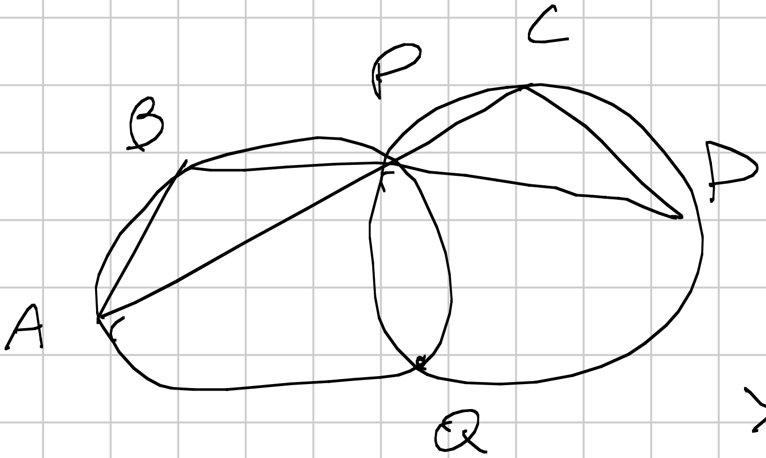
Titolo nota

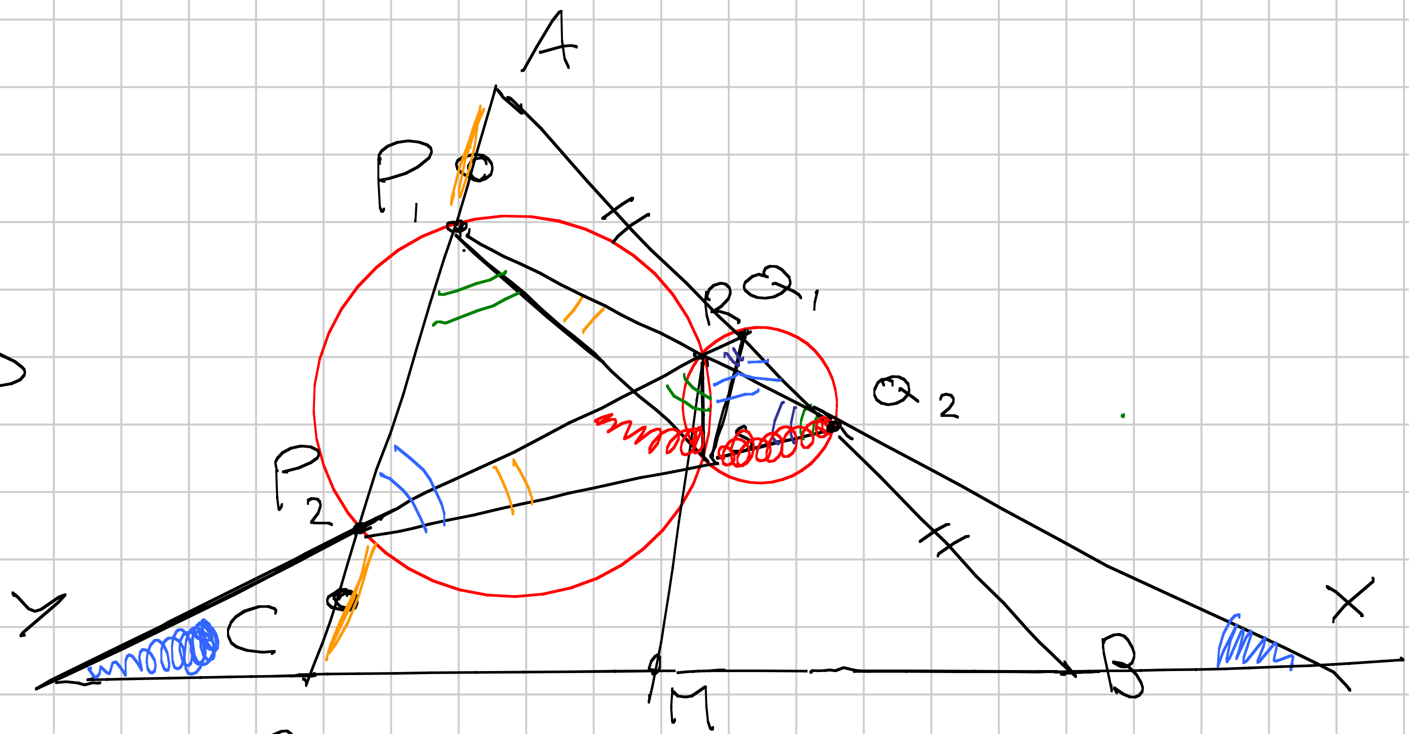
WC 2016 - GEOMETRIA (sintetica)

Problema 1.



