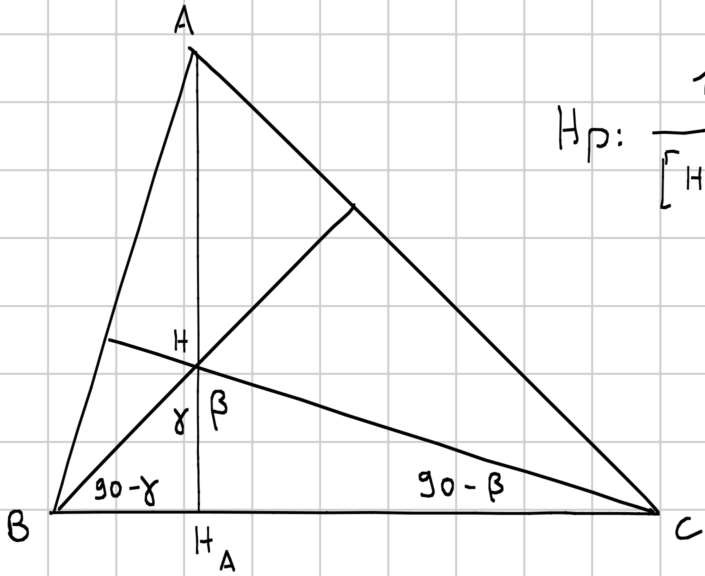
WC13 - B2

Titolo nota 01/02/2013

①



$$Hp: \frac{1}{[HAB]} + \frac{1}{[HAC]} = \frac{2}{[HBC]}$$

$$[HBC] = \frac{1}{2} \cdot \underbrace{BC}_{a} \cdot \underbrace{CH}_{\frac{a}{\sin\alpha}\cdot\cos\gamma = 2R\cos\gamma} \cdot \underbrace{\sin(90-\beta)}_{\cos\beta}$$

$$[HBC] = \frac{1}{2}\, a\ 2R\cos\gamma\cos\beta = \underbrace{R\cos\alpha\cos\beta\cos\gamma}_{k}\ \frac{a}{\cos\alpha} =$$

$$= k \cdot \frac{2abc}{b^2+c^2-a^2} = \lambda \cdot \frac{1}{b^2+c^2-a^2}$$

$$\frac{2(b^2+c^2-a^2)}{\lambda} = \frac{(a^2+c^2-b^2)}{\lambda} + \frac{(a^2+b^2-c^2)}{\lambda}$$

$$Hp \Leftrightarrow \boxed{b^2+c^2 = 2a^2}$$

$AG \perp GH \qquad (\vec{A}-\vec{G})(\vec{H}-\vec{G})=0$

$G = \frac{A+B+C}{3} \qquad H = A+B+C$

$A\cdot A = R^2 \qquad A\cdot B = R^2 - \frac{c^2}{2}$

$A - G = \frac{2A-B-C}{3} \qquad H - G = \frac{2}{3}(A+B+C)$

$Th \iff (2A-B-C)(A+B+C)=0$

$2\cancel{A\cdot A} + 2A\cdot B + 2A\cdot C - A\cdot B - \cancel{B\cdot B} - B\cdot C - A\cdot C - B\cdot C - \cancel{C\cdot C} = 0$

$A\cdot B + A\cdot C = 2\cdot B\cdot C$

$\cancel{R^2} - \frac{c^2}{2} + \cancel{R^2} - \frac{b^2}{2} = 2\cancel{R^2} - a^2$

$$\boxed{b^2+c^2=2a^2}$$

SOL 2 BARICENTRICHE

$G = [1,1,1] \qquad A = [1,0,0] \qquad H = [S_A, S_B, S_C]$

$r_1 \quad r_2 \qquad P_1 = r_1 \cap r_\infty \qquad P_2 = r_2 \cap r_\infty$

$r_\infty : x+y+z=0 \qquad P_1 = [\alpha_1, \beta_1, \gamma_1] \quad P_2 = [\alpha_2, \beta_2, \gamma_2]$

