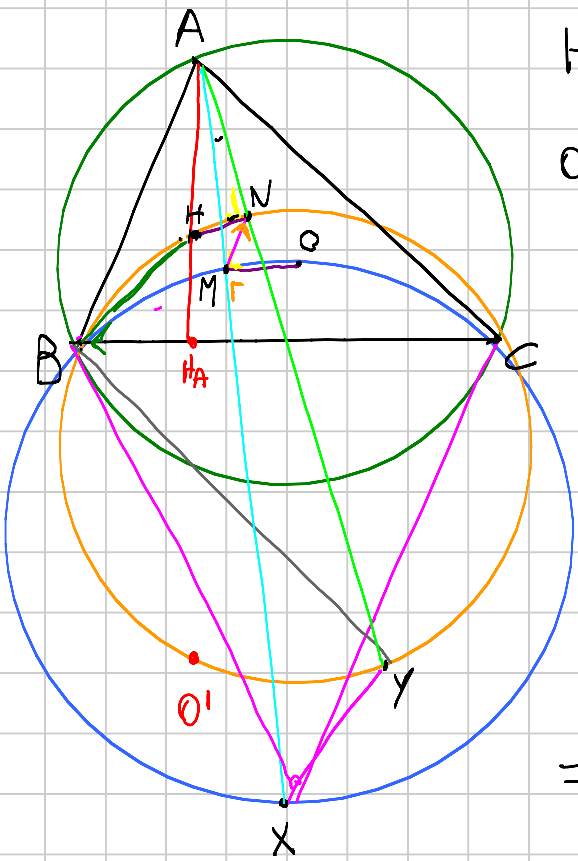
GEOMETRIA - SINTETICA

Titolo nota 30/01/2015



H diam. opposto di Y su ω_2

O " " " X su ω_1

AX simmediana $BX \perp BO$ $CX \perp CO$

$H\hat{B}Y = \frac{\pi}{2} \Rightarrow BY \parallel AC$

$H\hat{C}Y = \frac{\pi}{2} \Rightarrow CY \parallel AB$

ABYC parallelogramma.

$\Rightarrow$ AY mediana

Inversione in A raggio $\sqrt{bc}$ + simm. rispetto AI

$\overset{\triangle}{A}NM \sim \overset{\triangle}{A}YX \Leftrightarrow NM \parallel XY$ (perché $N\hat{A}M = Y\hat{A}X$)

$$\frac{AN}{AM} = \frac{AY}{AX} \Leftrightarrow AM \cdot AY = AN \cdot AX$$

$$AO' = \frac{bc}{AO} = \frac{bc}{\frac{b}{2\sin B}} = 2c\sin B = 2AH_A$$

$B \leftrightarrow C$ $\quad O \rightarrow O'$ $\quad AO \cdot AO' = bc$

O' = simm. di A rispetto BC. $\quad O' \in \omega_2$ (perché ω_2 è simm di Γ_{ABC} risp. BC)

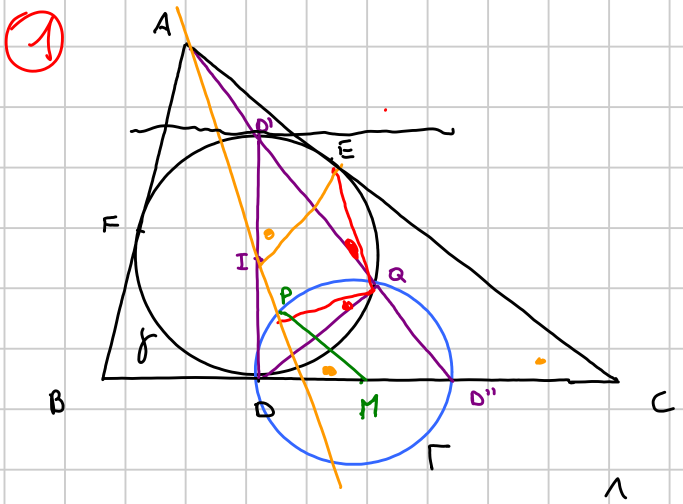
$\omega_1 \leftrightarrow$ circ. per $B'C'O' = \omega_2$. $\quad M = \omega_1 \cap$ simm. $\rightarrow \omega_2 \cap$ med $= Y \Rightarrow AM \cdot AY = AB \cdot AC$