A → C | A → B

B → D | C → D

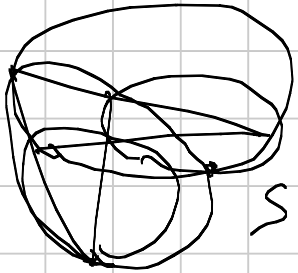


$\triangle P_2 S Q_1 \sim \triangle P_1 S Q_2$

$P_2 S P_1 \sim Q_1 S Q_2$

$P_2 S Q_1 A$

$Q_2 S P_1 A$



$P_2 S P_1 \longrightarrow Q_1 S Q_2$

$$\frac{P_2 S}{S Q_1} = \frac{\sin \widehat{S Q_1 P_2}}{\sin \widehat{Q_1 P_2 S}}$$

$$\boxed{\boxed{\frac{\sin \widehat{P_2 R S}}{\sin \widehat{S R Q_2}}} = \frac{P_2 S}{P_1 S} = \frac{Q_1 P_2}{P_1 Q_2}}$$

$$\frac{A P_1}{P_1 C} \frac{C X}{X B} \frac{B Q_2}{Q_2 A} = 1$$

$$\frac{A Q_1}{Q_1 B} \frac{B Y}{Y C} \frac{C P_2}{P_2 A} = 1$$

$$\frac{A P_1}{P_1 C} \cdot \frac{B Q_2}{Q_2 A} = \frac{C P_2}{P_2 A} \frac{A Q_1}{Q_1 B}$$

$$\frac{yC}{yB} = \frac{xB}{xC}$$

$$xC = xB + BC$$
$$yB = yC + BC$$

$$\frac{yB}{yC} = \frac{xC}{xB} \iff \frac{yC + BC}{yC} = \frac{xB + BC}{xB}$$

$$\cancel{1} + \frac{BC}{yC} = \cancel{1} + \frac{BC}{xB}$$

M pto medio di xy

T Seni :

$$\widehat{XBQ_2} \Rightarrow \frac{XB}{BQ_2} = \frac{\sin \widehat{XQ_2B}}{\sin \widehat{BXQ_2}}$$

$$\widehat{AP_1Q_2} \Rightarrow \frac{P_1Q_2}{P_1A} = \frac{\sin \widehat{P_1AQ_2}}{\sin \widehat{P_1Q_2A}}$$

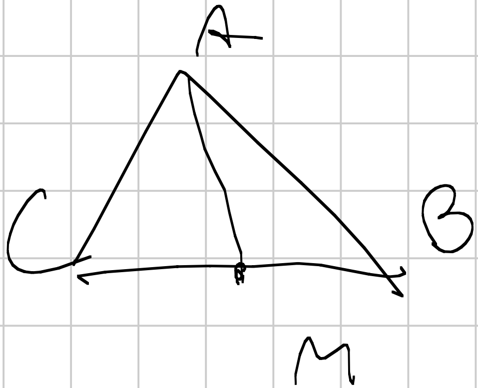
$$\widehat{P_1Q_2A} = \widehat{XQ_2B}$$

$$\frac{\sin \widehat{P_1AQ_2} \cdot P_1A}{P_1Q_2} = \sin \widehat{P_1Q_2A} = \sin \widehat{XQ_2B} = \frac{\sin \widehat{BXQ_2}\; XB}{BQ_2}$$

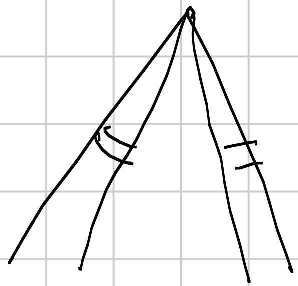
$$\frac{\sin \widehat{Q_1 A P_2} \cdot Q_1 A}{Q_1 P_2} = \frac{\sin \widehat{C Y P_2} \cdot YC}{C P_2}$$

$$\frac{\sin \widehat{P_1 A Q_2} \cdot P_1 A \cdot Q_1 P_2}{\sin \widehat{Q_1 A P_2} \cdot Q_1 A \cdot P_1 Q_2} = \frac{\sin \widehat{B X Q_2} \cdot XB \cdot C P_2}{\sin \widehat{C Y P_2} \cdot YC \cdot B Q_2}$$

$$\boxed{\frac{\sin \widehat{B X Q_2}}{\sin \widehat{C Y P_2}}} = \frac{Q_1 P_2}{P_1 Q_2} = \boxed{\frac{\widehat{P_2 R S}}{\widehat{S R Q_2}}}$$



$$\frac{\sin \widehat{CAM}}{\sin \widehat{MAB}} = \frac{AB}{AC} = \frac{\sin \gamma}{\sin \beta}$$



