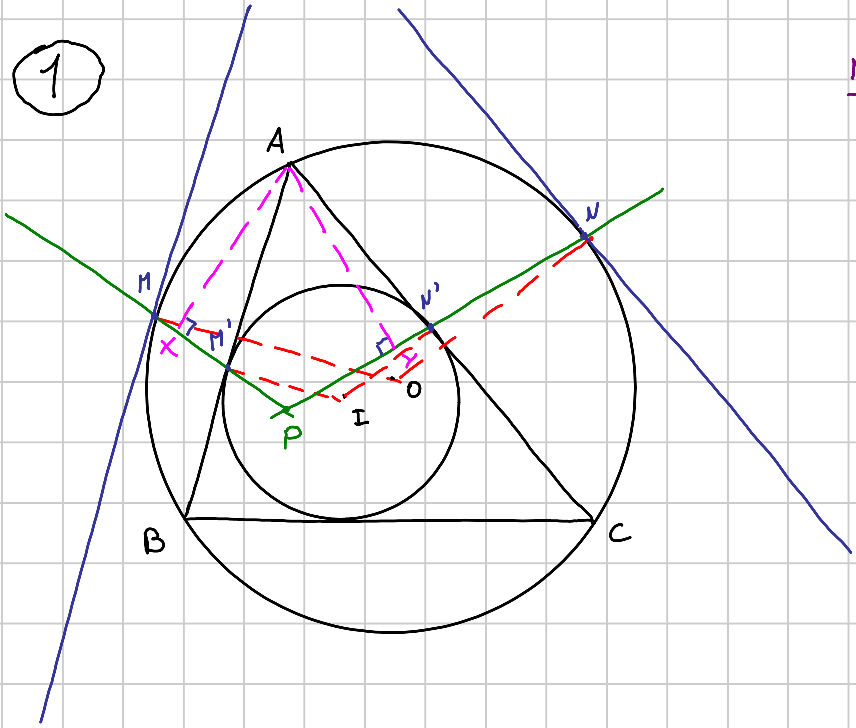
WC 14 – G2

Titolo nota 30/01/2014

1



NOTAZIONI

$\omega = (AXY)$

$P = MM' \cap NN'$

O = circocentro ABC

OSS. 1

$P \in \omega$ e AP diametro

OSS. 2

$AXIY$ ciclico $\Leftrightarrow$ $PI \perp AI$

OSS. 3

$M'IN'$ e MON sono simili

non solo!

hanno i lati a 2 a 2 paralleli

QUINDI $\rightarrow$ sono omotetici!

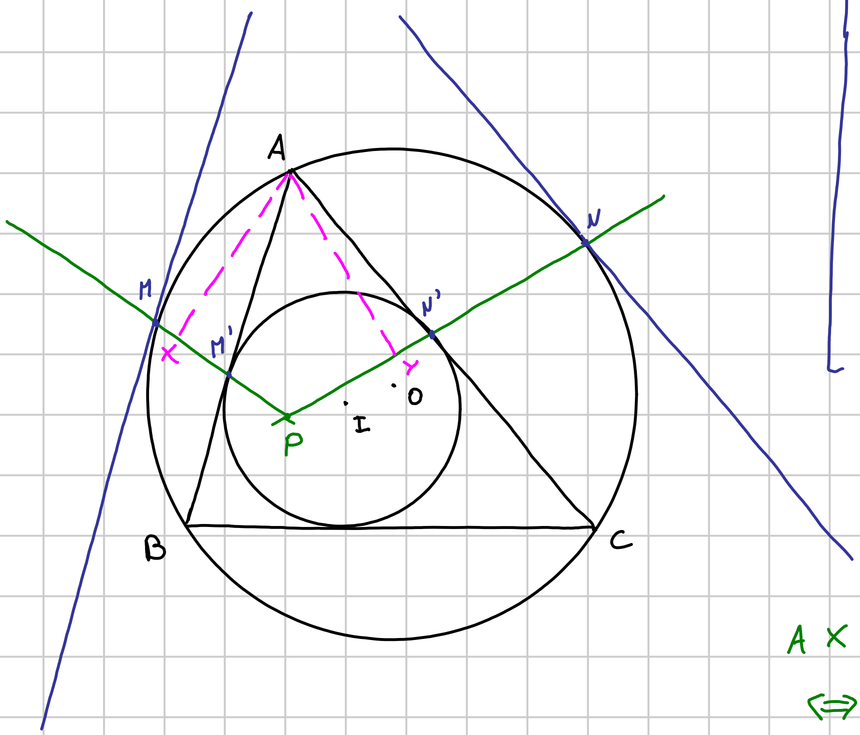
DIM

cosa fa omotetia di centro che manda $M' \rightarrow M$?