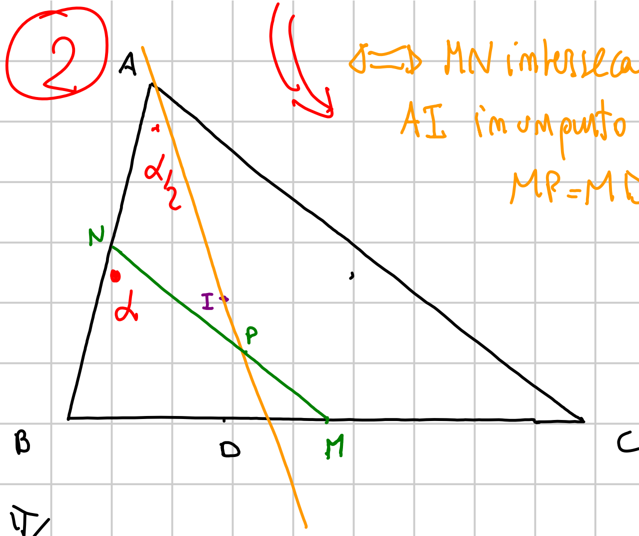
$N = \omega_2 \cap$ med $\rightarrow \omega_1 \cap$ simm $= X \Rightarrow AN \cdot AX = AB \cdot AC \Rightarrow AM \cdot AY = AN \cdot AX$

Con rif. alla fig. 2

$\overset{\triangle}{ANP}$ è isoscele $\Longrightarrow AN = NP = \frac{c}{2} \Longrightarrow MP = MN - NP = \frac{b}{2} - \frac{c}{2}$

Quanto vale MD? $MD = BM - BD = \frac{a}{2} - \frac{a+c-b}{2} = \frac{b}{2} - \frac{c}{2}$





⟺ MN interseca AI in un punto P t.c. MP = MD

Voglio mostrare che $P\hat{Q}E = \pi/2$

$AQ \cap \gamma = D'$ è il diam-opposto di D in γ

chi è $AQ \cap BC = ?$

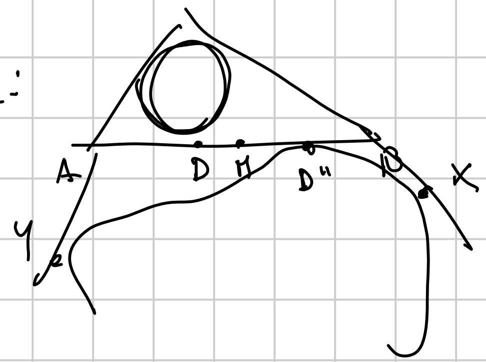
traccio a // BC pass. per D'

γ è exinscritta al triangolo che viene a formarsi,
con OMOTETIA

$D' \longrightarrow AD' \cap BC = D'' \Longrightarrow$ il p.to di tangenza dell'exinscritta relativa ad A con BC

$\Longrightarrow$ DD'' ha come p.to medio M

RECALL:



$AD = ?$ $\quad p - a = \frac{b+c-a}{2}$

$BD'' = ?$

$\|$

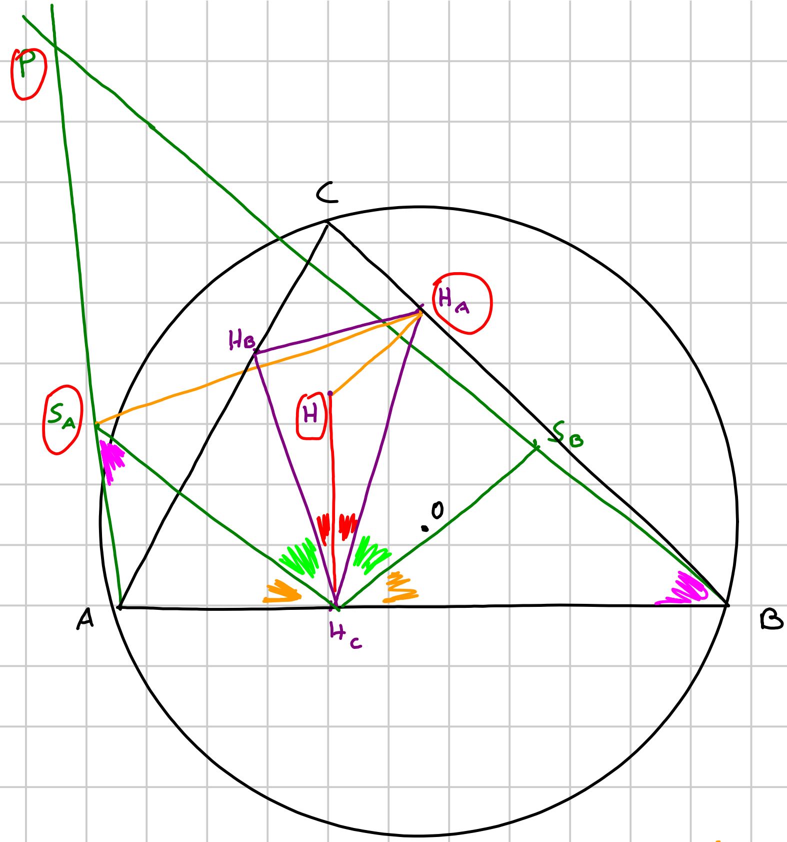
$BX = CX - CB = p - a$

$\Downarrow$ $BD'' = AD \longrightarrow DD''$ ha come p-to medio M

Su Γ ci sono D, D'', P, Q
perché $MQ = \frac{DD''}{2}$ essendo il triangolo DQD'' rettangolo

Th $\iff D'\hat{Q}E \cong P\hat{Q}D \iff P\hat{M}D \cong E\hat{I}D' \iff P\hat{M}D \cong \gamma \iff PM \parallel AC$

$\|$ (γ per ciclici)



Premesse:

- quadrilateri ciclici...
- $A\hat{H}_B H_C \sim A\hat{B}C$ ...
- H incentro di $H_A\hat{H}_B H_C$

$\boxed{1}$ ANGOLI, ANGOLI, ...

1a) $A\overset{\triangle}{H}_C S_A \sim S_B \overset{\triangle}{H}_C B$

$$\frac{AH_C}{H_CH_A} = \frac{H_CH_B}{BH_C}$$

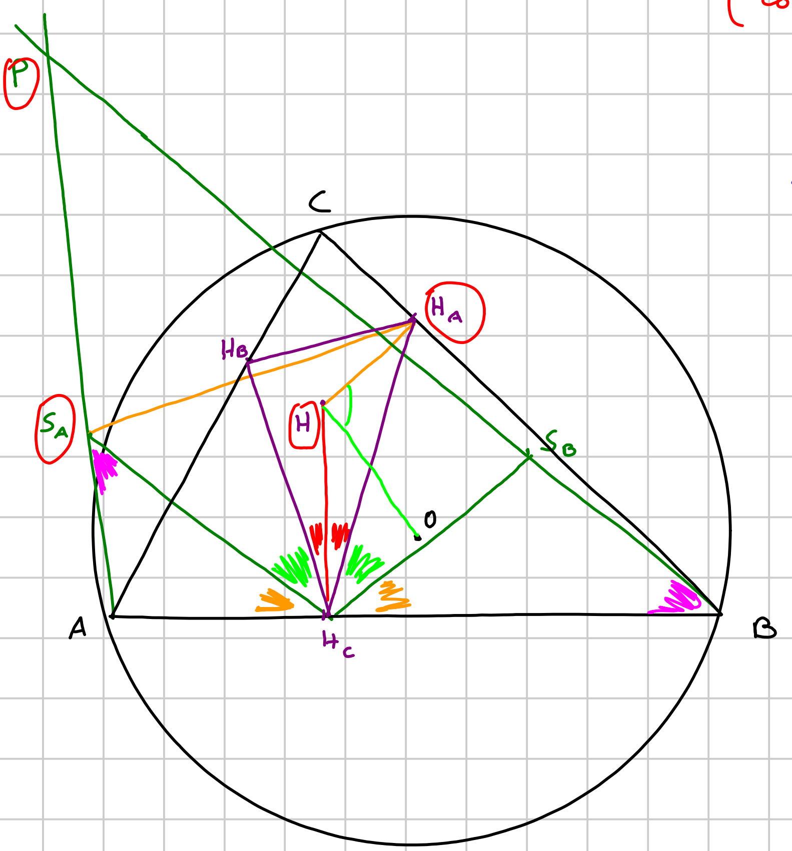
$H_CH_A \rightarrow S_AH_C$, $H_CH_B \rightarrow H_CS_B$

1b) PS_AH_CB ciclico

1c) PS_AHH_A ciclico

$$AH\cdot AH_A = AH_C\cdot AB = AS_A\cdot AP$$

(contenti perché togliamo S_B)