SOLUZIONE ALTERNATIVA

RS e RM sono simmetriche rispetto ad una bisettrice

RP_2Q_2 $\quad a = P_2Q_2 \quad b = RQ_2 \quad c = RP_2$

$\alpha = RQ_1 \quad Q_1 = (c+\alpha, -\alpha, 0)$

$\beta = RP_1 \quad P_1 = (b+\beta, 0, -\beta)$

$P_1 P_2 : \; x\beta + z(\beta + b) = 0$

$Q_1 Q_2 : \; x\alpha + y(c + a) = 0$

$$A = \left(-(b+\beta)(c+\alpha), \; \alpha(b+\beta), \; \beta(c+\alpha) \right)$$

$$\sum_A x = \alpha\beta - bc$$

$$A + B = Q_1 + Q_2$$

$$B = \left(\beta(c+\alpha)^2, \; -\alpha\beta(c+\alpha), \; -c^2(b+\beta) \right)$$

$$C = \left(\alpha(b+\beta)^2, -\beta^2(c+\alpha), -\alpha\beta(b+\beta) \right)$$

$$M = \left(b\beta(c+\alpha)^2 + \alpha c(b+\beta)^2, -b(c+\alpha)(\alpha\beta+bc), \right.$$
$$\left. , -c(b+\beta)(\alpha\beta+bc) \right)$$

$$RM:\ zb(c+\alpha) - yc(b+\beta) = 0$$

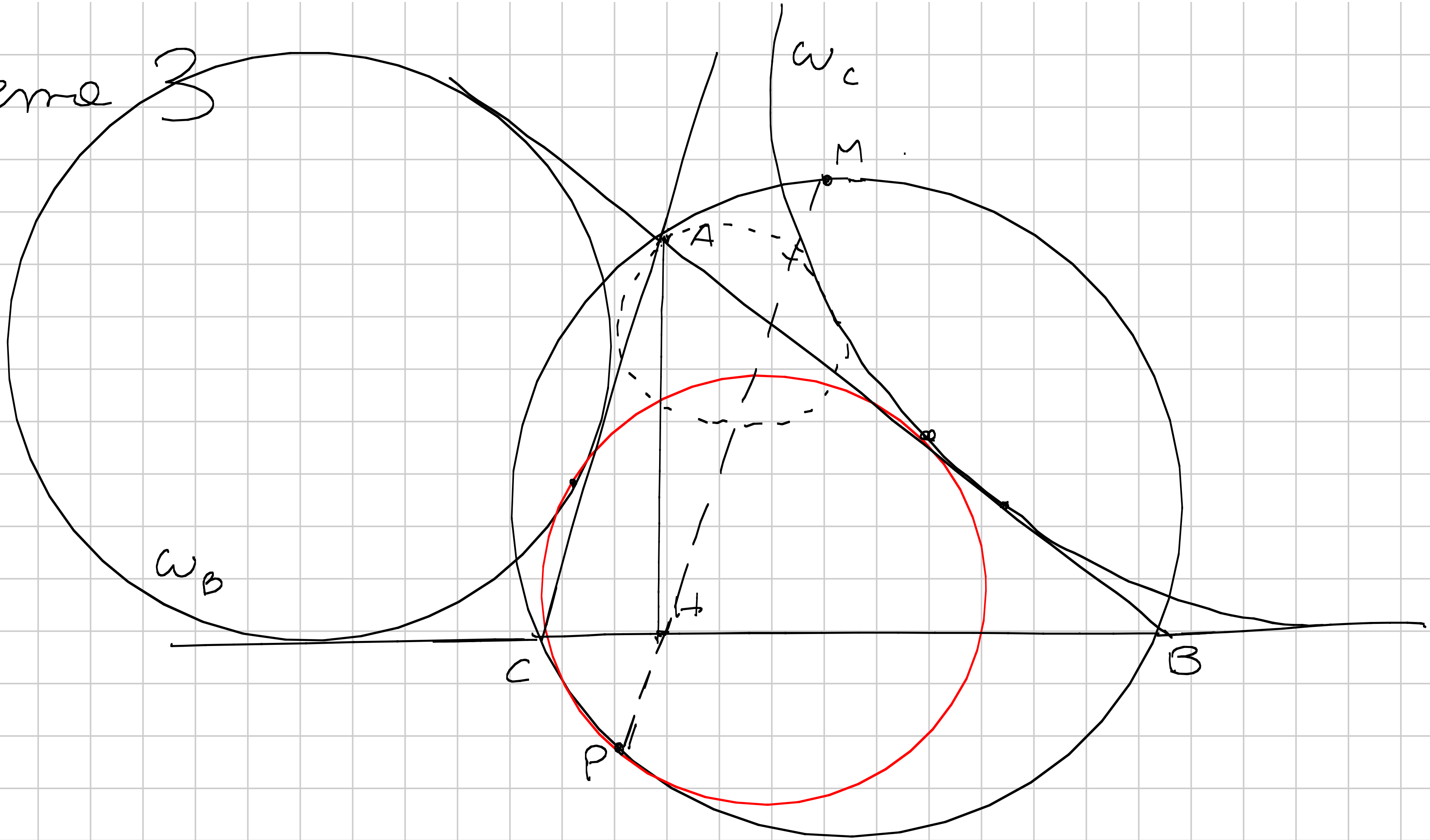
$$\sum_{cyc} a^2yz = (x+y+z)(bz)(b+\beta)$$