- $\ell(M'N') \rightarrow \ell(MN)$
- $N' \rightarrow N$
- $\ell(M'I) \rightarrow \ell(MO)$
- $\ell(N'I) \rightarrow \ell(NO)$
- $I \rightarrow O$

$\Downarrow$

P, I, O allineati!



$AXIY$ ciclico $\Leftrightarrow$ $PI \perp AI$ $\Leftrightarrow$

$\Leftrightarrow$ $IO \perp AI$

$$IO \perp AI \quad \Leftrightarrow \quad (\vec{I} - \vec{O}) \cdot (\vec{A} - \vec{I}) = 0$$

FATTO GENERALE

$$\vec{S} = \frac{[BCS]\vec{A} + [ACS]\vec{B} + [BAS]\vec{C}}{[ABC]}$$

$$\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$$

ALTRO RICHIAMO

origine in O

$\vec{A} \cdot \vec{A} = R^2$

$\vec{A} \cdot \vec{B} = R^2 - \frac{c^2}{2}$

$$\frac{aA + bB + cC}{a+b+c} \cdot \left(A - \frac{aA + bB + cC}{a+b+c} \right) = 0$$

SVOLGO I CONTI

$$(aA + bB + cC)\left((b+c)A - bB - cC\right) = 0$$

$$A \cdot A\,(ab + ac) + B \cdot B\,(-b^2) + C \cdot C\,(-c^2) +$$

$$+ A \cdot B\,(-ab + b^2 + bc) + A \cdot C\,(-ac + bc + c^2) + BC\,(-2bc) = 0$$

$$R^2(\cancel{ab} + \cancel{ac} - \cancel{b^2} - \cancel{c^2} - \cancel{ab} + \cancel{b^2} + \cancel{bc} - \cancel{ac} + \cancel{bc} + \cancel{c^2} - 2\cancel{bc}) +$$

$$- \frac{c^2}{\cancel{2}}(-ab + b^2 + bc) - \frac{b^2}{\cancel{2}}(-ac + bc + c^2) - \frac{a^2}{\cancel{2}}(-2bc)$$

$$-abc^2 + b^2c^2 + bc^3 - ab^2c + b^3c + b^2c^2 - 2a^2bc = 0$$

$$-\underline{ac} + \underline{bc} + \underline{c^2} - \underline{ab} + \underline{b^2} + \underline{bc} - 2\underline{a^2} = 0$$

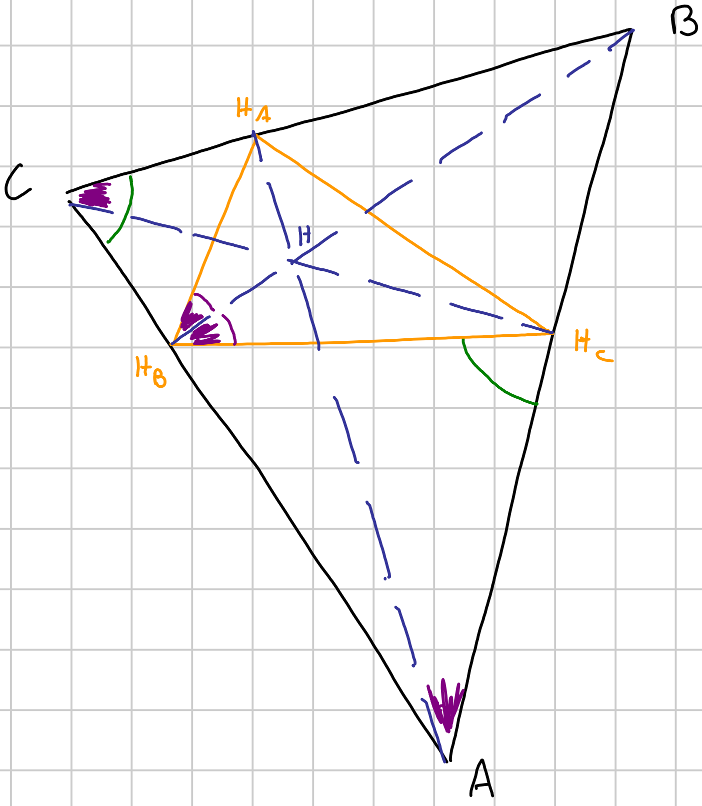
$$(b+c)^2 - a^2 - a(a+b+c) = 0$$

$$(b+c-a)(b+c+a) - a(a+b+c) = 0$$

$$(a+b+c)\underbrace{(b+c-2a)}_{=0} = 0$$

CVD

(2)