$r_1 \perp r_2 \iff \frac{\alpha_1\alpha_2}{S_A} + \frac{\beta_1\beta_2}{S_B} + \frac{\gamma_1\gamma_2}{S_C} = 0$

$r_{AG}: \quad y - z = 0 \qquad\qquad r_{\infty}: \quad x + y + z = 0$

$P_1 = (-2, 1, 1)$

$r_{GH}: \quad (s_B - s_C)x + (s_C - s_A)y + (s_A - s_B)z = 0$

$P_2 = (s_B + s_C - 2s_A,\ s_A + s_C - 2s_B,\ s_A + s_B - 2s_C)$

$$\frac{\cancel{4 s_A} - 2s_B - 2s_C}{s_A} + \frac{s_A + s_C - \cancel{2s_B}}{s_B} + \frac{s_A + s_B - \cancel{2s_C}}{s_C} \stackrel{?}{=} 0$$

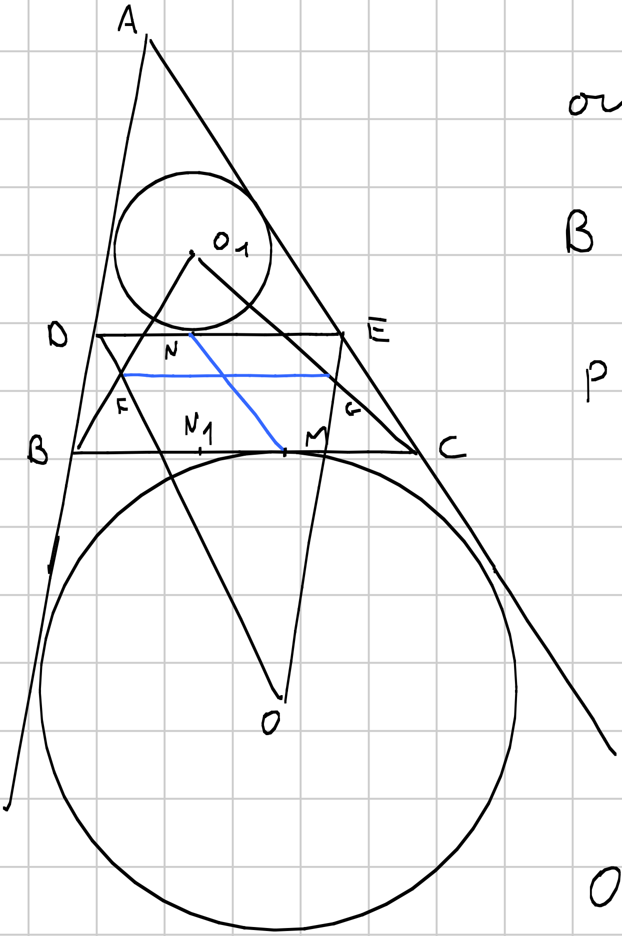
$$\frac{2}{s_A}(s_B + s_C) = \frac{1}{s_B}(s_A + s_C) + \frac{1}{s_C}(s_A + s_B)$$

$s_A + s_B + s_C = 1$

$$\frac{2(1 - s_A)}{s_A} = \frac{(1 - s_B)}{s_B} + \frac{(1 - s_C)}{s_C}$$

$$\frac{2}{s_A} = \frac{1}{s_B} + \frac{1}{s_C}$$

2



origine in A

$B \qquad C \qquad D = tB \qquad E = tC$

$$P = [x, y, z]_{ABC}$$

$$P = \frac{a x A + b y B + c z C}{ax + by + cz}$$

$$O = [-1, 1, 1]_{ABC}$$

$$O = \frac{bB + cC}{b + c - a}$$

$$O_1 = [1, 1, 1]_{ADE} \qquad O_1 = \frac{tbD + tcE}{ta + tb + tc} = \frac{tbB + tcC}{a + b + c}$$