RS : $bz(b+\beta) - cy(c+\alpha) = 0$

$dz + ey = 0$

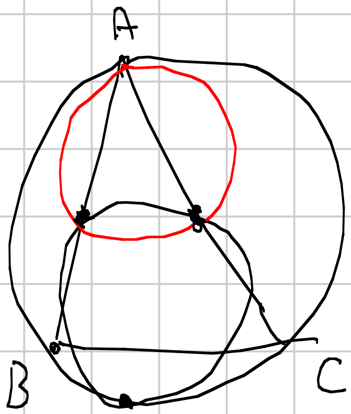
$b^2 e z + c^2 d y = 0$

Probleme 3



SIMILE NELLO SPIRITO:

(ma non nella soluzione)

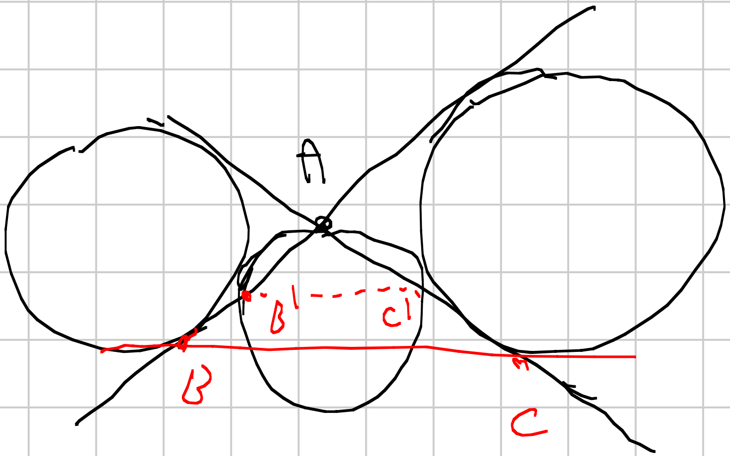


consideriamo la cfr. γ che tange (ABC), ω_B, ω_C

(l'altra)

Claim: tange (ABC) in A

parallel tangent LEMMA



γ': passa per A e tange ω_B, ω_C

voglio che γ' tanga (ABC)

se $\gamma' \cap AB, AC = B', C'$ basta che $BC \parallel B'C'$

DIM.: esiste un'inversione + simmetria nella bis. $B\hat{A}C$ che scambia ω_B e ω_C; sia essa φ

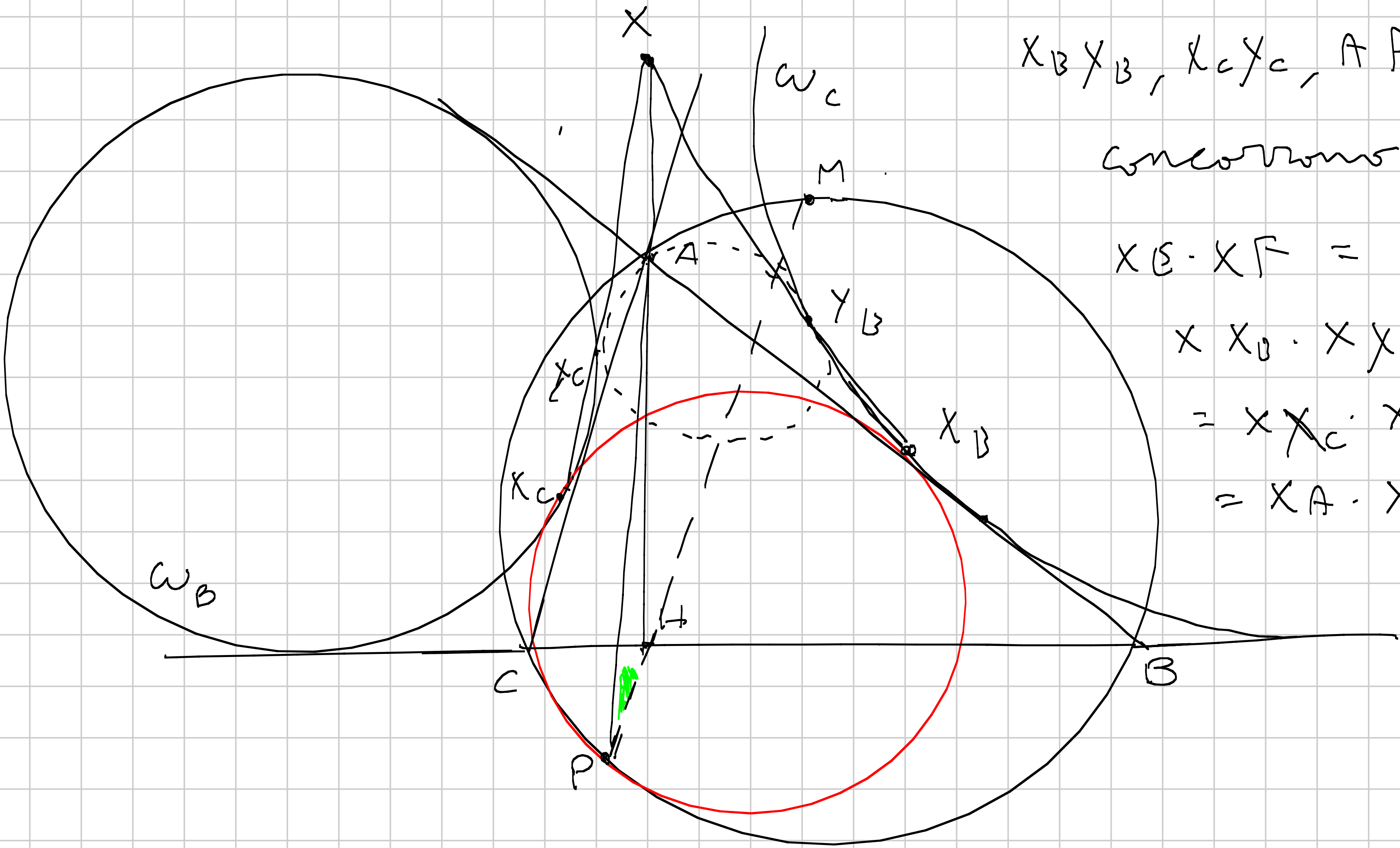
chi è $\varphi(\gamma')$? è una retta tangente a $\varphi(\omega_B)$, $\varphi(\omega_C)$, ovvero ω_B, ω_C