OSSERVAZIONI

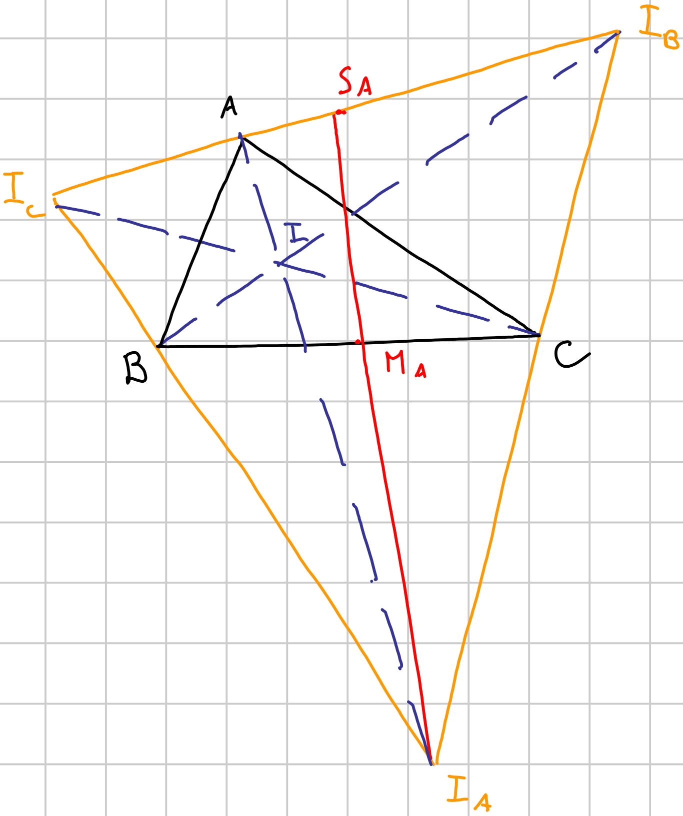
- $C H_A H H_B$ ciclico e così via
- H incentro del tr. ortico
- A, B, C excentri
- $C H_B H_C B$ ciclico e così via
- $A\hat{H_B}H_C \sim A\hat{B}C$

COSA FARE ORA??

BARICENTRICHE!!

alcuni punti:

- vertici $\rightarrow (1,0,0)$ ecc...
- $H = \left(\frac{a}{\cos\alpha}, \frac{b}{\cos\beta}, \frac{c}{\cos\gamma}\right)$
- $H_A = \left(0, \frac{b}{\cos\beta}, \frac{c}{\cos\gamma}\right)$ e simili
- $I = (a,b,c)$ - $I_A = (-a,b,c)$ e simili
- $G = (1,1,1)$
- coniug izog di $(x,y,z) \longrightarrow \left(\frac{a^2}{x}, \frac{b^2}{y}, \frac{c^2}{z}\right)$
- $K = (a^2, b^2, c^2)$



RIFORMULIAMO IL PROBLEMA

ABC triangolo

I incentro

I_A, I_B, I_C excentri

K pt. di Lemoine di ABC

K' pt. di Lemoine di $I_A I_B I_C$

Th $\rightarrow$ I, K, K' allineati

chi è K'?

chi sono le simmediane di $I_A I_B I_C$?

