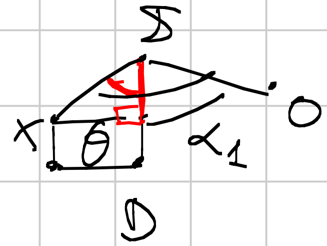
$$\mathcal{U} = \left\{ \frac{1}{x} = \frac{1}{y} + \frac{1}{z} \right\}.$$

↳ Trilineari.

G9) 1. Trigonometria



$$O\hat{I}F = \alpha_3$$
$$O\hat{I}E = \alpha_2$$
$$O\hat{I}D = \alpha_1$$
$$O\hat{I}X = \theta$$



$$X\hat{I}D = \theta - \alpha_1$$

$$ID - XX_1 = IX \cdot \cos(\theta - \alpha_1)$$
$$IE - XX_2 = IX \cdot \cos(\theta - \alpha_2)$$
$$IF - XX_3 = IX \cdot \cos(\theta - \alpha_3)$$

IO retta di Eulero di DEF

$r \cdot \sin\alpha_j =$ dist. con segno di D da IO

$$\sum_i (\qquad) = IX \sum_{j=1}^{3} \cos(\theta - \alpha_j)$$

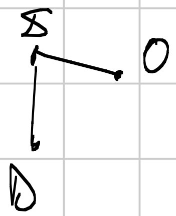
$$3r - S' = IX \sum_{j=1}^{3} \cos(\theta - \alpha_j)$$

$\Rightarrow \sum r \sin\alpha_j = 0$ perché il baricentro sta su IO (di DEF)

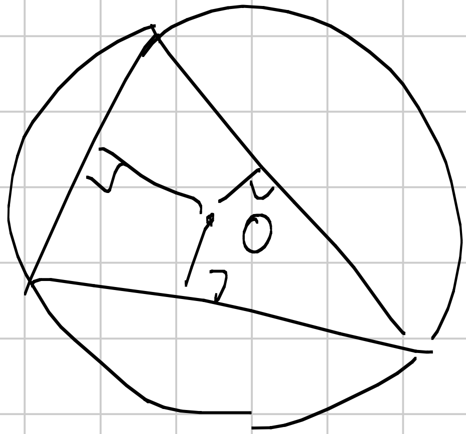
$$S' = 3r - IX \cdot \sum_{j=1}^{3} \cos(\theta - \alpha_j)$$

$$\| $$

$$\cos\theta \sum \cos\alpha_j + \sin\theta \underbrace{\sum \sin\alpha_j}_{= 0}$$

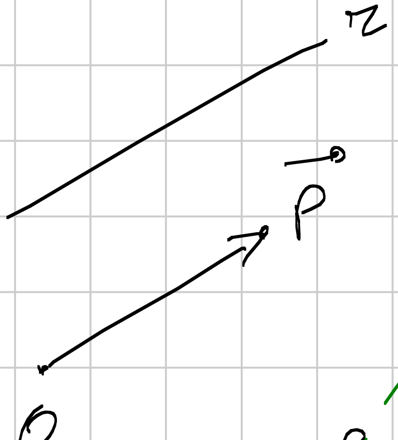


$$OI \sum \cos\alpha_j$$

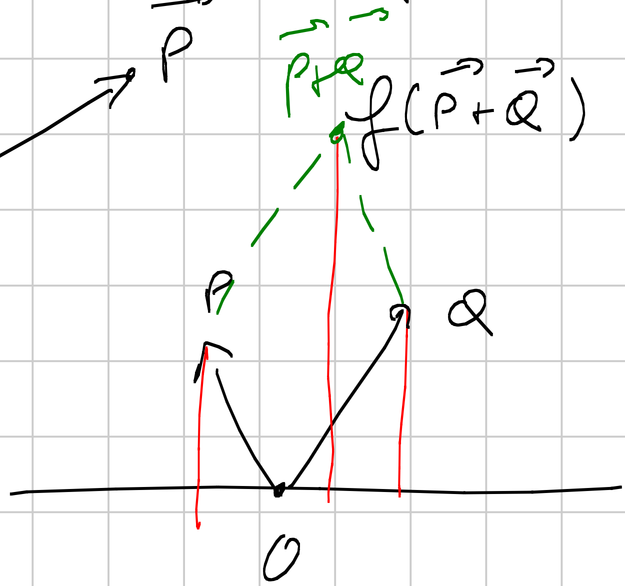


$$\cos\alpha + \cos\beta + \cos\gamma = 1 + \frac{r}{R}$$

•)