si vede che è BC.

allora $C = \varphi(B')$ e $B = \varphi(C')$

e dunque $AB' \cdot AC = AC' \cdot AB$

$$\Rightarrow \frac{AB'}{AB} = \frac{AC'}{AC}$$ come voluto $\square$



X_BY_B, X_CY_C, AP

concorrono in X

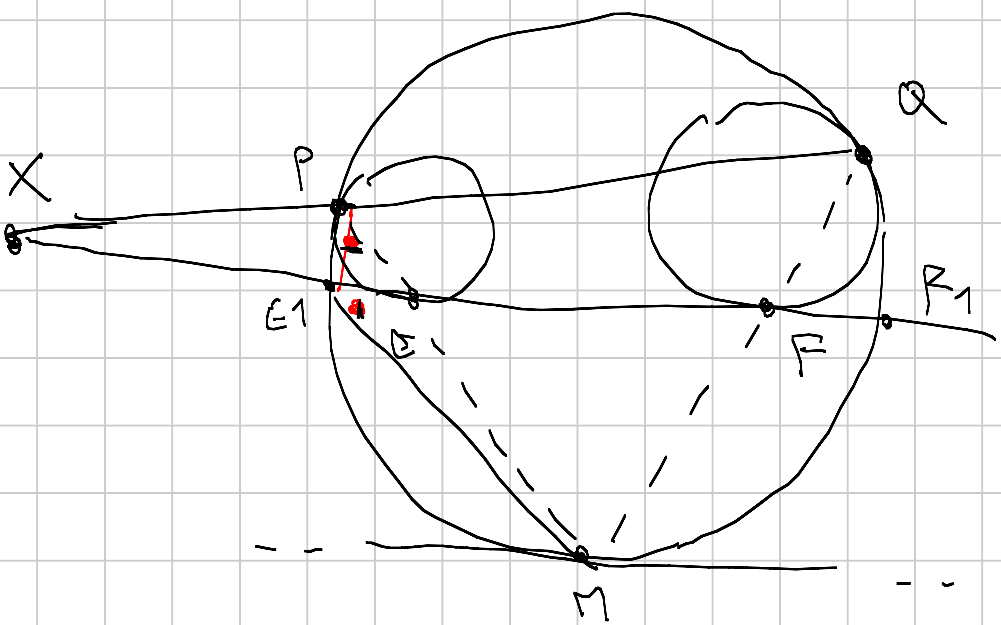
$XE \cdot XF =$

$XX_B \cdot XY_B$

$= XX_C \cdot XY_C$

$= XA \cdot XP$

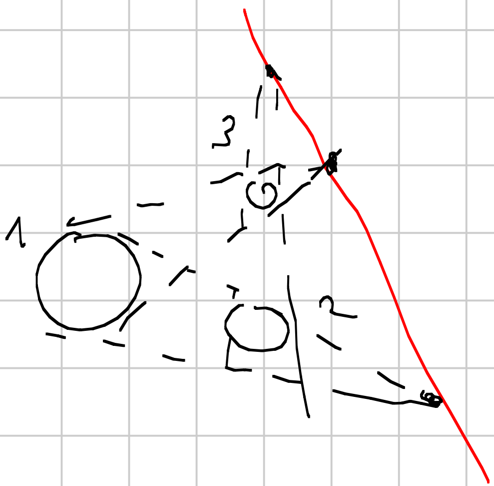
LEMMA (2) :



PQFE è ciclico

$\widehat{ME_1E} = \widehat{E_1PE} \Rightarrow ME_1^2 = ME \cdot MP = MF_1^2 = MF \cdot MQ$