$$\left[\frac{D}{t} = B \qquad \frac{E}{t} = C \right]$$

$$P \text{ t.c } \frac{BP}{PC} = \frac{\lambda_1}{\lambda_2} \rightarrow P = \frac{\lambda_2 B + \lambda_1 C}{\lambda_1 + \lambda_2}$$

$$\frac{BM}{MC} = \frac{a + b - c}{a + c - b}$$

$$M = \frac{(a + c - b)B + (a + b - c)C}{2a} = \frac{B + C}{2} - \frac{b - c}{2a}(B - C)$$

$$\frac{DN}{EN} = \frac{a + c - b}{a + b - c} \rightarrow N = \underbrace{t\,\frac{B + C}{2} + t\,\frac{(b - c)}{2a}(B - C)}_{\lambda(B + C) + \mu(bB + cC)}$$

$$F = \lambda B + (1-\lambda) O_1 = \mu D + (1-\mu) O$$

$$\lambda B + \frac{(1-\lambda) t b B}{a+b+c} + \frac{(1-\lambda) t c C}{a+b+c} =$$

$$= \mu t B + \frac{(1-\mu) b B}{b+c-a} + \frac{(1-\mu) c C}{b+c-a}$$

$$\lambda = \mu t \qquad \frac{(1-\mu t)\, t}{a+b+c} = \frac{(1-\mu)}{b+c-a}$$

$$r = b+c-a \qquad R = a+b+c$$

$$\mu = \frac{tr - R}{t^2 r - R}$$

$$\frac{F+G}{2} = \frac{tr-R}{t^2 r - R}\, t \cdot \frac{B+C}{2} + \frac{t(t-1)}{t^2 r - R} (bB + cC)$$

N, M, pt medio di FG allineati

$$\Updownarrow$$

$$(N-M) \times \left(\frac{F+G}{2} - M\right) = 0$$

$$(N-M) \times (F+G) = 2 N \times M$$

$$A \times A = 0$$

$$A \times B = - B \times A$$

$$\left[\alpha(B+C)+\beta(B-C)\right]\times\left[\gamma(B+C)+\delta(B-C)\right]=$$

$$=\alpha\delta\,(B+C)\times(B-C)+\beta\gamma\,(B-C)\times(B+C)=$$

$$=(\beta\gamma-\alpha\delta)\underbrace{(B-C)\times(B+C)}_{B\times C-C\times B=2B\times C}$$

$$2\,N\times M=2t\,\frac{b-c}{a}\,B\times C$$

$$(N-M)\times(F+G)=B\times C\,\frac{(b-c)t}{a(t^2 2-R)}\Big(-a(t-1)^2+(t+1)(t2-R)+$$