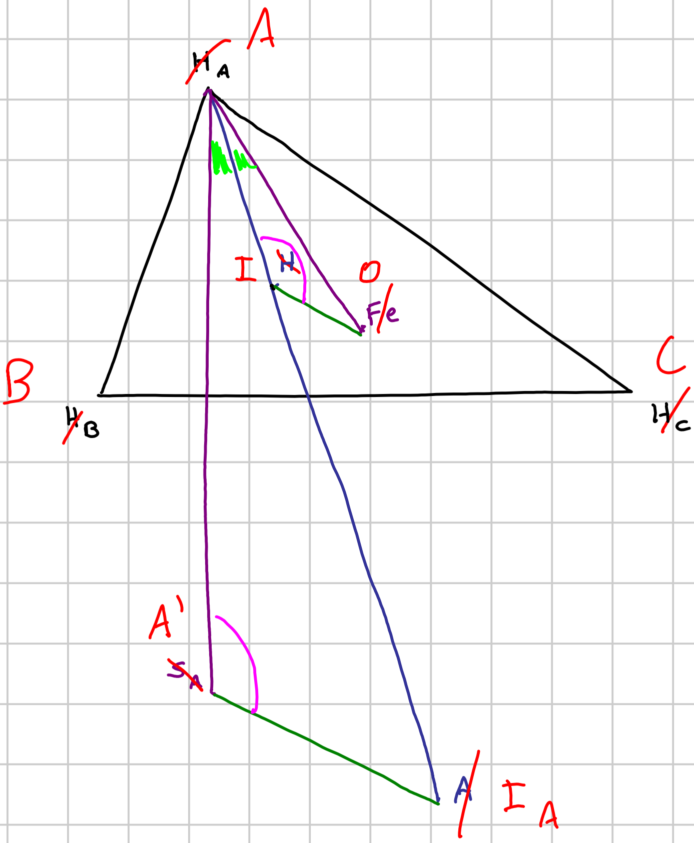


1d) Th1 $\Leftrightarrow$ $A\hat{S}_A H_A = O\hat{H}H_A$

$$O\hat{H}H_A \overset{?}{=} 180 - H_A\hat{H}P =$$

$$= 180 - H_A\hat{S}_AP = A\hat{S}_AH_A$$

(contenti perché togliamo P)



$\boxed{2}$ CAMBIARE TRIANGOLO

Th1 $\iff A\hat{A}'I_A = A\hat{I}O$

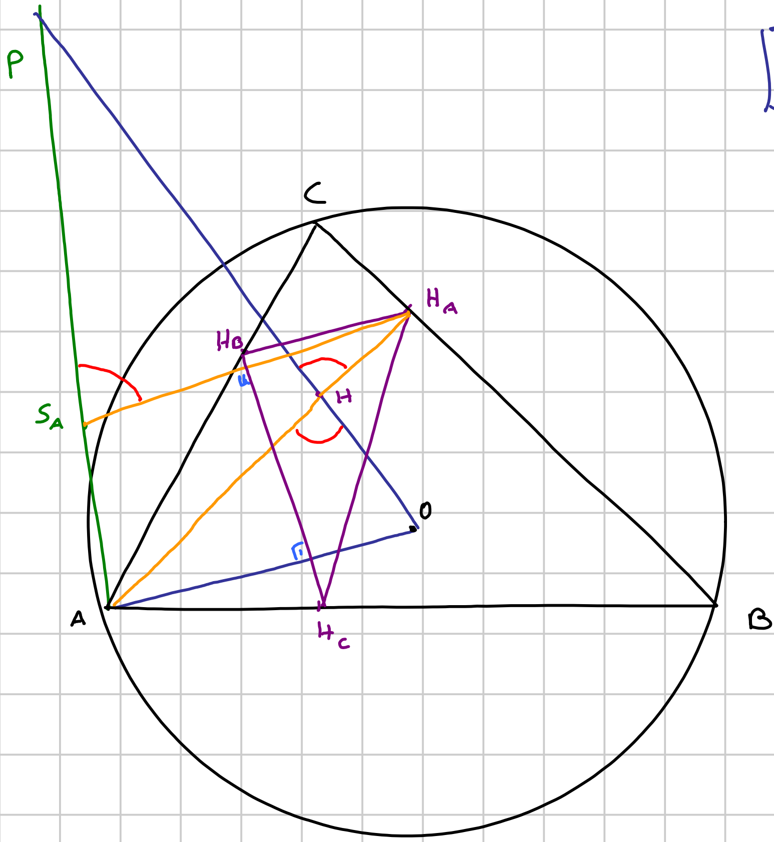
$\iff \overset{\triangle}{AA'I_A} = \overset{\triangle}{AIO} \iff$