MONGE :



ex(O1,O2)
ex(O2,O3) o in(O2,O3)
ex(O1,O3) in(O1,O3)

allineati

□

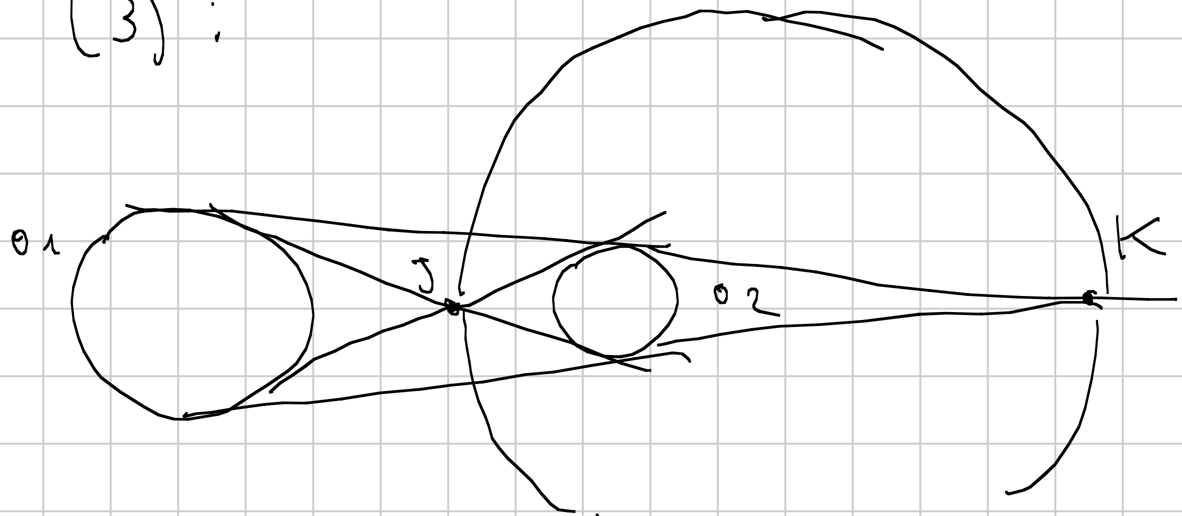
AX_BY_BP, AX_CY_CP sono ciclici

OSS: X sarà il centro radicale di $\omega_B, \omega_C, (ABC)$

in particolare sta sull'asse radicale di ω_B, ω_C

<u>IDEA</u>: costruiamo un cerchio coassiale a ω_B, ω_C che passa per A, P

LEMMA (3):



il cerchio (JK), (o1), (o2)

il luogo dei P t.c. $\dfrac{pow_{(o1)}(P)}{pow_{(o2)}(P)} = cost.$

è un cerchio coassiale a (o1), (o2)

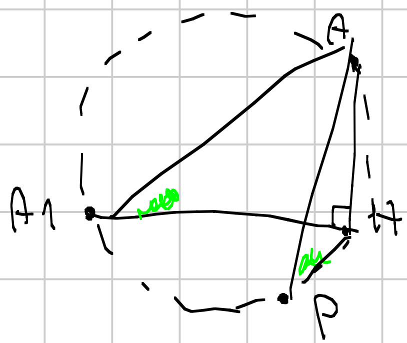
(nel caso $cost. = \left(r_1/r_2\right)^2$) □

il centro di simm. esterna di ω_B, ω_C appartiene a BC e alla bisettrice esterna di $B\hat{A}C$.

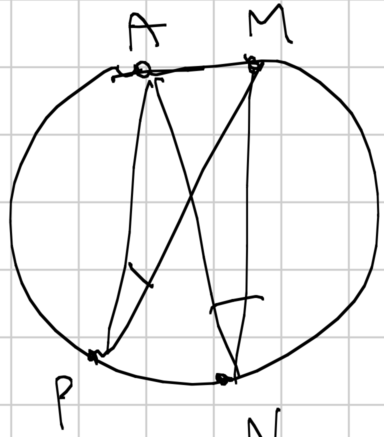
Sia esso A_1. Allora (AA_1), ω_B, ω_C sono coassiali $\Rightarrow P \in (A, A_1)$.

Ma di certo $H \in (A, A_1)$.