$$+(t^2-1)(b+c)\Big)$$

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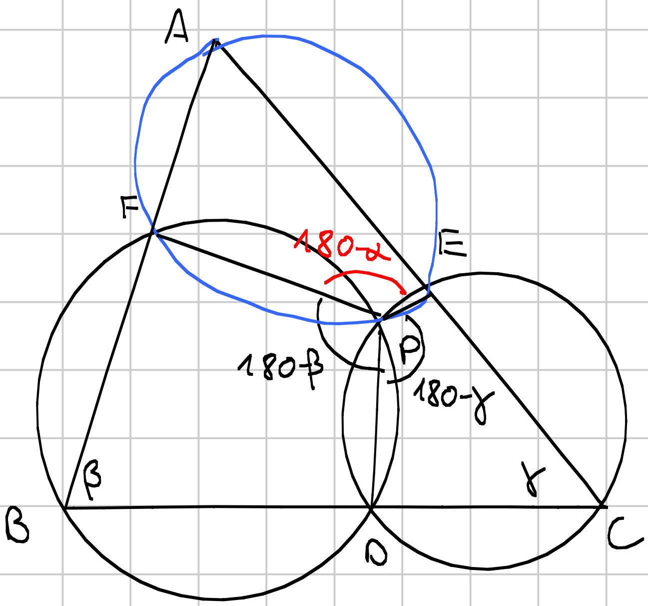
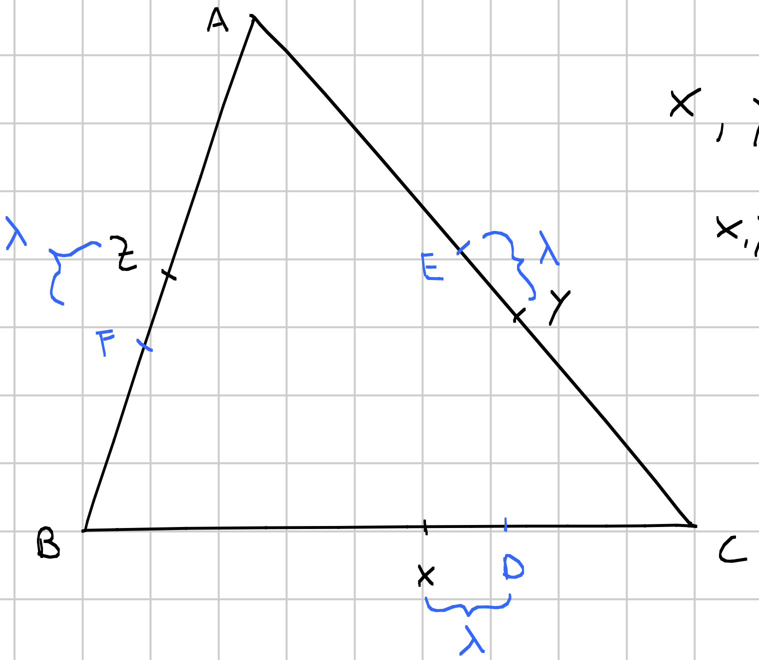


teo Miquel

Oss 1 le ipotesi sono "cicliche"