$\iff \dfrac{AA'}{AI_A} = \dfrac{AI}{AO} \iff$

$\iff \underbrace{AA' \cdot AO}_{= bc} = \underbrace{AI \cdot AI_A}_{= bc}$

$\overset{\triangle}{ABA'} \sim \overset{\triangle}{AOC}$

$\overset{\triangle}{ABI} \sim \overset{\triangle}{AI_AC}$



$\boxed{3}$ THE END

Th 2 $\iff OH \cdot OP = OA^2$

$\iff O\hat{H}A = O\hat{A}P$

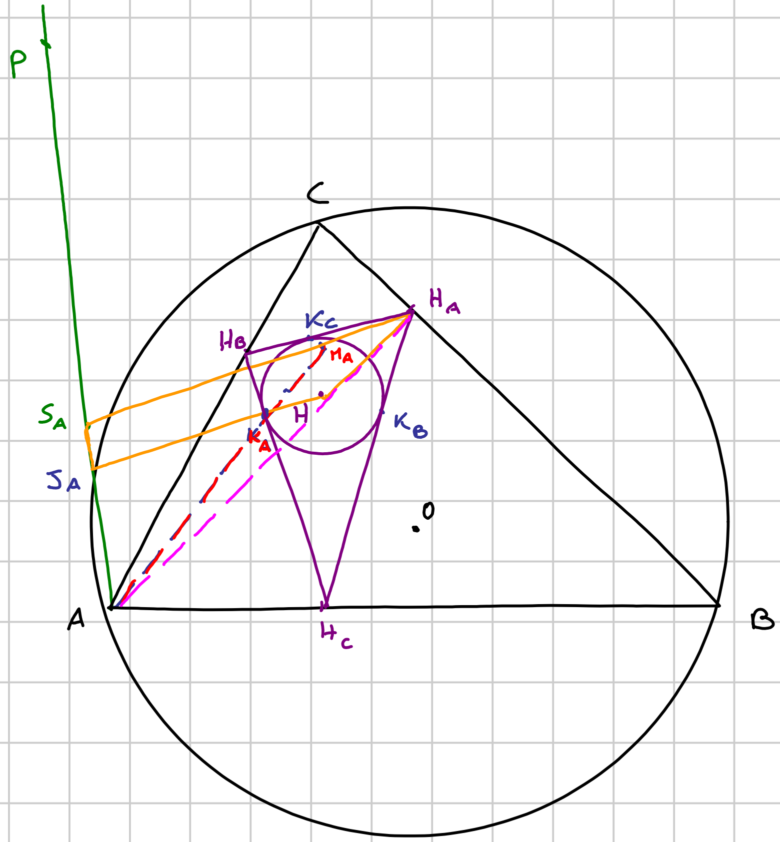
$O\hat{H}A = H_A\hat{H}P = H_A\hat{S_A}P$

$H_A\hat{S_A}P = O\hat{A}P$

$\Updownarrow$

$AO \parallel S_A H_A$

vero perché $\perp H_B H_C$



- $K_A K_B K_C$ omotetico a ABC
- M_A pt medio altezza del triangolo ortico
- M_A, K_A, A allineati
- $J_A = HK_A \cap AP$
- $\dfrac{HJ_A}{HK_A} = \dfrac{H_A M_A}{H_A S_A} = 4$
- $J_A J_B J_C$ omotetico di ABC
- P centro omotetia

– O, H "circocentri"

– P, O, H allineati

R, r raggi di $H_A H_B H_C$

$$Th \Leftrightarrow OH \cdot OP = 4R^2$$

$$\frac{PH}{PO} = \frac{2r}{R}$$

$$\frac{OH}{OP} = 1 - \frac{2r}{R} = \frac{R-2r}{R}$$

$$Th \Leftrightarrow OH^2 \cdot \frac{R}{R-2r} = 4R^2$$

$$\Updownarrow$$

$$\left(\frac{OH}{2}\right)^2 = R(R-2r)$$

HF_e^2

OI^2 nel triangolo ortico