$\Rightarrow H\hat{P}A = H\hat{A_1}A$



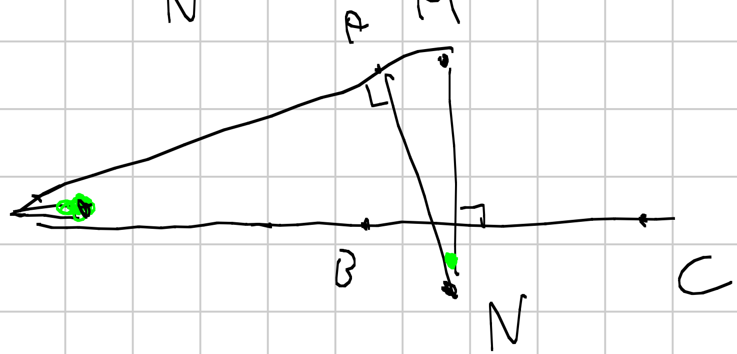
Se N è il punto medio dell'arco BC non contenente A, vale $M\hat{P}A = M\hat{N}A$

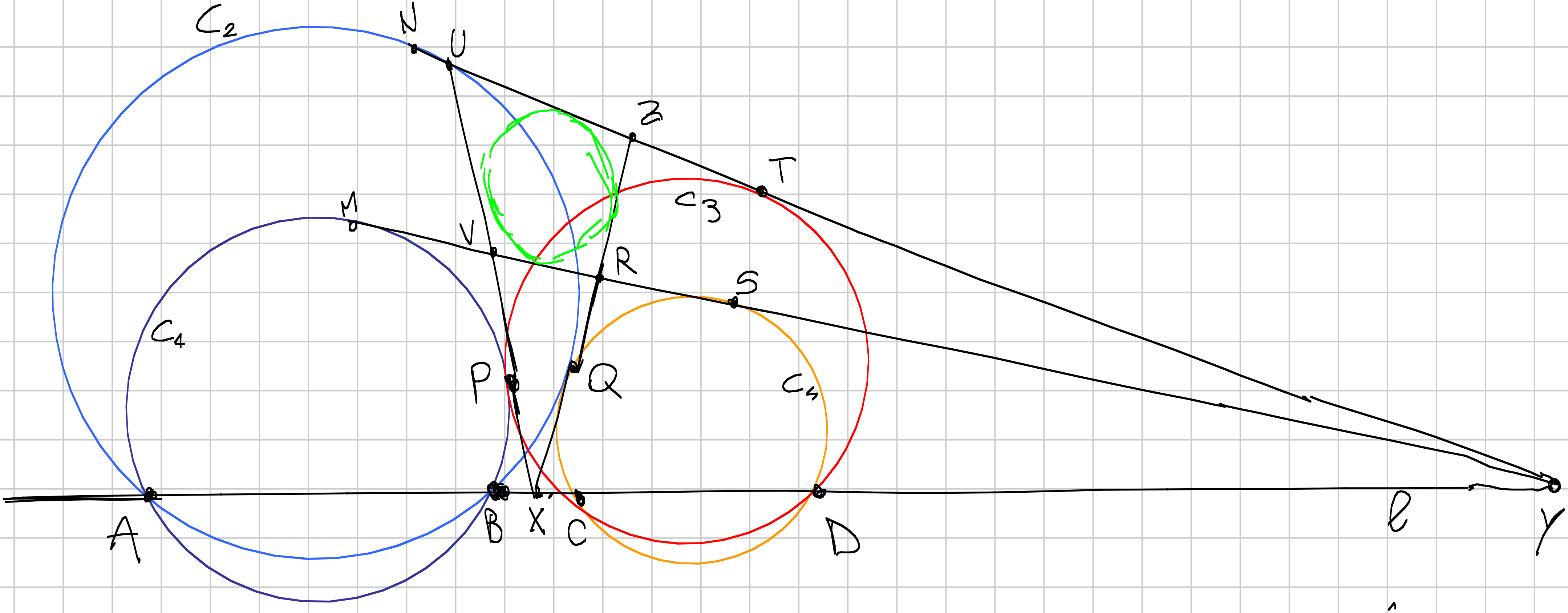


Ma $MN \perp BC$ (è l'asse)

$\Rightarrow M\hat{N}A = H\hat{A_1}A$

$\Rightarrow$ Ts.





$X = \text{tang}(C_1, C_3) \cap \ell$

$\text{pow}_{C_1}(X) = \text{pow}_{C_3}(X)$

$\text{pow}_{C_2}(X) = \text{pow}_{C_1}(X)$

$$\mathrm{pow}_{c_3}(X) = \mathrm{pow}_{c_2}(X)$$

$$\begin{cases} XP = XQ \\ YS = YT \end{cases} \qquad Y = \mathrm{tang}(c_2, c_3) \cap \ell$$

$$XU - UP = XR - RQ$$

$$YU - UT = YR - RS$$

$$XU - YU = XR - YR \Rightarrow \boxed{XU + YR = YU + XR}$$

Si conclude che $ZUVR$ è circoscrivibile a una circ.

dal lemma finale.