S_A piede simmediana $(\in I_B I_C)$

M_A pt medio di BC

I_A, M_A, S_A allineati

$I_A = (-a, b, c) \qquad M_A(0,1,1)$

$$\det\begin{pmatrix} -a & b & c \\ 0 & 1 & 1 \\ x & y & z \end{pmatrix} = 0$$

$\ell_A : (b-c)x + (a)y + (-a)z = 0$ simmediana

$\ell_B : (-b)x + (c-a)y + (b)z = 0$

$K' = \ell_A \cap \ell_B$

$$\underbrace{\begin{matrix} b-c & a & -a \\ -b & c-a & b \end{matrix}}$$

$\Downarrow$

$$K' = \left(ab + a(c-a),\ ab - b(b-c),\ (b-c)(c-a) + ab\right)$$

$$K' = \left(a(b+c-a),\ b(a+c-b),\ c(a+b-c)\right)$$

$$\det\begin{pmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ \underbrace{a(b+c-a)}_{\cdot\frac{1}{a}} & \underbrace{b(a+c-b)}_{\cdot\frac{1}{b}} & \underbrace{c(a+b-c)}_{\cdot\frac{1}{c}} \end{pmatrix} = 0$$

I, K

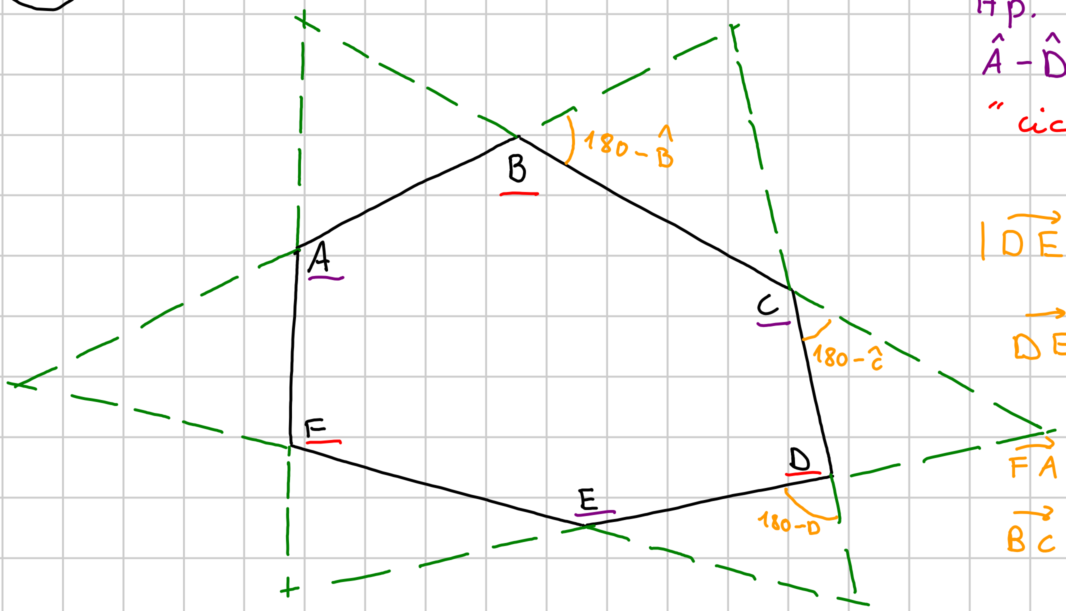
K'

$$\det\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c-a & a+c-b & a+b-c \end{pmatrix} = 0$$

$$\boxed{R_3 = (a+b+c)\cdot R_1 - 2R_2}$$

FINE!

③



Hp.
$\hat{A} - \hat{D} = \hat{C} - \hat{F} = \hat{E} - \hat{B}$
"ciclicità" ipotesi

$|\vec{DE}| = |\vec{AB}|$

$\vec{DE} = R_{(\cancel{x})}(\vec{AB})$

$\vec{FA} = R_{(\cancel{y})}(\vec{CD})$

$\vec{BC} = R_{(\cancel{z})}(\vec{EF})$

$x = (180 - B) + (180 - C) + (180 - D) = 540 - (\underline{B + C + D})$

$y = \text{— — — — — — —} = 540 - (\underline{D + E + F})$

$z = \text{— — — — — — —} = 540 - (F + A + B)$

$B + C + \cancel{D} \stackrel{?}{=} \cancel{D} + E + F$ sì!