$$f(P) = \text{dist}(P, r)$$



$$f(\vec{p} + \vec{q}) = f(\vec{p}) + f(\vec{q})$$

$$f(x,y) = ax + by$$

$$f(x,y) = ax + by + c$$

$$f(\vec{p} + \vec{q}) = f(\vec{p}) + f(\vec{q}) - f(\vec{o})$$

$f, g \qquad h = f + g$

$$h(p+q) = f(p) + f(q) - f(o) + g(p) + g(q) - g(o) =$$
$$= h(p) + h(q) - h(o).$$

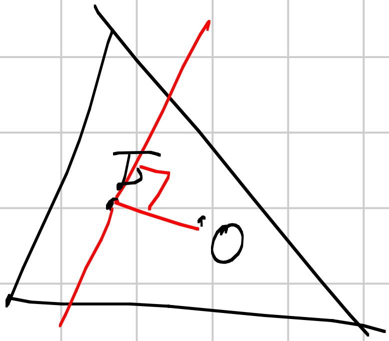
$$F(X) = d(X, AB) + d(X, BC) + d(X, CA)$$

$$F(x,y) = \alpha x + \beta y + \gamma$$

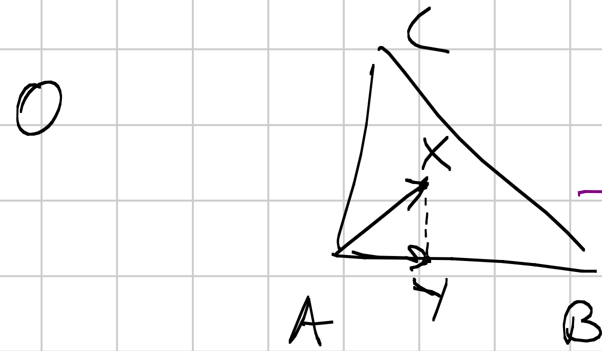
$$F(x,y) = \alpha x + \beta y + \gamma = k$$

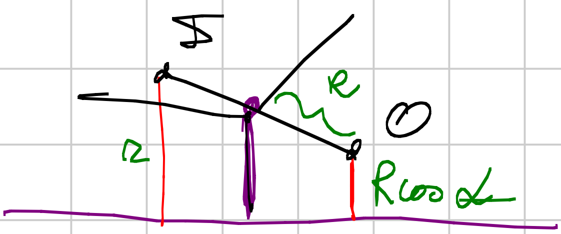
$$(\alpha, \beta)$$

$$F(X) = 3r$$



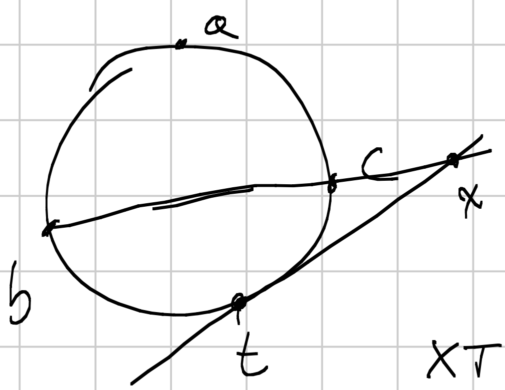
$X \longrightarrow X'$ proiez. di X su IO

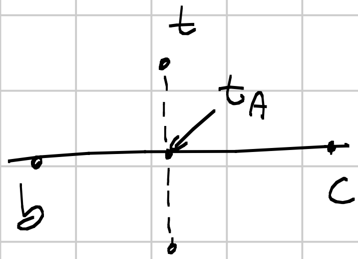




G10)