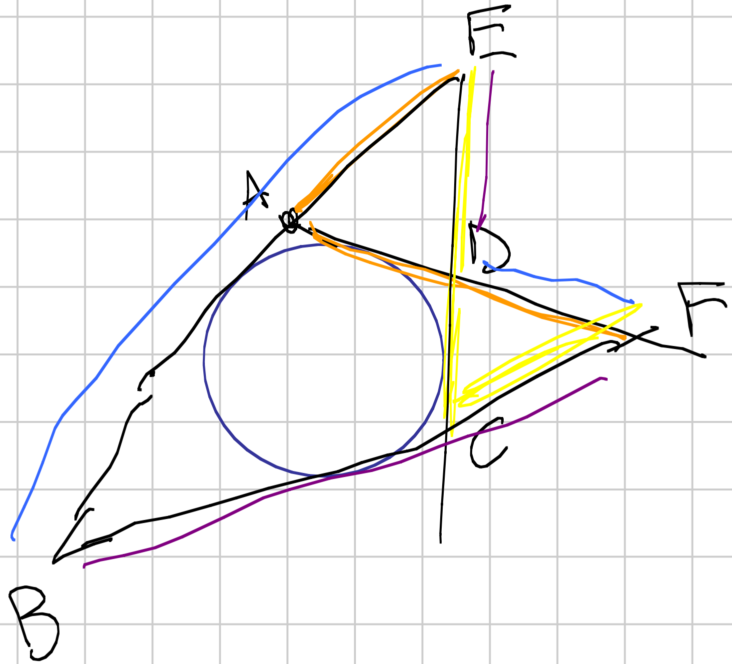
$YZ + ZN = YV + VM$

$ZQ = ZN$

$VP = VM$

$XZ - ZQ = XV - VP$

$$\boxed{YZ + XZ = YV + XV}$$



$EA + AF = EC + CF \iff$

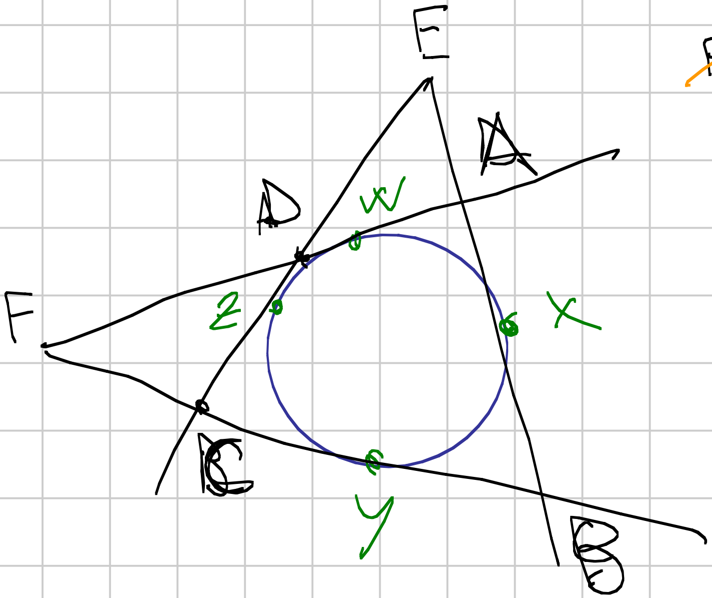
ABCD è circoscrivibile a $\bigcirc$

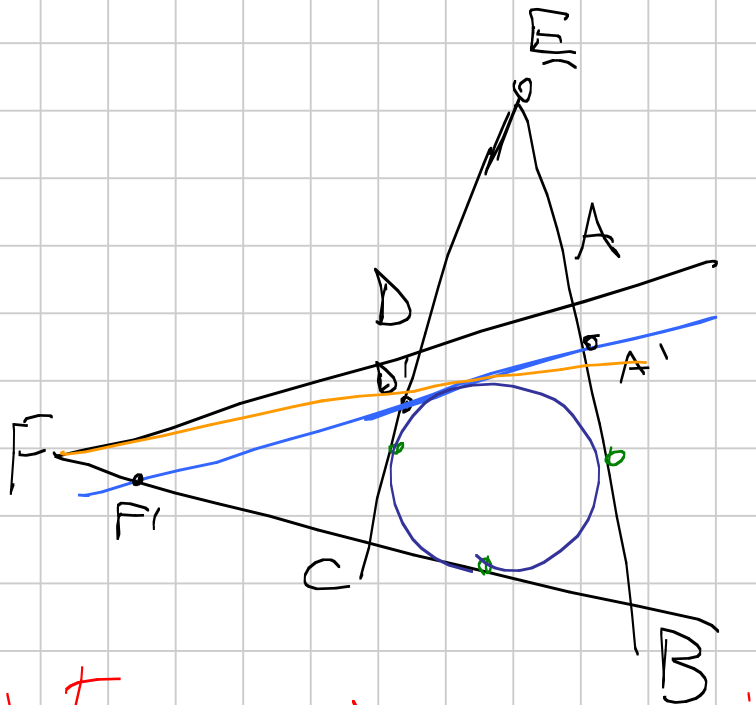
$\Updownarrow$

$EB + DF = DE + BF$

$$EA + AF = FC + CE$$

$$\cancel{EX} - \cancel{XA} + \cancel{AW} + \cancel{WF} = \cancel{FY} - \cancel{YC} + \cancel{CZ} + \cancel{ZE}$$





$$EA' + A'F' = F'C + CE$$

$$EA + AF = FC + CF$$

Non c'è tempo ⇒ esercizio.