QUINDI $\longrightarrow$ $x = y = z$

$\vec{DE} = R(\vec{AB})$ $\quad \vec{FA} = R(\vec{CD})$ $\quad \vec{BC} = R(\vec{EF})$

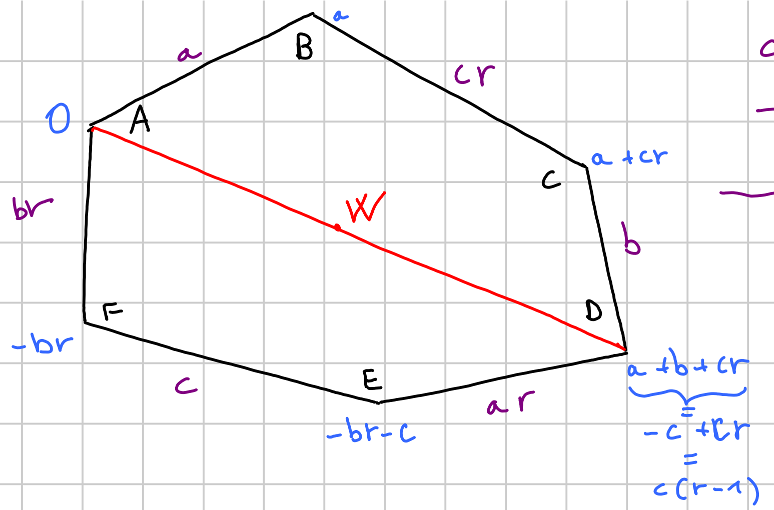
$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EF} + \vec{FA} = 0$

$\vec{AB} + \vec{CD} + \vec{EF} + \underbrace{R(\vec{AB}) + R(\vec{CD}) + R(\vec{EF})}_{R(\vec{AB} + \vec{CD} + \vec{EF})} = 0$

$\vec{x} = \vec{AB} + \vec{CD} + \vec{EF}$

$\vec{x} + R(\vec{x}) = 0 \longrightarrow$ R rotazione di $180°$
(esagono con lati opposti uguali e paralleli $\rightarrow$ FINE!)

$\searrow \vec{x} = 0$



c'è una rotazione →

→ numeri complessi

$a + b + c = 0$

r rotazione

$|r| = 1$

$$W = c(r-1)\lambda \qquad \lambda \in \mathbb{R}$$

Th : $W \in BE \iff W \in CF$

condizione allineamento B, W, E

$$\frac{\overbrace{c(r-1)\lambda}^{W} - \overbrace{a}^{B}}{\underbrace{-br-c}_{E} - \underbrace{a}_{B}} \in \mathbb{R}$$

$$\frac{c(r-1)\lambda - a}{-br+b} = \frac{c(r-1)\lambda - a}{-b(r-1)} = -\frac{c\lambda - \frac{a}{r-1}}{b} =$$

$$= \frac{b\lambda + a\left(\lambda + \frac{1}{r-1}\right)}{b} = \underbrace{\lambda}_{\in\mathbb{R}} + \underbrace{\frac{a}{b}\left(\lambda + \frac{1}{r-1}\right)}_{\alpha}$$

condizione allineamento C, W, F

$$\frac{\overbrace{c(r-1)\lambda}^{W} - \overbrace{(-br)}^{F}}{\underbrace{a+cr}_{C} - \underbrace{(-br)}_{F}} \in \mathbb{R}$$

$$\frac{c(r-1)\lambda + br}{a + cr + br} = \underbrace{\lambda}_{\in\mathbb{R}} + \underbrace{\frac{b}{a}\left(\lambda - \frac{r}{r-1}\right)}_{\beta}$$

Th: $\alpha \in \mathbb{R} \iff \beta \in \mathbb{R}$

cosa mi basta? per esempio posso dim. che $\forall \lambda$ vale $\alpha \cdot \beta \in \mathbb{R}$

HOPE: $\left(\lambda + \frac{1}{r-1}\right) \cdot \left(\lambda - \frac{r}{r-1}\right) \overset{?}{\in} \mathbb{R}$

$$\lambda^2 + \lambda\left(\frac{1}{r-1} - \frac{r}{r-1}\right) - \frac{r}{(r-1)^2} \overset{?}{\in} \mathbb{R}$$

$$\underbrace{\lambda^2 - \lambda}_{\in \mathbb{R}} - \frac{r}{(r-1)^2} \overset{?}{\in} \mathbb{R}$$

$$\frac{(r-1)^2}{r} \overset{?}{\in} \mathbb{R}$$

$$\frac{r^2 - 2r + 1}{r} = \underbrace{r + \frac{1}{r}} \underbrace{-2}_{\text{ok!}} \overset{?}{\in} \mathbb{R}$$

$$|r| = 1 \Rightarrow \frac{1}{r} = \bar{r} \Rightarrow r + \frac{1}{r} = r + \bar{r} \in \mathbb{R}$$

FINE!