• num complessi





$(|t| = |b| = |c| = 1)$

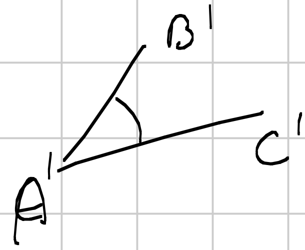
$$t_A = b + c - \frac{bc}{t}$$

$$XT^2 = XC \cdot XB$$

$$x = \frac{bc - tt_A}{b + c - t - t_A}$$

$$a' = (A') = a + \frac{bc(t-a)^2}{t^2(b+c)}$$

$$\frac{a'-b'}{a'-c'} = \frac{a^2-b^2}{a^2-c^2}$$



cf. per A, B, C $\{|z|=1\}$ è tg. alla cf. per A', B', C' $\ni P$

$$\bar{z} = \frac{1}{z}$$

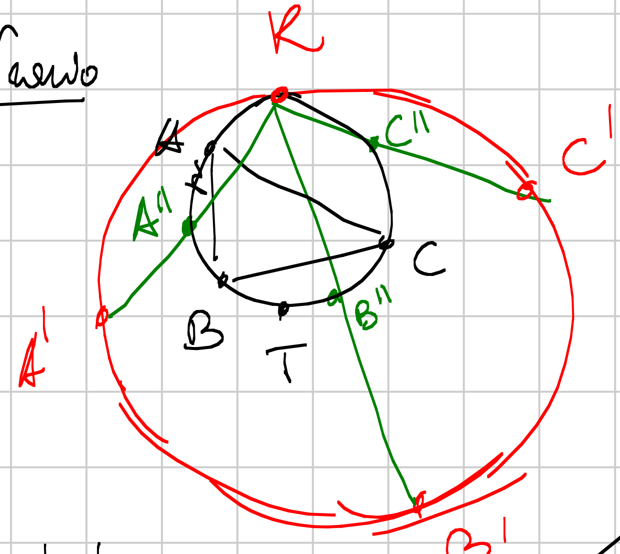
$$\Leftarrow \quad \frac{p-b'}{p-c'} \Big/ \frac{a'-b'}{a'-c'} \in \mathbb{R}.$$

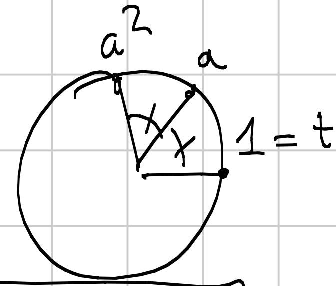
$$\frac{p-b'}{p-c'} \cdot \underbrace{\frac{a'-c'}{a'-b'}}_{\frac{a^2-c^2}{a^2-b^2}} = \frac{\frac{1}{p}-\bar{b}'}{\frac{1}{p}-\bar{c}'} \cdot \underbrace{\frac{\bar{a}'-\bar{c}'}{\bar{a}'-\bar{b}'}}_{\frac{c^2-a^2}{b^2-a^2}\cdot\frac{b^2}{c^2}}$$

$$\Downarrow$$

$$\alpha p^2 + \beta p + \gamma = 0 \qquad \beta^2 - 4\alpha\gamma = 0$$

- all'incontrario





$$\boxed{\widehat{TA} = \widehat{AA''}} \quad (*)$$

$A''C'' \,//\, A'C'$ = simm della tang. t risp. ad AC.

Costruisco A'', B'', C'' che soddisfano (✱). $\Rightarrow$ $A''B''C''$, $A'B'C'$ hanno i lati paralleli.

- $A''A'$, $B''B'$, $C''C'$ concorrono in K
- $K \in \omega$.

AA', BB', CC' bisett. di A', B', C' concorrono in I (inc. di $A'B'C'$).