$BD + BF = AC$ e $CD + CE = AB$ $\Rightarrow$ $AF + AE = BC$

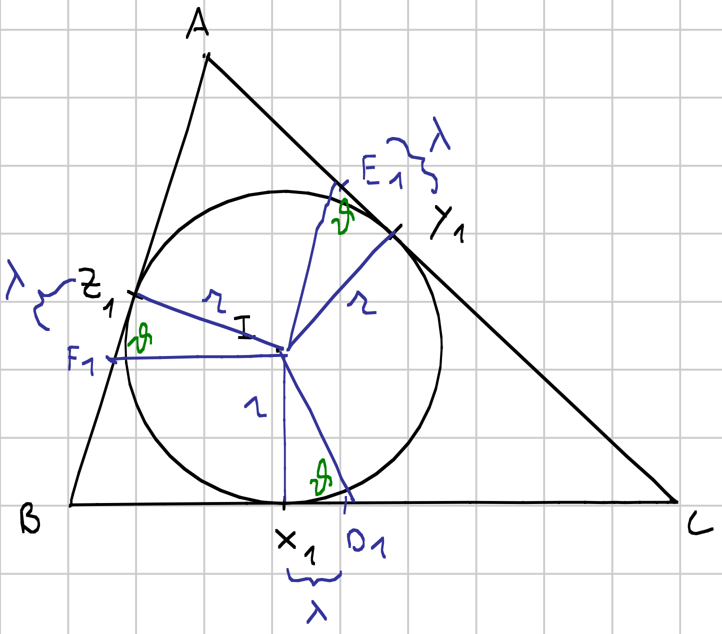


X, Y, Z pt tangenza excerchi

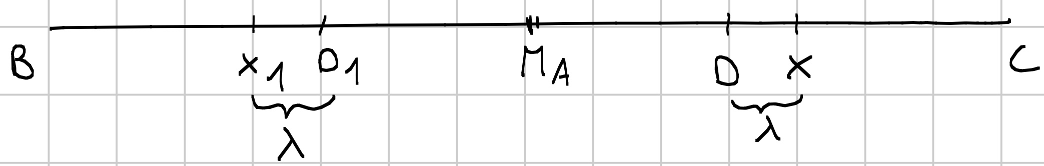
⇓

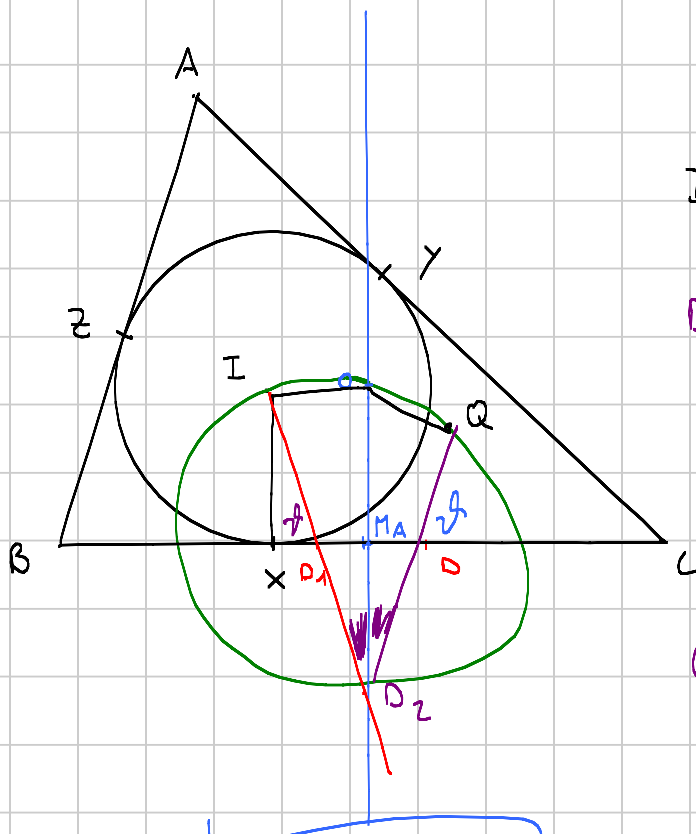
X, Y, Z soddisfano le ipotesi

$XD = \lambda$ $CX + CY = CD + CE$ $\Rightarrow$ $YE = \lambda$



$I\hat{D}_1B = I\hat{E}_1C = I\hat{F}_1A = \vartheta$





$I\hat{D}_1B = \vartheta$

$D_2 = ID_1 \cap OM_A$

$I\hat{D}_2O = 90 - \vartheta = I\hat{E}_2O = I\hat{F}_2O$

O, I, D_2, E_2, F_2 conciclici su ω

$Q \in \omega$ $\quad \boxed{IO = OQ}$ $\quad Q \neq I$

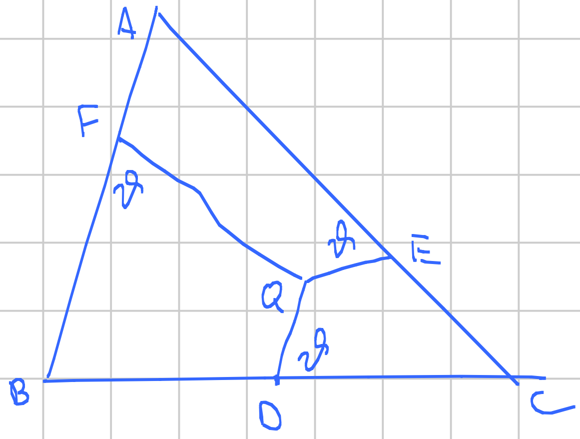
$O\hat{D}_2Q = I\hat{D}_2O = 90 - \vartheta$

D_2Q è simm di ID_2

rispetto all'asse di BC

D_2 D Q allineati

$Q\hat{D}C = \vartheta = Q\hat{E}A = Q\hat{F}B$



$\boxed{Q \equiv P}$