- $I \in \omega$

$k = B'B'' \cap \omega$ $\qquad$ $KB''CIBC''$

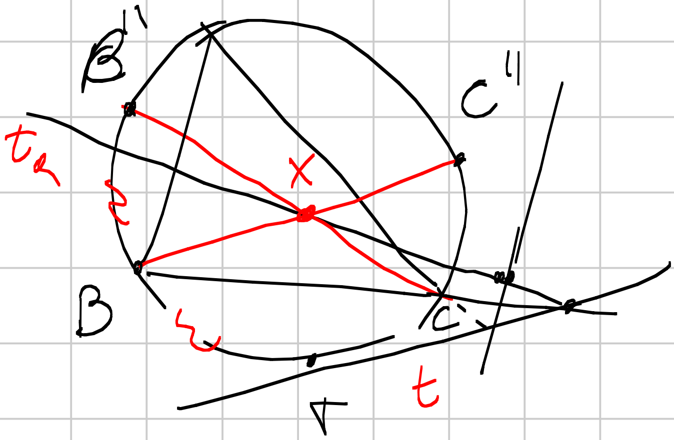
$\Downarrow$

$B' = KB'' \cap IB$, $B''C \cap BC''$, $CI \cap C''K$

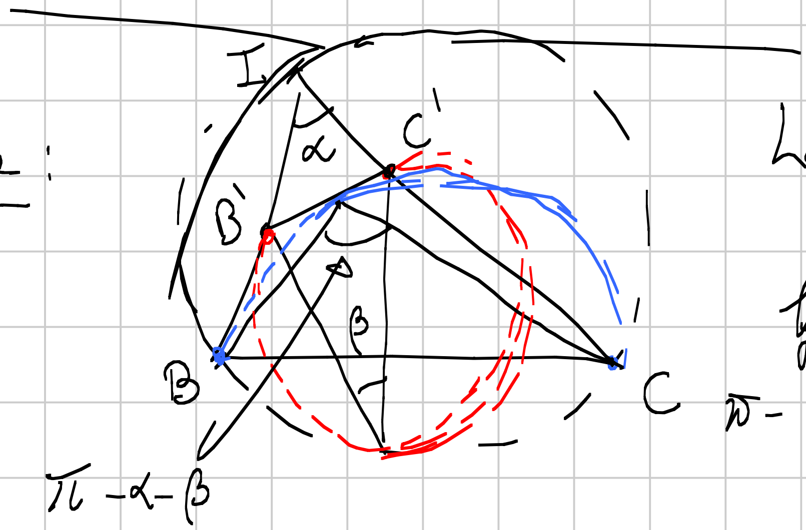
$X'' \in t_a$ $\qquad$ C''

$\Downarrow$

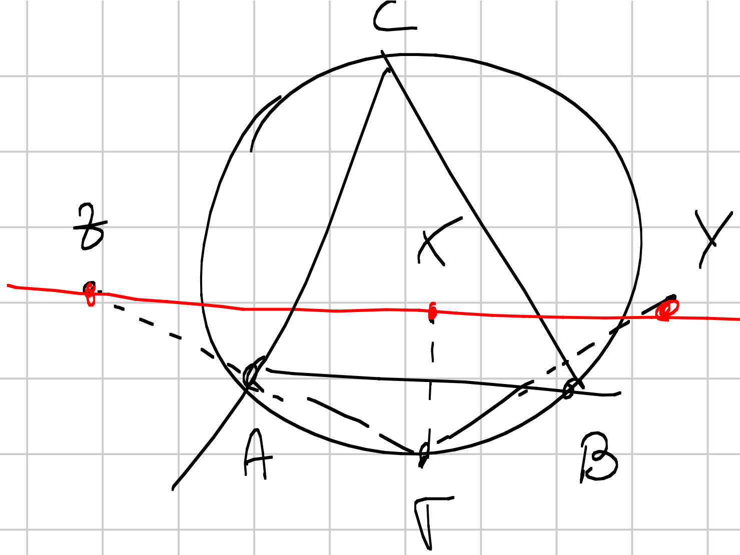
C', C'', K allineati.



Lemma:



La circ. per B e C che tange B'C' fa un angolo di $\pi - (\beta + \alpha)$



X, Y, C, C'

conciclici

X, Z, B, B'

Y, Z, A, A'